Appendix B Decision Making with Analytical Hierarchy Process

Making good decisions is a crucial skill at every level.—Peter Drucker

A hallmark of design is that many decisions must be made throughout the process. Typically, there are alternative solutions to a problem from which the best one relative to some criteria is selected. This appendix presents the Analytical Hierarchy Process (AHP) approach to decision making. AHP is a flexible quantitative and qualitative method, applicable to many problems, which provides a numerical score for the alternatives considered. Different aspects of AHP are applied throughout the text, and this appendix is structured to teach AHP via example. A summary of AHP is provided at the conclusion of the appendix.

To apply AHP there must be a decision to be made, criteria against which the decision is based, and a set of competing decisions from which one must be selected. This process is encapsulated in a decision matrix (shown in Table B.1) and therefore is sometimes referred to as the ***decision-matrix method***. The row headings are the criteria against which the decision is made and the column headings represent the alternatives. The criteria can have differing levels of importance and their relative weightings are reflected by w_i values in the matrix. The entries in the matrix α_{ij} are ratings for each jth alternative relative to ith criterion. Each alternative receives a score S_j, which is a weighted sum of the ratings computed as

$$S_j = \sum_{i=1}^{m} \omega_i \alpha_{ij}. \tag{1}$$

Table B.1 A decision matrix.

		Alternative 1	Alternative 2		Alternative n
Criteria 1	ω_1	α_{11}	α_{12}	$\cdots$	α_{1n}
Criteria 2	ω_2	α_{21}	α_{22}	$\cdots$	α_{2n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
Criteria m	ω_m	α_{m1}	α_{m2}	$\cdots$	α_{mn}
Score		$S_1 = \sum_{i=1}^{m} \omega_i \alpha_{i1}$	$S_2 = \sum_{i=1}^{m} \omega_i \alpha_{i2}$	$\cdots$	$S_n = \sum_{i=1}^{m} \omega_i \alpha_{in}$

The steps of AHP are to:

1. Determine the selection criteria.
2. Determine the criteria weightings.
3. Identify and rate alternatives relative to the criteria.
4. Compute scores for the alternatives.
5. Review the decision.

AHP is demonstrated through two examples in which the decision to purchase a car is considered. This example has the benefit of being relatively easy to understand, has readily available public data for supporting the decision, demonstrates the principles of AHP clearly, and is extensible to design problems. The first example demonstrates a straightforward set of criteria, while the second extends the first to include hierarchical criteria.

B.1 Applying AHP for Car Selection

AHP is demonstrated in this section by examining the decision to purchase an automobile.

Step 1: Determine the Selection Criteria

The first step is to brainstorm to identify the criteria against which the decision is made—ideally it is done prior to identification of the alternatives. The pitfall of identifying alternatives first is that the criteria may be selected to bias toward a particular alternative. Assume that the criteria determined are:

- Purchase cost
- Safety
- Design styling
- Brand-name recognition

Step 2: Determine the Criteria Weightings

To determine the weights ω_i, a method known as ***pairwise comparison*** is applied, where each criterion is systematically compared to all others. For example, the purchase cost is compared to safety, design, and brand name. Likewise, safety is compared to the remaining criteria and so on. A common practice in AHP to apply the following scale for pairwise comparison, and it is used throughout the book for consistency:

1 = equal, 3 = moderate, 5 = strong, 7 = very strong, 9 = extreme.

For example, if one criterion is deemed strongly more important than another, it is assigned a score of 5, while if it is deemed strongly less important, it is assigned the reciprocal value 1/5.

Example comparisons are captured in the comparison matrix in Table B.2. For each cell, the corresponding row criterion is compared to the column criterion. From the first row of the table it is apparent that purchase cost is considered of equal importance to safety, moderately more important than design, and very strongly more important than brand name. By definition, the diagonal elements are assigned values of 1 since each is equally important to itself. The matrix should have the following relationship about the diagonal: $x_{ij} = 1/x_{ji}$.

Table B.2 Pairwise comparison of the selection criteria.

	Purchase cost	Safety	Design	Brand name
Purchase cost	1	1	3	7
Safety	1	1	5	9
Design	1/3	1/5	1	3
Brand name	1/7	1/9	1/3	1

People often make comparisons that are inconsistent. Look at the first row—purchase cost and safety are deemed to be equally important, while cost is moderately more important than design (factor of 3). Yet, in the second row safety is seen as strongly more important than design (by a factor of 5). This is inconsistent, since if we are to believe the first row, then safety would be only moderately more important than design (by a factor of 3)— just as the purchase cost was compared relative to safety in the first row.

An intuitive approach for computing the weight for each criterion is to sum each row. Since a given row represents the comparison of a single criterion to all others, the larger a row sum is the more important it is and the higher the weight it achieves. However, the problem of inconsistency needs to be addressed. There are a number of approaches that can be shown mathematically to reduce the inconsistency in the matrix. A simple method is to take the geometric mean of each row. The geometric mean of a series of numbers, $a_1, \ldots, a_n$, is computed as

$$\text{Geometric mean} = \sqrt[n]{a_1 a_2 \cdots a_n}\,. \tag{2}$$

The geometric mean is often used to reduce bias in skewed data. Table B.3 demonstrates how the weights are computed. First, the geometric mean of each row is computed and then the sum of the geometric means is found. The mean values are divided by the sum to produce a normalized set of weights; that is,

$$\sum_i \omega_i = 1.$$

No matter what method is applied to find the weights, they should be normalized to a sum of one.

Table B.3 Weight values computed from the pairwise comparison.

	Purchase cost	Safety	Design	Brand name	Geometric Mean	Weights
Purchase cost	1	1	3	7	2.1	0.37
Safety	1	1	5	9	2.6	0.46
Design	1/3	1/5	1	3	0.7	0.12
Brand name	1/7	1/9	1/3	1	0.3	0.05

The criteria have the following weights $\omega_1 = 0.37$, $\omega_2 = 0.46$, $\omega_3 = 0.12$, and $\omega_4 = 0.05$. These calculations are easily automated with spreadsheet software.

Step 3: Identify and Rate Alternatives Relative to the Criteria

The three competing alternatives to be evaluated are the 2006 model year Honda CR-V, Hyundai Tucson, and Toyota RAV4, which are all small sport-utility vehicles. The ratings of each alternative relative to each criterion, a_{ij}, that make up the body of the decision matrix are determined next. Ideally, quantitative ratings are determined, but in many cases it is necessary to use a more qualitative approach. Higher ratings should reflect a better match to the criteria. Creativity and ingenuity are often needed to determine a proper metric.

Let's examine purchase cost. The vehicle costs are \$21,026 (Honda), \$18,183 (Hyundai), and \$21,989 (Toyota). The purchase cost itself cannot be used directly for a rating metric, since the highest cost would achieve the highest rating, whereas the objective is to minimize cost and reward the lowest cost with the highest rating. An alternative metric is needed. A metric that works when the objective is to minimize a criterion is to compare it to the minimum of all values, using the following ratio

$$\alpha = \frac{\min\{\text{cost}\}}{\text{cost}}. \tag{3}$$

This assigns a maximum value of 1 to the lowest cost option, which in this case is the Hyundai. The cost ratings are computed to be $\alpha_{11} = 0.86$, $\alpha_{12} = 1$, and $\alpha_{13} = 0.83$. It is important that the ratings relative to each criteria be normalized so that their sum is one. If not, the sum of ratings for each criterion will be different. This would introduce bias by altering the relative weights of the criteria. The normalized ratings are $\alpha_{11} = 0.32$, $\alpha_{12} = 0.37$, and $\alpha_{13} = 0.31$.

Next, a metric for safety is needed. Fortunately, there is real data to draw upon from the U.S. National Highway Transportation Safety Association (*www.safercar.gov*) which rates vehicles in multiple categories on a 5-point scale. The average rating for each car is $\alpha_{21} = 4.8$ (Honda), $\alpha_{22} = 4.8$ (Hyundai), and $\alpha_{23} = 4.6$ (Toyota), producing the following normalized values $\alpha_{21} = 0.34$, $\alpha_{22} = 0.34$, and $\alpha_{23} = 0.32$.

The rating of the cars relative to the design styling criterion is considered next. This requires a more subjective approach than the previous two criteria. To quantify the subjective evaluation pairwise comparison is again applied to determine the relative value of one automobile's design to another. Pairwise comparison for the design styling criteria is shown in Table B.4. The 2006 CR-V has older styling, resulting in a much lower design styling rating than the others.

Table B.4 Pairwise comparison of design styling to determine ratings.

	Honda CRV	Hyundai Tucson	Toyota RAV4	Design Rating
Honda CRV	1	1/3	1/5	0.11
Hyundai Tucson	3	1	1/2	0.31
Toyota RAV4	5	2	1	0.58

Finally, the ratings for brand-name recognition are determined by pairwise comparison as shown in Table B.5.

Table B.5 Pairwise comparison of brand name to determine ratings.

	Honda CRV	Hyundai Tucson	Toyota RAV4	Brand name Rating
Honda CRV	1	4	1	0.44
Hyundai Tucson	1/4	1	1/4	0.12
Toyota RAV4	1	4	1	0.44

Note that the pairwise comparison method can be somewhat time-consuming since many comparisons must be made. For making subjective estimates, a faster approach is to use scoring rubric that reflects how well each of the alternatives meet the criterion; for example,

1 = does not meet criterion, 5 = partially meets criterion, 9 = completely meets criterion.

Keep in mind that pairwise comparison has the advantage of a systematic comparison of alternatives, and this reduces inconsistency in ratings.

Step 4: Compute Scores for the Alternatives

The decision matrix is built and the overall weighted scores for the alternatives are computed as shown in Table B.6.

Table B.6 The decision matrix.

		Honda CR-V	Hyundai Tucson	Toyota RAV4
Cost	0.37	0.32	0.37	0.31
Safety	0.46	0.34	0.34	0.32
Design styling	0.12	0.11	0.31	0.58
Brand name	0.05	0.44	0.12	0.44
Score		0.31	0.34	0.35

Step 5: Review the Decision

The result is a set of numerical scores for the alternatives, and if all work is done properly the final scores should sum to one. In this case there is not much difference between the scores, and a simple decision based upon the maximum value would lead to selection of the RAV4. Since there is not much of a difference, all three are good choices according to the selection criteria. The decision matrix allows the examination of different scenarios, as the weights and ratings can be varied to see how they affect the overall scores. The RAV4 beats the Tucson primarily because of styling and brand name, which are both subjective ratings. This might warrant revisiting those ratings to see how they affect the decision. The RAV4 barely beats out the Tucson, so if cost and safety are truly more important, the Tucson is a better decision, as it beats the RAV4 in both of those categories.

B.2 Hierarchical Decision Criteria

The example in the previous section had fairly simple selection criteria and it arguably missed some important ones, such as the operating costs. Further, when considering operating cost it becomes apparent that it has multiple dimensions, or subcriteria, such as fuel and insurance costs. Similarly, the design styling criterion can be subdivided into interior and exterior design. Thus there is a hierarchy in the criteria, giving rise to the name analytical hierarchy process. The following extends the previous example to include the use of hierarchical criteria.

Step 1: Determine the Selection Criteria

Assume that the criteria have been expanded to be more realistic as shown below:

- Purchase cost
- Operating costs
    - Fuel; miles per gallon (MPG)
    - Insurance
- Safety
- Design styling
    - Interior
    - Exterior
- Brand-name recognition

Step 2: Determine the Criteria Weightings

Pairwise comparison is again used to determine the weights. The difference now is that this comparison is done at each level in the hierarchy. First, purchase cost, operating costs, safety, design, and brand-name recognition are compared. Then fuel and insurance costs are compared to each other since they are at the same level and grouping in the hierarchy, as are exterior and interior design. Assume that the pairwise comparisons produce the following criteria weights:

- Purchase cost (0.33)
- Operating costs (0.11)
  - MPG (0.67)
  - Insurance (0.33)
- Safety (0.40)
- Design Styling (0.12)
  - Interior (0.50)
  - Exterior (0.50)
- Brand-name recognition (0.04)

Step 3: Identify and Rate Alternatives Relative to the Criteria

To utilize the results from the previous section, it is only necessary to compute scores relative to the new criteria. Using mileage per gallon as a metric for ratings fuel costs is straightforward, since the two are directly proportional. This data is readily available and is 24, 23, and 25 miles per gallon for the CR-V, Tucson, and RAV4 respectively. This produces normalized ratings of 0.33, 0.32, and 0.35 respectively.

In terms of insurance costs, the National Highway Transportation Safety Institute publishes the relative average loss per insured vehicle, a rating that is used by insurance companies to determine insurance costs. A score of 100 is the industry average—those exceeding 100 are above the average and those less than 100 are below it. The publicly available ratings are 93, 100, and 112 for the CR-V, Tucson, and RAV4 respectively. Again, the objective is to minimize these values so the metric in equation (3) is used. This produces normalized ratings of 0.36, 0.34, and 0.30 respectively.

Styling is subjective and ratings could be determined using a scoring rubric or the pairwise comparison method. Assume that this produces the scores shown in Table B.7.

Step 4: Compute Scores for the Alternatives

The decision matrix shown in Table B.7 reflects the hierarchical criteria. The scores are computed slightly differently when subcriteria are involved. For example, the score for operating costs of the Honda is based upon the two subcriteria as $(0.67 \times 0.33 + 0.33 \times 0.36) = 0.34$. The value of 0.34 is then the rating reflected in the table for the operating costs of this vehicle. Then to get the weighted score for operating costs, this result of 0.33 is multiplied by the weight of 0.11, exactly as in the previous example.

Table B.7 The decision matrix.

		Honda CR-V	Hyundai Tucson	Toyota RAV4
Purchase Cost	0.33	0.32	0.37	0.31
Operating Cost	0.11	0.34	0.32	0.33
Fuel	0.67	0.33	0.32	0.35
Insurance	0.33	0.36	0.34	0.30
Safety	0.40	0.34	0.34	0.32
Design Styling	0.12	0.20	0.40	0.40
Interior	0.60	0.20	0.40	0.40
Exterior	0.40	0.20	0.40	0.40
Brand-name	0.04	0.44	0.12	0.44
Score		0.32	0.35	0.33

Step 5: Review the Decision

With this expanded set of criteria, there is a shift in the overall ranking and the Hyundai Tucson edges out the Toyota RAV4. Frankly, all three are good choices and reflect the highly competitive car market. A comparison of vehicles across classes, such as sport-utility, mid-size, and luxury cars would likely result in a much larger differential in the scoring on the basis of the weighted criteria used here.

B.3 Summary and Further Reading

The analytical hierarchy process is an effective tool for comparing competing alternatives that integrates both quantitative and qualitative judgments. In order to compare alternatives, criteria for comparison are selected and the relative weights of the criteria assigned. In terms of selecting the criteria and their weightings, remember the following:

- Ideally, the criteria are selected prior to identification of the alternatives. This avoids the trap of selecting the criteria to support a preconceived decision.
- There is no single correct way to compute the criterion weights. However, the pairwise comparison approach is well accepted. It can be a bit time-consuming, particularly when a large number of criteria are to be compared. If used properly, pairwise comparison reduces bias in the decision.
- There are alternatives to pairwise comparison. One is to rank the criterion from most important to least important and then assign a relative weighting based upon the ranking. Another alternative is to utilize a scoring rubric using a semiqualitative scale, as 1 = low importance, 5 = medium importance, and 9 = extremely important.
- Regardless of how the weights are selected, the sum of the weights should be normalized to a value of one.

Once the criteria and weights are found, ratings for each option relative to the criteria are determined. When computing the ratings, remember the following:

- If quantitative data is available, it should be used. It is admittedly hard to determine for many problems.
- Appropriate metrics need to be used for rating quantitative data.
- If quantitative data is unavailable, qualitative judgments must be made and quantified. Again, the pairwise comparison is a good tool for quantifying such judgments. Alternatively, a rubric for rating the alternatives against the criteria can be used.
- The ratings relative to each criterion need to be normalized so that their sum is equal to one.

The final result is a decision matrix and scores for each of the alternatives. This matrix shows the quantified criteria, allowing a rational approach to decision making and minimizing emotional factors in the decision.

AHP was originally developed by Thomas Satay [Sat88] and has found wide acceptance since. There are software packages available, such as Expert Choice, that allow for rapid comparison of alternatives and evaluation.