Camera Calibration

1.1 Linear camera calibration

Procedures of calculating P matrix

- x = PX, x is the homogenous coordinates of 2D points; X is the homogenous coordinates of 3D points

$$\mathbf{P}\,\mathbf{X}_i = \begin{bmatrix} \mathbf{p}_1^T\mathbf{X}_i \\ \mathbf{p}_2^T\mathbf{X}_i \\ \mathbf{p}_3^T\mathbf{X}_i \end{bmatrix} \qquad \mathbf{x}_i \times \mathbf{P}\,\mathbf{X}_i = \begin{bmatrix} v'_i\,\mathbf{p}_3^T\mathbf{X}_i - w'_i\,\mathbf{p}_2^T\mathbf{X}_i \\ w'_i\,\mathbf{p}_1^T\mathbf{X}_i - u'_i\,\mathbf{p}_3^T\mathbf{X}_i \\ u'_i\,\mathbf{p}_2^T\mathbf{X}_i - v'_i\,\mathbf{p}_1^T\mathbf{X}_i \end{bmatrix} \begin{bmatrix} \mathbf{0}_4^T & -w'_i\,\mathbf{X}_i^T & v'_i\,\mathbf{X}_i^T \\ w'_i\,\mathbf{X}_i^T & \mathbf{0}_4^T & -u'_i\,\mathbf{X}_i^T \\ -v'_i\,\mathbf{X}_i^T & u'_i\,\mathbf{X}_i^T & \mathbf{0}_4^T \end{bmatrix} \begin{bmatrix} \mathbf{p}_1 \\ \mathbf{p}_2 \\ \mathbf{p}_3 \end{bmatrix} = 0$$

- Since the third matrix is a rank2 matrix, so it can be rewritten as

$$\begin{bmatrix} \mathbf{0}^T & -X_i^T & y_i X_i^T \\ X_i^T & \mathbf{0}^T & -x_i X_i^T \end{bmatrix} \begin{bmatrix} P^1 \\ P^2 \\ P^3 \end{bmatrix} = \mathbf{0}$$

- We name the first matrix as A, so the equation becomes Ap = 0 for each corresponding 2D and 3D data point.
- Stack A to be a 2n*12 matrix for all 2D & 3D data points
- Apply Singular Value Decomposition (svd) on A
- Get the last column of V (12*1) and reshape it into a 3*4 matrix P

Projective camera matrix

P =

```
   -0.2905   -0.0532    0.1866    0.6283
    0.0881   -0.3264    0.0881    0.6010
   -0.0002   -0.0002   -0.0002    0.0021
```

Decomposition of P

- K =

```
  1.0e+03 *

    1.1177   -0.0000    0.3005
         0    1.1177    0.3005
         0         0    0.0010
```

- R =

```
   -0.7071    0.0000    0.7071
    0.4150   -0.8097    0.4150
   -0.5725   -0.5869   -0.5725
```

- t =

```
   -0.0000
   -0.0810
    6.9276
```

- Intrinsic parameter:
  - Focal length of camera in X and Y direction is 1.1177*10^3
  - Principal point x = 300.5, y = 300.5
- Extrinsic parameter
  - Camera center:
    ```
    3.9999
    4.0000
    3.9999
    ```
  - R matrix:
    ```
    -0.7071    0.0000    0.7071
     0.4150   -0.8097    0.4150
    -0.5725   -0.5869   -0.5725
    ```

1.2 3D to 2D projection

As we can see from our output, it is very close to the given Points2D matrix. (Code is attached at Appendix)

The difference in x direction is 0.0026; The difference in y direction is 0.0027. Thus, we can see the differences between original 2D points and calibrated 2D points are quite small.

3D Points	(0,0,0)	(1,1,1)	(5,5,5)	(2,1,0.5)	(1,3,3)	(100,100,100)
2D Points	(300.5,287.4361)	(300.5,287.4363)	(300.5,287.4336)	(59.0176,333.9128)	(849.9493, -29.8094)	(300.5334,287.4356)

From the table, we can see that, the x value of (0,0,0), (1,1,1) and (100,100,100) are almost the same, and y value of these points are very close. Since (0,0,0) (1,1,1) (5,5,5) (100,100,100) are on the same ray from the camera center, they will be projected into a same point on the image plane. After transforming, point (1,3,3) has a negative y value.