STAT 673 name ______________________

December 2020

Final Exam

Show all work. 6 Problems. 100 points total.

(20pts) 1. Consider the model:

$$(1 - \phi B)(1 - B)Z_t = a_t$$

where a_t is a mean zero white noise process with constant variance σ_a^2.

(a) What is this time series model? Please give the order of the process.

(b) Show that this process can be written as:

$$Z_t = (1 + \phi)Z_{t-1} - \phi Z_{t-2} + a_t$$

(c) Let $|\phi| < 1$. Is the Z_t process stationary? Explain.

(d) Find the MA representation of this process. Give the ψ_j weights for $j = 0, 1, 2$.

(e) For $t = n$, find the l-step ahead forecast, $\hat{Z}_n(l)$ of Z_{n+l} for $l = 1, 2$. Give the recursion relationship for $l > 2$.

(f) Find the variance of the l-step ahead forecast error for $l = 1, 2$.

(g) Find the AR representation of this process. Give the π_j weights for $j = 0, 1, 2$.

(20pts) 2. Problems 2 is a continuation of Problem 1, Data: Monthly AA Railroad Bond Yields ($\% \times 100$), January 1968 through June 1976, n=102. Use the **R output** on the next page to answer the questions.

(a) What 2 models are fit to the data?

(b) Which model is the "best" based on AIC? Explain.

(c) In Problem 1 you found the l-step ahead forecast, $\hat{Z}_n(l)$ of Z_{n+l} for $l = 1, 2$ and the variance of the l-step ahead forecast error for $l = 1, 2$. Calculate the l-step ahead forecasts and the 95% forecast limits for $l = 1, 2$. Use the **R output** for estimates of ϕ and σ_a^2. The last 5 observations of the data are: $838, 828, 823, 814, 812$. Here, the last one is $Z_n = 812$, where $n = 102$.

(d) For $t = n$, suppose the observation at $t = 103$ turns out to be 813, find the updated forecast for Z_{104}.

(e) In R, how would you compute the l-step ahead forecasts and the 95% forecast limits for $l = 1, 2$. Use the R variable names given in the output.

```
> postscript("bondyields.ps", horizontal=FALSE)
> byields <- scan("http://www-rohan.sdsu.edu/~babailey/stat673/bondyields.dat")
Read 102 items
>
> byields <- ts(byields, start=1968)
>
> #Graph the data.
> par(mfrow=c(3,2))
> par(oma=c(2,2,8,2))
>
> plot(byields)
> title("AA Railroad Bond Yields")
> plot(diff(byields))
> title("Differenced")
> acf(byields)
> acf(diff(byields))
> acf(byields, type="partial")
> acf(diff(byields), type="partial")
>
> #Fit some models
> fit1 <- arima(byields, order=c(1,1,0))
> fit2 <- arima(byields, order=c(1,1,1))
>
> fit1

Call:
arima(x = byields, order = c(1, 1, 0))

Coefficients:
         ar1
      0.4778
s.e.  0.0865

sigma^2 estimated as 84.46:  log likelihood = -367.47,  aic = 738.94
> fit2

Call:
arima(x = byields, order = c(1, 1, 1))

Coefficients:
         ar1     ma1
      0.3560  0.1576
s.e.  0.2181  0.2405

sigma^2 estimated as 84.11:  log likelihood = -367.27,  aic = 740.54
>
> tsdiag(fit1)
> mtext("Fit 1", line=-1, outer=T)
> tsdiag(fit2)
> mtext("Fit 2", line=-1, outer=T)
>
> dev.off()
```

(18pts) 3. Consider the model:

$$Z_t = a_t - a_{t-1} + 0.6a_{t-2}$$

where a_t is a mean zero white noise process with constant variance σ_a^2.

(a) Is the model for Z_t stationary? Explain.
(b) Is the model for Z_t invertible? Explain.
(c) Calculate the values for the ACF ρ_k for $k = 0, 1, 2, 3$.
(d) Find the MA representation of this process. Give the ψ_j weights for $j = 0, 1, 2$.
(e) Find the AR representation of this process. Give the π_j weights for $j = 0, 1, 2$.
(f) Find the autocovariance generating function $\gamma(B)$ and spectrum $f(\omega)$ for the Z_t process. You do **not** need to plot the spectrum.

(12pts) 4. Consider the process:

$$(1 - B)Z_t = (1 - 0.5B)a_t$$

where a_t is a mean zero white noise process.

(a) What is the model for Z_t? Is the model for Z_t stationary? Explain.
(b) Let $W_t = (1 - B)Z_t$. Is the model for W_t stationary? Explain.
(c) Let $W_t = (1 - B)Z_t$. Is the W_t process invertible? Explain.

(15pts) 5. Consider the linear filter of Z_t:

$$W_t = (1 - B^2)Z_t$$

where $\alpha(B) = 1 - B^2$.

(a) Find the filter function $|\alpha(e^{i\omega})|^2 = \alpha(e^{i\omega})\alpha(e^{-i\omega})$.
(b) Plot the filter function for ω in $[0, \pi]$. Be sure to label your axes.
(c) Comment on what range of frequencies would be allowed to "pass" through this filter, i.e. which frequencies are preserved?

(15pts) 6. Suppose that Z_t is generated according to

$$Z_t = a_t + ca_{t-1} + \cdots + ca_1, \text{ for } t \geq 1,$$

where c is a constant and a_t is a mean zero white noise process with constant variance σ_a^2.

Let $W_t = (1 - B)Z_t$.

(a) Find the mean of W_t.
(b) Find the variance of W_t.
(c) Find the covariance of W_t and W_{t+k}.
(d) Is the W_t process (covariance) stationary? Explain.

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