Kryptologic: A Discrete Mathematics Gamified Educational Card Game

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***Abstract*— This paper presents the design and development of Kryptologic along with an artificial intelligence (AI) based from a random binary-permutation tree. Kryptologic is a logical game and makes use of logical operations instead of arithmetic operations. Nonetheless, the Kryptologic is still similar to Krypto with respect to rules and scoring system with the only difference being the cards. There will be 56 cards in the game, 28 of which are labeled as T (True) and 28 are labeled as F (False) with each card also including logical operators. This can serve as a gamified educational tool for propositional logic and operators to undergraduate students taking up the course Discrete Mathematics. Kryptologic's AI aims to presents a solution and determine if the current problem is solvable or not. Krypto AI's processing time was taken from the average time taken in solving a hand in 10 valid hands. For future developments, the researches will study if playing this game will lead to a positive result on the students' performance towards the subject, discrete mathematics.**

Keywords—gamification, Krypto, logic, propositional logic, card games, random binary-permutation tree

I. INTRODUCTION

Learners today are digital natives, growing up with digital technologies along with a different learning style. Because of this, different teaching methods and approaches are implemented by educators in order to implement active participation with strong motivation and engagement to learning among their students [1]. Likewise, one of these methods is through the gamification approach which primary intends to improve the student's engagement and motivation. Gamification is defined as using game-based mechanics, aesthetics and game thinking to engage people, motivate action, promote learning, and solve problems [2]. Furthermore, using game mechanics in education enhances engagement and improves motivation and learning in formal and informal conditions [3]. Consequently, the use of game mechanics in learning improves the abilities to learn new skills by 40% and ultimately leads to higher level of commitment and motivation among users as game mechanics are familiar mostly among consumers and this tackles one of the main problems in modern education: lack of engagement and motivation of students to participate actively in the learning process [4]. While gamification in learning certainly does not directly associate with the student's knowledge and skills, gamification ultimately affects students' behavior, commitment, and motivation, which can lead to improvement of the knowledge and skills [5].

In this paper, we present Kryptologic, a card game of logic for a gamified learning tool for logical operations in engineering education inspired by the mathematical card game Krypto. In the Philippines, Krypto along with other games such as Damath, SciDama, and Math Checkers, are being promoted by the Department of Education most notably by the Division of Quezon Province as an innovative enrichment strategy for improving students' mathematical thinking skills [6]. In standard Krypto game, a deck of 56-cards is being used wherein six cards will be dealt (Fig. 1)—each of the first five cards' number will be used exactly once, in which players may use any combination of arithmetic operations in order to arrive at the sixth card or the objective card's value (Fig. 2). The first player to come up with the correct expression will become the winner. Krypto, as a form of game-based learning material has shown significant effect in improving performance level of students in understanding concepts in basic arithmetic [6].

For most beginner students, manipulating logic operations and symbols is challenging as approaches are different compared to the traditional arithmetic mathematics they are used to doing. Furthermore, propositional logic is one of the topics covered in the course Discrete Mathematics, an essential course among computer engineering students.

With this, the main goal of Kryptologic is to enhance conceptual understanding of the topic at hand and improve the students' critical thinking, problem solving skills, and specifically, skills in manipulating logical operations. As students, we observed that there are plenty of students who do not practice solving as they opt for having an index card with solutions and formulas in steps as these are allowed in examinations, making the students miss the essence of learning and mastery. Researchers who have investigated expert and novice performance have uncovered important distinctions between deliberate practice and other activities, such as play or repetition. Although other factors such as intelligence and motivation affect performance, practice is necessary if not sufficient for acquiring expertise [7]. Finally, an artificial intelligence (AI) game assistant is developed for Kryptologic game utilizing a random binary-permutation tree in finding a correct solution asked by the game.

Fig. 1 A sample hand in standard Krypto game

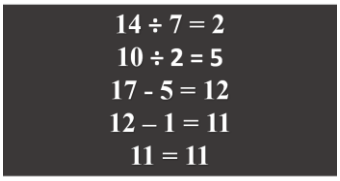

Fig. 2 A sample solution in standard Krypto game

II. METHODOLOGY

A. Game Design

In designing Kryptologic as a card game, the following has been considered in the game planning stages, namely:

1. Making the game simple yet challenging since the game aims to sharpen students' quick-thinking skills and attentiveness to details.
2. Encourage students to formulate strategies by studying different patterns i.e. logical equivalence laws and identifying unsolvable hands.
3. The gameplay and scoring system encourage students for delving themselves into the game as they are rewarded for playing well.

Kryptologic, as a variant of Krypto, is a non-sequential perfect information game wherein players will race in getting a correct logical expression that satisfies the corresponding given. In this game, instead of numerical values, truth values (**T** of **F**) will be presented in each card together with a logical operator $X \mid X \in \{\wedge, \vee, \uparrow, \downarrow, \rightarrow, \neg, \Leftrightarrow, \veebar\}$.

B. Game Mechanics Summary

The following is the general rules for Kryptologic:

1. The game is to be played by at least 2 players and 1 game moderator. The game moderator is responsible for dealing the cards and tallying the score.
2. Six cards will be dealt in succession— each of the first five cards' truth values together with its corresponding logical operator will be used exactly once in order to arrive at the sixth card or the objective card's value (Fig. 3).
3. The first player must shout "**Krypto!**" to tell everyone that he got the answer. The said player will have to state his solution for 1 minute to be checked by the game moderator. Complete answer must be written in a paper in order to assure that the players did solve the problem. Failure to show solution in the given time limit will result to the player's score to be deducted.
4. The game has total of 10 valid rounds (hereafter referred to as "hands"). A hand will be only valid if and only if it is solved by the players.
5. For scoring, players receive a point for each "Krypto". Players will receive double their previous hand each time they "Krypto" repetitively in sequence. Once the sequence is broken, the score will return to 1. Whenever a player's "Krypto" results in error, they will receive -1 point and they will not be allowed to score for that specific hand, and another hand will be dealt for the remaining players. In the next hand, all players are now eligible to play unless another error in "Kryptoing" occurs (Table I).
6. At the end of 10 valid hands, the player with the highest score wins the game. In case of a tie, the game moderator will deal more additional hands as needed in order to break the score tie.

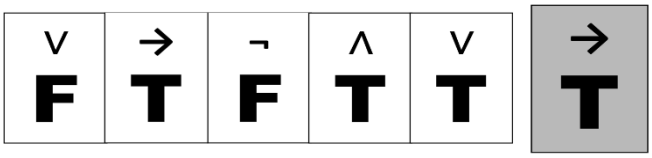

Fig. 3 A sample hand in Kryptologic game

Given hand in Fig. 3, here is an example solution:

Step 0.
Operators = $\vee \rightarrow \neg \wedge \vee$
Variables = FTFTT

Step 1.
$F \vee T = T$
Operator = $\rightarrow \neg \wedge \vee$
Variables = TFTT

Step 2.
$T \rightarrow F = F$
Operator = $\neg \wedge \vee$
Variables = FTT

Step 3.
$\neg F = T$
Operator = $\wedge \vee$
Variables = TTT

Step 4.
$T \wedge T = T$
Operator = $\vee$
Variables = TT

Step 5.
$T \vee T = T$
Operator = null
Variables = T

Since T = T
Therefore,
Solution is correct!

TABLE I. SAMPLE FINAL SCORE BOARD

	HAND										
PLAYER	1	2	3	4	5	6	7	8	9	10	TOTAL
ISAIAH	1						1	2	4	8	16
DING		1	2								3
TEDDY		-1		1	2	4					6

C. Krypto AI

In this section, the algorithm of Krypto AI is discussed. Krypto AI is a game assistant and a player in the game. Its role is to solve a given hand in a predetermined time and tell the players if the current hand is solvable or not. Krypto AI's algorithm takes an array of operators and operands as input and outputs a logical expression that is equal to the target truth value.

Krypto AI's structure follows a random binary-permutation tree which is illustrated in Figure 4.

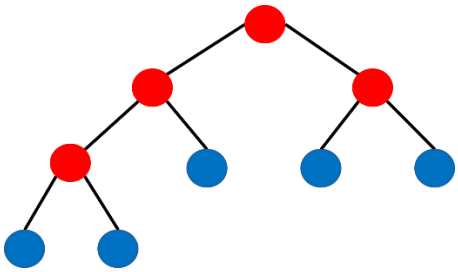

Fig. 4 Krypto AI random binary-permutation tree

Krypto AI's structure follows a random binary-permutation tree which is illustrated in Figure 4. The red nodes represent operator nodes, while the blue nodes represent operand nodes.

Algorithm 1. Getting a sample solution
Input: String[] operators, String[] operands
boolean negate, String target
Output: Logical expression in infix form, infix

1. Let elapseTime = 0
2. Let maxTime = 60000 ms
3. while elapseTime < maxTime
4. shuffle(operators)
5. shuffle(operands)
6. if(!negate) evaluateExpression(operators,operand)
7. else not.evaluateExpression(operators,operand)
8. infix = tree.inorder()
9. if checkTree(infix) != target **continue**
10. end while

Algorithm 2. Produce a logical expression from a tree (evalueateExpression)
Input: String[] operators, String[] operands
Output: tree.root()

1. tree.root = new node(operators[0])
2. tree.root.left = new node(operators[1])
3. tree.root.right = new node(operators[2])
4. tree.root.left.left = new node(operators[3])
5. tree.root.left.right = new node(operands[0])
6. tree.root.right.left = new node(operands[1])
7. tree.root.right.right = new node(operands[2])
8. tree.root.left.left.left = new node(operands[3])
9. tree.root.left.left.right = new node(operands[4])

Algorithm 3. Determine if the solution is correct (checkTree)
Input: tree.root()
Output: boolean **T** or **F**

1. postfix = toPostfix(infix)
2. Define stack
3. for(int i=0; i<postfix.length(); i++)
4. if !isOperation(postfix.charAt(i)) && Character.isLetter(postfix.charAt(i))
5. a = Character.toString(postfix.charAt(i));
6. operation.push(a)
7. end if
8. else if isOperation(postfix.charAt(i)) && !operation.isEmpty()
9. case infix.charAt(i) of
10. '¬' :
11. char a = stack.top()
12. char a2 = performNegate(a)
13. stack.pop()
14. stack.push(a2)
15. '∧' :
16. char b = stack.top()
17. stack.pop()
18. char c = stack.top()
19. stack.pop()
20. stack.push(performAnd(c,b))
21. '∨' :
22. char d = stack.top()
23. stack.pop()
24. char e = stack.top()
25. stack.pop()
26. stack.push(performOr(e,d))
27. '↑' :
28. char f = stack.top()
29. stack.pop()
30. char g = stack.top()
31. stack.pop()
32. stack.push(performNand(g,f))
33. '↓' :
34. char h = stack.top()
35. stack.pop()
36. char k = stack.top()
37. stack.pop()
38. stack.push(performNor(k,h))
39. '→' :
40. char l = stack.top()
41. stack.pop()
42. char m = stack.top()
43. stack.pop()
44. stack.push(performImplication(m,l))
45. '⇔' :
46. char n = stack.top()
47. stack.pop()
48. char o = stack.top()
49. stack.pop()
50. stack.push(performBiconditional(o,n))
51. '⊻' :
52. char r = stack.top()
53. stack.pop()
54. char s = stack.top()
55. stack.pop()
56. stack.push(performXor(s,r))
57. end for

III. RESULTS AND DISCUSSION

A. *Kryptologic Program*

In this section, the visualization of the game will be discussed. Figure 5 shows the main menu of the game. Figure 6 displays the actual game play wherein it showcases the actual Kryptologic gameplay. The program is written in Java programming language that made use of object-oriented programming.

Fig. 5 Kryptologic main menu

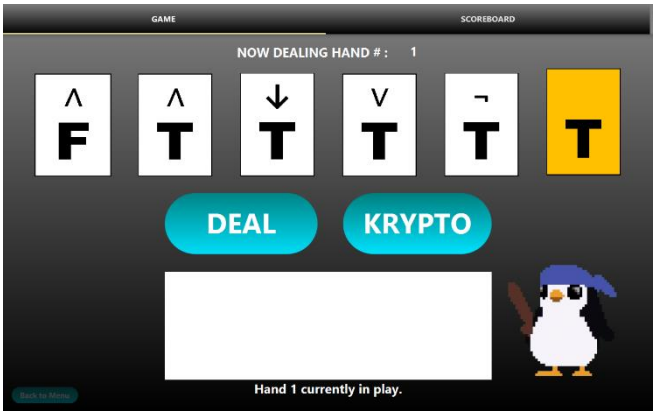

Fig. 6 Kryptologic actual gameplay

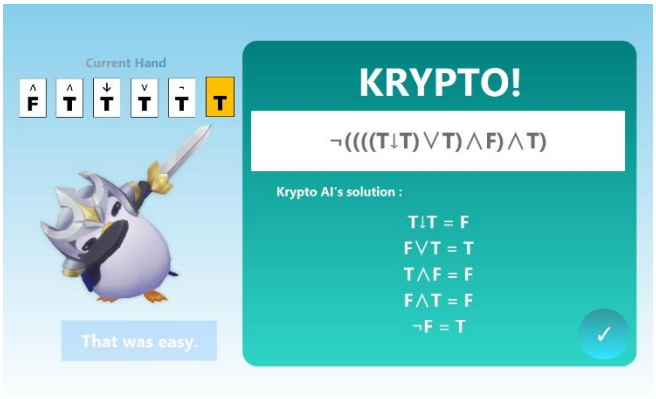

Fig. 7 Krypto AI sample solution

Fig. 8 Kryptologic sample scoreboard in hand 3

Figure 7 shows the display that shows Krypto AI solution while Figure 8 highlights the scoreboard of the game. The game will conclude after 10 valid hands. The player with the highest total score will win the game.

To decide the Krypto AI's processing time to generate a solution, the researchers tested solving 10 valid hands and averaged the time in doing so. Table II highlights the result of the test.

TABLE II. AVERAGE TIME IN GETTING A SOLUTION

ATTEMPT	TIME
1	78 secs
2	90 secs
3	89 secs
4	70 secs
5	90 secs
6	91 secs
7	89 secs
8	79 secs
9	94 secs
10	98 secs
AVERAGE	**86.8 secs**

Based from the 10 tests, an average of 86.8 seconds of solving was obtained. This will be the Krypto AI's time in getting a correct solution.

IV. CONCLUSION

In this paper, Kryptologic, a new gamified learning tool as an introduction to the topic of logical operation and manipulation in discrete mathematics. Moreover, this work presents the development of an AI game assistant for playing Kryptologic by using random binary-permutation tree. The average time in getting a solution in 10 attempts was found out to be 86.8 seconds in which became this game's AI processing time to get a correct solution. For future developments, the researches will study if playing this game will lead to a positive result on the students' performance towards the subject, discrete mathematics.

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