21EES101T-ELECTRICAL AND ELECTRONICSENGINEERING

UNIT 1

Unit-1 -**Electric Circuits**

Introduction to basic terminologies in DC circuit, Kirchhoff's Current law, Kirchhoff's Voltage law, Mesh Current Analysis, Nodal Voltage Analysis, Thevenin's Theorem, Maximum power transfer Theorem, Superposition Theorem.

Basic terminologies of AC -RMS and Average value of half wave and Full wave alternating quantity, Fundamentals of single-phase AC circuits-Analysis of R-L, R-C, R-L-C series circuits-Fundamentals of three phase AC system, Three-Phase Winding Connections, Relationship of Line and Phase Voltages, and Currents in a Delta and Star-connected System

Practice on Theorems, Halfwave, Full wave bridge rectifier circuits.

Introduction to basic terminologies in DC circuit

Electric circuits are broadly classified as direct current (dc) circuits and alternating current (ac) circuits. In both dc and ac circuits several two-terminal elements are interconnected. Table shows the elements used in dc circuits and ac circuits.

Direct current circuits		Alternating current circuits	
Elements	Representation	Elements	Representation
Voltage source (Battery)	or	Voltage source	
Current source		Current source	
Resistor	R	Resistor	R
		Inductor	L
		Capacitor	C

Active and Passive two terminal elements

Active Components

An **active** component is an electronic component which supplies energy to a circuit. Active elements have the ability to electrically control electron flow (i.e. the flow of charge). All electronic circuits must contain at least one active component.

Examples

Voltage sources, Current sources, Generators , transistors, Diodes

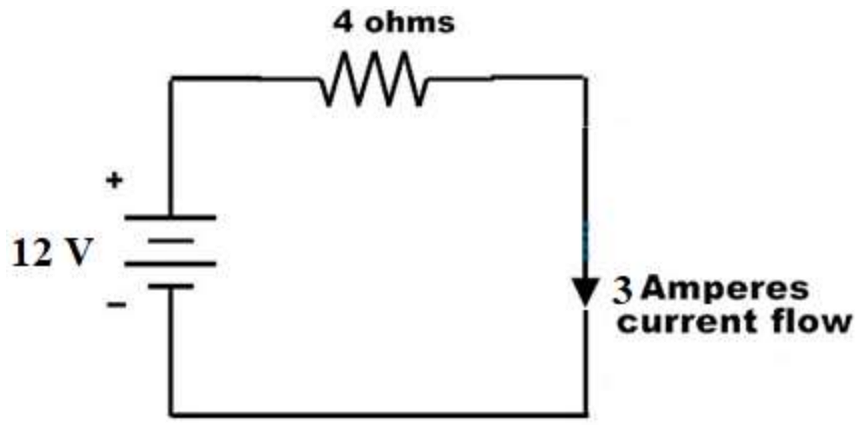
Passive Components

A **passive component** is an electronic component which can only receive energy, which it can either dissipate, absorb or store it in an electric field or a magnetic field

Examples

Resistors, Inductors, Capacitors, Transformers.

A simple DC circuit is given in below figure to get aware of DC circuit components and its parameters.

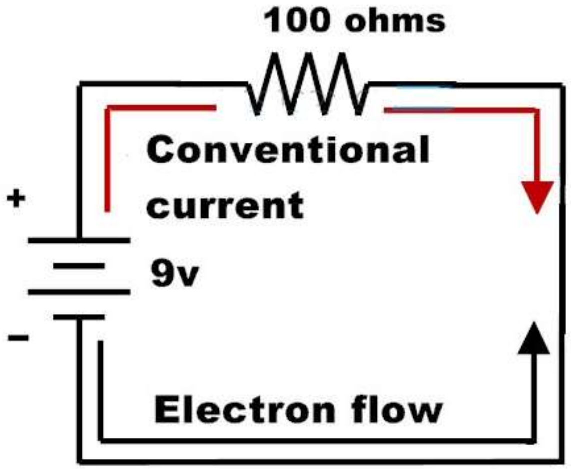


Electric Voltage: The potential difference between two points or voltage in an electric circuit is the amount of energy required to move a unit charge between two points. Unit: Volts

Electric Current

It is the flow of electrons or electric charge. Unit: Ampere

Difference Between Conventional and Electron Current Flow:



OHM'S LAW-STATEMENT

Statement The ratio of potential difference between any two points of a conductor to the current flowing between them is constant, provided the physical conditions (e.g. temperature, etc.) do not change.

i.e. $V/I = \text{constant}$ (or) $V/I = R$

Ohm's law can be expressed in three forms:

i.e. $V/I = R; \quad V = IR; \quad I = V/R$

Resistance:

The resistance of a conducting material opposes the flow of electrons. It is measured in ohms (**Ω**)

Electric Power (P)

The power is termed as the work done in a given amount of time. Unit : Watts

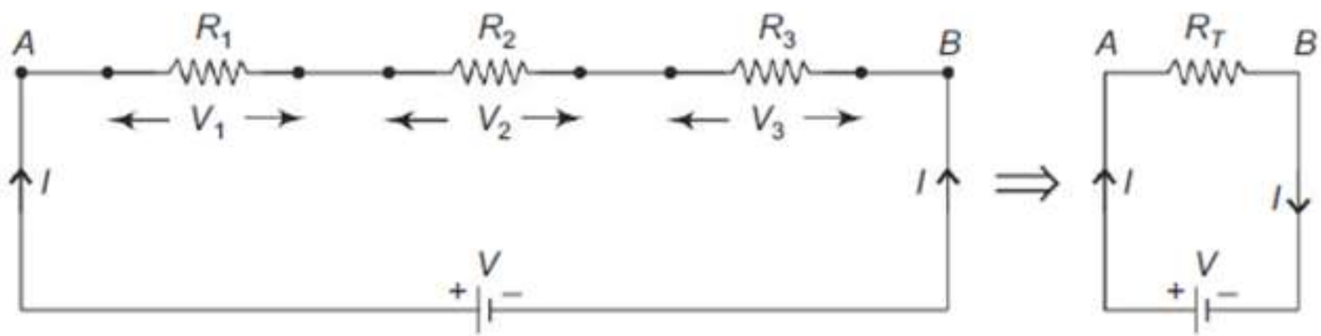
$$P = VI \text{ or } I^2R \text{ or } V^2/R$$

Electrical Energy

The rate at which electrical power consumed is generally referred as electrical energy. Unit: watt-seconds or watt-hr

$$E = P \times t$$

RESISTANCES IN SERIES



$$R_T = R_1 + R_2 + R_3$$

$$I = V / R_T$$

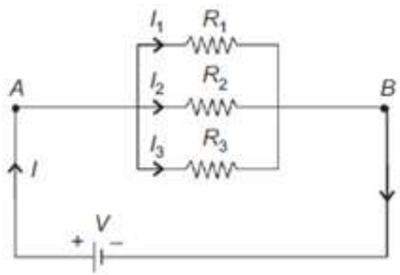
VOLTAGE DIVISION TECHNIQUE

$$V_1 = IR_1;\ V_2 = IR_2 \quad \text{and} \quad V_3 = IR_3$$

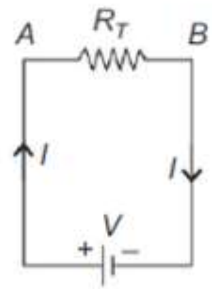
OR

$$V_1 = \frac{V}{R_T} R_1;\ V_2 = \frac{V}{R_T} R_2; \quad V_3 = \frac{V}{R_T}\ R_3$$

RESISTANCES IN PARALLEL



$\Rightarrow$



$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

When Two Resistances are in Parallel Two resistances R_1 and R_2 ohms are connected in parallel across a battery of V volts. Current through R_1 is I_1 and through R_2 is I_2 (Fig. 1.18).

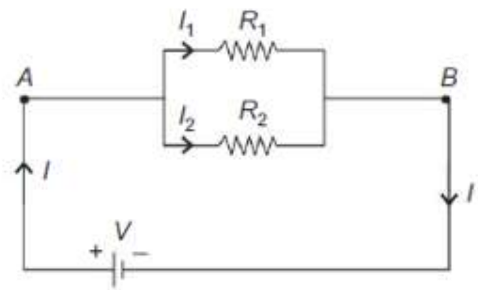
(i) Total resistance R_T

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

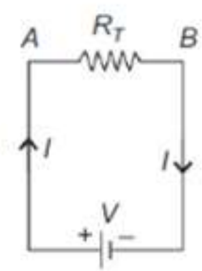
$$= \frac{R_2 + R_1}{R_1 R_2}$$

or

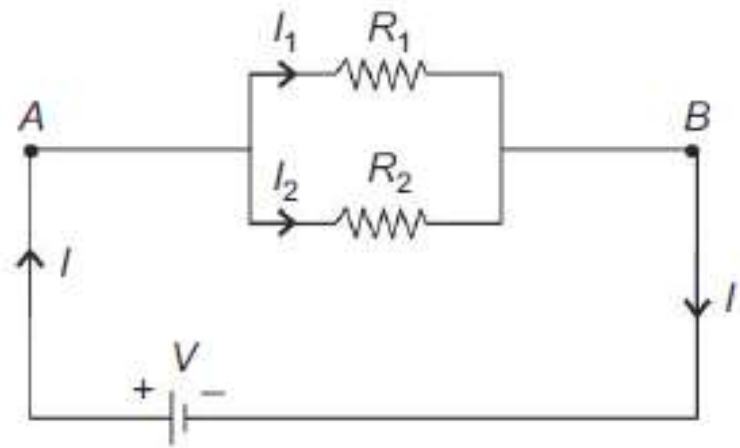
$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$



$\Rightarrow$



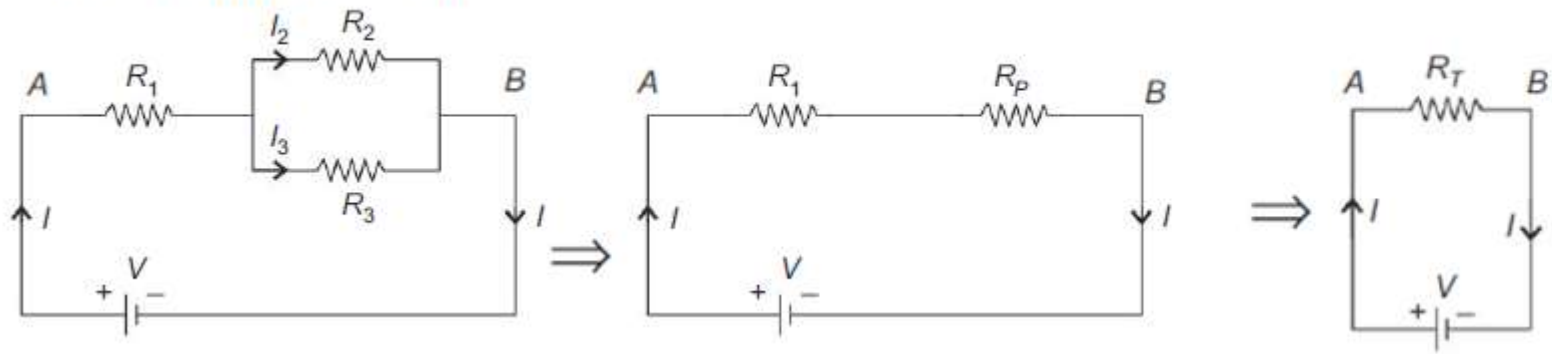
Current Division Technique



$$I_1 = I \frac{R_2}{R_1 + R_2}$$

$$I_2 = I \frac{R_1}{R_1 + R_2}$$

Series-Parallel Circuits



The equivalent resistance of parallel combination $= R_p = \dfrac{R_2 R_3}{R_2 + R_3}$

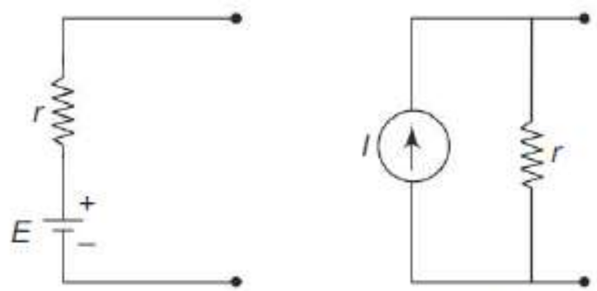
$$R_T = R_1 + R_P$$

$\therefore$ Total current $\qquad I = \dfrac{V}{R_T}$

Internal Resistance of Sources

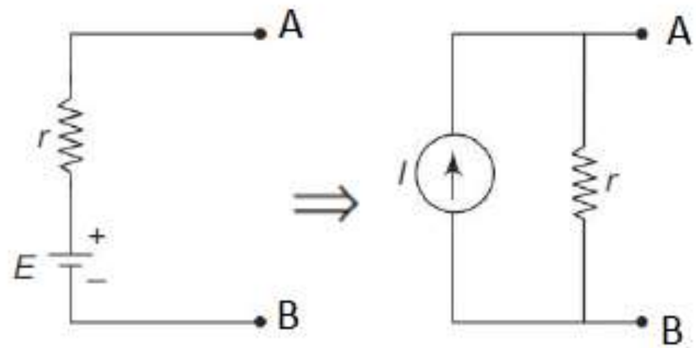
(i) Voltage Sources All voltage sources (battery, generators, etc.) must have some internal resistance (r) (very small in value). This is shown as a series resistor connected external to the source in Fig. (i).

(ii) Current Sources All current sources must have some internal resistance (r) (very high in value). This is connected externally across the source, as shown in Fig. (ii).

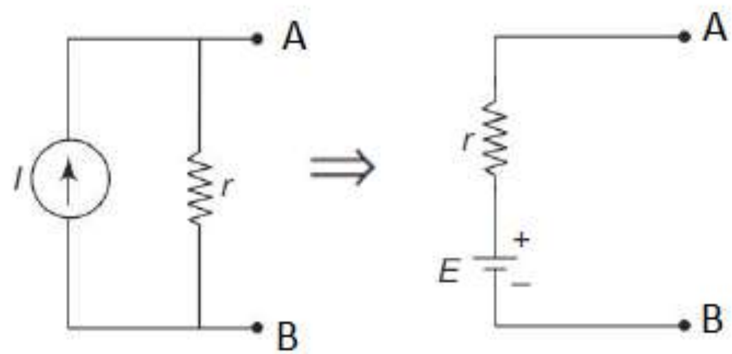


(i) (ii)

Source Transformation

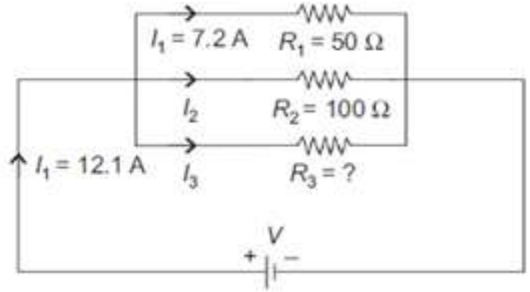


Where I= E/r



Where E=Ir

1. For the circuit shown below, find R_3



Sol:

This is a Pure Parallel circuit.

i.e. $V = V_{R_1} = V_{R_2} = V_{R3}$

$V_{R_1} = I_1 \times R_1$ (ohms Law).

$= 7.2 \times 50$

$= 360$ Volts.

$I_2 = \dfrac{V_{R2}}{R_2} = \dfrac{360}{100} = 3.6\,A$.

Apply KCL

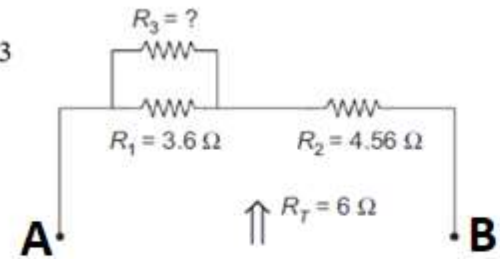
$I_T = I_1 + I_2 + I_3$

$12.1 = 7.2 + 3.6 + I_3$

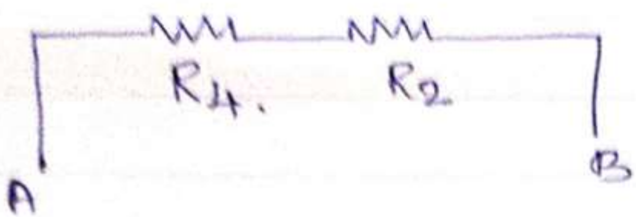
$I_3 = 1.3\,A$.

$R_3 = \dfrac{V_{R3}}{I_3} = \dfrac{360}{1.3} = 276.92\ \Omega$ Ans.

2. For the circuit shown below, find R_3



Sol: Given $R_{AB} = R_T = 6\,\Omega$.



where $R_4 = \dfrac{R_1 R_3}{R_1 + R_3} = \dfrac{3.6 R_1}{3.6 + R_3}$.

$R_T = R_4 + R_2$

$6 = \dfrac{3.6 R_3}{3.6 + R_2} + 4.56$ (or) $\dfrac{3.6 R_3}{3.6 + R_3} = 6 - 4.56$

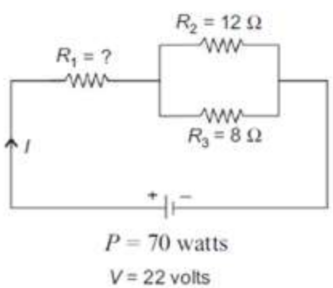
$$\frac{3.6R_3}{3.6+R_3} = 1.44 \Rightarrow 3.6R_3 = 1.44(3.6+R_3)$$

$$3.6R_3 = 5.184 + 1.44R_3$$

$$2.16R_3 = 5.184$$

$$R_3 = 2.4\,\Omega$$

3. For the circuit shown below, find R_1



Sol:

$$R_T = R_1 + \frac{R_2R_3}{R_2+R_3}$$

$$R_T = R_1 + \frac{12 \times 8}{20}$$

$$\boxed{R_T = R_1 + 4.8} \rightarrow (1)$$

$\cancel{I_T / 7}$

$$I = \frac{P}{V} = \frac{70}{22} = 3.18\,A$$

$$R_T = \frac{V}{I} = \frac{22}{3.18} = 6.92\,\Omega$$

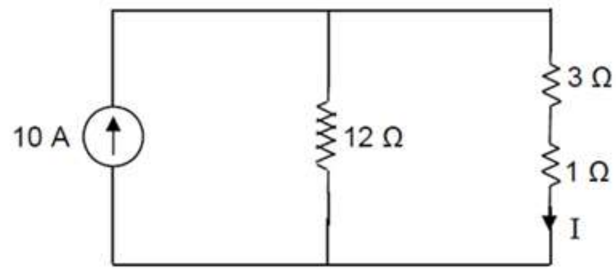
Sub R_T in eqn (1).

$$6.92 = R_1 + 4.8 \Rightarrow R_1 = 2.12\,\Omega$$

$\Rightarrow$ Ans

One mark solved sample problems

4 In the circuit shown, determine the value of current I.

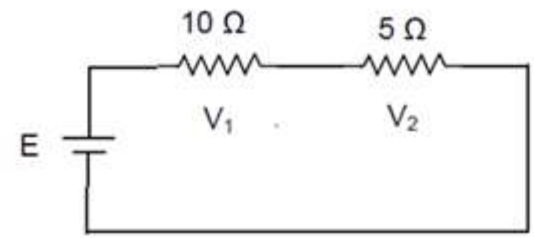


Sol: Applying Current division Rule.

$$I = \frac{I_T \times \text{opposite Resistance}}{\text{Total Resistance}}$$

$$= \frac{10 \times 12}{16} = 7.5\,A.$$

5 In the circuit shown, the ratio of $\frac{V_2}{V_1}$ is



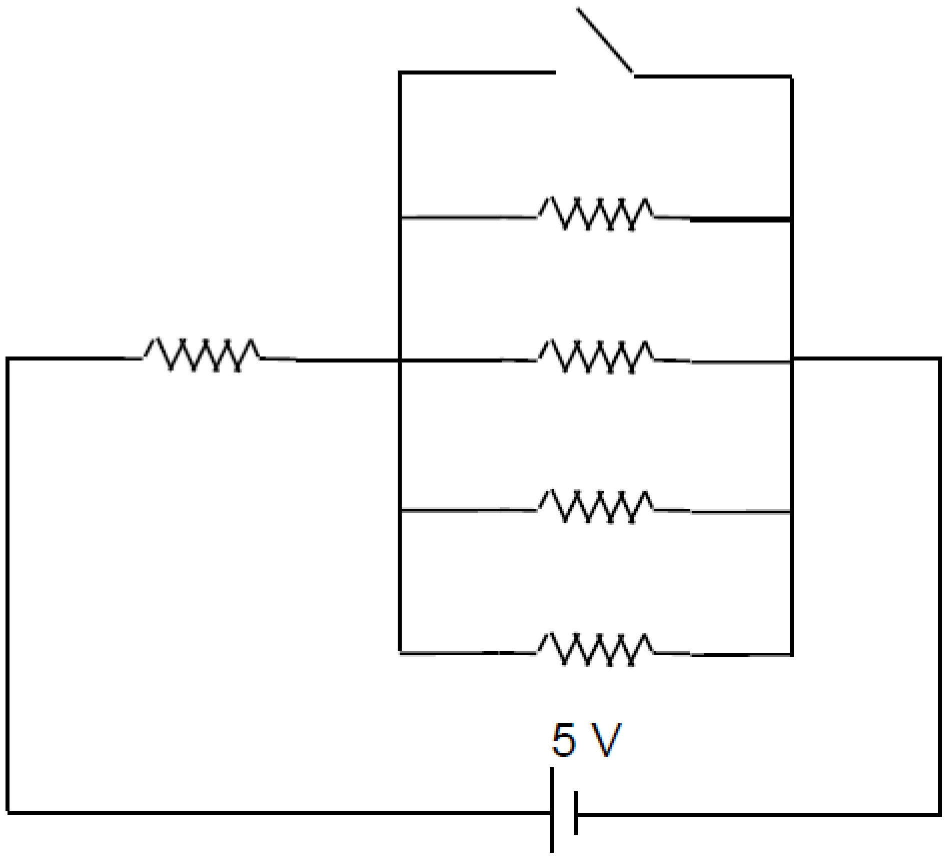
Sol:

Let I be the total current

$$V_1 = 10I \quad ; \quad V_2 = 5I$$

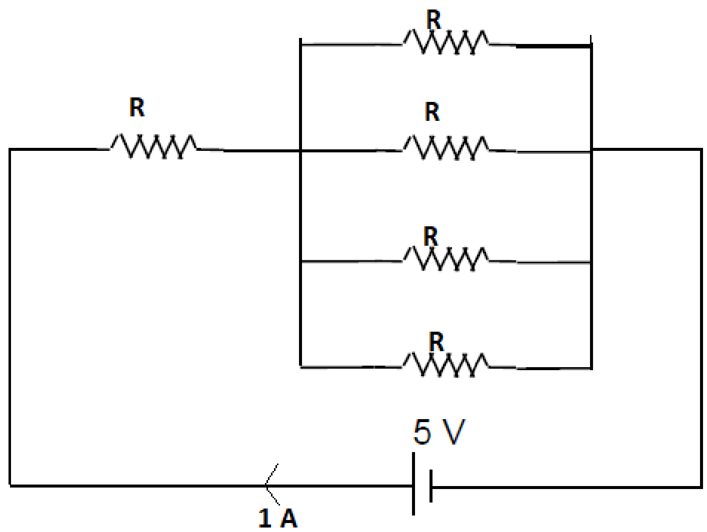
$$\frac{V_2}{V_1} = \frac{5I}{10I} = 0.5$$

6 In the circuit shown, resistors are of equal value of R.



When the switch is in open position battery current is 1 A. Value of R is

Sol:



Sol

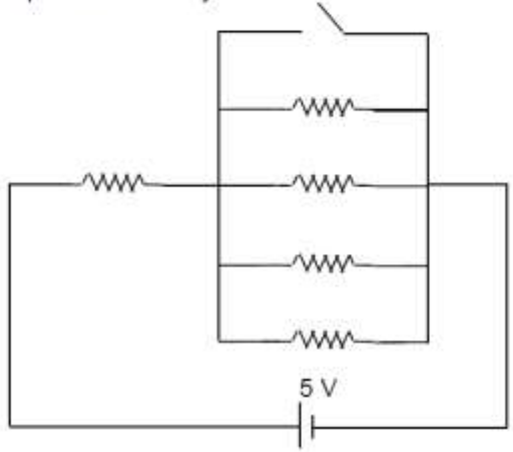
$$R_T = \frac{V}{I_T} = \frac{5}{1} = 5\,\Omega \quad (\text{ohms law}) \rightarrow ①$$

$$R_T = R + R/4$$

$$R_T = \frac{5R}{4} \rightarrow ②$$

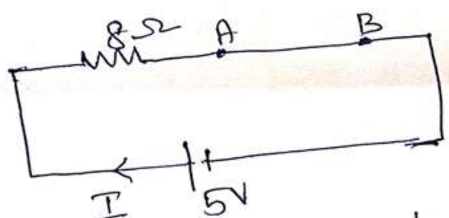
Equating ① & ② $\frac{5R}{4} = 5 \Rightarrow \frac{R}{4} = 1 \Rightarrow \boxed{R = 4\,\Omega}$

7 In the circuit shown, resistors are of equal value of 8 Ω. With the switch is in closed position battery current is



Sol.

when Switch is closed, it become short circuit Path [ie R=0] . ∴ The Remaining Resistors are useless. ie All current will flow through short circuit Path. The circuit becomes



$I = \frac{5}{8} = 0.625 A$

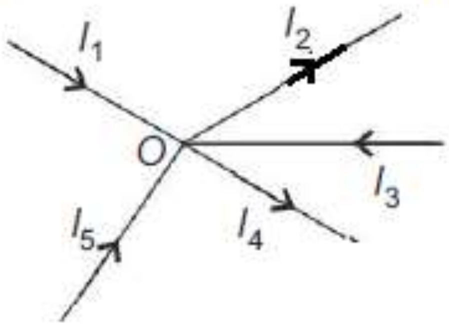
Note: Don't stop 5/8 ~~A~~. you have to solve and get 0.625 A.

Kirchoff 's Current Law (KCL)

Statement: The algebraic sum of currents meeting at a junction or node in an electrical circuit is zero. [OR]

Statement: The sum of the currents flowing towards any junction in an electric circuit is equal to the sum of the currents flowing away from that junction.

Explanation An algebraic sum is one in which the sign of the quantity is taken into account. Consider five conductors, carrying current, I_1, I_2, I_3, I_4 and I_5 meeting at point O as shown in Fig. If we assume the currents flowing towards point O as positive, then, the currents flowing away from point O will have negative sign. Now, applying Kirchoff's current law at junction O, we get



$$(+I_1) + (-I_2) + (+I_3) + (-I_4) + (+I_5) = 0$$

i.e. $$I_1 - I_2 + I_3 - I_4 + I_5 = 0$$

or $$I_1 + I_3 + I_5 = I_2 + I_4$$

i.e sum of incoming currents = sum of outgoing currents.

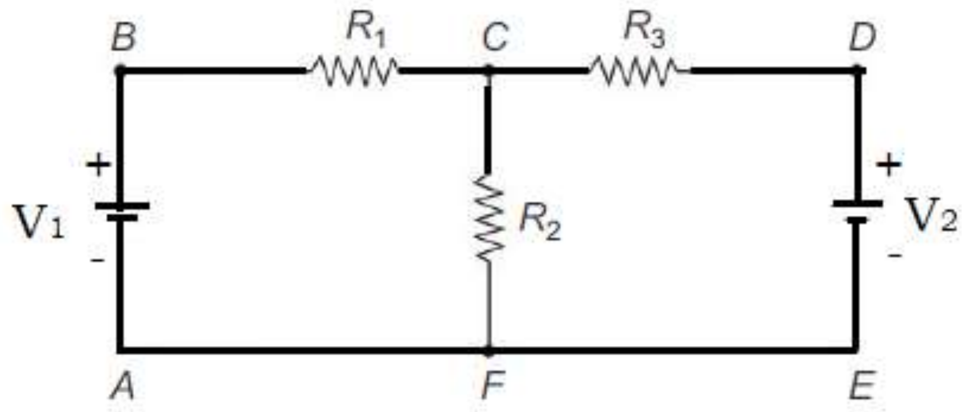
Kirchoff 's Voltage Law (KVL)

Statement: In any closed circuit or mesh or loop, the algebraic sum of all the voltages taken around is zero.

[OR]

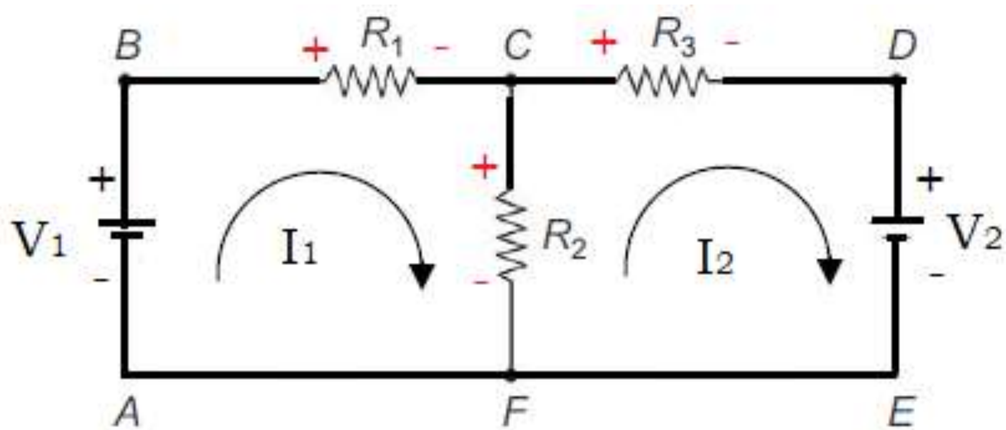
Statement: In any closed circuit or mesh or loop, sum of voltage drops equal sum of voltage rise.

Kirchoff's laws can be explained with the help of the circuit shown in Fig below



While applying KVL, algebraic sums are involved. So, it is necessary to assign proper signs to the voltage rises and voltage drops. The following sign convention may be used.

- Consider the above circuit. It has two Mesh. [Mesh 1-ABCFA, Mesh 2-CDEFC]
- Assume current direction for each Mesh. (Assume clockwise for all mesh so that analysis will be easy).
- Assume $I_1 > I_2$.



•+, - for voltage sources are known . We have to enter +, - for all resistor. In each resistor, current entering point is + and leaving point is - . [Since $I_1 > I_2$, in R_2, current flows from up to down.]

•Apply KVL for Mesh 1 [BCFAB].

•In the above Fig, for Mesh 1, Start from B-C-F-A-and end in B. While moving, if + comes first, it is potential drop. If - comes first, it is potential rise. [Put the sign which is coming first].

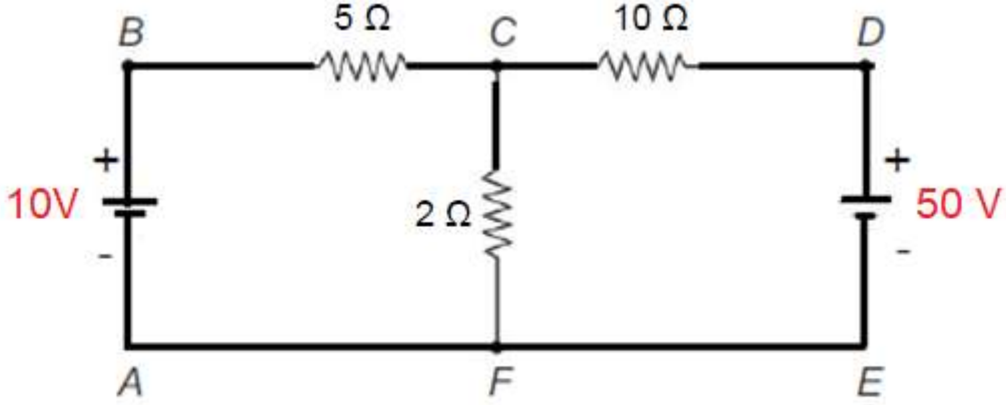
$$+ I_1R_1 + (I_1-I_2)R_2 - V_1 = 0 \quad \text{.........Equation -1}$$

•For Mesh 2, Start at C-D-E-F-and end in C

$$+I_2R_3 + V_2 - (I_1-I_2)R_2 = 0 \quad \text{.........Equation -2}$$

•V_1, V_2, R_1, R_2, R_3 are known quantity. So if we solve equation 1& 2, we get I_1 & I_2

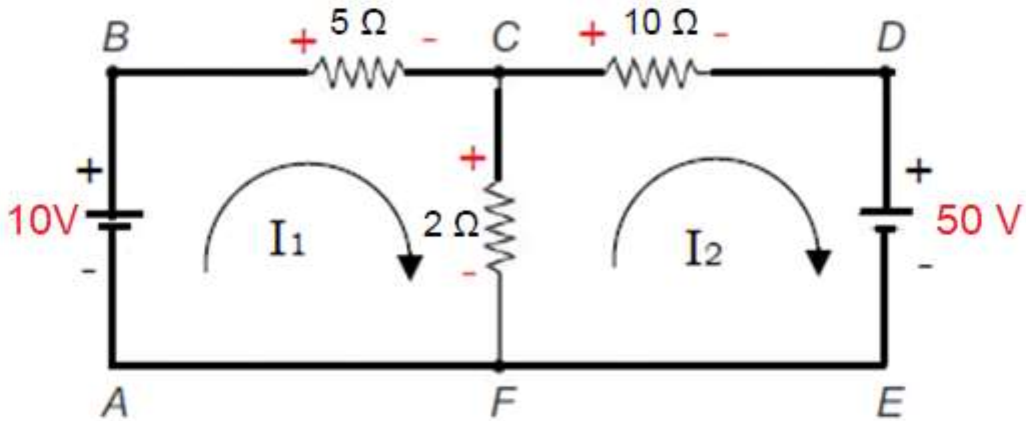
4. Apply KVL and find the current in each resistor



Sol :

•Assume current direction for each Mesh. (Assume clockwise for all mesh so that analysis will be easy).

•Assume $I_1 > I_2$.



•Enter +, - for all resistor. In each resistor, current entering point is + and leaving point is - .

•Apply KVL for Mesh 1 [BCFAB]

$+5I_1 + 2(I_1-I_2) - 10=0$ ....Equation -1

Apply KVL for Mesh 2 [CDEFC]

$+ 10I_2 + 50 -2(I_1-I_2) =0$ ....Equation -2

From Equ 1

$2I_1-2I_2 + 5I_1= 10$

$7I_1-2I_2 = 10$ ....Equation -3

From Equ 2

$10I_2 - 2I_1 + 2I_2 = - 50$

$– 2I_1 + 12I_2 = - 50$ ....Equation -4

Solving Equation using Calculator Casio fx 991 MS, we get $I_1= 0.25$ A ; $I_2= -4.125$ A [- ve I_2 indicates our assumption direction (clockwise) is wrong. So change its direction]

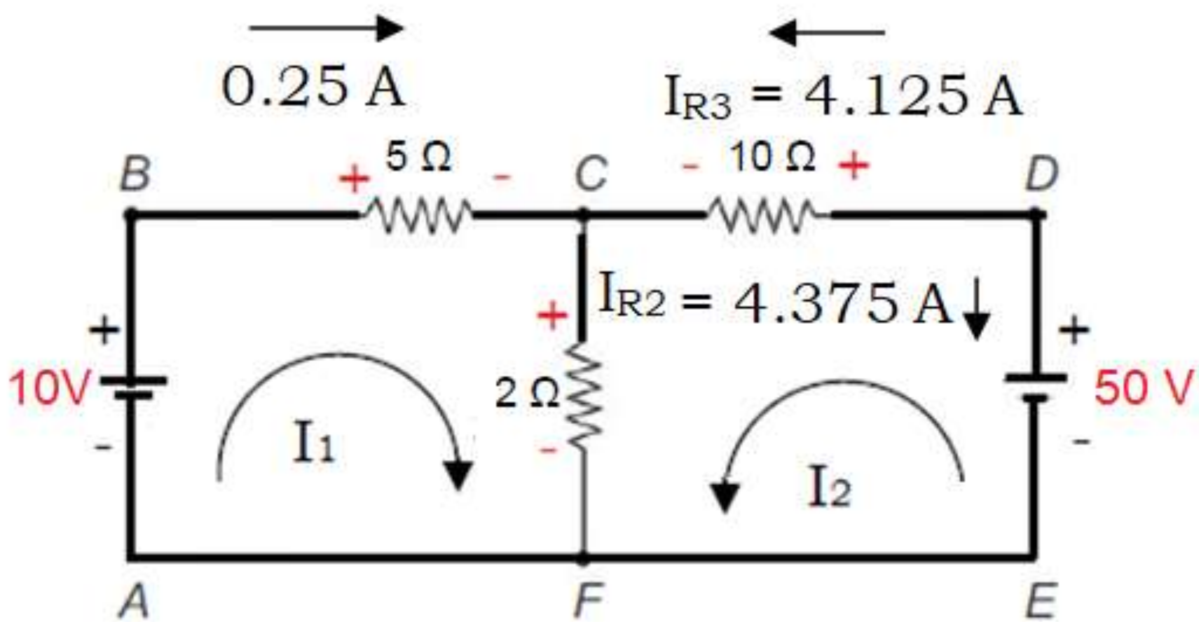
$I_1= 0.25$ A (clockwise) ; $I_2= 4.125$ A (anticlockwise)

$I_{R1} = 0.25\ A \longrightarrow$

$I_{R3} = 4.125\ A \longleftarrow$

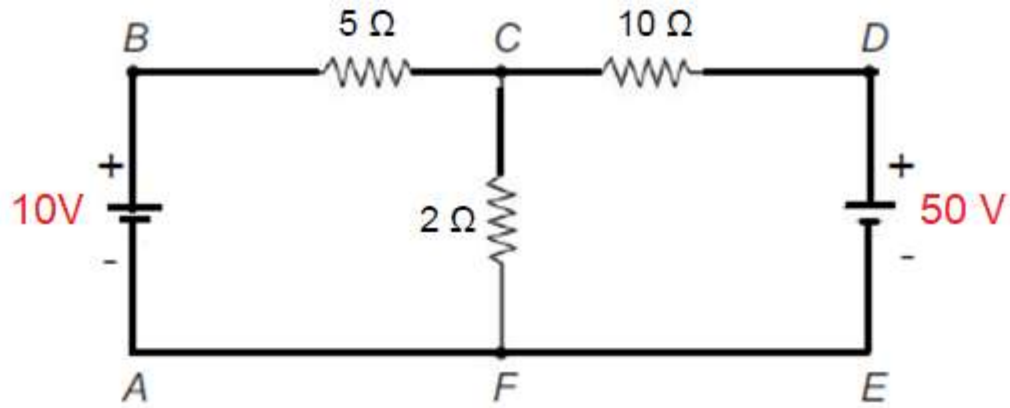
$I_{R2} = 0.25\ A + 4.125\ A$

$= 4.375\ A \downarrow$



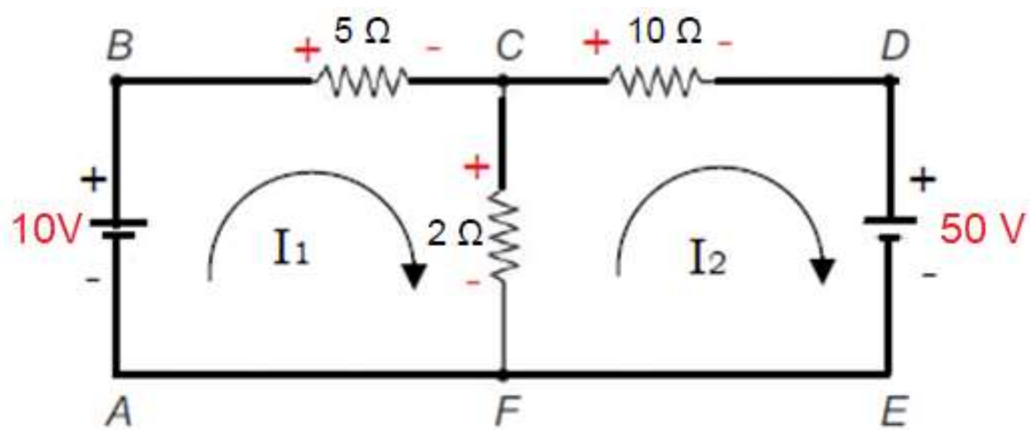
MESH CURRENT ANALYSIS OR MESH ANALYSIS

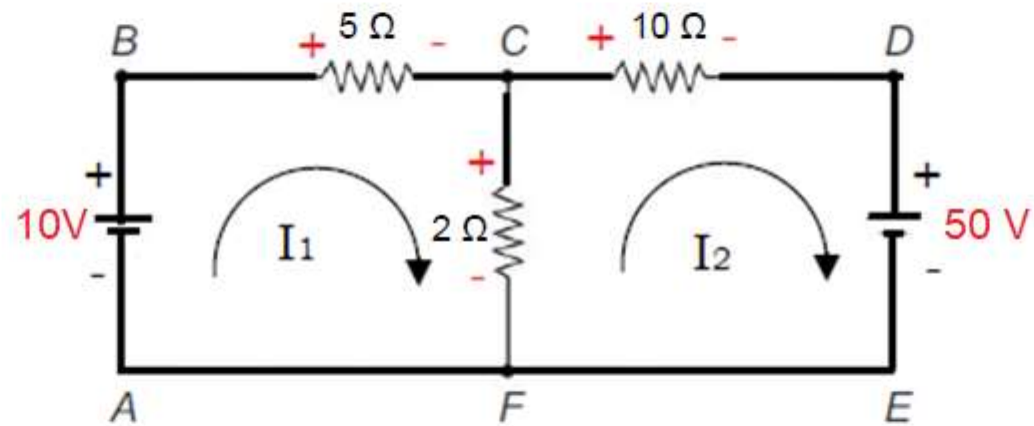
For the same problem, apply mech analysis and find the current in each resistor



Sol :

- Assume current direction for each Mesh. (Assume clockwise for all mesh so that analysis will be easy). .





We know that RI=V (Ohms law). This we are going to write in matrix form.

Size of ‘R’ matrix: No. of mesh x No. of mesh

Size of ‘I’ matrix: No. of mesh x 1

Size of ‘V’ matrix: No. of mesh x 1

$$\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Diagonal elements

Diagonal elements- Always positive

Off-Diagonal elements-,Positive if both currents are in same direction in the common resistor, negative if currents are in opposite direction in the common resistor.

Note: If you assume all mesh currents in clockwise, your Off-Diagonal elements will be always negative.

Voltage Matrix: If assumed mesh current and actual current [which flows from +ve to - ve] are same. V is + ve. If not. V is -ve

$$\begin{bmatrix} 7 & -2 \\ -2 & 12 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 10 \\ -50 \end{bmatrix}$$

$7I_1 - 2I_2 = 10$ ....Equation -1

$-2I_1 + 12I_2 = -50$ ....Equation -2

Solving Equation using Calculator Casio fx 991 MS, we get

I_1= 0.25 A ; I_2= -4.125 A [- ve I_2 indicates our assumption direction (clockwise) is wrong. So change its direction]

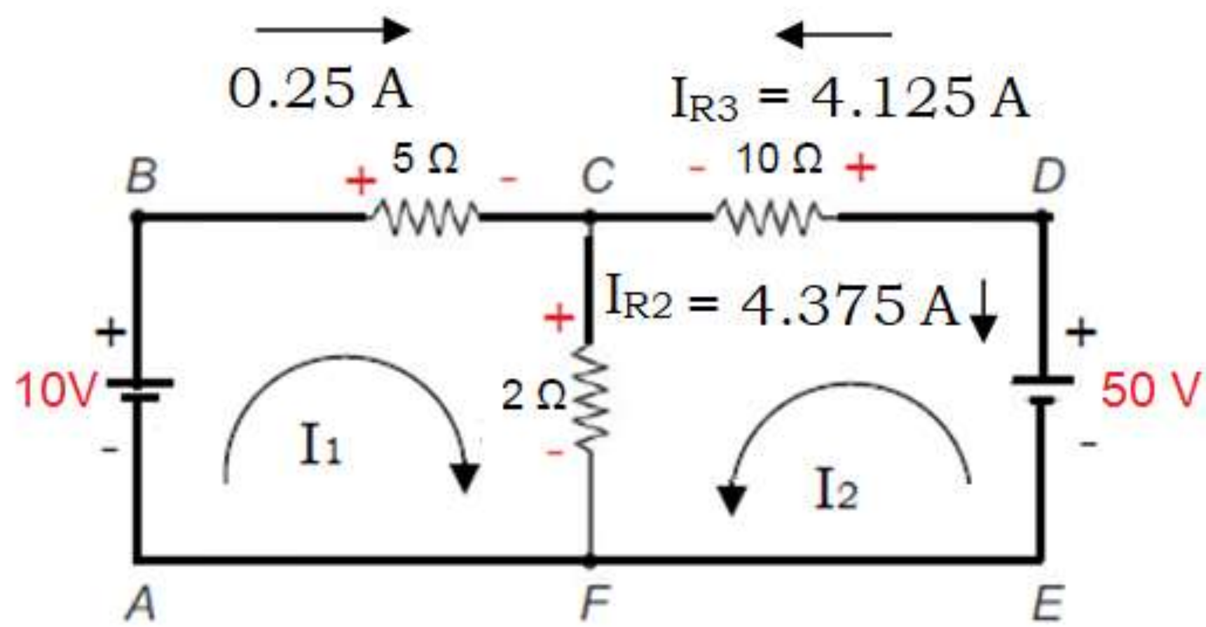
I_1= 0.25 A (clockwise) ; I_2= 4.125 A (anticlockwise)

I_{R1} = 0.25 A $\longrightarrow$

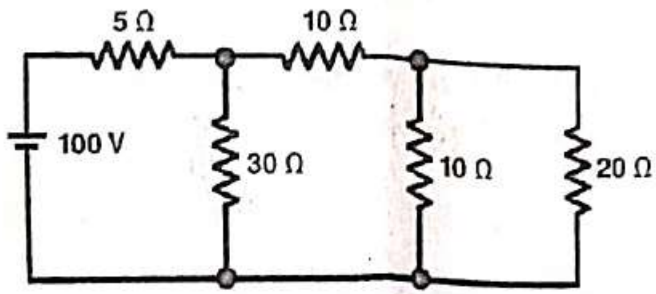
I_{R3} = 4.125 A $\longleftarrow$

I_{R2} = 0.25 A + 4.125 A

= 4.375 A $\downarrow$

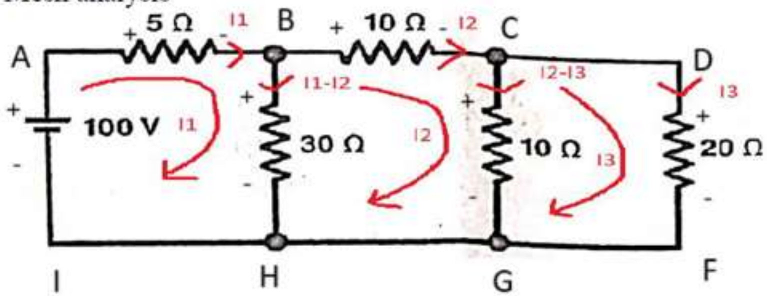


5 Apply mesh analysis to find current and power dissipation in each resistor for the given circuit.



Sol:

Mesh analysis



Step 1: Consider Mesh ABHIA

$$5I_1 + \cancel{B}\,30(I_1 - I_2) - 100 = 0.$$

$$5I_1 + 30I_1 - 30I_2 = 100$$

$$\boxed{35I_1 - 30I_2 = 100} \rightarrow (1)$$

Step2: Consider Mesh BCGHB

$$10I_2 + 10(I_2 - I_3) - 30(I_1 - I_2) = 0$$

$$10I_2 + 10I_2 - 10I_3 - 30I_1 + 30I_2 = 0$$

$$\boxed{-30I_1 + 50I_2 - 10I_3 = 0} \longrightarrow (2)$$

Step3.: Consider Mesh CDFGC.

$$20I_3 - 10(I_2 - I_3) = 0$$

$$20I_3 - 10I_2 + 10I_3 = 0$$

$$\boxed{-10I_2 + 30I_3 = 0} \longrightarrow (3)$$

Mesh equations

$35I_1 - 30I_2 = 100$ ---(Equation 1)

$-30I_1 + 50I_2 - 10I_3 = 0$ ---(Equation 2)

$-10I_2 + 30I_3 = 0$ ---(Equation 3)

After solving the above equation

$I_1 = I_{5\Omega} = 6.364A$ $\quad I_2 = I_{10\Omega} = 4.091A$ $\quad I_3 = I_{20\Omega} = 1.364A$

$I_{30\Omega} = I_1 - I_2 = 2.273A$ $\quad I_{10\Omega} = I_2 - I_3 = 2.727\ A$

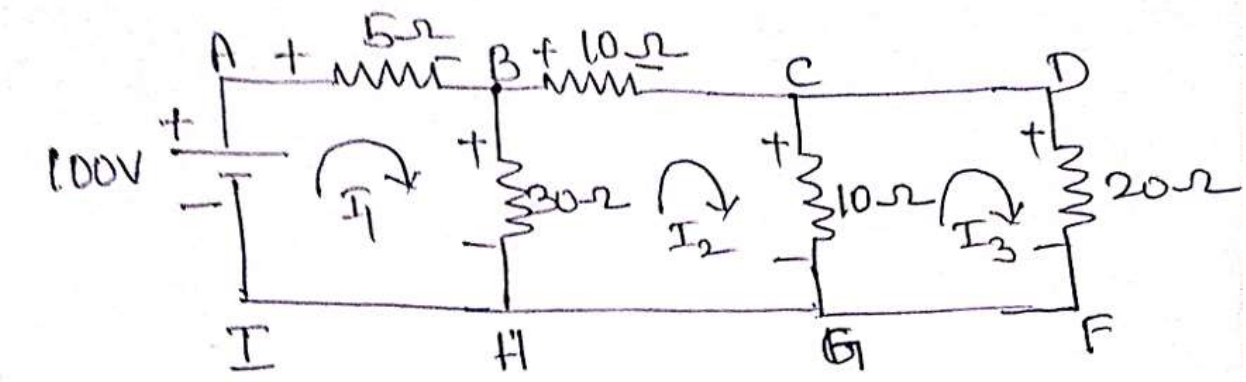
Apply $P = I^2R$

$P_{5\Omega} = 202.47W$ $\quad P_{10\Omega} = 167.35W$ $\quad P_{20\Omega} = 37.19W$

$P_{30\Omega} = 154.95W$ $\quad P_{10\Omega} = 74.38W$

Matrix Method.

Assume $I_1 > I_2 > I_3$



$$[R][I] = [V]$$ Ohms law

Size of 'R' matrix: No. of mesh x No. of mesh
Size of 'I' matrix: No. of mesh x 1
Size of 'V' matrix: No. of mesh x 1

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$\begin{bmatrix} 35 & -30 & 0 \\ -30 & 50 & -10 \\ 0 & -10 & 30 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix}$$

$$35I_1 - 30I_2 = 100 \rightarrow (1)$$

$$-30I_1 + 50I_2 - 10I_3 = 0 \rightarrow (2)$$

$$-10I_2 + 30I_3 = 0 \rightarrow (3)$$

$I_1 = I_{5\Omega} = 6.364A$ $\quad$ $I_2 = I_{10\Omega} = 4.091A$ $\quad$ $I_3 = I_{20\Omega} = 1.364A$

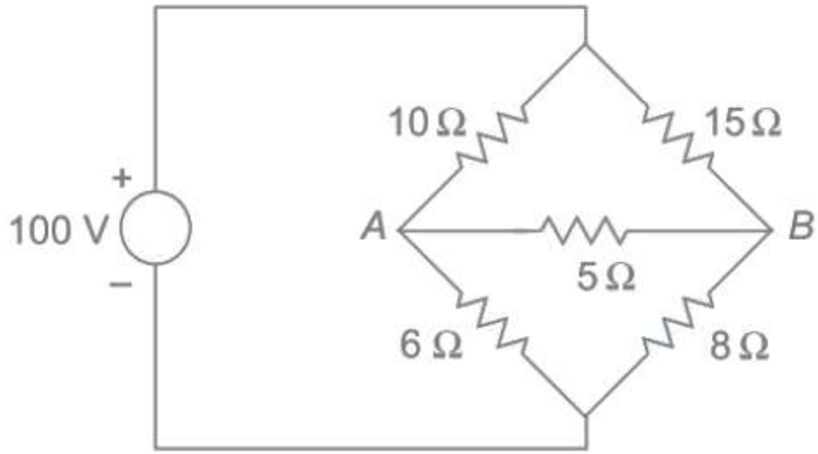
$I_{30\Omega} = I_1 - I_2 = 2.273A$ $\quad$ $I_{10\Omega} = I_2 - I_3 = 2.727\ A$

Apply $P = I^2R$

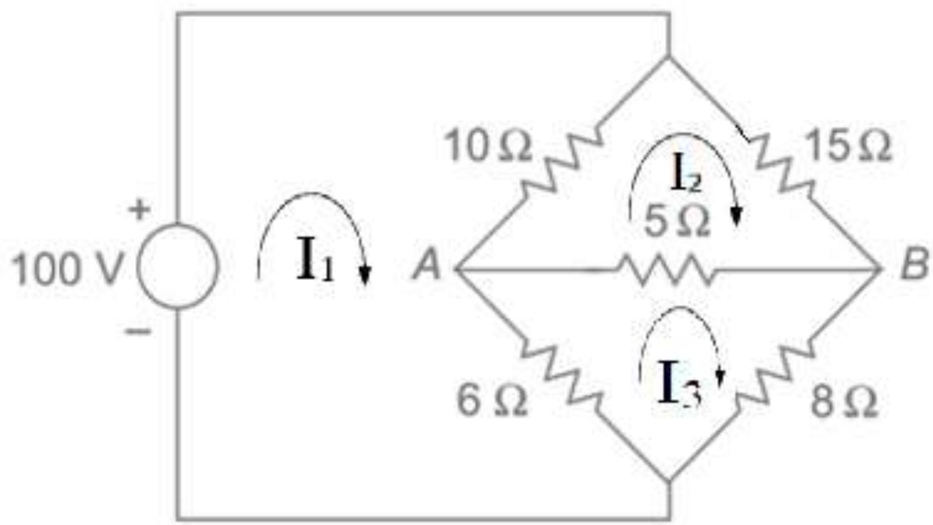
$P_{5\Omega} = 202.47W$ $\quad$ $P_{10\Omega} = 167.35W$ $\quad$ $P_{20\Omega} = 37.19W$

$P_{30\Omega} = 154.95W$ $\quad$ $P_{10\Omega} = 74.38W$

6 Find the power consumed by 5 Ω resistor



Sol:



$$\begin{bmatrix} 16 & -10 & -6 \\ -10 & 30 & -5 \\ -6 & -5 & 19 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} +100 \\ 0 \\ 0 \end{bmatrix}$$

$I_1 = 10.6$ A

$I_2 = 4.28$ A

$I_3 = 4.47$ A

$I_{5\Omega} = I_2 - I_3$

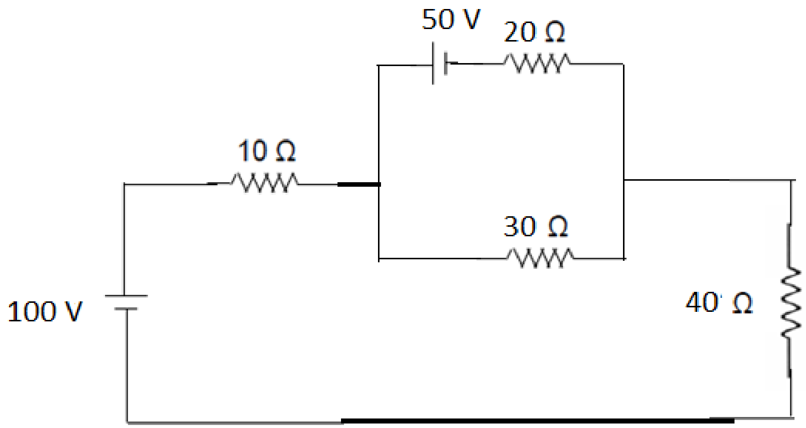
$= 4.28 - 4.47$

$= 0.19$ A

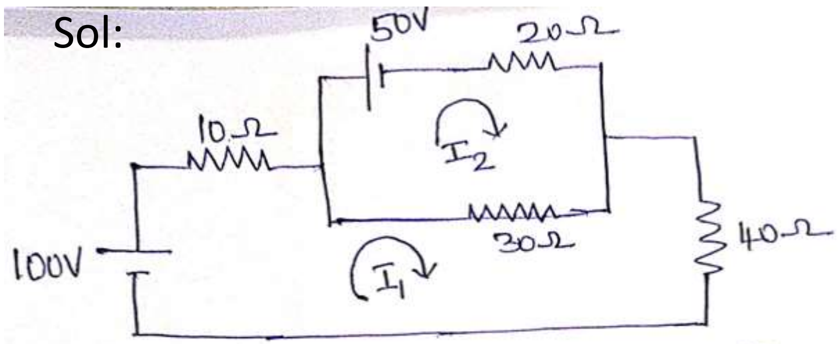
$P_{5\Omega} = 0.19^2 \times 5$

$= 0.18$ W

7 Find the power consumed by each resistor and the total power consumed by the circuit



Sol:



$$\begin{bmatrix} 80 & -30 \\ -30 & 50 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} +100 \\ -50 \end{bmatrix}$$

$I_1 = 1.13A$

$I_2 = -0.32A$ [-ve shows out Assumption direction is wrong]

$P_{10\Omega} = 1.13^2 \times 10 = 12.769 W$

$P_{40\Omega} = 1.13^2 \times 40 = 51.076 W$

$P_{30\Omega} = 1.45^2 \times 30 = 63.075 W$

$P_{20\Omega} = 0.32^2 \times 20 = 2.048 W$

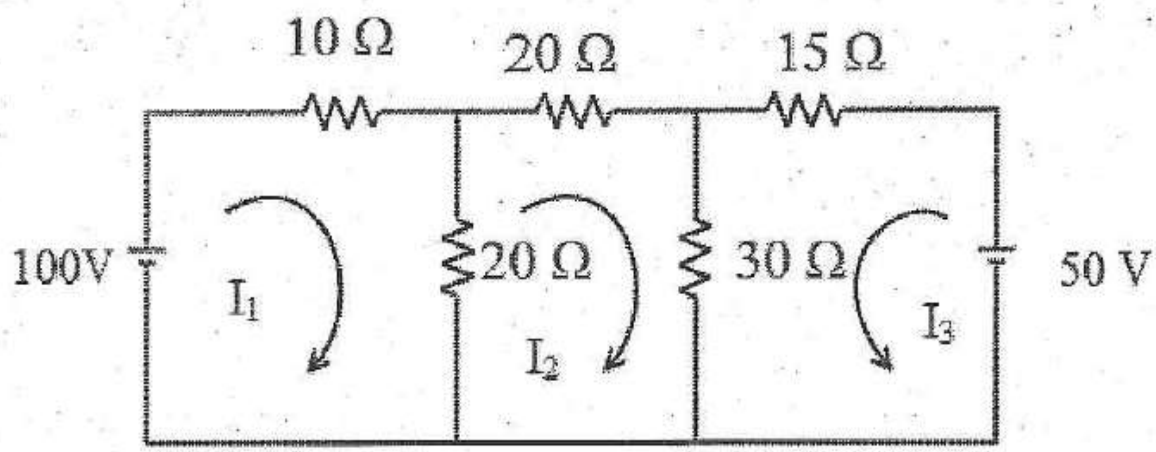
Total Power = 128.96 Watts

Total Power $= V_1 I_1 + V_2 I_2$

$= 100 \times 1.13 + 50 \times 0.32$

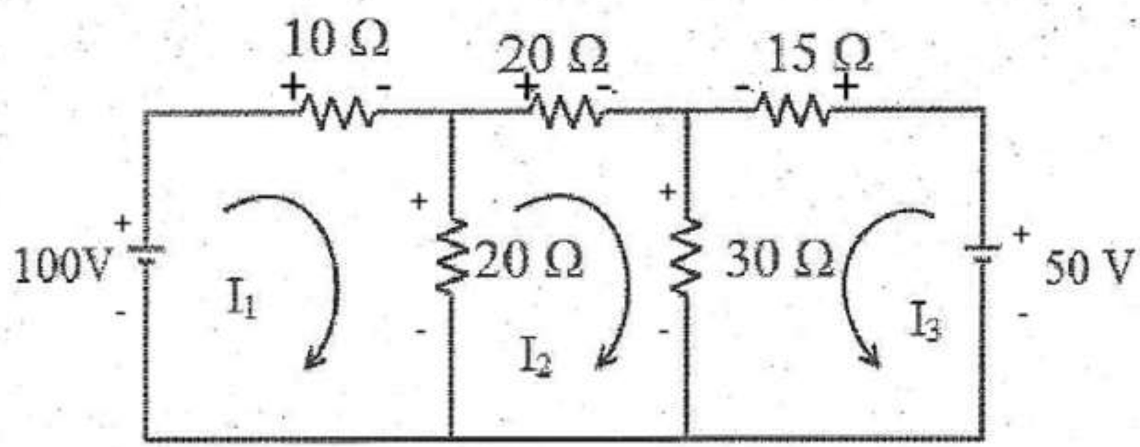
$= 129$ Watts

8 Using mesh analysis, determine mesh current in each loop given in the below circuit.



Sol:

Let us assume I_1 > I_2 > I_3



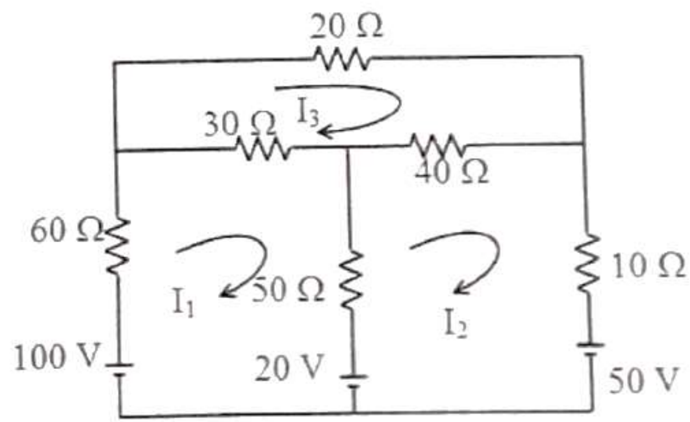
$$\begin{bmatrix} 30 & -20 & 0 \\ -20 & 70 & 30 \\ 0 & 30 & 45 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} +100 \\ 0 \\ +50 \end{bmatrix}$$

$$I_1 = 3.939 \text{ A}$$

$$I_2 = 0.909 \text{ A}$$

$$I_3 = 0.505 \text{ A}$$

9 Find the current that flows through the 50 Ω resistor for the circuit shown below using mesh analysis.



Sol:

Let us assume $I_1 > I_2 > I_3$

$$\begin{bmatrix} 140 & -50 & -30 \\ -50 & 100 & -40 \\ -30 & -40 & 90 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 100-20 \\ 20-50 \\ 0 \end{bmatrix}$$

$I_1 = 0.7$ A

$I_2 = 0.174$ A

$I_3 = 0.311$ A

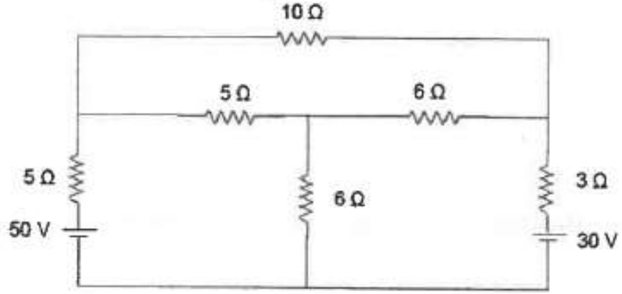
$I_{50\Omega} = I_1 - I_2$

$= 0.7 - 0.174$

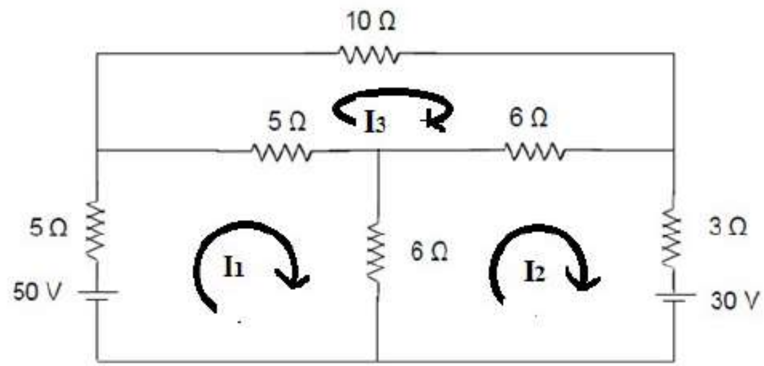
$= 0.526$ A

10

Find the current through 10 Ω resistor.



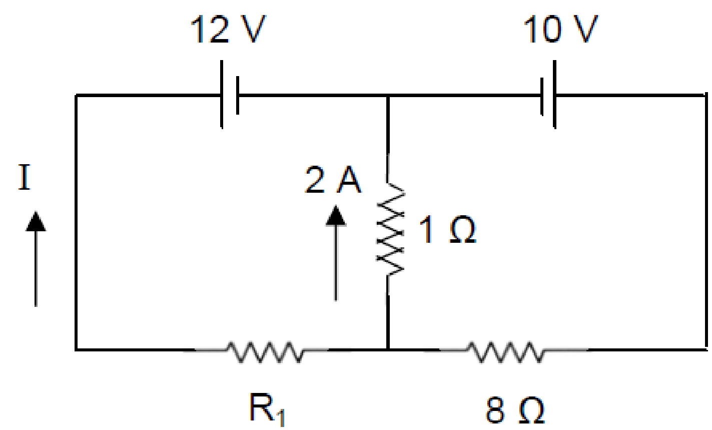
Sol:



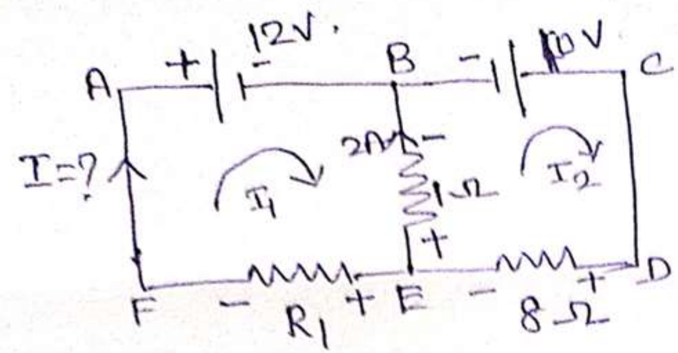
$$\begin{bmatrix} 16 & -6 & -5 \\ -6 & 15 & -6 \\ -5 & -6 & 21 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 50 \\ -30 \\ 0 \end{bmatrix}$$

$I_3 = I_{10\Omega} = 0.595\ A$

11 Determine I and R_1 in the circuit



Sol:



Apply KVL for Mesh 1. (i.e ABEFA)

$$+12 - 2\times 1 + I_1 R_1 = 0.$$

$$\boxed{I_1 R_1 = -10} \rightarrow (1)$$

Apply KVL for Mesh 2 (ii BCDEB)

$$-10 + 8I_2 + 2 = 0$$

$$8I_2 = 8$$

$$I_2 = 1A.$$

Also $I_2 - I_1 = 2A$

$$1 - I_1 = 2A$$

$$-I_1 = 1A \Rightarrow \boxed{I_1 = -1A}$$

We know $I_1 R_1 = -10$

$$-R_1 = -10$$

$$\boxed{R_1 = 10\Omega}$$

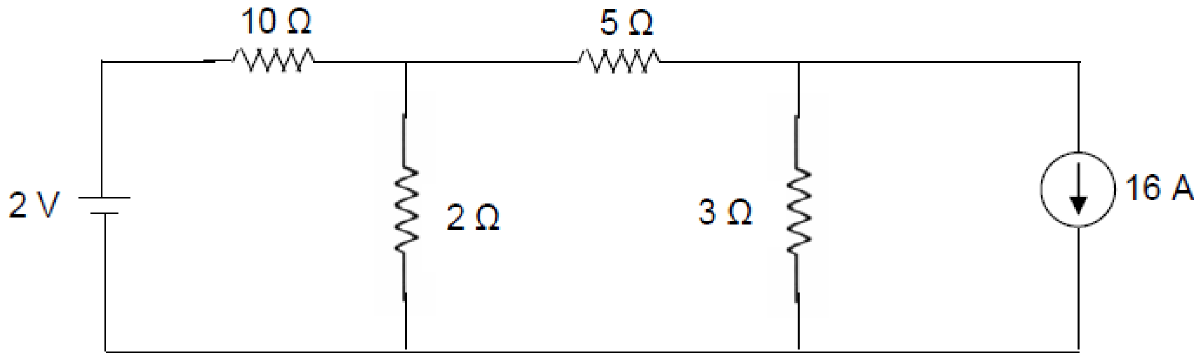
MESH ANALYSIS WITH VOLTAGE & CURRENT SOURCES

Method 1: KVL Method

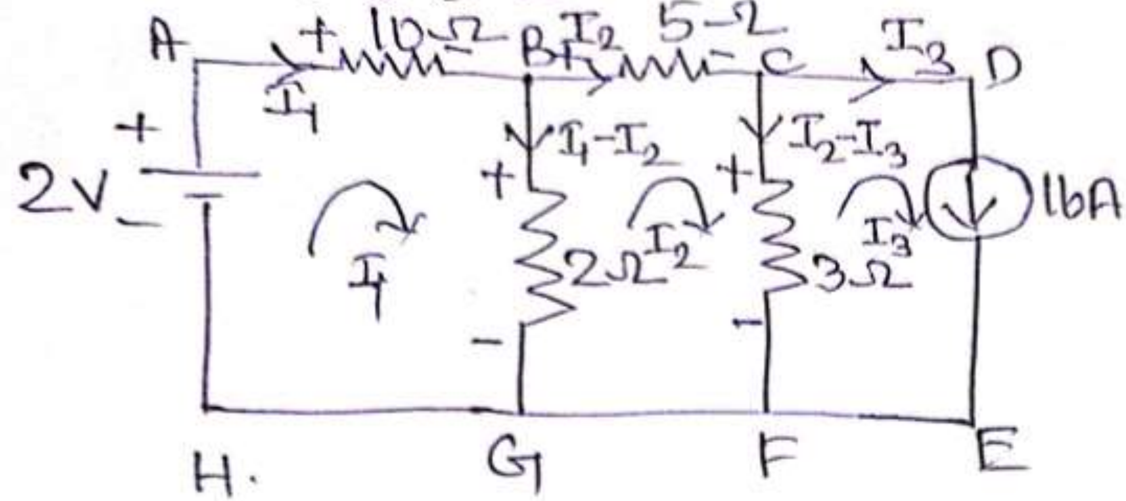
Method 2: Source transformation and then mesh analysis

Note: If load [where current/voltage/power is asked] is internal resistance of current source, you have solve only by KVL Method

12 find the power supplied by the 2 V source.



Method:1 :- KVL method.



(X) $\boxed{I_3 = 16A}$

Apply KVL for Mesh1 (ABGHA)

$$10I_1 + 2(I_1 - I_2) - 2 = 0$$

$$10I_1 + 2I_1 - 2I_2 = 2$$

$$\boxed{12I_1 - 2I_2 = 2} \longrightarrow (1)$$

Apply KVL for Mesh 2 (BCFGB).

$$5I_2 + 3(I_2 - I_3) - 2(I_1 - I_2) = 0.$$

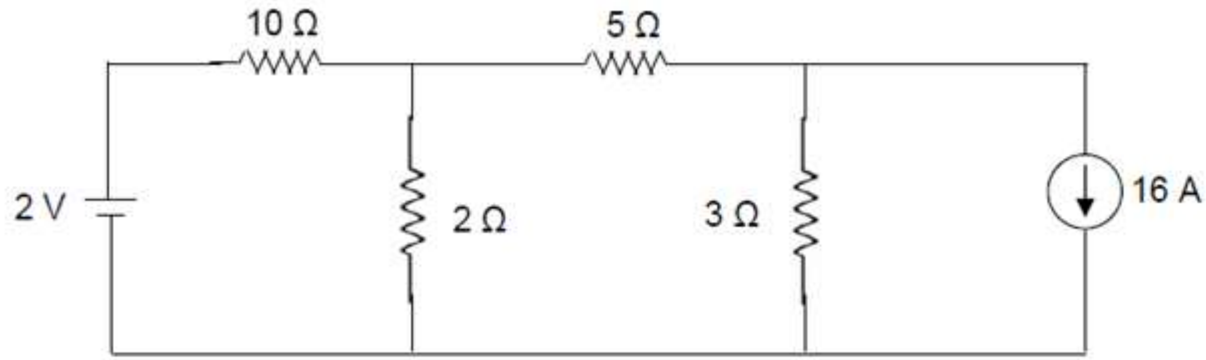
$$5I_2 + 3I_2 - 3I_3 - 2I_1 + 2I_2 = 0$$

$$\boxed{-2I_1 + 10I_2 = 48} \rightarrow (2)$$

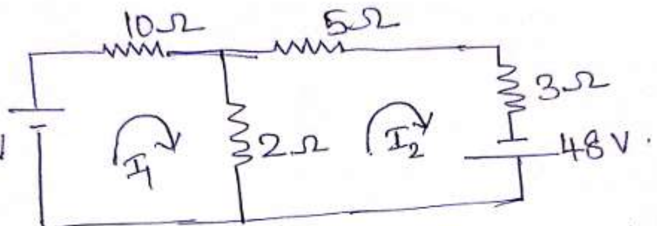
Solving 1 & 2 $I_1 = 1A$; $I_2 = 5A$

$$P_1 = V_1 I_1 = 2 \times 1$$
$$= 2 \text{ Watts.}$$

Method 2: Source Transformation method



Sol.



[Use Source Transformation]

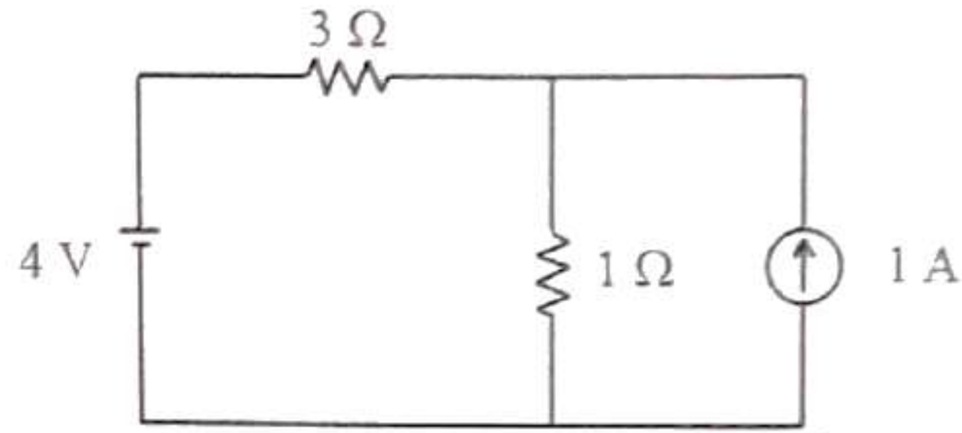
$$\begin{bmatrix} 12 & -2 \\ -2 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 48 \end{bmatrix}$$

$I_1 = 1A$; $I_2 = 5A$.

$P_1 = V_1 I_1 = 2 \times 1$

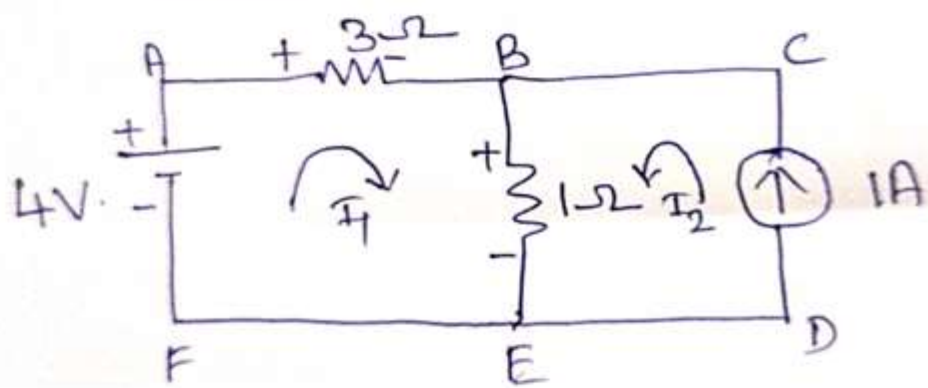
$= 2$ Watts.

13 For the circuit shown, find the voltage across the 1 ohm resistors.



Sol

Note: Since Load is 1Ω Resistor, you Should not use Source transformation



Apply KVL for mesh ABEFA.

$$+3I_1 + 1(I_1+I_2) - 4 = 0.$$

$$3I_1 + I_1 + I_2 = 4$$

$$\boxed{4I_1 + I_2 = 4} \rightarrow (1)$$

$$I_2 = 1 \rightarrow (2)$$

Sub (2) in (1)

$$4I_1 + 1 = 4$$

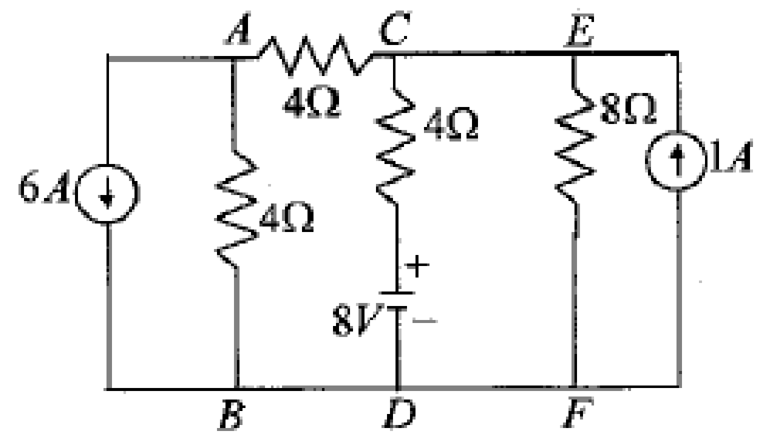
$$4I_1 = 3$$

$$I_1 = 0.75\,A.$$

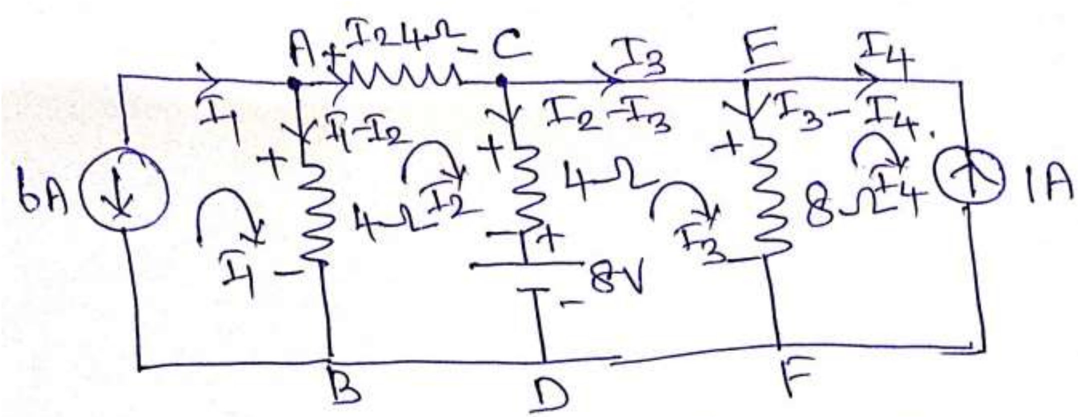
$$I_{1\Omega} = I_1 + I_2$$
$$= 1.75\,A.$$

$$V_{1\Omega} = 1.75 \times 1$$
$$= 1.75\,V.$$

14 For the circuit shown below, find currents I_{AB}, I_{AC}, I_{CD} and I_{EF} using mesh current method.



Source transformation is not possible since R_{AB} & R_{EF} are loads



Given $I_1 = -6A$; $I_4 = -1A$.

Apply KVL for Mesh HCDBH

$$+4I_2 + 4(I_2 - I_3) + 8 - 4(I_1 - I_2) = 0.$$

$$4I_2 + 4I_2 - 4I_3 + 8 - 4(-6) + 4I_2 = 0.$$

$$\boxed{12I_2 - 4I_3 = -32} \rightarrow ①$$

Apply KVL for Mesh CEFDC.

$$+8(I_3 - I_4) - 8 - 4(I_2 - I_3) = 0.$$

$$8I_3 - 8I_4 - 4I_2 + 4I_3 = 8$$

$$12I_3 - 4I_2 - 8(-1) = 8$$

$$\boxed{-4I_2 + 12I_3 = 0}$$

$$I_2 = -3A; \quad I_3 = -1A.$$

$\therefore$ $I_1 = 6A$; $I_2 = 3A$; $I_3 = 1A$

$I_4 = 1A$

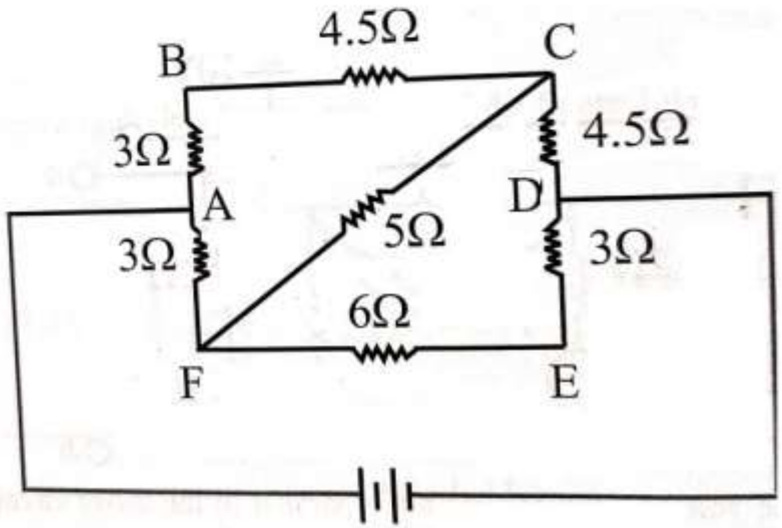
$I_{AB} = I_1 - I_2 = 3A \uparrow$ | $I_{EF} = I_3 - I_4 = 0A.$

$I_{AC} = I_2 = 3A \leftarrow$

END OF MESH ANALYSIS

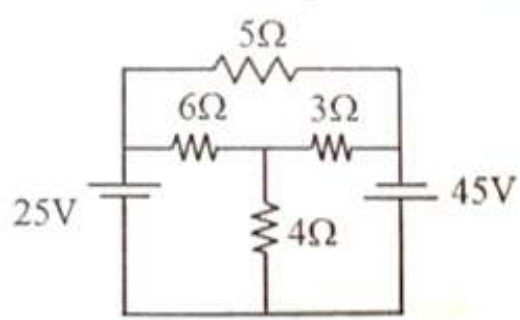
Assignment problems

1 **Find the current flowing through 5Ω resistor**



V=24 Volts

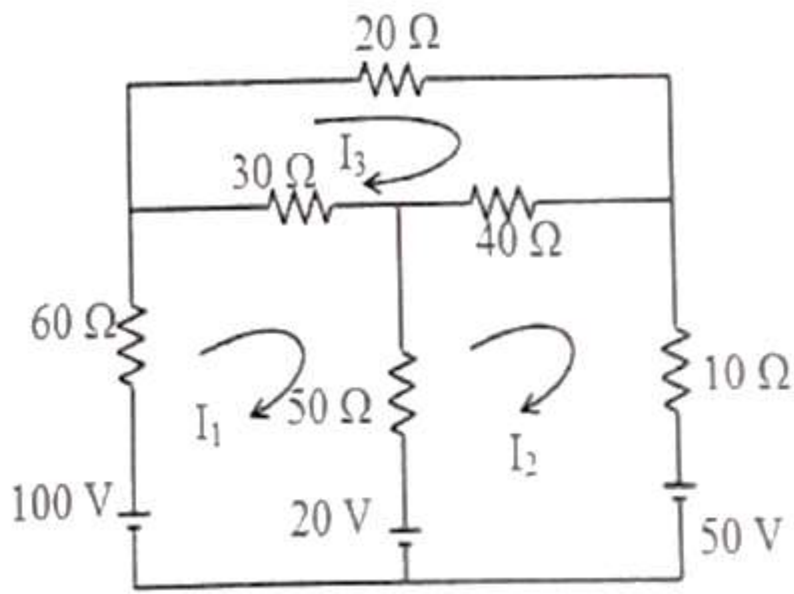
2 Find the current in 5Ω resistor from the given circuit using Mesh analysis.



Superposition Theorem

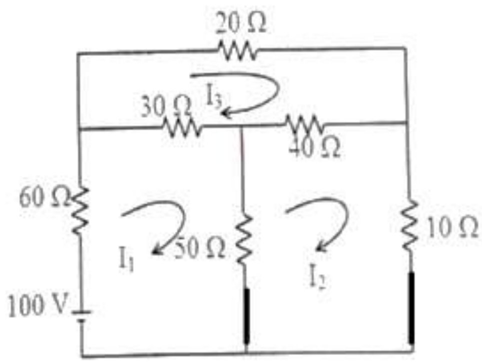
Superposition theorem states the following:
"In any linear and bilateral network or circuit having multiple independent sources, the response of an element will be equal to the algebraic sum of the responses of that element by considering one source at a time."

1 Find the current that flows through the 50 Ω resistor for the circuit shown below using Superposition Theorem



Note: In super position theorem, while considering one source, we have to kill the remaining sources. Killing means: If the source is a **voltage source**, remove the source and **short circuit** it. If the source is a **current source**, remove the source and **open circuit** it.

Step 1: Considering 100 V source only



$$\begin{bmatrix} 140 & -50 & -30 \\ -50 & 100 & -40 \\ -30 & -40 & 90 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \\ 0 \end{bmatrix}$$

$I_1 = 1.23$ A

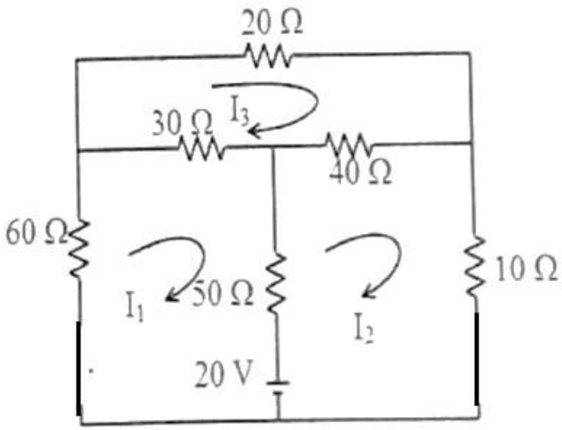
$I_2 = 0.94$ A

$I'_{50\Omega} = I_1 - I_2$

$I'_{50\Omega} = 1.23 - 0.94$

$I'_{50\Omega} = 0.29$ A $\downarrow$

Step 2: Considering 20 V source only



$$\begin{bmatrix} 140 & -50 & -30 \\ -50 & 100 & -40 \\ -30 & -40 & 90 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -20 \\ 20 \\ 0 \end{bmatrix}$$

$I_1 = -0.056$ A

$I_2 = 0.2$ A

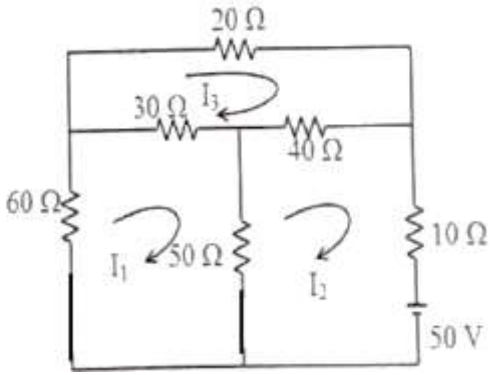
$I''_{50\Omega} = I_1 - I_2$

$I''_{50\Omega} = -0.056-0.2$

$I''_{50\Omega} = -\ 0.256$ A $\downarrow$

$I''_{50\Omega} = 0.256$ A $\uparrow$

Step 3: Considering 50 V source only



$$\begin{bmatrix} 140 & -50 & -30 \\ -50 & 100 & -40 \\ -30 & -40 & 90 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 0 \\ -50 \\ 0 \end{bmatrix}$$

$I_1 = -0.474$ A

$I_2 = -0.973$ A

$I_{50\Omega}''' = I_1 - I_2$

$I_{50\Omega}''' = -0.474 - (-0.973)$

$I_{50\Omega}''' = \underline{\underline{0.499}}$ A $\downarrow$

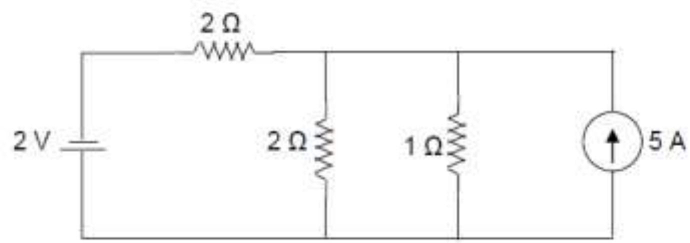
Step 4: Apply superposition theorem

$I_{50\Omega} = I_{50\Omega}' + I_{50\Omega}'' + I_{50\Omega}'''$

$= 0.29 \text{ A} \downarrow + 0.256 \text{ A} \uparrow + 0.499 \text{ A} \downarrow$

$= 0.533 \text{ A} \downarrow$

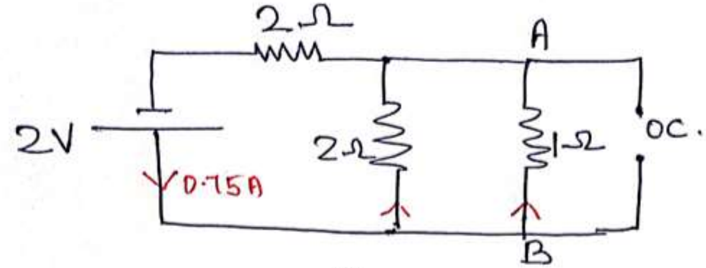
2 Calculate the current through the 1 Ω resistor in the circuit shown

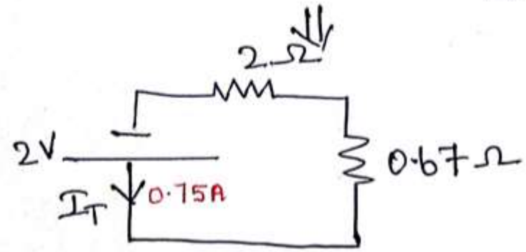


Sol:

Super Position theorem is the easiest method to solve the above Problem.

Step I : Consider 2V Source only



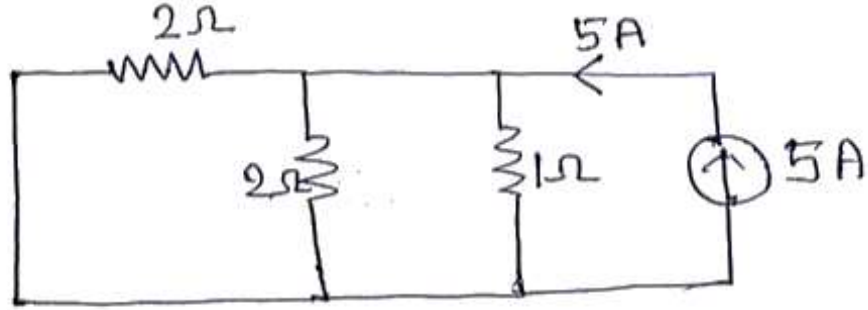


$\Rightarrow I_T = 0.75\,A.$

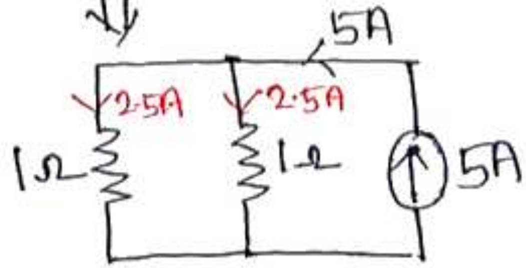
$$I_{1\Omega} = \frac{I_T \times \text{Opp. Res}}{\text{Total Res}}$$ [current division Rule]

$$I_{1\Omega}' = \frac{0.75 \times 2}{3} = 0.5\,A \uparrow$$

Step 2: Consider 5A Source only



$\Downarrow$



$\Rightarrow I''_{1\Omega} = 2.5A \downarrow$

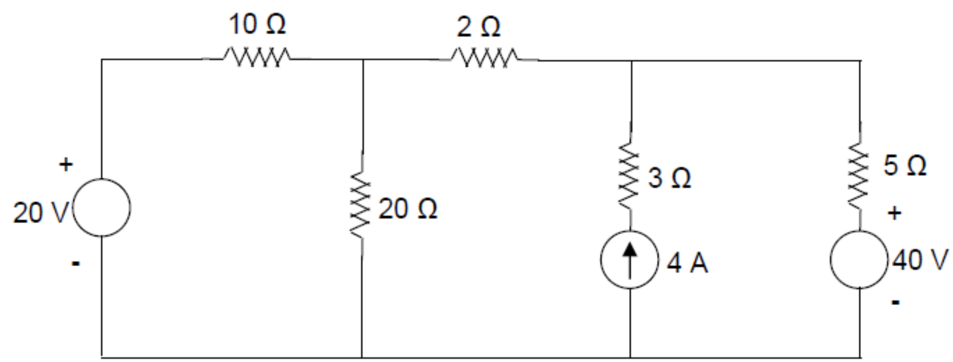
Step 3: Apply superposition theorem

$$I_{1\Omega} = I'_{1\Omega} + I''_{1\Omega}$$

$$= 0.5A\uparrow + 2.5A\downarrow$$

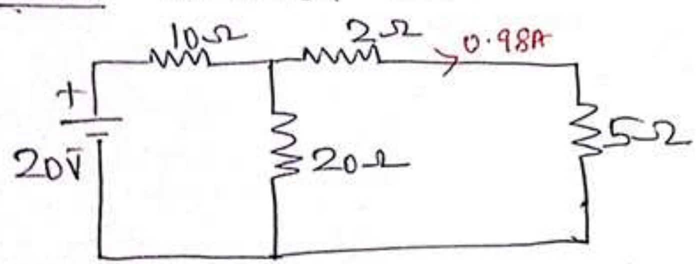
$$I_{1\Omega} = 2A\downarrow$$

3 Find the power consumed by 2 Ω resistor using Superposition theorem. **UQ-15 MARKS**



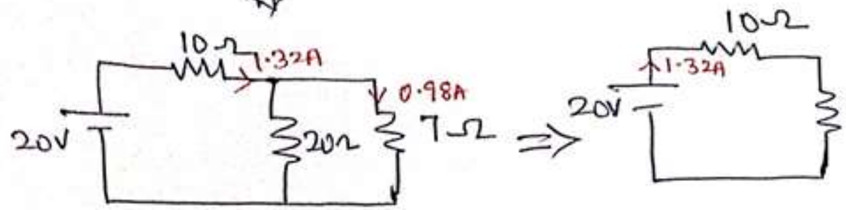
Sol:

Step 1: Consider 20V Source only.



[Note: No use of 3Ω Resistor]

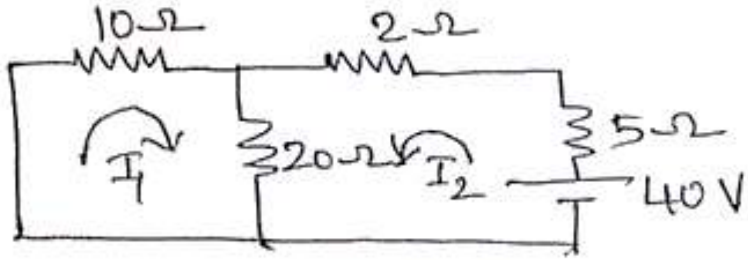
⇓



⇒ $I_T = 1{\cdot}32A$

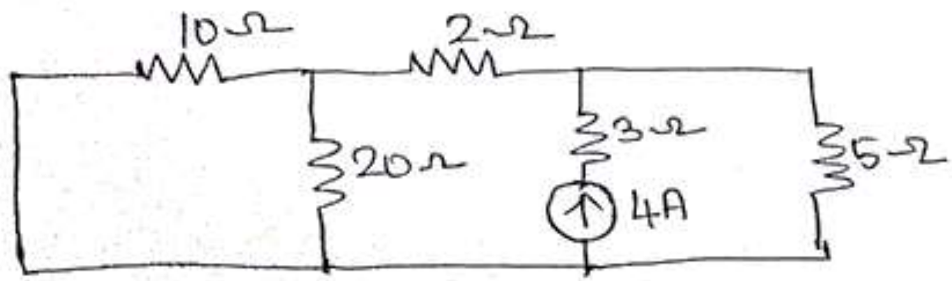
$$I_{7\Omega} = \frac{1{\cdot}32 \times 20}{27} = 0{\cdot}98A \Rightarrow \boxed{I'_{2\Omega} = 0{\cdot}98A \rightarrow}$$

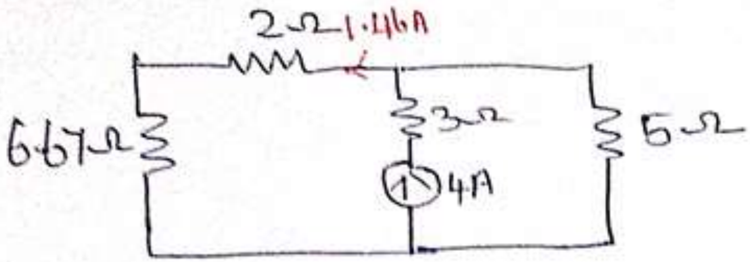
Step2: Consider 40V source only



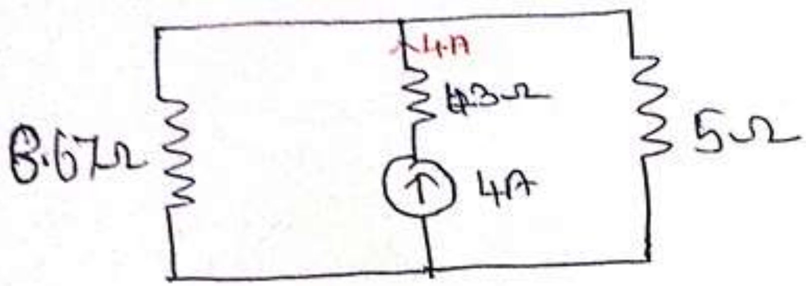
$$\begin{bmatrix} 30 & 20 \\ 20 & 27 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 40 \end{bmatrix} \Rightarrow \boxed{I''_{2\Omega} = 2.92A}$$

Step 3.: Consider 4A source only





⇓



8.67 Ω and 5 Ω Resistors are connected in Parallel.

$$I_{8.67\Omega} = \frac{4 \times 5}{13.67} = 1.46\ A \downarrow$$

$$\boxed{I'''_{2\Omega} = 1.46A \leftarrow}$$

Step 4: Apply Superposition theorem

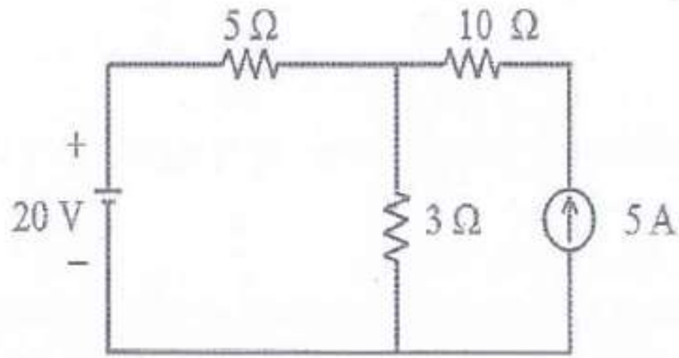
$$I_{2\Omega} = I_{2\Omega}' + I_{2\Omega}'' + I_{2\Omega}'''$$

$$= 0.98A \rightarrow + 2.92A \leftarrow + 1.46A \leftarrow$$

$$I_{2\Omega} = 3.4A \leftarrow$$

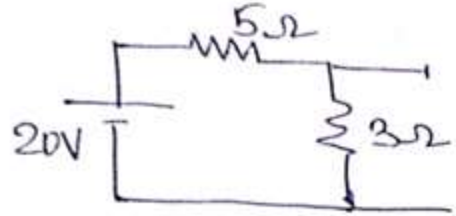
Ans

4 Identify the current through 3Ω resistor in the circuit shown below, using superposition theorem.



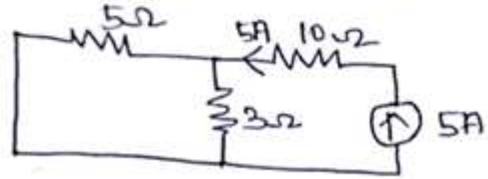
Sol:

Step1: Consider 20V Source only.



$\Rightarrow I'_{3\Omega} = \frac{20}{8} = 2.5A\downarrow$

Step2: Consider 5A Source only



$I''_{3\Omega} = \frac{5\times5}{8}$

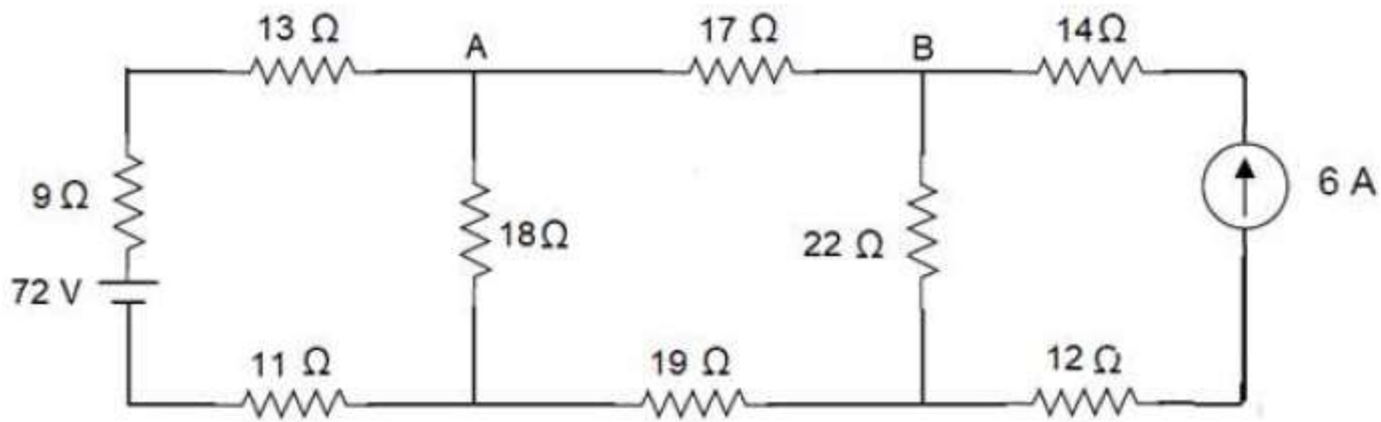
$= 3.125A\downarrow$

Step3: Apply Superposition theorem

$I_{3\Omega} = 2.5A\downarrow + 3.125A\downarrow$

$= 5.625A\downarrow$ Ans.

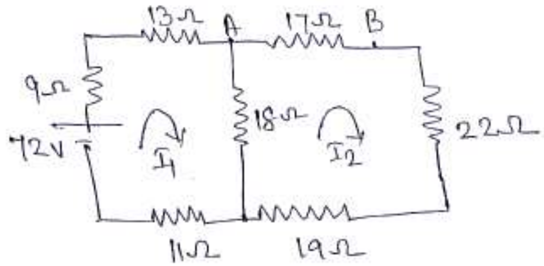
5 Using Superposition theorem, calculate the current through the 17 Ω resistor.



Sol:

Note: Resistor connected in series with current source and resistor connected in parallel with voltage source are useless.

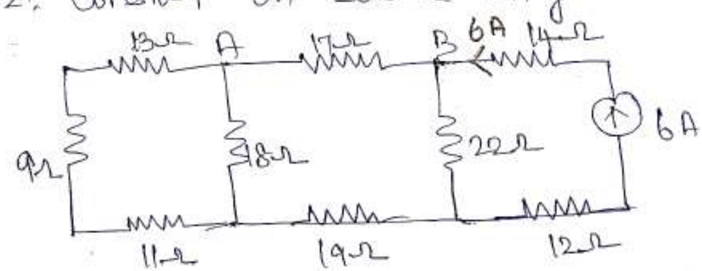
Step1: Consider 72V source only.

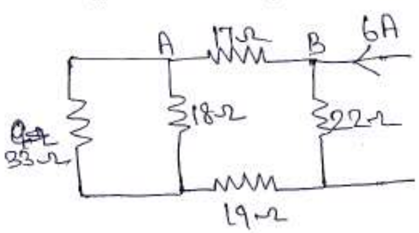


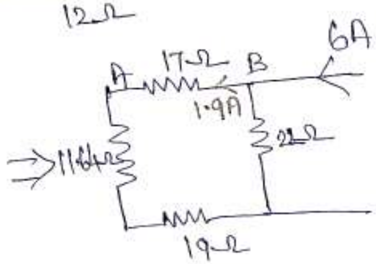
$$\begin{bmatrix} 51 & -18 \\ -18 & 76 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 72 \\ 0 \end{bmatrix} \Rightarrow I_2 = 0.36A$$

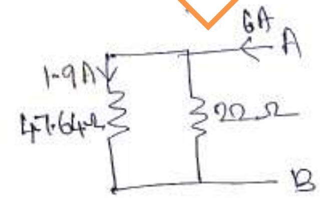
$$\boxed{\therefore I'_{17\Omega} = 0.36A \rightarrow \text{ A to B}}$$

Step2: Consider 6A source only.









$$I_{47.64\Omega} = \frac{6 \times 22}{(47.64 + 22)} = 1.9A$$

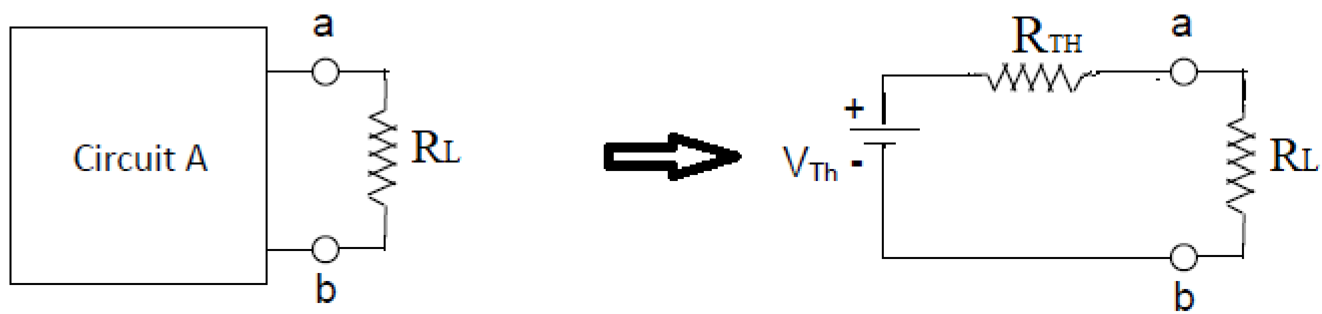
$$\boxed{I''_{17\Omega} = 1.9\text{ Amps} \leftarrow \text{ B to A}}$$

Step 3

$$I_{17\Omega} = I'_{17} + I''_{17}$$
$$= 1.9 - 0.36$$
$$= 1.54A \leftarrow \text{ B to A}$$

Ans

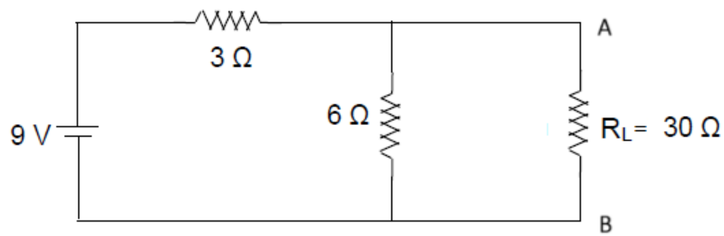
Thevenin's Theorem



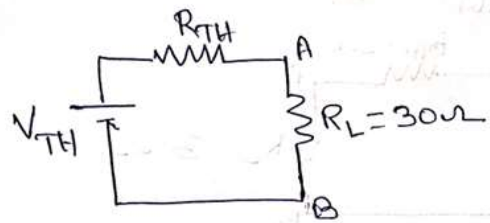
V_{TH} = Open circuit voltage across ab (After removing load)

R_{TH} = Equivalent resistance across ab (After removing load. Kill the source also)

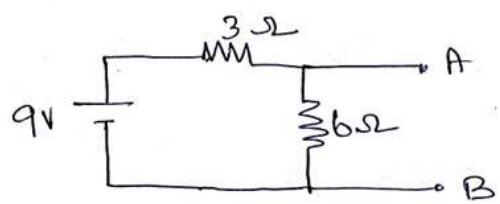
1 Find the current flowing through 30 Ω using Thevenin's theorem



Step1: Thevenin's equivalent ckt.



Step2: To find V_{TH} [Remove the Load and Find voltage across AB]

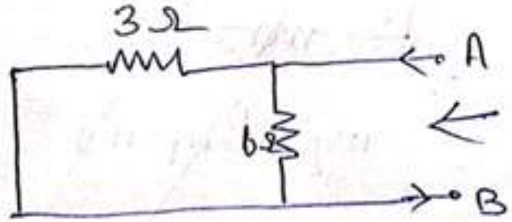


$V_{TH} = V_{AB} = V_{6\Omega}$

$I_T = \frac{9}{9} = 1A \Rightarrow V_{6\Omega} = 6\,V$

Step3: To find R_{TH} [Looking back resistance]

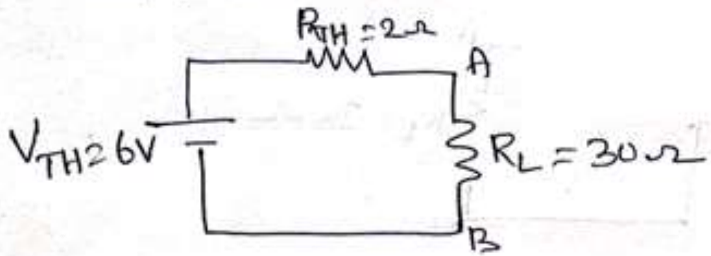
[Remove the load and Resistance across AB, Kill the Source(S)]



Killing means: If the source is a **voltage source**, remove the source and **short circuit** it. If the source is a **current source**, remove the source and **open circuit** it.

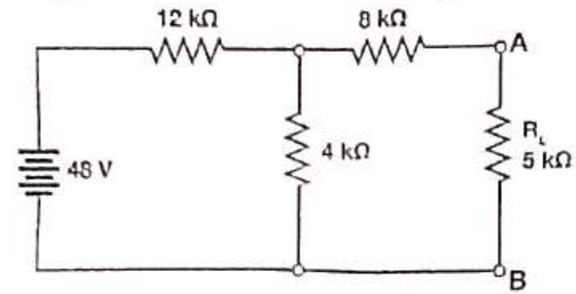
$$R_{TH} = \frac{3\times 6}{9} = 2\Omega$$

Step4: Sub V_{TH} and R_{TH} in Thevenin's equivalent ckt.

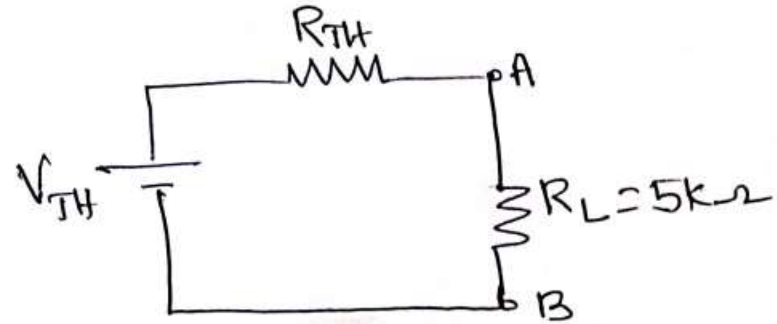


$$I_L = \frac{6}{32} = 0.187A.$$

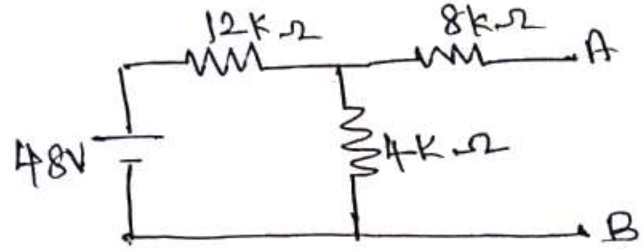
2 Find the current through load resistor 5 kΩ using Thevenin's theorem.



Step 1: Thevenin's equivalent ckt.



Step2 : To find V_{TH}

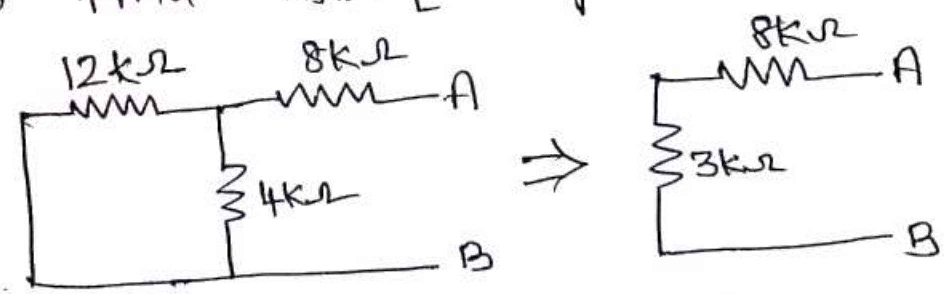


$$V_{TH} = V_{AB} = V_{4k\Omega}.$$

$$I_T = \frac{48}{16\times10^3} = 3mA.$$

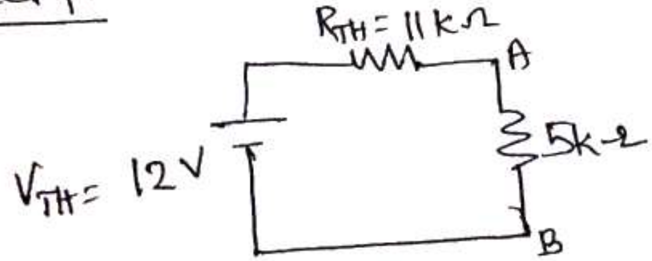
$$V_{TH} = 4\times10^3 \times 3\times10^{-3}$$
$$= 12V$$

Step3 : To find R_{TH}. [Looking Back Resistance]



$\therefore R_{TH} = 11k\Omega$

step4: Sub V_{TH} and R_{TH} in (i).



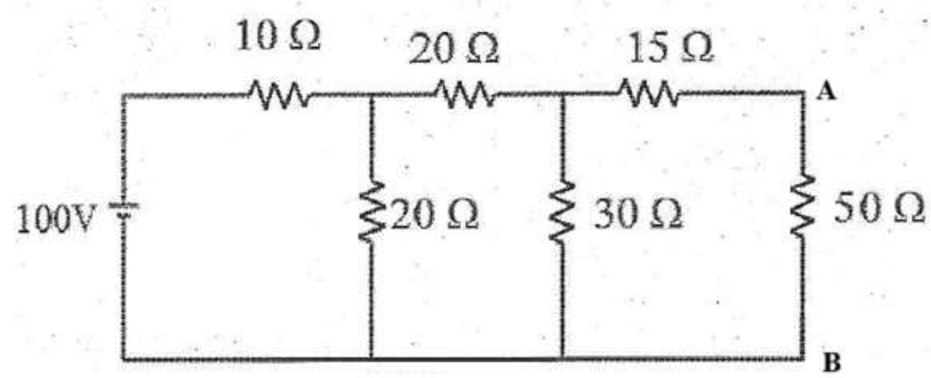
$$\Rightarrow I_L = \frac{12}{16 \times 10^{3}}$$

$$= 0.75 mA$$

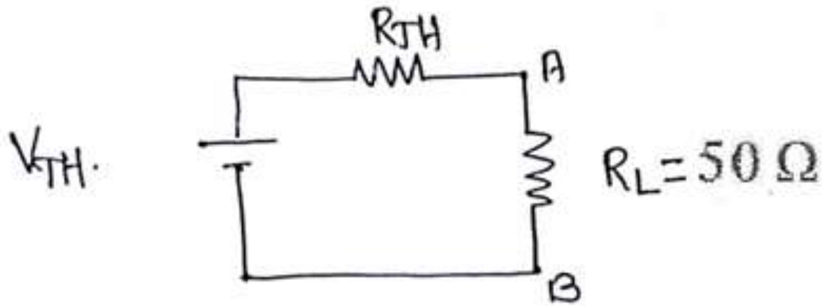
$\longrightarrow$ Ans.

3 Find the current flowing through 50 Ω resistor using Thevenin’s Theorem

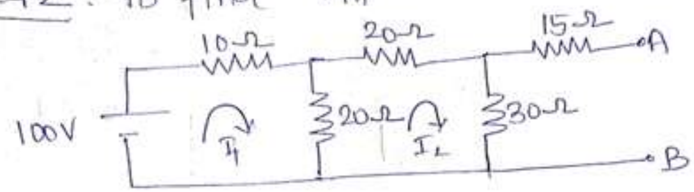
UQ-15 MARKS



<u>Step I</u>. Thevenin's equivalent ckt.



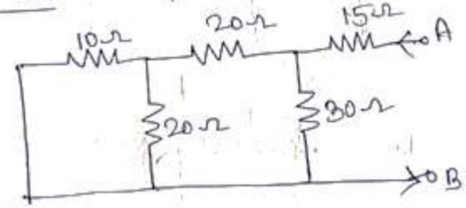
Step 2 : To find V_{TH}



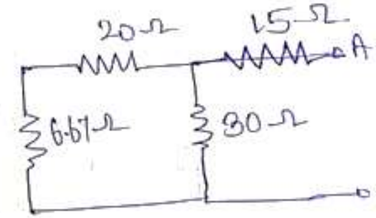
$V_{TH} = V_{AB} = V_{30\Omega}$

$$\begin{bmatrix} 30 & -20 \\ -20 & 70 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \end{bmatrix} \Rightarrow \begin{aligned} I_2 &= I_{30\Omega} = 1.17 \\ V_{30\Omega} &= 1.176 \times 30 \\ &= 35.28V \end{aligned}$$

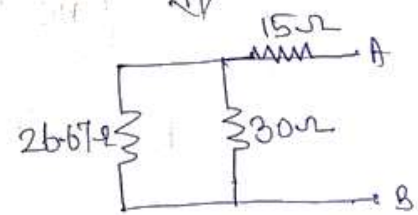
Step 3 : To find R_{TH}



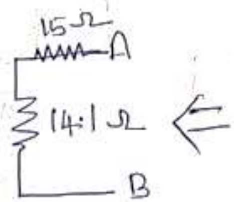
$\Rightarrow$



$\Downarrow$



$\Leftarrow$



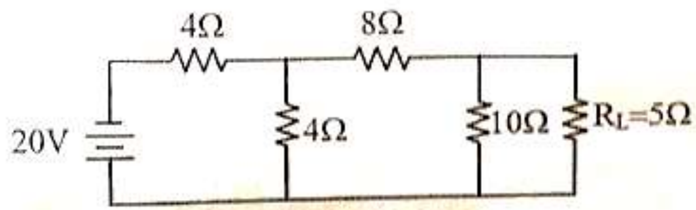
$R_{TH} = 29.1\Omega \Leftarrow$

Step 4 : Sub V_{TH} and R_{TH} in step 1.

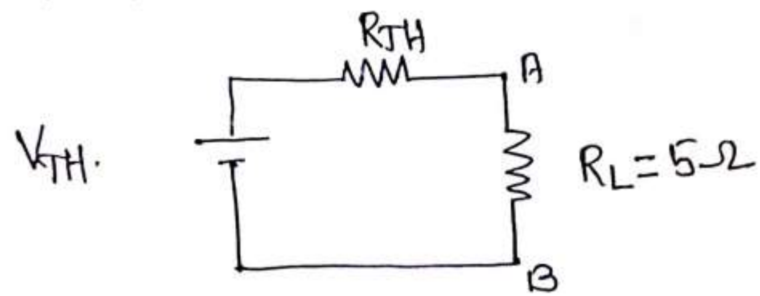
$$\Rightarrow I_L = \frac{35}{29.1+50} = 0.44A$$

UQ-12 Marks

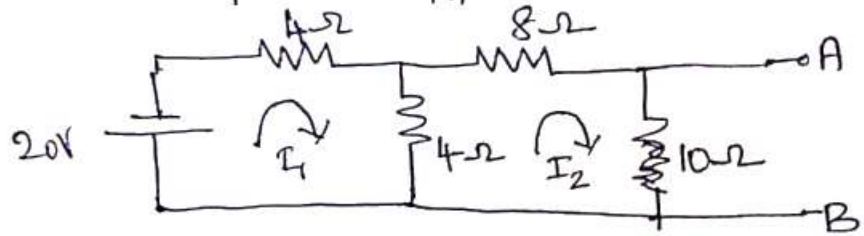
4 Calculate the current through the load resistance (R_L=5Ω) using Thevenin's theorem.



Step I - Thevenin's equivalent Ckt.



Step2 To find V_{TH}

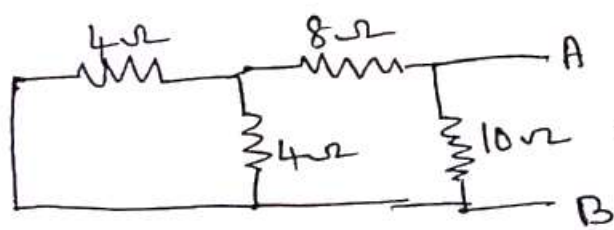


$$\begin{bmatrix} 8 & -4 \\ -4 & 22 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -20 \\ 0 \end{bmatrix} \Rightarrow \quad I_2 = -0.5A$$

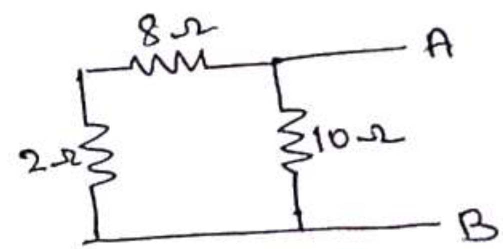
$$V_{TH} = +10 \times -0.5$$

$$\boxed{V_{TH} = -5V}$$

Step3: To find R_{TH}

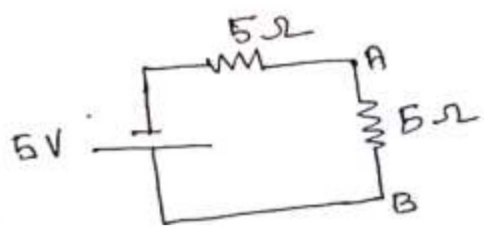


$\Rightarrow$



$R_{TH} = 5\Omega$

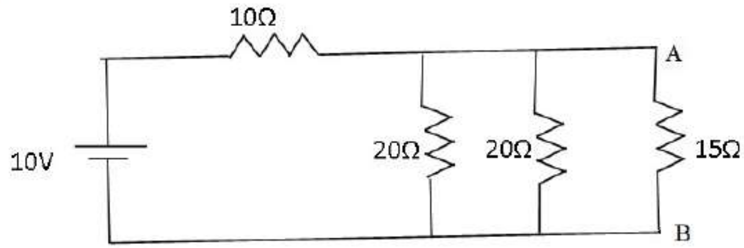
step4: Sub V_{TH} and R_{TH} in ①



$\Rightarrow I_L = 0.5A \uparrow$

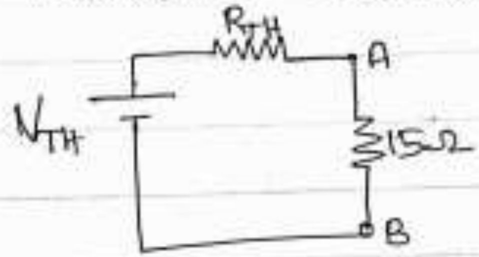
[i.e. B to A]

5 Determine the current through the 15 ohm resistor for the network given below using Thevenin's theorem.

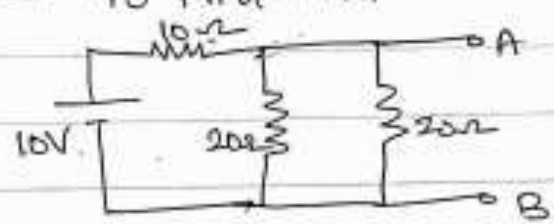


Step 1

Thevenin's equivalent ckt.

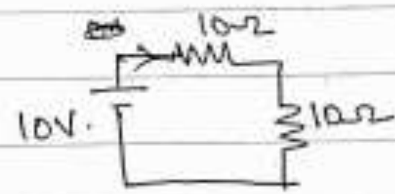


Step2 To Find V_{TH} -



where

$V_{TH} = V_{AB} = V_{20\Omega}$



$\Rightarrow \boxed{V_{AB} = 5V = V_{TH}}$

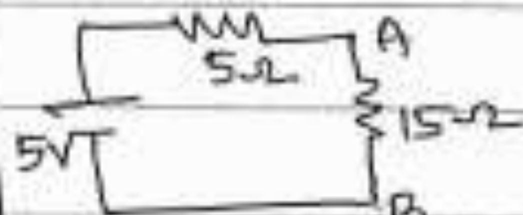
Step3 To find R_{TH}

$$R_{TH} = \left[10^{-1} + 20^{-1} + 20^{-1}\right]^{-1}$$
$$= 5\Omega$$

where $R_{TH} = R_{AB}$

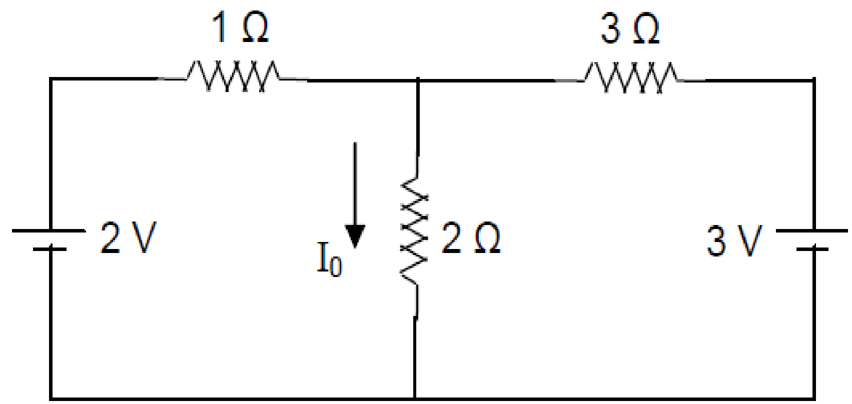
Step4:

sub ② & ③ in ①

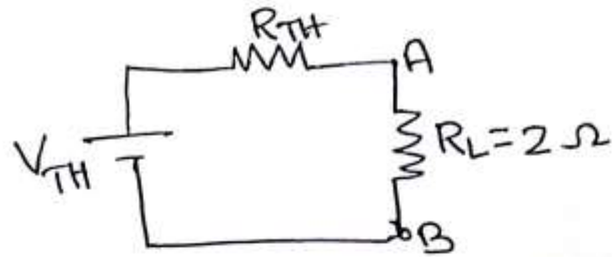


$$I_{AB} = \frac{5}{20} = 0.25A$$

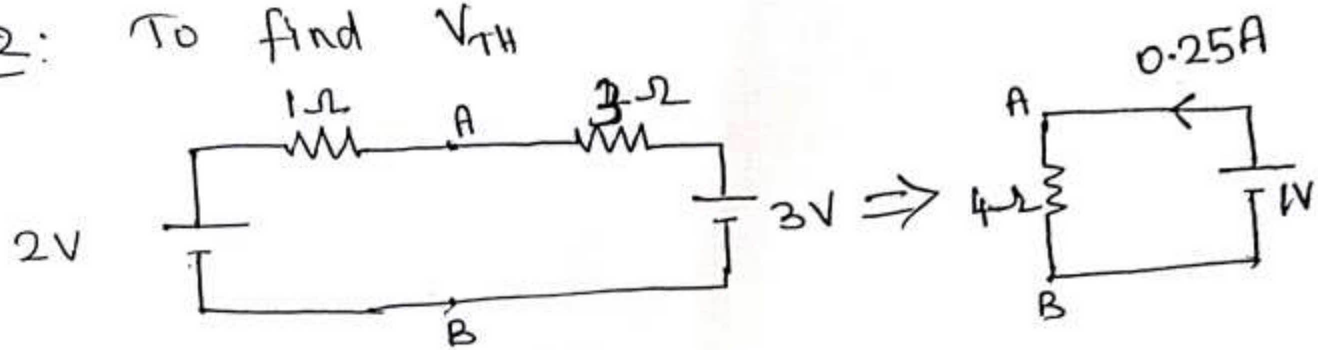
6 Using Thevenin's equivalent circuit, calculate the current I_0 through the 2 Ω resistor in the circuit

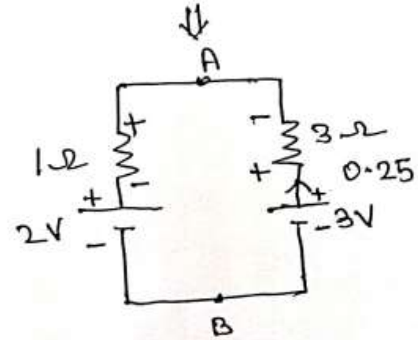


Step1: Thevenin's equivalent ckt



Step2: To find V_{TH}





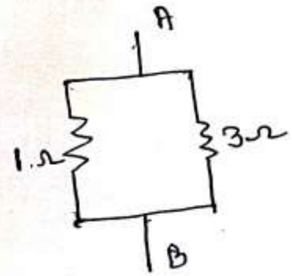
APPLY KVL

$V_{TH} = V_{AB} = 1 \times 0.25 + 2 = 2.25\ V$

(or)

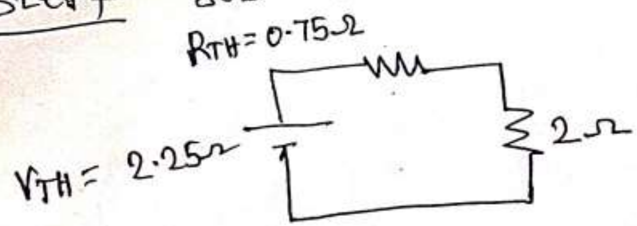
$V_{TH} = V_{AB} = -3 \times 0.25 + 3 = 2.25\ V$

Step3 : To find R_{TH}.



$\Rightarrow \quad R_{TH} = \frac{3}{4} = 0.75\ \Omega$

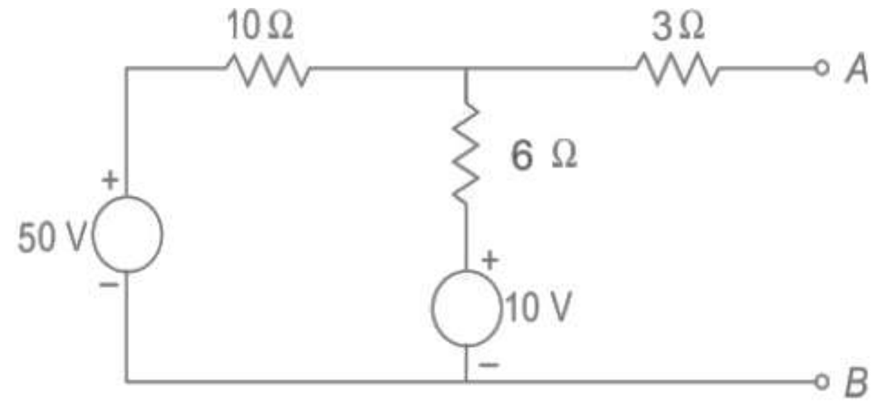
Step4 Sub V_{TH} and R_{TH} in ①



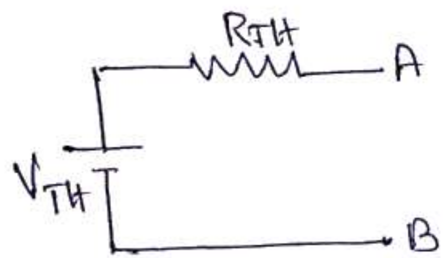
$I_L = \frac{2.25}{2.75}$

$= 0.818A$

7 Find Thevenin's equivalent circuit for the circuit shown

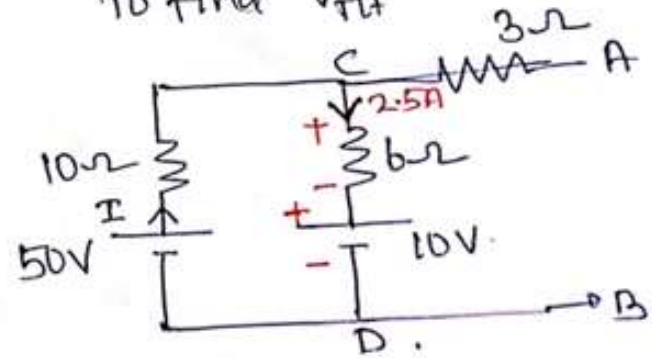


Step1. Thevenin's equivalent ckt.



Step2.

To find V_{TH}



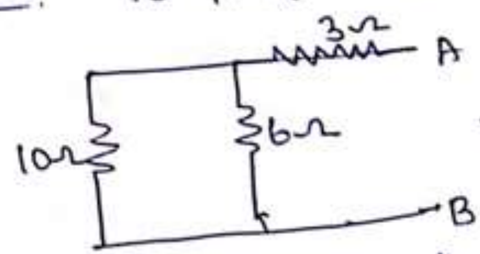
where $V_{TH} = V_{AB} = V_{CD}$.

$$I = \frac{50-10}{16} = \frac{40}{16} = 2.5A$$

Apply KVL

$$V_{CD} = +2.5 \times 6 + 10 = 25V.$$

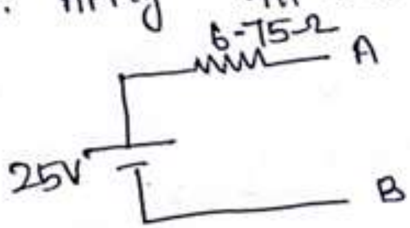
Step3: To find R_{TH}



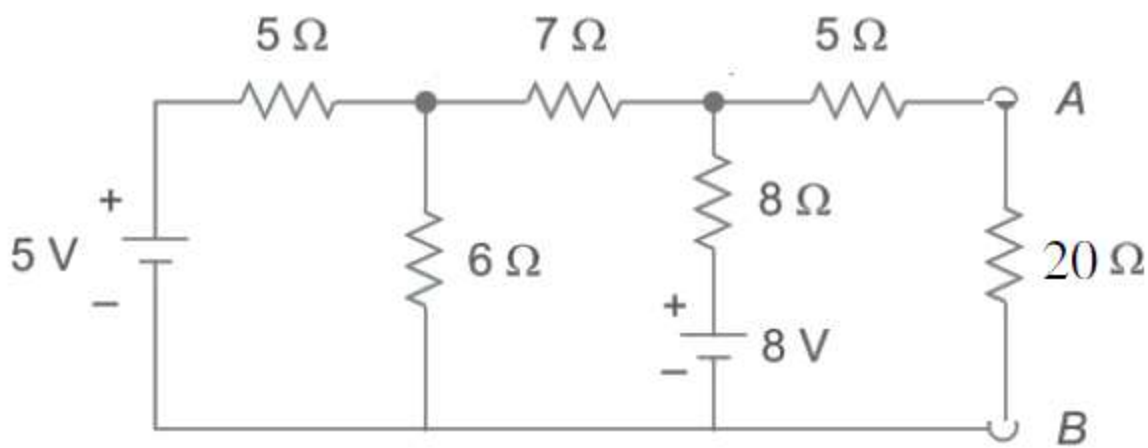
$\Rightarrow$ $R_{TH} = \frac{6 \times 10}{16} = 3.75\Omega + 3\Omega$

$= 6.75\Omega$

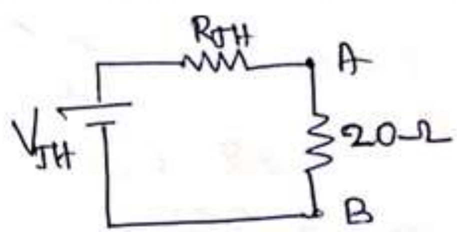
Step4: Apply V_{TH} and R_{TH} in (1)



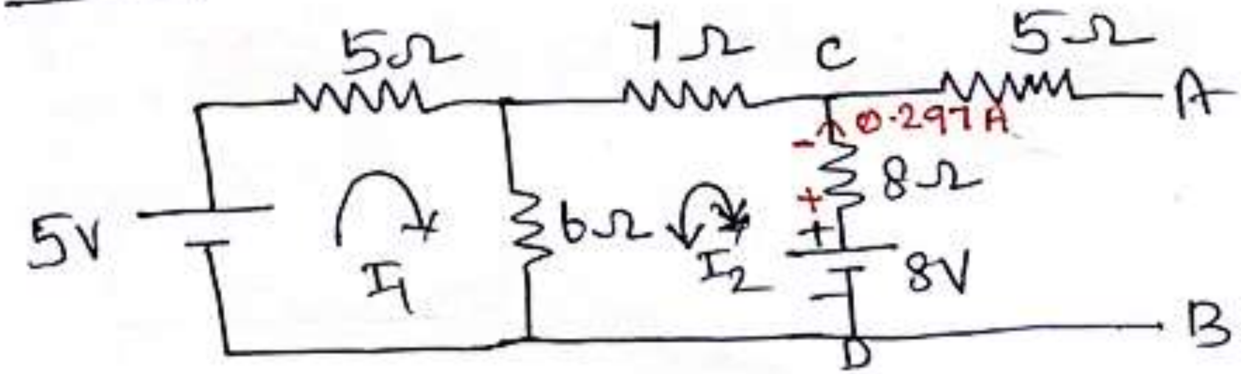
8 Find the current flowing through 20 Ω resistor using Thevenin's Theorem



Step1 Thevenin's equivalent ckt.



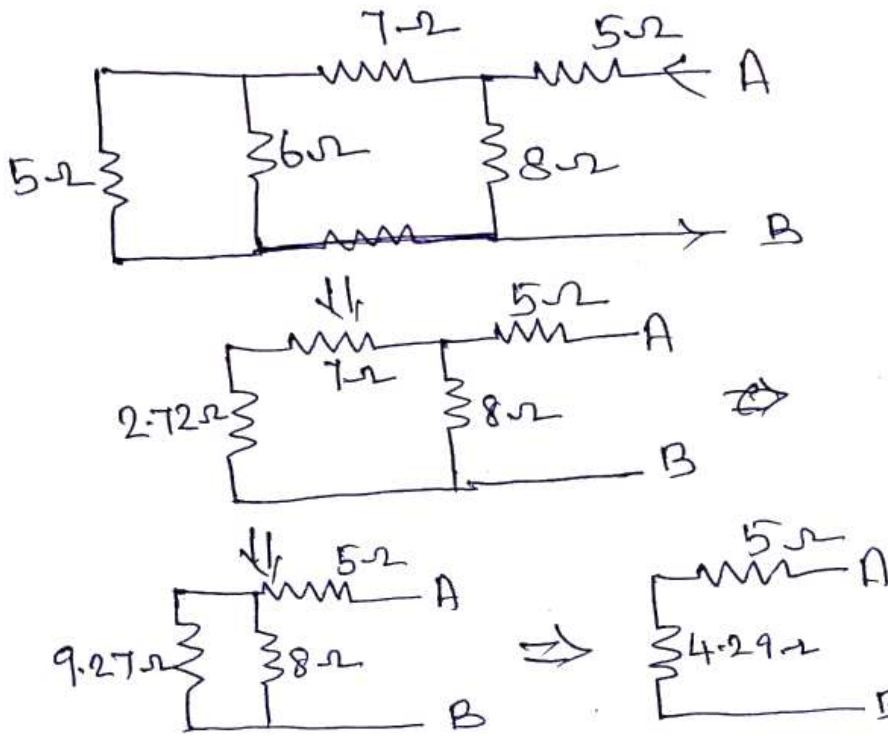
Step 2 To find V_{TH}



$$\begin{bmatrix} 11 & -6 \\ -6 & 21 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 5 \\ -8 \end{bmatrix} \Rightarrow I_2 = -0.297\,A$$

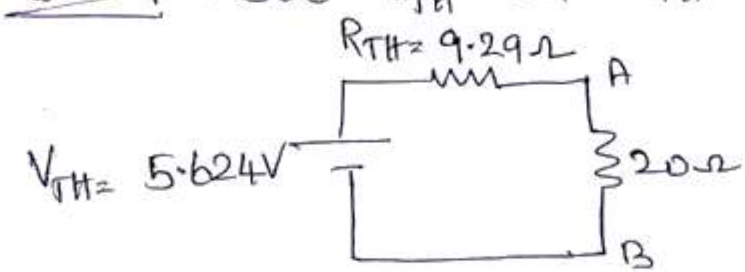
$$V_{TH} = V_{AB} = V_{CD} = -0.297 \times 8 + 8 = 5.624\,V.$$

Step3 To find R_{TH} (Looking Back Resistance)



$\Rightarrow R_{TH} = 9.29\Omega$

Step4 : Sub V_{TH} and R_{TH} in CKT 1



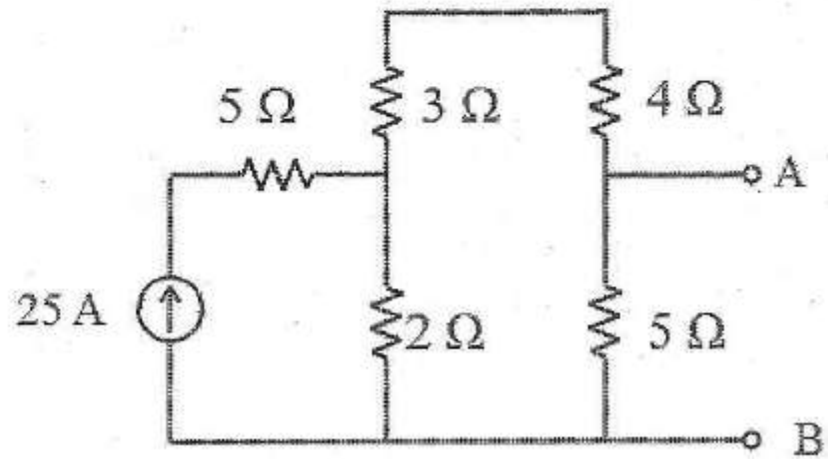
$$I_L = \frac{5.624}{9.29+20}$$

$$= 0.192A$$

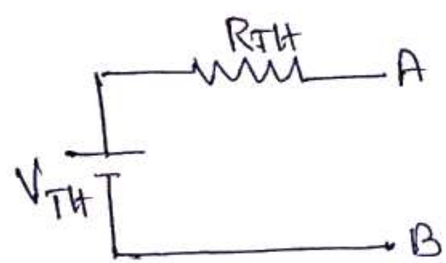
Ans

9 Simplify the given circuit into

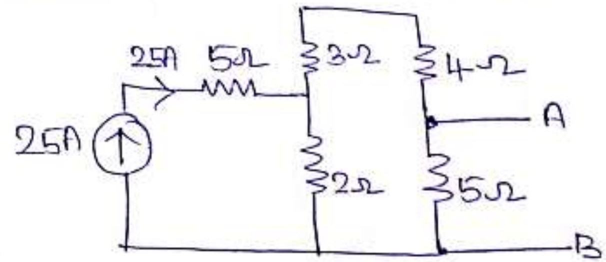
(i) Thevenin's equivalent circuit and



Step1. Thevenin's equivalent ckt.



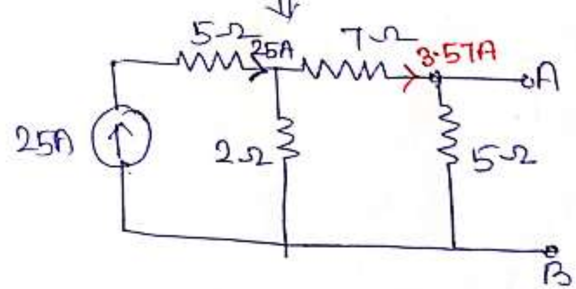
Step2 : To find V_{TH}.



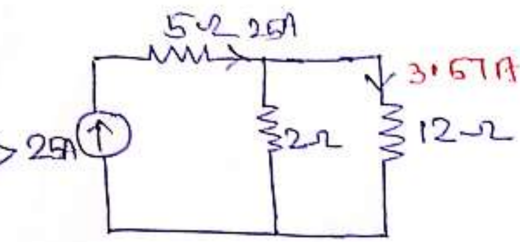
Here

$$V_{TH} = V_{AB} = V_{5\Omega}$$

$\Downarrow$



$\Rightarrow$



$\Downarrow$

$$I_{12\Omega} = \frac{25 \times 2}{14}$$

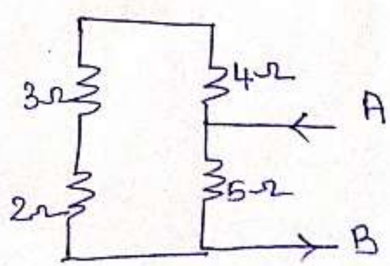
$$= 3.57\,A$$

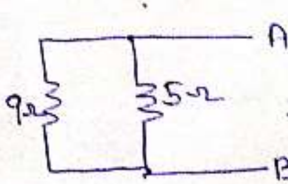
$\Leftarrow$

$$V_{5\Omega} = 3.57 \times 5$$

$$= 17.85V.$$

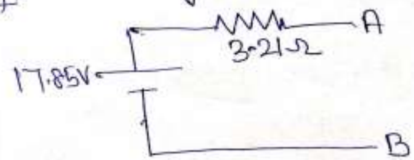
Step3: To find R_{TH}. [Looking Back Resistance]



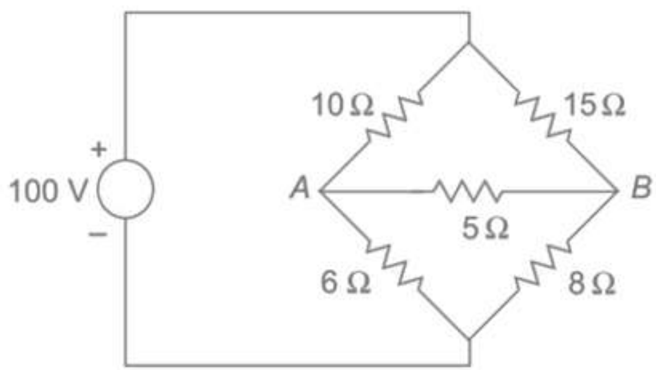


$\Rightarrow R_{AB} = 3.21\Omega$

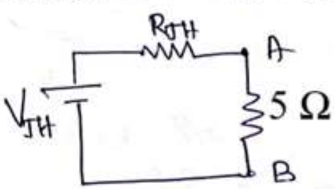
Step4: Apply V_{TH} and R_{TH} in Step ①



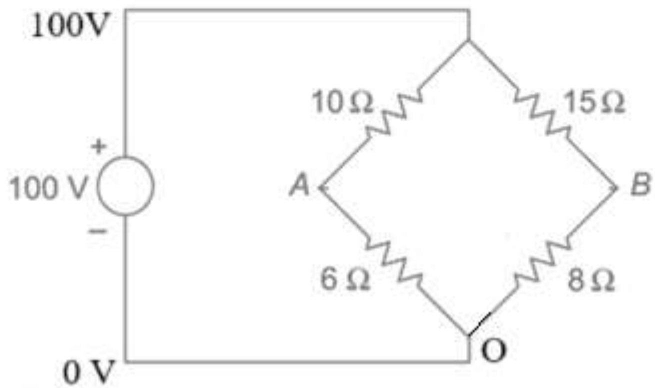
10 Using Thevenin's Theorem, find the current flowing through 5 Ω resistor



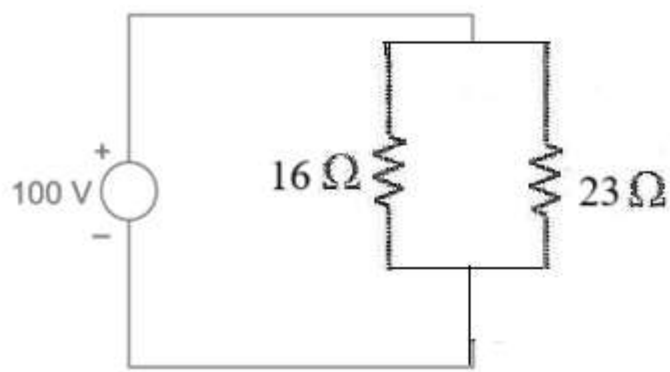
Step1 Thevenin's equivalent ckt

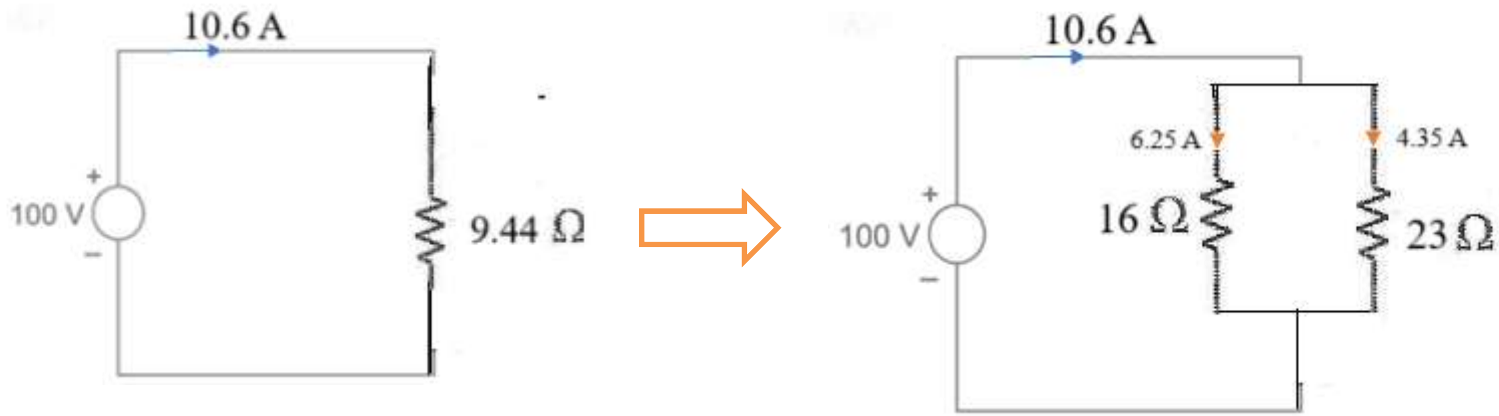


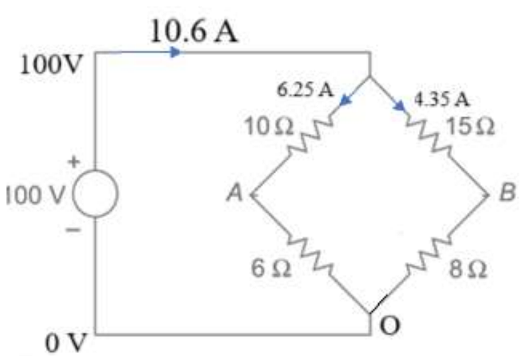
Step 2: To find V_{TH}



$$V_{TH} = V_{AB} = V_{AO} - V_{BO}$$







$V_{TH} = V_{AB} = V_{AO} - V_{BO}$

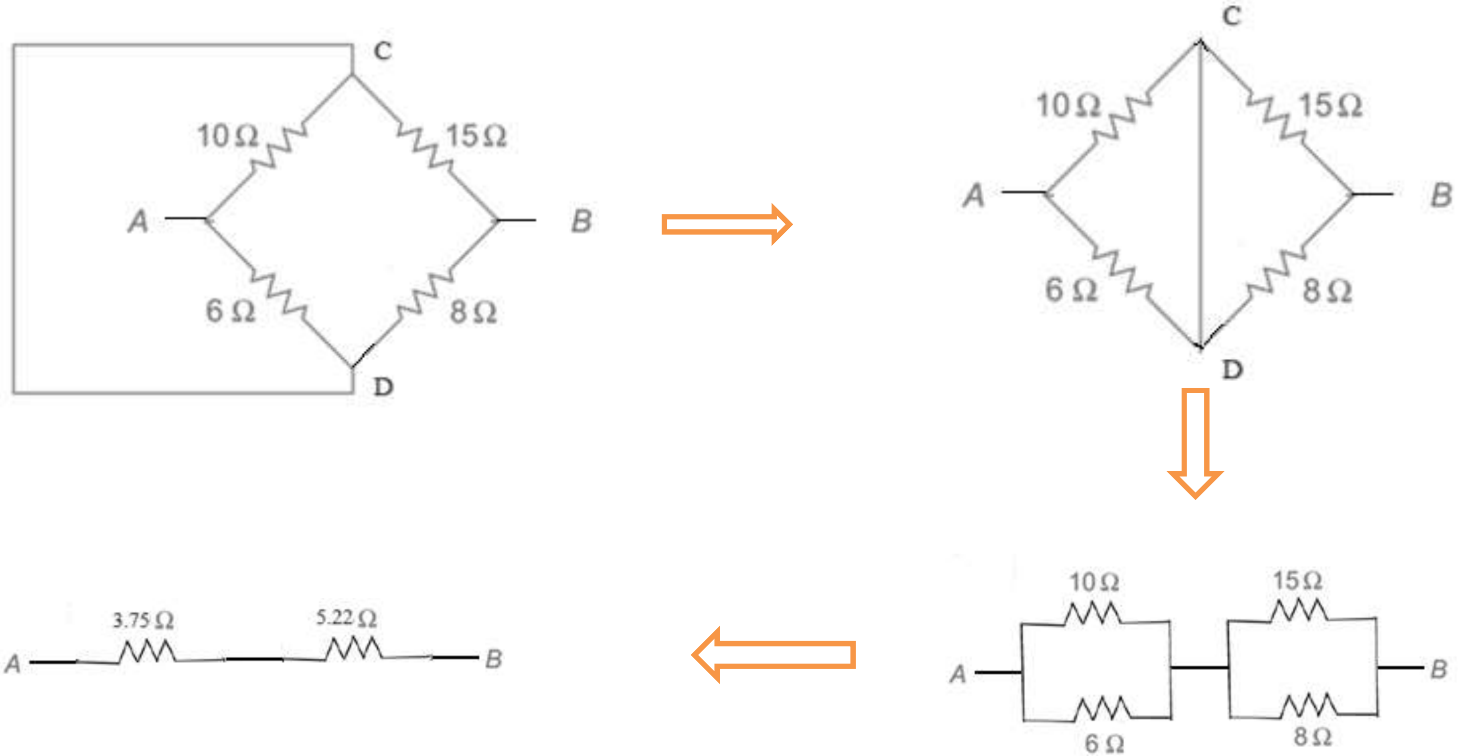
$V_{AB} = V_{AO} - V_{BO}$

$= 6.25 \times 6 - 4.35 \times 8$

$= 37.5 - 34.8$

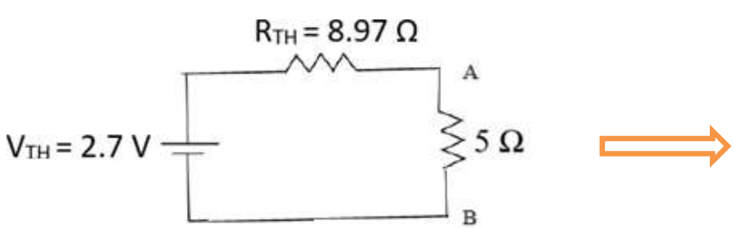
$= 2.7$ Volts

Step 3: To find R_{TH}



$R_{AB} = 8.97\ \Omega$

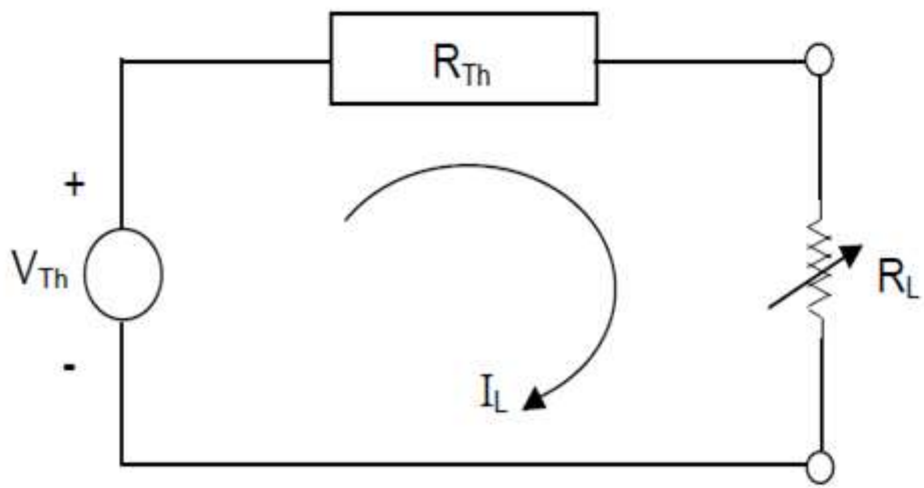
Step 4: Sub 2 & 3 in 1



$I_L = 0.19$ A

Maximum Power Transfer Theorem

Load is a variable resistance R_L [For DC Supply]



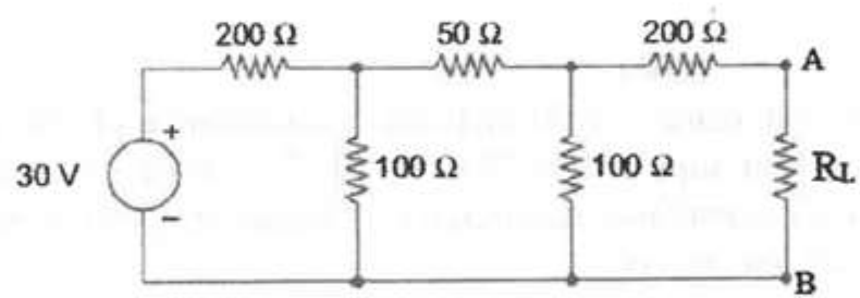
For Maximum power Through Load R_L

$$R_L = R_{TH}$$

and Max Power is

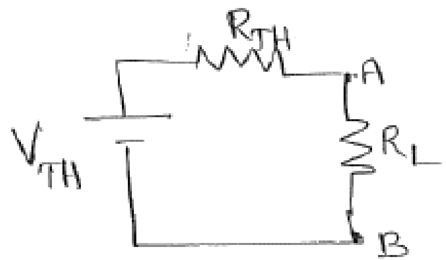
$$P_{max} = \frac{V_{Th}^2}{4 R_{Th}}$$

1 Using maximum power transfer theorem, find the maximum power consumed by load R_L.



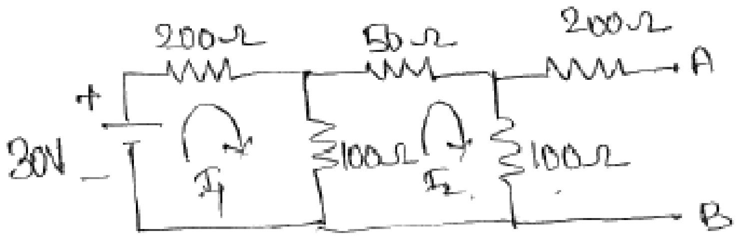
Step i

Thevenin's equivalent circuit



Step ii

Finding V_{TH}.



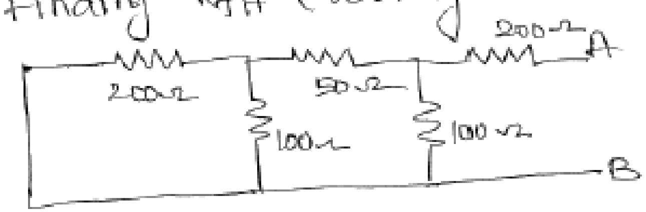
$$\begin{bmatrix} 300 & -100 \\ -100 & 250 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 30 \\ 0 \end{bmatrix}$$

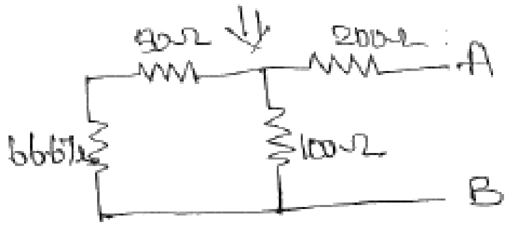
$$I_2 = 0.0462\,A \Rightarrow V_{TH} = 0.0462 \times 100$$

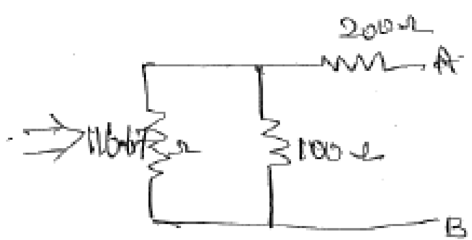
$$\boxed{V_{TH} = 4.62 \text{ Volts}}$$

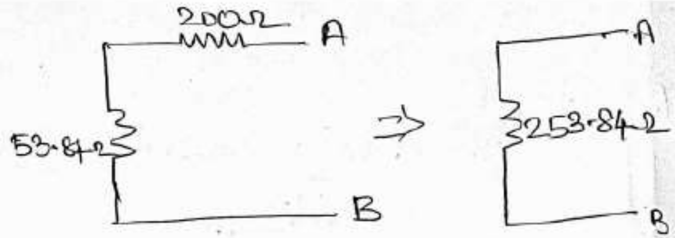
Step iii

Finding R_{TH} (Looking Back Resistance)





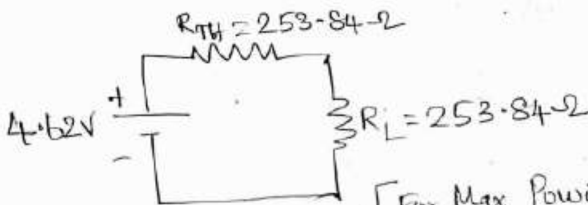




$$\boxed{R_{TH} = 253.84\ \Omega}$$

Step iv

Substitute V_{TH} & R_{TH} in Step (i)



[For Max Power, $R_L = R_{TH}$]

Step v

$$P_{Max} = \frac{V_{TH}^2}{4R_{TH}} = \frac{4.62^2}{4 \times 253.84} = 20.9\ \text{mWatts}$$

(or)

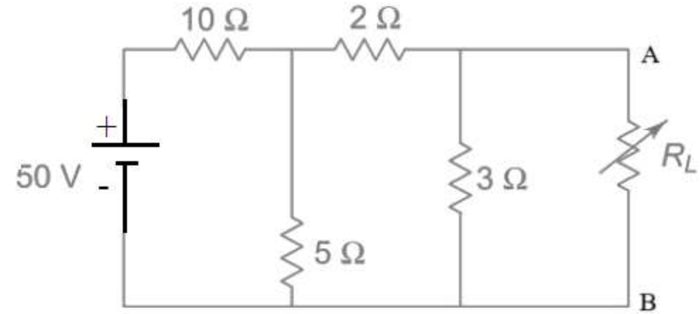
$$I_L = \frac{4.62}{253.84 + 253.84} = 9.1\ \text{mA}$$

$$P_{max} = \left(9.1 \times 10^{-3}\right)^2 \times 253.84$$

$$= 21\ \text{mWatts}$$

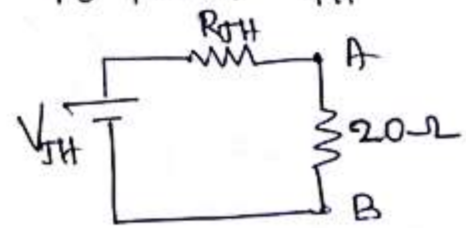
[Step 1: **1 M;** Step 2: **4 M**; Step 3: **4 M;** Step 4: **2 M;** Step 5: **4 M;**]

2 Determine the maximum power delivered to the load in the circuit

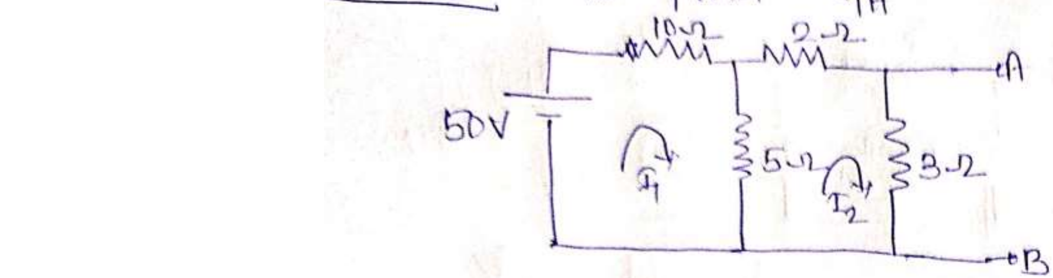


Step1 Thevenin's equivalent Ckt.

To find V_{TH}



Step 2. To find V_{TH}



$V_{TH} = V_{AB} = V_{3\Omega}$

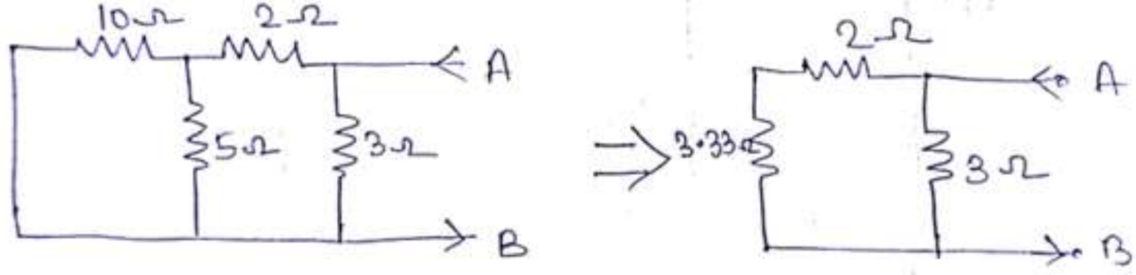
$$\begin{bmatrix} 15 & -5 \\ -5 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 50 \\ 0 \end{bmatrix} \Rightarrow$$

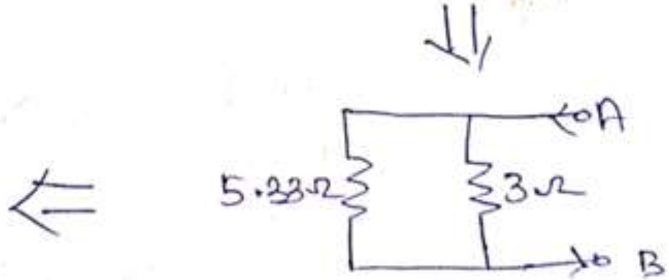
$I_2 = 2\,A$

$V_{3\Omega} = 2 \times 3 = 6$ Volts

Step 3

To find R_{TH}



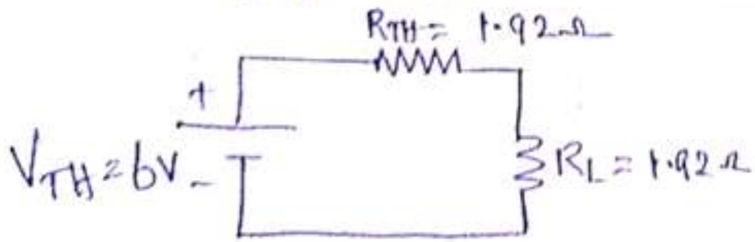


$R_{TH} = 1.92\,\Omega$

Step 4:

For Max Power thro load, $\boxed{R_L = R_{TH} = 1.92\,\Omega}$

Sub this in step 1.



$$P_{Max} = \frac{V_{TH}^2}{4R_{TH}} = \frac{6^2}{4\times 1.92}$$

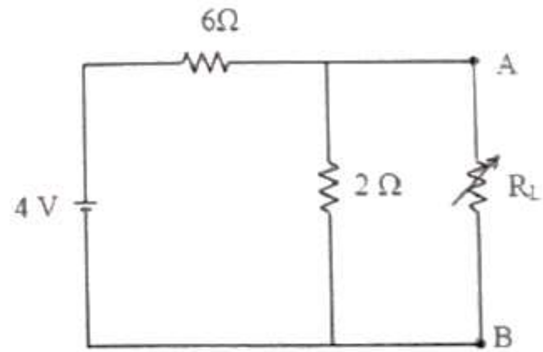
$$= 4.68 \text{ Watts}$$

(or)

$$I_L = \frac{6}{1.92+1.92} = 1.5625A$$

$$P_{Max} = 1.5625^2 \times 1.92 = 4.68 \text{ Watts}$$

3 Determine the value of load resistance R_L when it is dissipating maximum power. Also find the maximum power dissipated in the load resistance for the circuit given below.



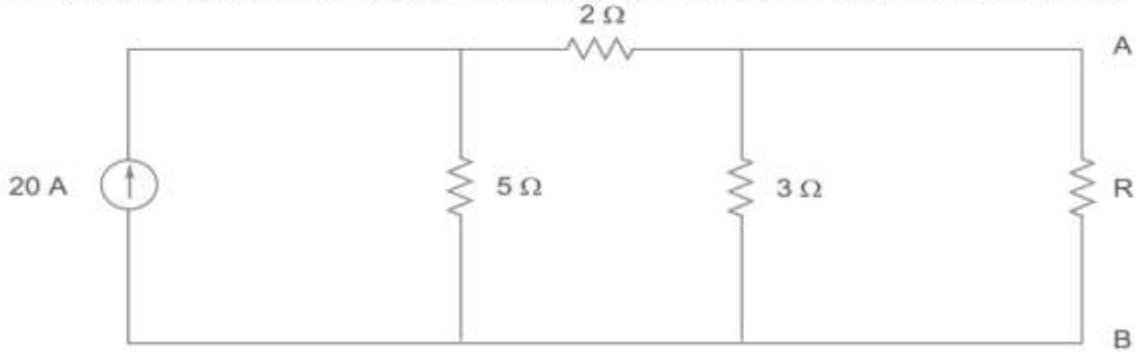
① $I_{2\Omega} = \frac{4}{8} = 0.5A \Rightarrow V_{TH} = 2 \times 0.5 = 1\,\text{Volt}.$

② $R_{TH} = \frac{6 \times 2}{8} = 1.5\,\Omega \Rightarrow R_L = 1.5\,\Omega$

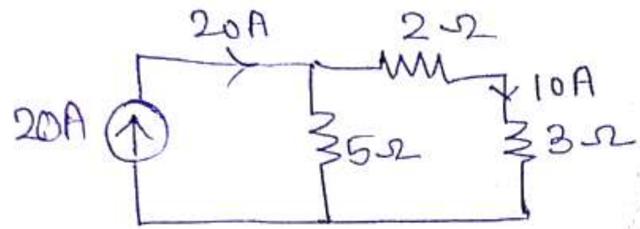
$$P_{Max} = \frac{V_{TH}^2}{4R_{TH}} = \frac{1^2}{4 \times 1.5} \doteq 0.167\,\text{watts}.$$

4

The circuit below has resistance R which absorbs maximum power. Compute the value of R and maximum power.

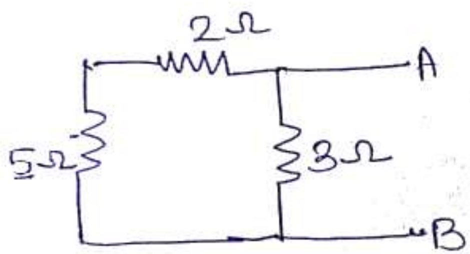


1)



$$\Rightarrow V_{TH} = 3\times10 = 30V.$$

2)

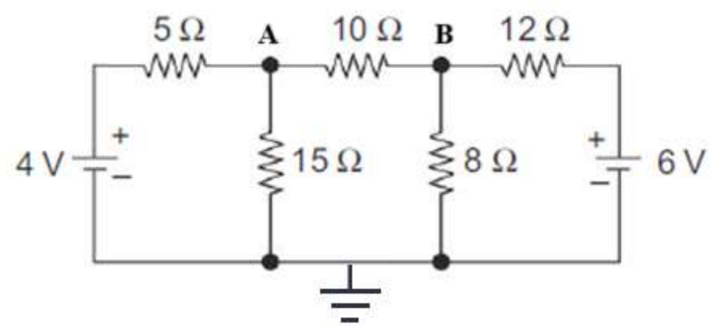


$$\Rightarrow R_{TH} = \frac{7\times3}{10} = 2.1\Omega = R_L$$

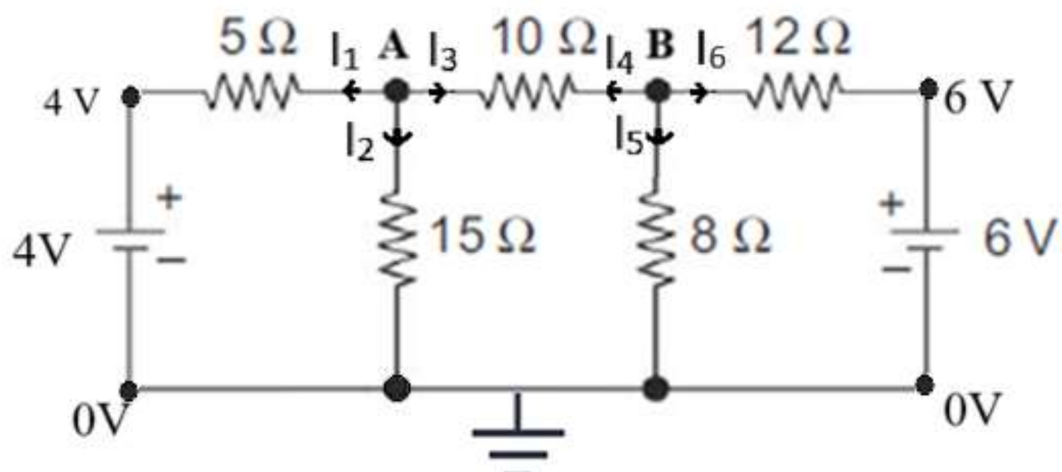
$$P_{Max} = \frac{V_{TH}^2}{4R_{TH}} = \frac{30^2}{4\times2.1} = 107.1 \text{ Watts}$$

Nodal Analysis or Node Voltage method or KCL Method

1. Find the current flowing through all resistors using Nodal Analysis



Solution:



Step 1: Apply KCL at node A [Sum of all outgoing currents will be zero]

$$I_1 + I_2 + I_3 = 0$$

$$\frac{V_A - 4}{5} + \frac{V_A - 0}{15} + \frac{V_A - V_B}{10} = 0$$

$$\frac{6V_A - 24 + 2V_A + 3V_A - 3V_B}{30} = 0$$

$$\boxed{11V_A - 3V_B = 24} \rightarrow (1)$$

Step2

Apply KCL at node B

$$I_4 + I_5 + I_6 = 0$$

$$\frac{V_B - V_A}{10} + \frac{V_B - 0}{8} + \frac{V_B - 6}{12} = 0$$

$$\frac{12V_B - 12V_A + 15V_B + 10V_B - 60}{120} = 0$$

$$\boxed{-12V_A + 37V_B = 60} \rightarrow (2)$$

Solving equation 1 & 2

$$V_A = 2.878V; \quad V_B = 2.555V.$$

$$I_{5\Omega} = \frac{V_A - 4}{5} = \frac{2.878 - 4}{5} = -0.2244A \leftarrow \Rightarrow 0.2244A \rightarrow$$

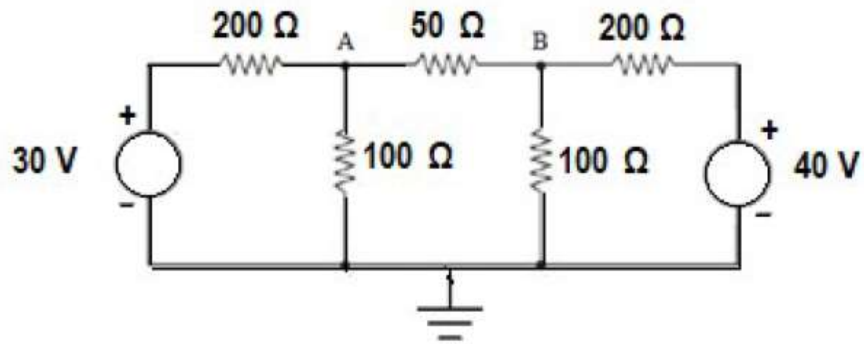
$$I_{15\Omega} = \frac{V_A}{15} = \frac{2.878}{15} = 0.192A \downarrow$$

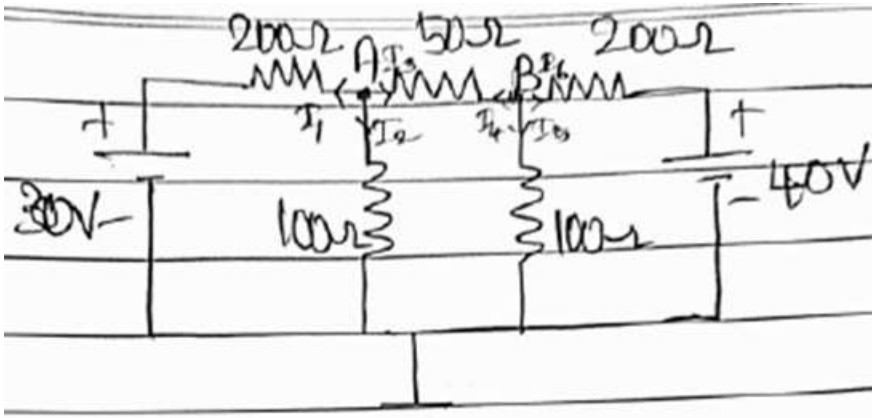
$$I_{10\Omega} = \frac{V_A - V_B}{10} = \frac{2.878 - 2.555}{10} = 0.0323A \rightarrow$$

$$I_{8\Omega} = \frac{V_B}{8} = \frac{2.555}{8} = 0.32A \downarrow$$

$$I_{12\Omega} = \frac{V_B - 6}{12} = \frac{2.555 - 6}{12} = -0.287A \rightarrow \Rightarrow 0.287A \leftarrow$$

Find the current flowing through 50 Ω resistor in the circuit shown below using Nodal Analysis.





Apply KCL at node A

$I_1 + I_2 + I_3 = 0$ [All outgoing $I = 0$]

$$\frac{V_A-30}{200}+\frac{V_A}{100}+\frac{V_A-V_B}{50}=0$$

$$\frac{V_A-30+2V_A+4V_A-4V_B}{200}=0$$

$$\boxed{7V_A-4V_B=30} \rightarrow 1$$

Apply KCL at node B.

$$I_4+I_5+I_6=0$$

$$\frac{V_B-V_A}{50}+\frac{V_B}{100}+\frac{V_B-40}{200}=0$$

$$\frac{4V_B-4V_A+2V_B+V_B-40}{200}=0$$

$$\boxed{-4V_A + 7V_B = 0} \rightarrow (2)$$

Solving 1 & 2 we get

$V_A = 11.21$ Volts

$V_B = 12.12$ Volts.

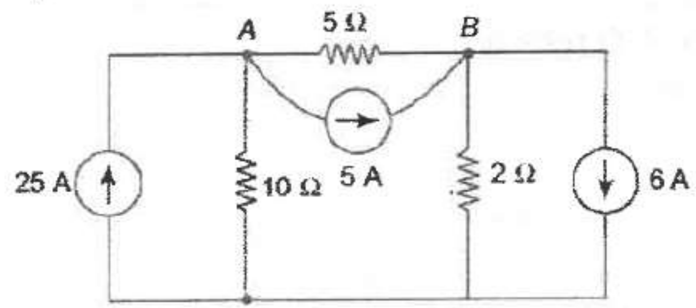
$$I_{50\Omega} = \frac{12.12 - 11.21}{50}$$

$= 18.2\,mA$

Nodal Analysis with current sources

Using nodal analysis, find the node voltage V_A and V_B. Also find the power observed by each resistor.

UQ-JAN- 15 Marks



Apply KCL @ node A.

$$I_1 + I_2 + I_3 + I_4 = 0$$

$$-25 + \frac{V_A}{10} + 5 + \frac{V_A - V_B}{5} = 0.$$

$$\frac{-250 + V_A + 50 + 2V_A - 2V_B}{10} = 0$$

$$-250 + V_A + 50 + 2V_A - 2V_B = 0$$

$$\boxed{3V_A - 2V_B = 200} \rightarrow ①$$

Apply KCL @ node B.

$$I_5 + I_6 + I_7 + I_8 = 0$$

$$\frac{V_B - V_A}{5} - 5 + \frac{V_B}{2} + 6 = 0$$

$$\frac{2V_B - 2V_A - 50 + 5V_B + 60}{10} = 0$$

$$\boxed{-2V_A + 7V_B = -10} \rightarrow (2)$$

$$\boxed{V_A = 81.18\,V} \quad \boxed{V_B = 21.76\,V}$$

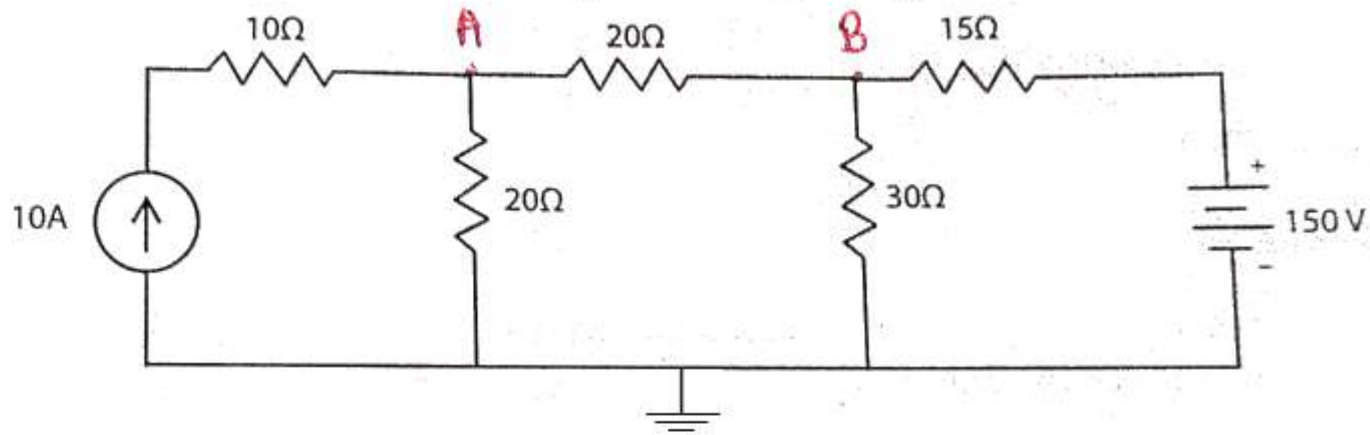
$I_{10\Omega} = \frac{V_A}{10} = \frac{81.18}{10} = 8.118A$; $P_{10\Omega} = 8.118^2 \times 10 = 659W$

$I_{5\Omega} = \frac{V_A - V_B}{5} = \frac{81.18 - 21.76}{5} = 11.88A$; $P_{5\Omega} = 11.88^2 \times 5 = 705.6W$

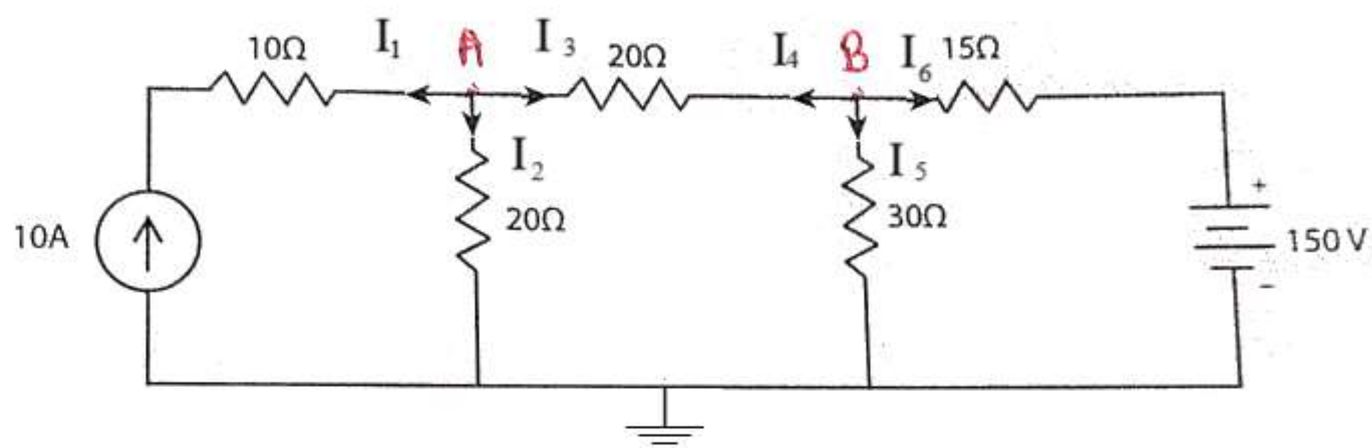
$I_{2\Omega} = \frac{V_B}{2} = \frac{21.76}{2} = 10.88A$; $P_{2\Omega} = 10.88^2 \times 2 = 236.7W$

Nodal Analysis with voltage & current sources

Using nodal analysis, find all node voltages in the given fig



Solution:



Apply KCL at node A.

$$I_1 + I_2 + I_3 = 0.$$

$$-10 + \frac{V_A}{20} + \frac{V_A - V_B}{20} = 0$$

$$\frac{V_A + V_A - V_B}{20} = 10$$

$$2V_A - V_B = 200 \rightarrow (1)$$

Apply KCL at node B

$$I_4 + I_5 + I_6 = 0.$$

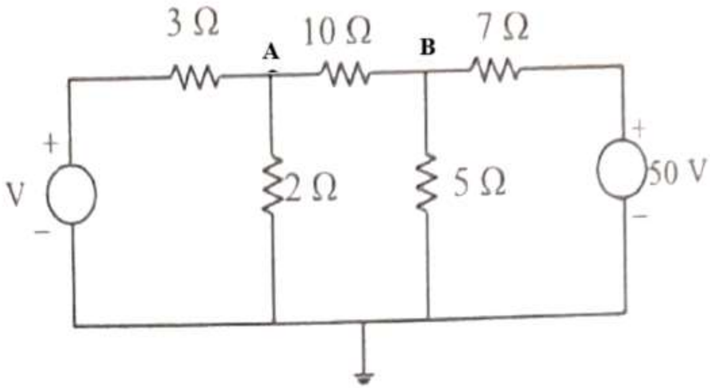
$$\cancel{\frac{V_B - 150}{15} +} \frac{V_B - V_A}{20} + \frac{V_B}{30} + \frac{V_B - 150}{15} = 0$$

$$\frac{3V_B - 3V_A + 2V_B + 4V_B - 600}{60} = 0$$

$$-3V_A + 9V_B = 600 \rightarrow (2)$$

$$\boxed{V_A = 160V ; \; V_B = 120V.}$$

Find the voltage 'V' in the circuit shown below which makes the current in the 10 Ω resistor zero by using nodal analysis.



Apply KCL at node A

$$I_{3\Omega} + I_{2\Omega} + I_{10\Omega} = 0$$

Given $I_{10\Omega} = 0$

$$\therefore I_{3\Omega} + I_{2\Omega} = 0$$

$$\frac{V_A - V}{3} + \frac{V_A}{2} = 0$$

$$\frac{2V_A - 2V + 3V_A}{6} = 0 \Rightarrow \boxed{5V_A - 2V = 0} \rightarrow (1)$$

Apply KCL at node B

$$\cancel{I_{10\Omega}}^{0} + I_{5\Omega} + I_{7\Omega} = 0$$

$$\frac{V_B}{5} + \frac{V_B - 50}{7} = 0$$

$$\frac{7V_B + 5V_B - 250}{35} = 0$$

$$12V_B = 250$$

$$V_B = 20.83V.$$

Given $I_{10\Omega} = 0$ i.e. $V_A = V_B = 20.83V \rightarrow (2)$
[when both Potential are same, then no current will flow]

Sub (2) in (1)

$$5(20.83) - 2V = 0$$

$$104.15 = 2V$$

$$\boxed{V = 52 \text{ Volts}}$$

Same Problem by Mesh Analysis

$$\begin{bmatrix} 5 & -2 & 0 \\ -2 & 17 & -5 \\ 0 & -5 & 12 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V \\ 0 \\ -50 \end{bmatrix}$$

Given $I_2 = 0$.

i.e. $\dfrac{\Delta_2}{\Delta} = 0 \Rightarrow \boxed{\Delta_2 = 0}$

$$\Delta_2 = \begin{vmatrix} 5 & V & 0 \\ -2 & 0 & -5 \\ 0 & -50 & 12 \end{vmatrix} = 0.$$

$$5(0-250) - V(-24+0) = 0.$$

$$-1250 + 24V = 0$$

$$24V = 1250$$

$$\boxed{V = 52 \text{ Volts}}$$

Problems seen so far:

- Resistor connected in series
- Resistor connected in parallel
- Resistor connected in series & Parallel
- KVL or Mesh analysis
- Superposition Theorem
- Thevenin's Theorem
- Maximum Power Transfer Theorem
- KCL or Nodal analysis

GENERATION OF ALTERNATIVE EMF

Sinusoidal emf or Sinusoidal voltage

Consider a coil of n turns placed in a magnetic field of maximum value ϕ_m Webers [see Fig. 4.1 (a)]. The coil is initially along the reference axis. In this position, the field is perpendicular to the plane of the coil.

Let the coil be rotated in the anticlockwise direction with an angular velocity of ω rad/sec.

When the coil is along the reference axis at $\omega t = 0$, it is called as zero e.m.f. position. This is because the movement of the coil at this instant $\omega t = 0$ is along the field.

Let at any instant t sec. the coil takes a position as shown in Fig. 4.1(b).

At this instant, the coil makes an angle $\theta = \omega t$ with the reference axis.

At this position, the normal component of the mangnetic flux with respect to the plane of the coil is equal to

$$\phi_m \cos \theta \qquad (\therefore \quad \theta = \omega t)$$

The normal component $= \phi_m \cos \omega t$

Flux linkages (ψ) at this instant (y) is equal to $N\phi_m \cos \omega t$. According to Faraday's law.

The emf induced in the coil at the instant under consideration.

$$e = -\frac{d\psi}{dt} = \frac{-d}{dt}(N\phi_m \cos \omega t)$$

$$= -N\phi_m\omega(-\sin \omega t)$$

$$e = (N\phi_m\omega) \sin \omega t \tag{4.1}$$

With the above expression, we can calculate the emf induced at various instants.

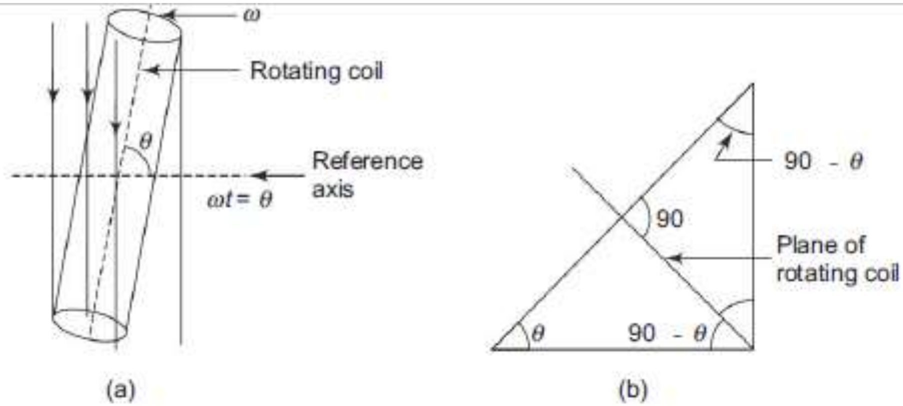


Fig. 4.1

When $\omega t = 0$ or $180°$

$$= (N\phi_m \omega) \sin 0 = (N\phi_m\, \omega) \sin 180°$$

$$e = 0$$

when $\omega t = 90°$ $e = (N\phi_m\omega) \sin 90°$

when $\omega t = 270°$ $e = N\phi_m\omega \sin (270°)$

$$e = -N\phi_m\omega$$

Let $N\phi_m\omega = E_m$ denote the maximum value of induced emf then from Eq. (4.1) we can write,

Instantaneous emf $\quad e = E_m \sin \omega t$ (Refer Fig. 4.2) $\quad$ (4.2)

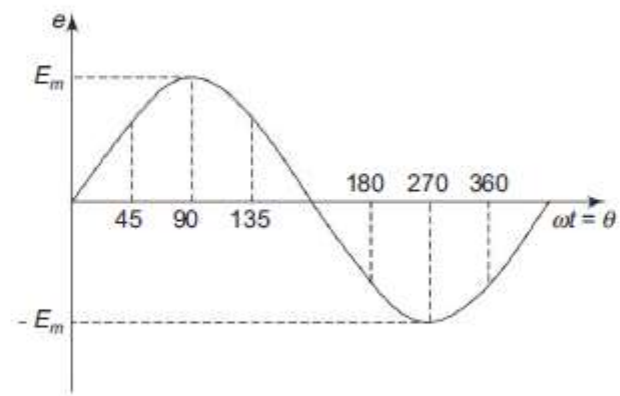


Fig. 4.2 Alternating emf wave for one complete cycle

4.2 TERMINOLOGY

1. Waveform A waveform is a graph in which the instantaneous value of any quantity is plotted against time. Examples of waveforms are shown in Fig. 4.3.

2. Alternating Waveform This is a wave which reverses its direction at regularly recurring intervals, e.g. Fig. 4.3(a).

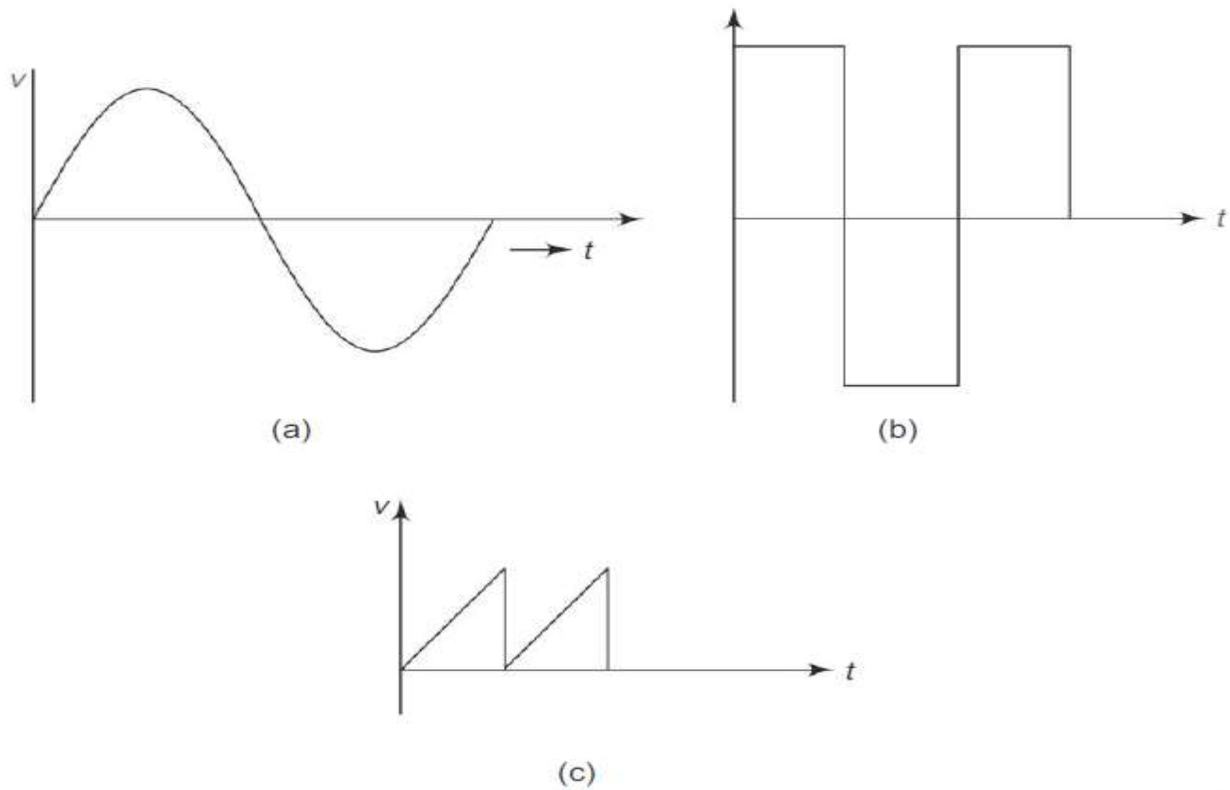


Fig. 4.3 (a) Sinusoidal waveform (b) Rectangular waveform (c) Sawtooth waveform

3. *Periodic Waveform* Periodic waveform is one which repeats itself after definite time intervals.

4. *Sinusoidal and Non-Sinusoidal Waveform*

Sinusoidal waveform It is an alternating waveform in which sine law is followed.

Non-sinusoidal waveform It is an alternating waveform in which sine law is not followed.

5. *Cycle* One complete set of positive and negative halves constitute a cycle.

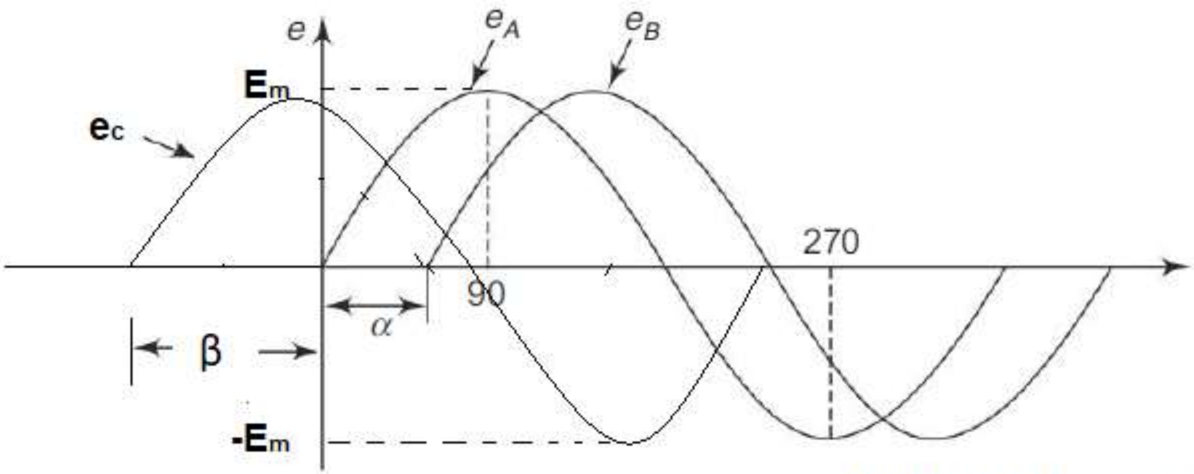
6. *Amplitude* The maximum positive or negative value of an alternating quantity is called the amplitude.

7. *Frequency* The number of cycles per second of an alternating quantity is known as frequency. Unit for frequency is expressed as c/s or Hertz (Hz).

8. *Period (T)* Time period of an alternating quantity is the time taken to complete one cycle. Time period is equal to the reciprocal of frequency. Time period is expressed in secs.

9. *Phase* The phase at any point on a given wave is the time that has elapsed since the quantity has last passed through zero point of reference and passed positively.

10. *Phase Difference* The term is used to compare the phase of two waveforms or alternating quantities.



$e_A = E_m \sin \omega t$ **Voltage A is reference**

$e_B = E_m \sin(\omega t - \alpha)$ **Voltage B lags voltage A by an angle α**

$e_c = E_m \sin(\omega t + \beta)$ **Voltage C leads voltage A by an angle β**

Definition Effective or RMS value of an alternating current is deifned by that steady value of current (dc) which when flowing in a given circuit for a given time produces the same heat as would be produced by the alternating current flowing in the same circuit for the same time.

Method to Obtain the RMS Value for Sinusoidal Currents

Let the alternating current be represented by

$$I_{\text{RMS}} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt}$$

$$i = I_m \sin \omega t$$

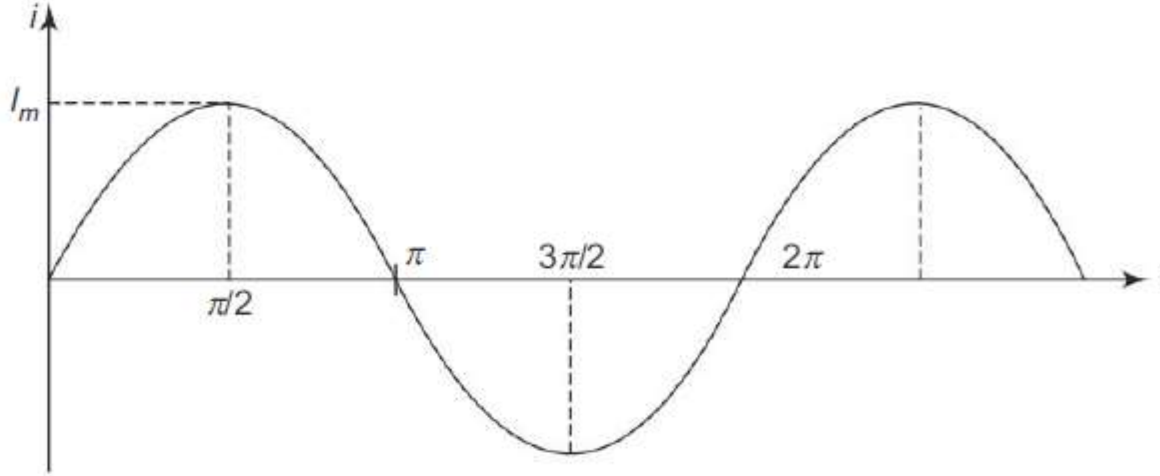
$$= I_m \sin \theta \,(\theta = \omega t)$$

$$i^2 = I_m^2 \sin^2 \theta$$

$$\text{Mean square of AC} = \int_0^{2\pi} \frac{I_m^2 \sin^2 \theta}{2\pi} d\theta$$

$$= \frac{I_m^2}{2\pi}\int_0^{2\pi} \sin^2 \theta \, d\theta$$

$$= \frac{I_m^2}{2\pi}\int_0^{2\pi} \frac{I - \cos 2\theta}{2} d\theta$$



$$= \frac{I_m^2}{2\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{2\pi}$$

$$= \frac{I_m^2}{2\pi}\frac{2\pi}{2} = \frac{I_m^2}{2}$$

RMS value of the alternating sinusoidal current is

$$I = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}} = 0.707\, I_m$$

$$I_{\text{RMS}} = 0.707\, I_m$$

Similarly, For a sinusoidal voltage

$$V_{\text{RMS}} = \frac{V_m}{\sqrt{2}} = 0.707\, V_m$$

AVERAGE VALUE OF AC

Definition The average value of an ac is given by that steady current which transfers across a circuit the same charge as would be transferred by the ac across the same circuit in the same time.

Method to Obtain the Average Value for Sinusoidal Current

Let $$i = I_m \sin\theta$$

Since this is a symmetrical wave it has two equal half cycles namely positive and negative halves.

Considering one half cycle for this symmetrical wave the average value is obtained by

$$I_{av} = \frac{1}{T}\int_0^T i\,dt$$

$$I_{av} = \frac{1}{\pi}\int_0^{\pi} i\,d\theta = \frac{1}{\pi} I_m \sin\theta\, d\theta$$

$$= \frac{I_m}{\pi}(-\cos\theta)_0^{\pi}$$

$$= \frac{I_m}{\pi}(1+1) = \frac{I_m}{\pi}\times 2$$

$$\boxed{I_{av} = \frac{2\,I_m}{\pi}}$$

$$I_{av} = 0.637\, I_m$$

where I_m is the maximum value of current.

For a sinusoidal voltage wave,

$$V_{av} = 0.637\, V_m.$$

RMS and average value of half wave rectified sine wave

RMS value

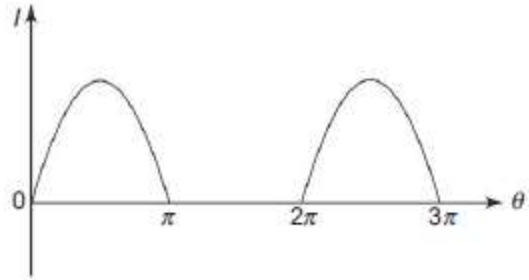


Fig. E.4.6

$$i = I_m \sin\theta \quad \text{for } 0 < \theta < \pi$$
$$= 0 \qquad \pi < \theta < 2\pi$$

$$\text{Mean square value} = \frac{1}{2\pi}\left[\int_0^{\pi} I_m^2 \sin^2\theta + \int_{\pi}^{2\pi} \theta d\theta\right]$$

$$= \frac{1}{2\pi}\int_0^{\pi} I_m^2 \sin^2\theta d\theta$$

$$= \frac{I_m^2}{2\pi}\int_0^{\pi} \frac{1-\cos^2\theta}{2} d\theta = \frac{I_m^2}{4\pi}\int_0^{\pi} (1-\cos 2\theta) d\theta$$

$$= \frac{I_m^2}{4\pi}\left(\theta - \frac{\sin 2\theta}{2}\right)_0^{\pi}$$

$$= \frac{I_m^2}{4\pi}\left(\pi - \frac{\sin 2\pi}{2} - 0 - \frac{\sin 2\times 0}{2}\right)$$

$$= \frac{I_m^2}{4\pi}\times\pi = \frac{I_m^2}{4}$$

$$\text{RMS value } \sqrt{\frac{I^2 m}{4}} = \frac{Im}{2}$$

RMS and average value of half wave rectified sine wave

Average value

Average value Half-wave rectified wave is an unsymmetrical wave.

$$\text{Average value} = \frac{\text{Area under the curve for one complete cycle}}{\text{Period}}$$

$$\text{Area under one complete cycle} = \int_0^{\pi} I_m \sin\theta \, d\theta + \int_{x}^{2\pi} 0 \, d\theta$$

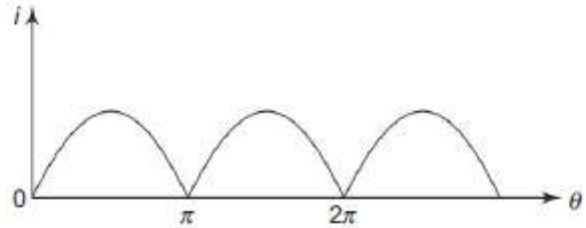
$$= I_m (-\cos\theta)_0^{\pi}$$

$$= I_m (-\cos\pi + \cos 0) = I_m(1+1) = 2I_m$$

$$\text{Average value} = \frac{2I_m}{2\pi} = I_m/\pi$$

RMS and average value of full wave rectified sine wave

RMS Value



$$\text{Mean square value} = \frac{\text{Area under one squared curve}}{\text{Period}}$$

$$\text{Mean square value} = \frac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\theta \, d\theta$$

$$= \frac{I_m^2}{\pi}\int_0^{\pi} \frac{1-\cos 2\theta}{2} d\theta = \frac{I_m^2}{2\pi}\left(\theta - \frac{\sin 2\theta}{2}\right)_0^{\pi}$$

$$= \frac{I_m^2}{2\pi}\left(\pi - \frac{\sin 2r}{2} - 0 + \frac{\sin 0}{2}\right)$$

$$= \frac{I_m^2}{2\pi} \times \pi = \frac{I_m^2}{2}$$

$$\boxed{\text{RMS value} = \frac{I_m}{\sqrt{2}}}$$

Average Value

$$\text{Average value} = \frac{\text{Area under the curve for one complete cycle}}{\text{Period}}$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta \, d\theta \qquad (\because \text{the given wave is symmetrical})$$

$$= \frac{I_m}{\pi}(-\cos\theta)_0^{\pi} = \frac{I_m}{\pi}(1+1) = \boxed{\frac{2I_m}{\pi}}$$

	Sine wave and full wave rectified sine wave	Half wave rectified sine wave
RMS Value	$I_{\mathrm{RMS}} = \frac{I_m}{\sqrt{2}}$	$I_{\mathrm{RMS}} = \frac{I_m}{2}$
Average Value	$I_{\mathrm{av}} = \frac{2\,I_m}{\pi}$	$I_{\mathrm{av}} = \frac{I_m}{\pi}$

Same applicable to voltage also

ANALYSIS OF SINGLE PHASE AC CIRCUITS

1. Real power / True power / Average power/ Power

$$P = V\,I\,\cos\Phi \quad \textbf{or}$$

$$P = V_{RMS}\, I_{RMS} \cos\Phi$$

$$P = |V|\,|I| \cos\Phi$$

$$P = |I|^2 R$$

Unit: Watts

Where Φ Angle between voltage and current

2. Reactive Power

$$Q = V\,I\,\sin\Phi$$

$$Q = V_{RMS}\, I_{RMS} \sin\Phi$$

Unit: VAR

3. Apparent power

$$S = VI$$

$$S = \sqrt{P^2 + Q^2}$$

Unit : VA

4. Power Factor = $\cos\Phi$

$$= \frac{R}{|Z|}$$

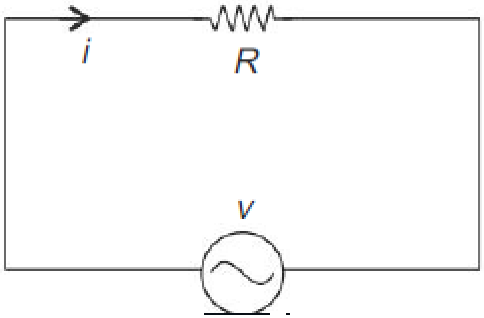
No Unit

Where Φ Angle between voltage and current

Where $|Z| = \sqrt{R^2 + (X_L - X_C)^2}$

ANALYSIS OF AC CIRCUIT

The response of electric circuits to alternating current can be studied by passing an alternating current through the basic circuit elements resistor (R), inductor (L) and capacitor (C).



1. Pure Resistive Circuit

Let the sinusoidal voltage applied across the resistance be

$$v = V_m \sin \omega t$$

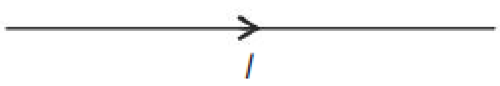
The resulting current has an instantaneous value, i By Ohm's law,

$$i = I_m \sin \omega t$$

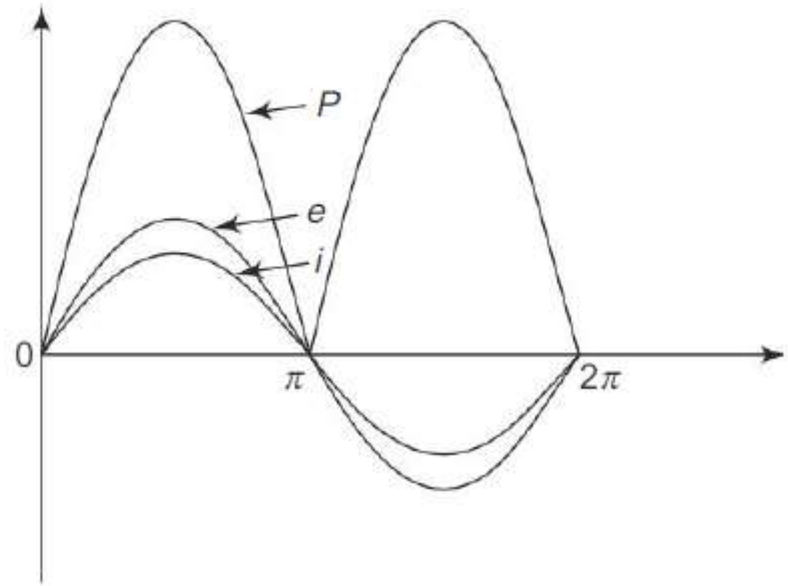
$$\text{where } I_m = \frac{V_m}{R}$$

Phasor Representation

In a pure resistive circuit,there is no phase difference between the voltage applied and the resulting current, i.e the phase angle $\phi = 0$. .If the voltage is taken as the reference phasor, thephasor representation for voltage and current in a pure resistive circuit is given in Fig.



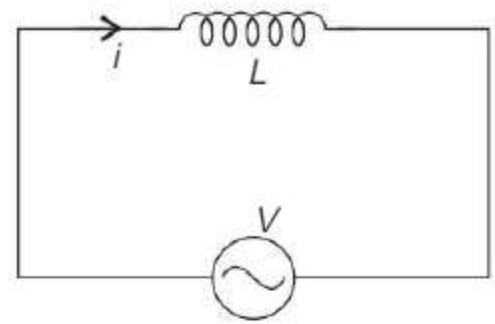
Waveform Representation



Power Factor It is the cosine of the phase angle between voltage and current
$\cos \phi = \cos 0 = 1$ (unity)

2. Pure Inductive Circuit

Consider the circuit of Fig. (4.16). In this circuit, an alternating voltage is applied across a pure inductor of self inductance L Henry.



Let the applied alternating voltage be

$$v = V_m \sin \omega t$$

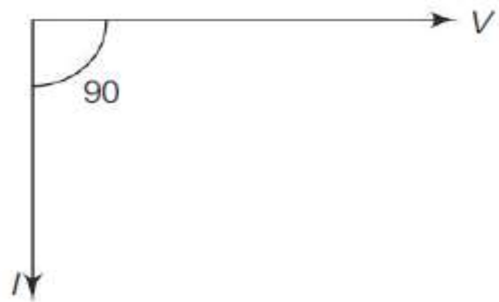
We know that the self induced emf always opposes the applied voltage.

$$v = L\frac{di}{dt}$$

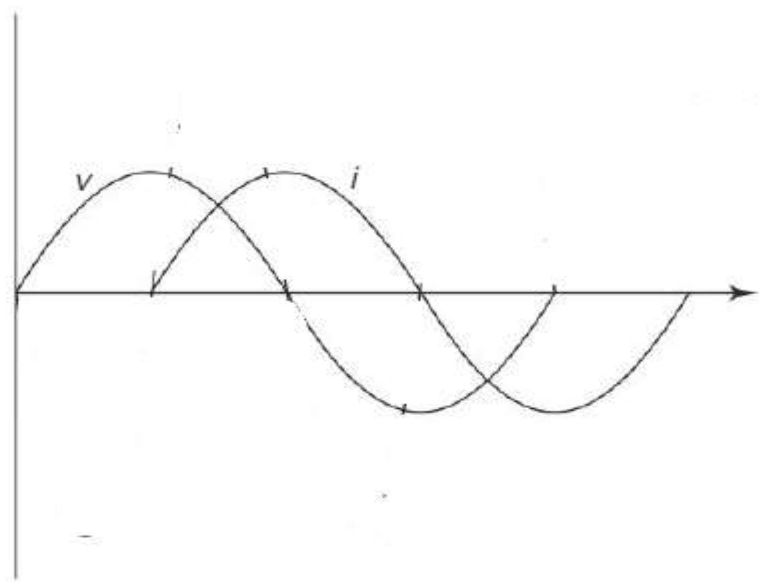
$$\therefore \quad i = \frac{1}{L}\int \cdot v\, dt = I_m \sin(\omega t - \pi/2)$$

we can say that the current through an inductor lags the applied voltage by 90°.

Phasor Representation Taking the voltage phasor as reference, the current phasor is shown to lag the voltage by 90° (Fig. 4.18).



Waveform Representation The current waveform is lagging behind the voltage waveforms by 90°.



Since $\phi = 90°$ Real power P=0

The pure inductor does not consume any real power

Power Factor In a pure inductor the phase angle between the current and the voltage phasors is 90°.

i.e. $\phi = 90°$; $\cos\theta = \cos 90° = 0$

Thus the power factor of a pure inductive circuit is zero lagging.

3 Pure Capacitive Circuit

Consider the circuit of Fig. 4.19 in which a capacitor of value C Farad is connected across an alternating voltage source.

Let the sinusoidal voltage applied across the capacitance be

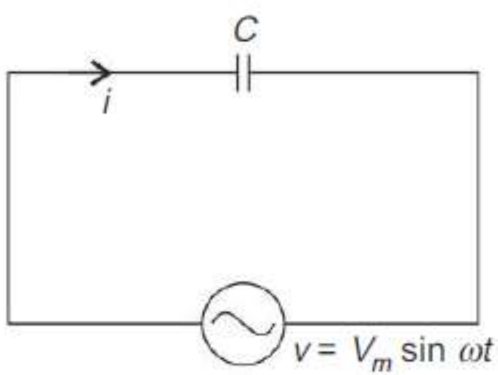
$$v = V_m \sin \omega t$$

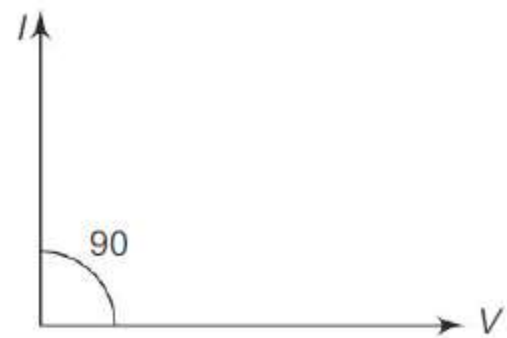
The characteristic equation of a capacitor is

$$V = \frac{1}{C}\int i dt$$

$$i = I_m \sin (\omega t + 90)$$

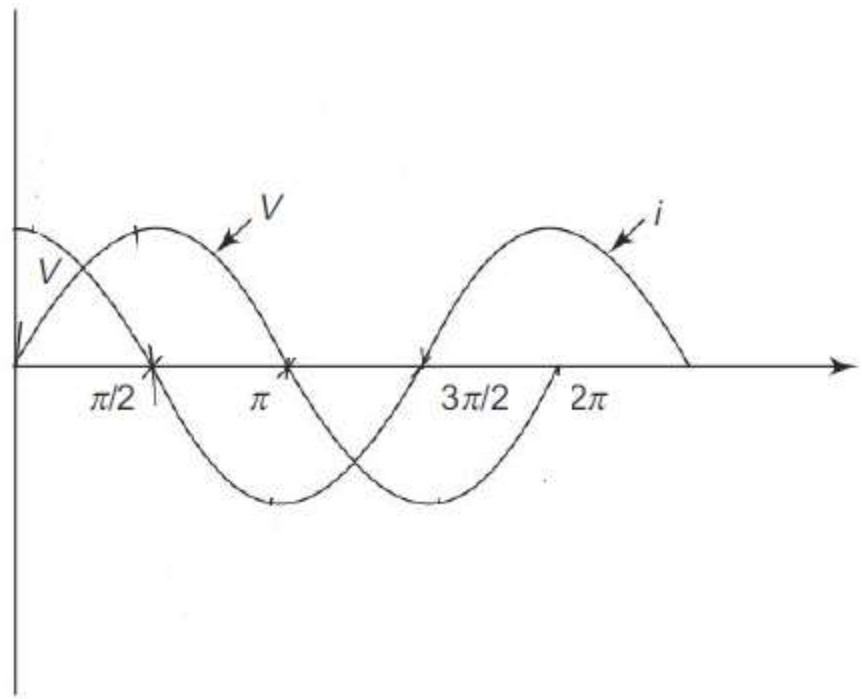
The current in a pure capacitor leads the applied voltage by an angle of 90°.



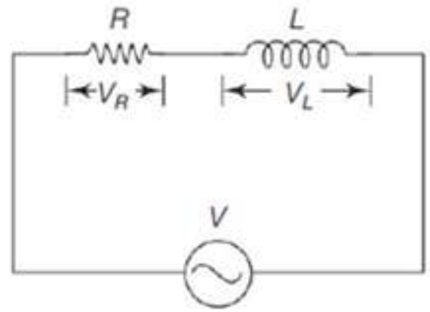


Phasor Representation In the phasor representation, voltage phasor is taken as the reference. The current phasor leads an angle of 90°

Waveform Representation The current waveform is ahead of the voltage waveform by an angle of 90°.



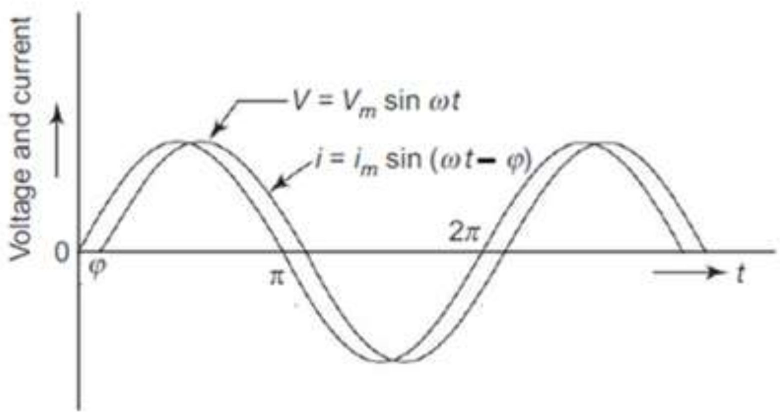
4 R–L SERIES CIRCUIT

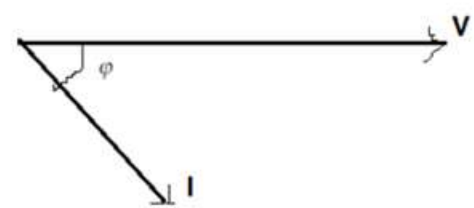


Let $v = V_m \sin \omega t$ be the applied voltage
then the current equation is

$$i = I_m \sin(\omega t - \phi)$$

Waveform





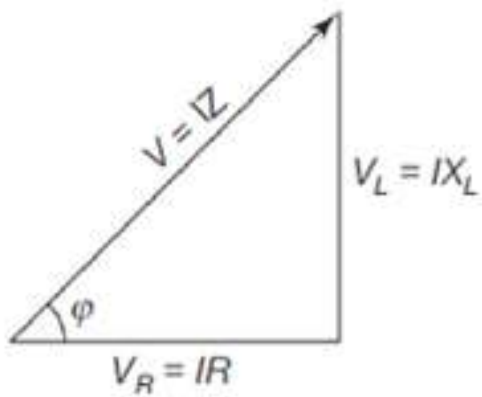
The current I lag Voltage V by an angle φ

Impedance of circuit $Z = V/I = R + jX_L = |Z|\angle\phi$

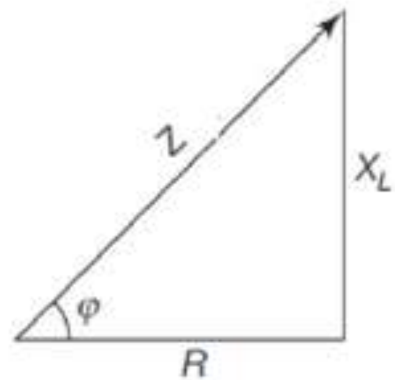
Here R is resistance and X_L is called inductive reactance

Where $X_L = 2\pi f L \quad \Omega$

Voltage Diagram



Impedance Diagram



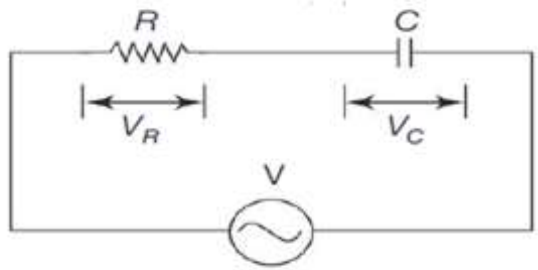
Magnitude of $V = \sqrt{V_R^2 + V_L^2}$

Magnitude of impedance $|Z| = \sqrt{R^2 + X_L^2}$

Where Voltage across resistor $V_R = |I|\ R$

Voltage across inductor $V_L = |I|\ X_L$

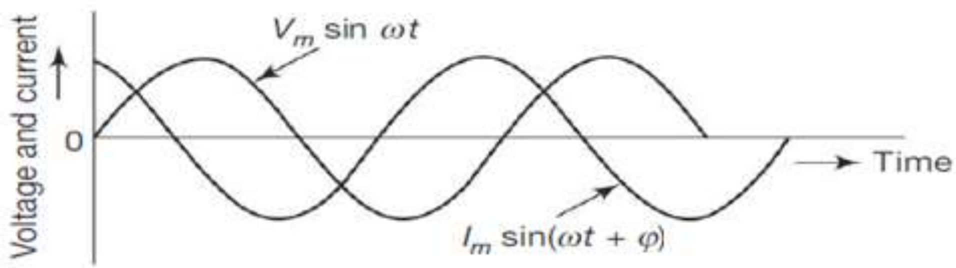
5 R-C SERIES CIRCUIT



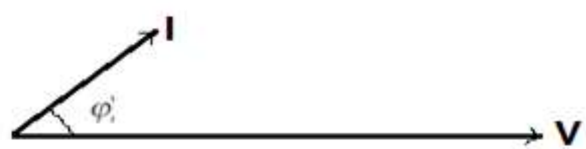
Let $v = V_m \sin \omega t$ be the applied voltage

Then the current equation is $i = I_m \sin (\omega t + \phi)$

Waveform



Phasor

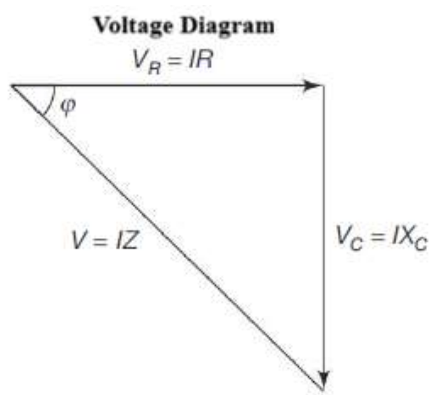


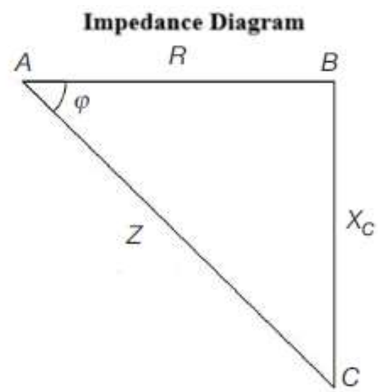
The current I leads the voltage V by an angle φ

Impedance of the circuit Z = $R - jX_C = |Z| \angle -\phi$

Here R is resistance and X_C is called capacitive reactance

Where $X_C = \dfrac{1}{2\pi f C}\ \Omega$





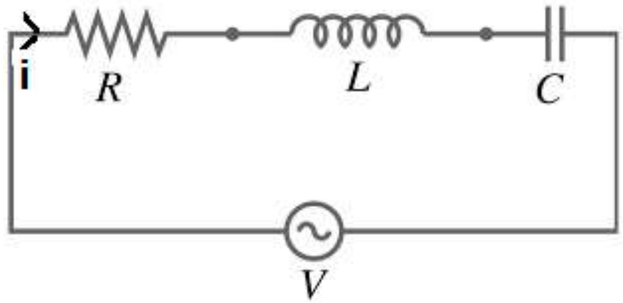
Magnitude of $V = \sqrt{V_R^2 + V_C{}^2}$

$|Z| = \sqrt{R^2 + X_C^2};$

Where Voltage across resistor $V_R = |I|\ R$

Voltage across capacitor $V_C = |I|\ X_C$

6. R-L-C SERIES CIRCUIT



Depends upon the value of X_L and X_C , the circuit behaves

If $X_L > X_C$, then the circuit behaves as RL circuit

If $X_L < X_C$, then the circuit behaves as RC circuit

Impedance $Z = R + j\,X_L - j\,X_C$ Unit: Ω

or

$Z = R + j(X_L - X_C)$

Where i or j = $\sqrt{-1}$

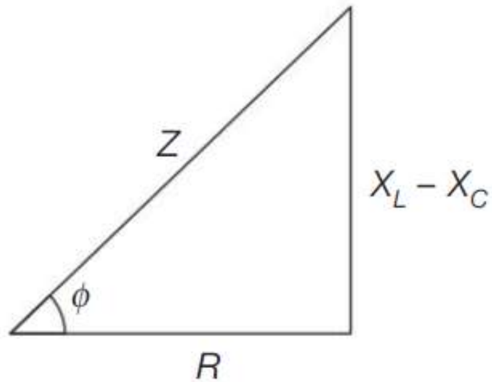
Inductive Reactance $X_L = 2\pi fL$ Unit: Ω

Capacitive Reactance $X_C = \dfrac{1}{2\pi fC}$ Unit: Ω

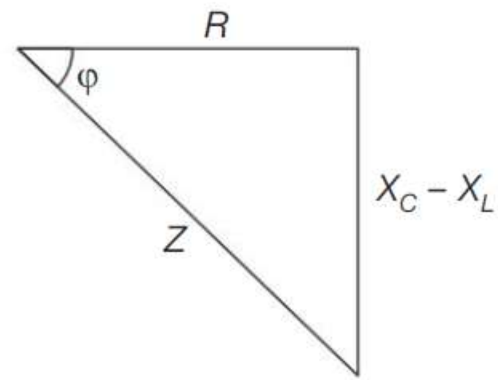
f-Supply Frequency in Hz

Impedance Diagram

Case (i) If $X_L > X_C$



Case (ii) If $X_C > X_L$



$$|\overline{Z}| = \sqrt{R^2 + (X_L - X_C)^2}$$

UQ 8Marks

A resistor of 10 Ω, an inductance of 50 mH and a capacitance of 150 μF are connected in series and supplied by 200 V, 50 Hz supply. Calculate (i) inductive reactance (ii) capacitive reactance (iii) impedance (iv) current (v) power factor and (vi) real power.

Also find the voltage across R, L and C

(i) $X_L = 2 \Pi f L \quad \Omega$

$= 2 \Pi \times 50 \times 50 \times 10^{-3}$

$= 15.7 \ \Omega$ (1 Mark)

(ii) $X_C = 1 / 2 \Pi f C \quad \Omega$

$= 1 / (2 \Pi \times 50 \times 150 \times 10^{-6})$

$= 21.2 \ \Omega$ (1 Mark)

(iii) $Z = R + j (X_L - X_C) \ \Omega$

$= 10 + j (15.7 - 21.2) \ \Omega$

$= (10 - j 5.5) \ \Omega$ OR $11.4 \ \Omega$ (2 Marks)

(iv) $I = V / Z$

$= 200 / (10 - j\,5.5)$

$= (15.35 + 8.45i)$ A OR $17.5\angle 28.8$ A **(2 Marks)**

(v)

Power factor $= \cos \Phi$

$= \cos 28.8$

$= 0.87$ (leading)

Power factor $= \dfrac{R}{|Z|}$

$= 10 / 11.4$

$= 0.87$ (leading)

(1 Mark)

(vi) $P = V\,I \cos \Phi$

$= 200 \times 17.5 \times 0.87$

$= 3045$ Watts

$P = I^2 R$

$= 17.5^2 \times 10$

$= 3062.5$ Watts

(1 Mark)

(Power value Approx 3000-3100 W)

Voltage across resistor $V_R = |I|\, R \quad =17.5 \times 10 = 175$ V

Voltage across inductor $V_L = |I|\, X_L \quad =17.5 \times 15.7 = 274.75$ V

Voltage across capacitor $V_C = |I|\, X_C \quad =17.5 \times 21.2 = 371$ V

UQ 8Marks

An inductive coil takes 10A and dissipates 1000W when connected to a supply of 250V, 25Hz. Calculate the impedance, resistance, reactance and the power factor.

$P = |I|^2 R$; Resistance $R = \frac{1000}{100} = 10\ \Omega$

$|Z| = \frac{250}{10} = 25\Omega$; From impedance triangle $X = \sqrt{25^2 - 10^2} = 22.9128\Omega$

Thus impedance $Z = (10 + j\ 22.9128)\ \Omega = 25 \angle 66.42^{0}\ \Omega$

Resistance $R = 10\ \Omega$ Reactance $X = 22.9128\ \Omega$

From impedance triangle, power factor $= \frac{R}{|Z|} = \frac{10}{25} = 0.4$ lagging

A resistance of 100 Ω is connected in series with a 50μF capacitor to a supply at 200 V, 50 Hz. Determine the

(i) Current
(ii) Power factor
(iii) Voltage across resistor and capacitor

Given: $R = 100\,\Omega$; $C = 50\,\mu F$; $V = 200\angle 0^\circ\,V$; $f = 50\,Hz$

$$Z = R - jX_C\,\Omega\,;\quad X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}}$$

$$\boxed{X_C = 63.66\,\Omega}$$

For RC Ckt $\Rightarrow \boxed{Z = R - jX_L}\,\Omega$

$\therefore Z = 100 - 63.66i\,\Omega$

(i) $I = \frac{V}{Z} = \frac{200}{100 - 63.66i} = 1.42 + 0.91i$ Amps

$= 1.68\angle 32.5^\circ$ Amps.

(ii) Power factor $= \cos\phi = \cos 32.5 = 0.84$ (leading)

iii) $V_R = |I|R = 1.68 \times 100 = 168$ Volts.

$V_C = |I|X_C = 1.68 \times 63.66 = 106.9$ Volts.

PROBLEMS

1 A voltage 100 sin ωt is applied to a 10-ohm resistor. Find the instantaneous current, the current (rms) and the average power.

Solution: V or $e = 100 \sin \omega t$

$R = 10$ ohms

$i = e/R = 100/10 \sin \omega t = 10 \sin \omega t$ A

$I_{rms} = I_m / \sqrt{2}$

$= 10 / \sqrt{2}$

$= 7.07$ A

$P = V I \cos \phi$

$= \frac{100}{\sqrt{2}} \times \frac{10}{\sqrt{2}} \times \cos 0$

= 500 Watts

2. A voltage v = 340 sin 314t is applied to a circuit and the resulting current, i = 42.5 sin 314t. Identify and hence find the values of the component. Find the value of power consumed.

Solution: $v = 340 \sin 314t = \frac{340}{\sqrt{2}} \angle 0$ Volts $= 240 \angle 0$ Volts

$i = 42.5 \sin 314\,t = \frac{42.5}{\sqrt{2}} \angle 0$ Amps $= 30 \angle 0$ Amps

From the above voltage and current equations, we find that they are in phase with each other. Thatis angle between V and I is 0. Hence, the basic component connected in the circuit must be resistor.

$$R = V / I$$
$$= 240.4 / 30$$
$$= 8\ \Omega$$

Note

$$V_{rms} = |V| = V$$

$$P = VI \cos \Phi$$
$$= 240. \times 30 \times \cos 0$$
$$= 7200 \text{ Watts}$$

$$P = I^2 R$$
$$= 30 \times 30 \times 8$$
$$= 7200 \text{ Watts}$$

3 In a series circuit containing pure resistance and pure inductance. the current and voltage expressed as $i\,(t) = 5 \sin\,(314\,t + 2\,\pi/3)$ and $V\,(t) = 20\,(314\,t + 5\,\pi/6)$.

(1) What is the impedance of the circuit?

(ii) What are the values of resistance, inductance and power factor?

(iii) What is the average power drawn by the circuit?

Solution

$$v(t) = 20 \sin\,(314t + 5\,\pi/6)$$

$$i(t) = 5 \sin\,(314t + 2\,\pi/3)$$

$$\text{Phase angle of voltage} = \frac{5\pi}{6}\text{ radians} = 5 \times \frac{180^\circ}{6} = 150^\circ$$

$$\text{Phase angle of current} = \frac{2\pi}{3}\text{ radians} = 2 \times \frac{180^\circ}{3} = 120^\circ$$

$$V = \frac{20}{\sqrt{2}} \angle 150^\circ \text{ Volts} = 14.14 \angle 150^\circ \text{ Volts}$$

$$I = \frac{5}{\sqrt{2}} \angle 120^\circ \text{ Amps} = 3.53 \angle 120^\circ \text{ Amps}$$

Current lags the voltage by $150 - 120^\circ = 30^\circ$.

Lagging p.f. means that it is an R–L circuit.

$$Z = V / I$$

$$= \frac{14.14 \angle 150^\circ}{3.53 \angle 120^\circ} = 3.46 + j\,2 \text{ Ohms}$$

$$= R + j\,X_L$$

Therefore R= 3.46 Ohms

X_L = 2 Ohms

X_L = 2 Ohms

$\omega L =$ 2 Ohms

314L = 2 Ohms

ω = 314 rad/s [given]

L= 2/314

=6.36 mH

Power Factor = COS Φ

= COS 30

= 0.866 [LAGGING]

Average Power P = VI COS Φ

= 14.14 x 3.53 x cos 30

= 43.22 Watts

P= I^2 R

= 3.53^2 X 3.46

= 43.22 Watts

4. Find the circuit constants of a two element series circuit which consumes 700 W with 0.707 leading p.f. The applied voltage is $V = 141.4 \sin 314\, t$.

Solution

$$v = 141.4 \sin 314t$$

$$P = 700 \text{ W}, \quad \text{p.f.} = 0.707 \text{ leading}$$

Leading p.f. means R–C circuit

Max. value of supply voltage = 141.4 V

R.M.S. value of supply voltage $= \dfrac{141.4}{\sqrt{2}} = 99.98$ V

$$\cos\phi = 0.\ 707 \text{ leading; Power} = VI\cos\phi$$

$$700 = 99.98 \times I \times 0.707; \quad I = 9.9 \text{ A}$$

Impedance $\quad |Z| = \dfrac{V}{I} = \dfrac{99.98}{9.9} = 10.09$ ohms

$$\cos\phi = \frac{R}{|Z|} \Rightarrow R = |Z|\cos\phi$$

$$R = 10.09 \times 0.707 = 7.13\ \Omega$$

$$X_C = \sqrt{Z^2 - R^2} = \sqrt{10.09^2 - 7.13^2} = 7.13 \text{ ohms}$$

$$\frac{1}{\omega C} = 7.13; \quad \frac{1}{314 \times C} = 7.13 \Rightarrow C = \frac{1}{314 \times 7.13}$$

$$C = 4.466 \times 10^{-4} \text{ F}$$

$$= 446.6 \times 10^{-6} \text{ F}$$

$$C = 446.6 \, \mu\text{F}$$

THREE PHASE AC CIRCUITS

Three Phase Connections There are two possible connections in 3-phase system. One is star (or wye) connection and the other is delta (or mesh) connection. Each type of connection is governed by characteristic equations relating the currents and the voltages. Phasor diagrams plays a vital role in this analysis.

Star Connection

Here three similar ends of the three phase coils are joined together to form a common point. Such a point is called the starpoint or the neutral point. The free ends of the three phase coils will be operating at specific potentials with respect to the potential at the star point.

It may also be noted that wires are drawn from the three free ends for connecting loads. We actually have here three phase four wire system (Fig. 5.32) and three phase three wire system (Fig. 5.33).

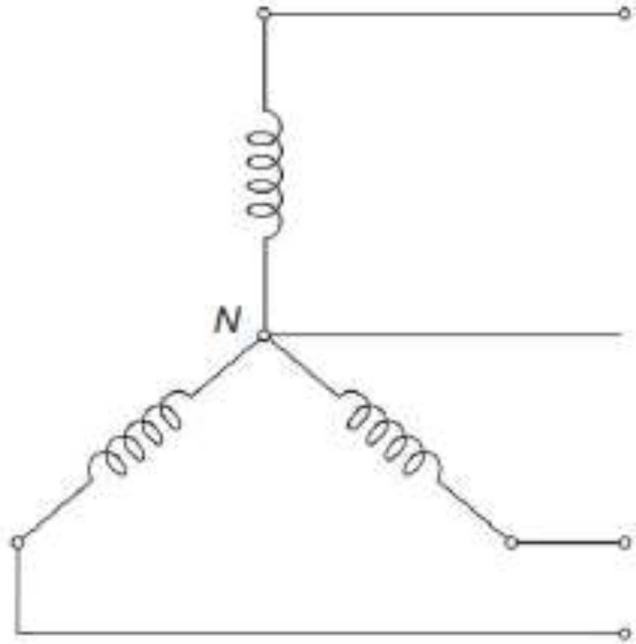


Fig. 5.32

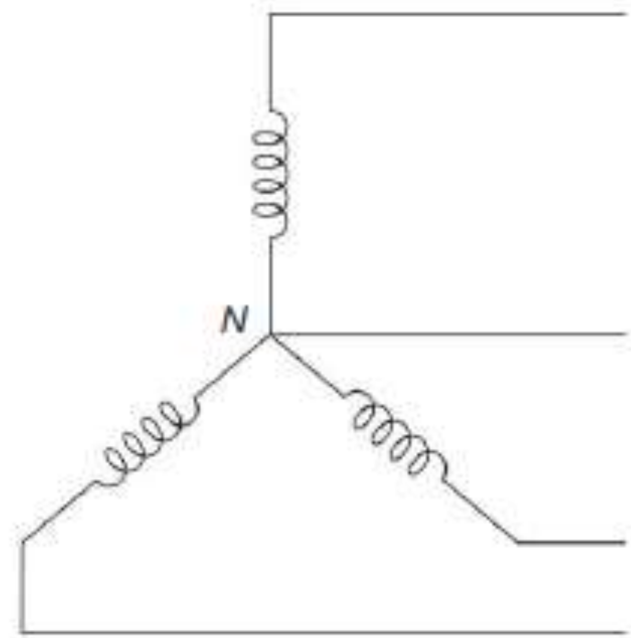


Fig. 5.33

Analysis Let us analyse the relationship between currents and relationship between voltages. We also arrive at the power equations.

Notations Defined

E_R, E_Y, E_B : Phase voltages of R, Y and B phases
I_R, I_Y, I_B : Phase currents
V_{RY}, V_{YB}, V_{BR} : Line voltages
I_{L1}, I_{L2}, I_{L3} : Line currents

In a balanced system,

$$E_R = E_Y = E_B = E_P \qquad V_{RY} = V_{YB} = V_{BR} = V_L$$
$$I_R = I_Y = I_B = I_P \qquad I_{L1} = I_{L2} = I_{L3} = I_L$$

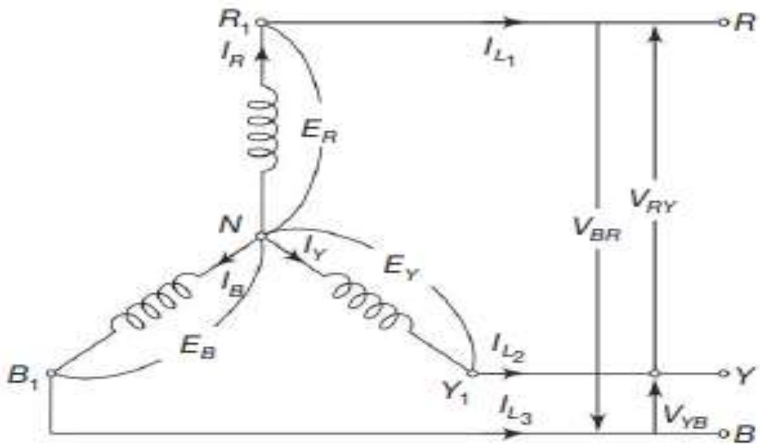


Fig. 5.34

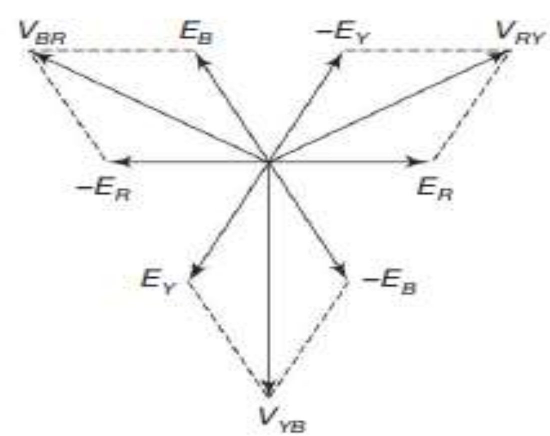


Fig. 5.35

Current Relationship Applying Kirchhoff's current law at nodes R_1, Y_1, B_1 we get $I_R = I_{L1}$; $I_Y = I_{L2}$; $I_B = I_{L3}$.

This means that in a balanced star connected system, phase current equals the line current

$$I_P = I_L.$$

Voltage Relationship Let us apply Kirchhoff's voltage law to the loop consisting of voltages E_R, V_{RY} and E_Y. We have

$$\overline{E}_R - \overline{E}_Y = \overline{V}_{RY}$$

Using law of parallelogram,

$$|\overline{V}_{RY}| = V_{RY} = \sqrt{E_R^2 + E_Y^2 + 2\,E_R\,E_Y \cos 60^\circ}$$

$$= \sqrt{E_P^2 + E_P^2 + 2\,E_P\,E_P\,(\%0)} = E_P\sqrt{3}$$

Similarly,

$$\overline{E}_Y - \overline{E}_B = \overline{V}_{YB} \quad \text{and} \quad \overline{E}_B - \overline{E}_R = \overline{V}_{BR}$$

$\therefore$ $$\overline{V}_{YB} = E_P\sqrt{3} \text{ and } \quad V_{BR} = E_P\sqrt{3}$$

Thus, $$V_L = \sqrt{3}\ E_P$$

Line voltage = $\sqrt{3}$ phase voltage

Power Relationship Let $\cos\phi$ be the power factor of the system.

Power consumed in one phase = $E_P\,I_P \cos\phi$

Power consumed in three phases = $3E_P\,I_P \cos\phi$

$$= 3\,\frac{V_L}{\sqrt{3}}\,I_L \cos\phi$$

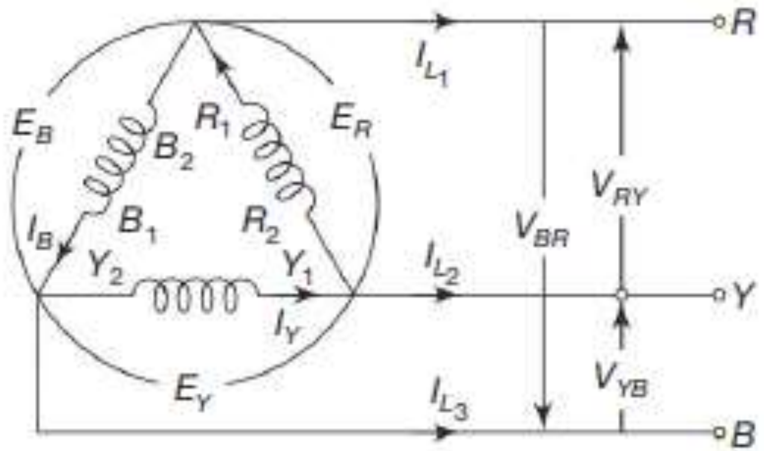
$$= \sqrt{3}\ V_L\,I_L \cos\phi \text{ watts}$$

Delta Connection

Here the dissimilar ends of the three phase coils are connected together to form a mesh. Wires are drawn from each junction for connecting load. We can connect only three phase loads as there is no fourth wire available.

Let us now analyse the above connection.

The system is a balanced one. Hence the currents and the voltages will be balanced. Notations used in the star connection are used here.



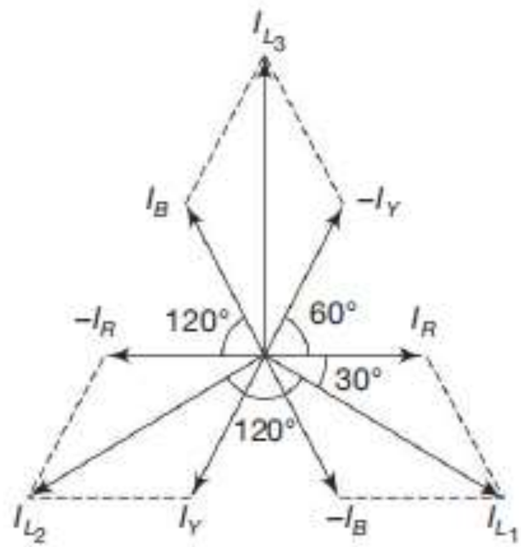


Fig. 5.37

$E_R = E_Y = E_B = E_P$, phase voltage
$I_R = I_Y = I_B = I_P$, phase voltage
$V_{RY} = V_{YB} = V_{BR} = V_L$, line voltage
$I_{L1} = I_{L2} = I_{L3} = I_L$, line voltage

Voltage Relationship Applying Kirchhoff's voltage law to the loop consisting of E_R and V_{RY}, we have $E_R = V_{RY}$.

Similarly, $E_Y = V_{YB}$ and $E_B = V_{BR}$,

Thus $E_P = V_L$.

Phase Voltage = Line Voltage

Current Relationship Applying Kirchhoff's current law at the junction of R_1 and B_2, we have $\bar{I}_R - \bar{I}_B = \bar{I}_L 1$.

Referring to the phasor diagram and applying the law of parallelogram, we have

$$I_L I = \sqrt{I_R^2 + I_B^2 + 2 I_R I_B \cos 60^\circ}$$
$$= \sqrt{I_P^2 \, I_P^2 + 2 I_P \, I_P (\%0)}$$
$$= I_P \sqrt{3}$$

Similarly, we have

$$\bar{I}_Y - \bar{I}_R = \bar{I}_L 2 \quad \text{and} \quad \bar{I}_B - \bar{I}_Y = \bar{I}_L 3$$

Hence, $\qquad I_L 2 = I_P \sqrt{3}$ and $I_L^3 = I_P \sqrt{3}$

Thus, line current $= \sqrt{3}$ phase current

$$I_L = \sqrt{3}\, I_P$$

Power Relationship Let $\cos \phi$ be the power factor of the system.

$$\text{Power per phase} = E_P \, I_P \cos \phi$$

$$\text{Total power for all the three phases} = 3\, E_P \, I_P \cos \phi$$
$$= 3\, V_L \frac{I_L}{\sqrt{3}} \cos \phi$$
$$= \sqrt{3}\, V_L \, I_L \cos \phi \text{ watts}$$

Sl. No.	Star (Y) Connected System	Delta (Δ) Connected System
1.	In star connected system there is common point known as neutral 'n' or star point. It can be earthed.	There is no neutral point in delta connected system
2.	In star connected system we get 3-phase, three wire system and also 3-phase, 4 wire system is taken out.	Only 3-phase, 3 wire system is possible in delta connected system
3.	Line voltage $V_L = \sqrt{3}\, V_{ph}$ or, $V_{ph} = \frac{1}{\sqrt{3}} V_L$	Line voltage = Phase voltage $V_L = V_{ph}$
4.	Line current = Phase current $I_L = I_{ph}$	Line current $I_L = \sqrt{3}\, I_{ph}$ $I_{ph} = \frac{1}{\sqrt{3}} I_L$

END OF UNIT 1

Additional Problems (Solved and unsolved)

1. For a three phase star connected system with a line voltage of 400 V, calculate the value of phase voltage.

A. 400 V

B. 692.8 V

C. 331.33 V

D. 230.94 V

ANSWER: D

2. Phase voltages of the windings of a 3-wire star-connected machine are 2 kV. Line voltages of the machine is

a.1732.05 V

b.1154.70 V

c.2309.4 V

d.3464.10 V

ANSWER: (d)

3. For a three phase delta connected system with a line voltage of 400 V and line current is 100 A, calculate the value of phase voltage and phase current.

A. 400 V, 173.2 A

B. 400 V, 57.7 A

C. 230.9 V, 100 A

D. 230.9 V, 57.7 A

ANSWER:B

In a series RC circuit, 12 V is measured across the resistor and 15 V is measured across the capacitor. The source voltage is

(A) 3 V
(B) 27 V
(C) 19.2 V
(D) 12 V

Krichoff's voltage law is based on

(A) Law of conservation of energy
(B) Law of conservation of charge
(C) Faraday's law of electromagnetic induction
(D) Fleming's right hand rule

Superposition theorem is applied to

(A) Only linear circuit
(B) Only non linear circuit
(C) Either on linear or non linear circuit
(D) Only on DC circuit

Three equal resistances of 3 Ω are connected in star. What is the resistance in one of the arms in an equivalent delta circuit?

(A) 10 Ω
(B) 27Ω
(C) 9 Ω
(D) 3Ω

In a certain series RC circuit, the true power is 2W, and the reactive power is 3.5 VAR. What is the apparent power?

(A) 3.5 VA
(B) 2.03 VA
(C) 4.03 VA
(D) 3 VA

A power factor of '0' indicates

(A) Purely resistive element
(B) Purely inductive element
(C) Combination of both (A) and (B)
(D) Purely capacitive element and resistive element

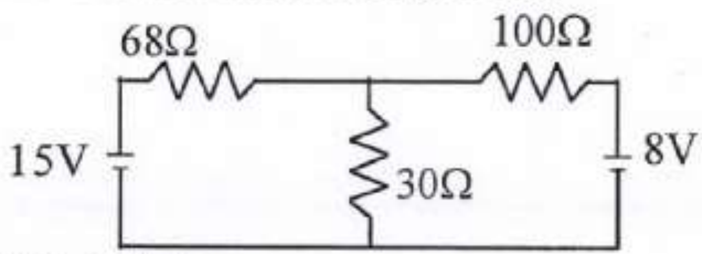
A 10Ω resistor is connected in parallel with a 15Ω resistor and the combination in series with a 12Ω resistor. The equivalent resistance of the circuit is

(A) 37Ω (B) 18Ω
(C) 27Ω (D) 4Ω

A current is said to be direct current when its

(A) Magnitude remains constant with time (B) Magnitude changes with time
(C) Direction changes with time (D) Magnitude and direction changes with time

What is the current through 30Ω?



(A) 3.19 A (B) 319 mA
(C) 1.73 mA (D) 173 mA

When an additional resistor is connected across an existing parallel circuit, the total resistance

(A) Remains the same (B) Decreases by the value of the added resistor
(C) Increases by the value of the added resistor (D) Decreases

When a fourth resistor is connected in series with three resistor, the total resistance

(A) Increase by one-fourth
(B) Increases
(C) Decreases
(D) Remains the same

A circuit consists of three resistors in parallel, when one resistor is removed the circuit current,

(A) Decreases
(B) Increases by one third
(C) Decreases by one-third
(D) Decrease by the amount of current through the removed resistor.

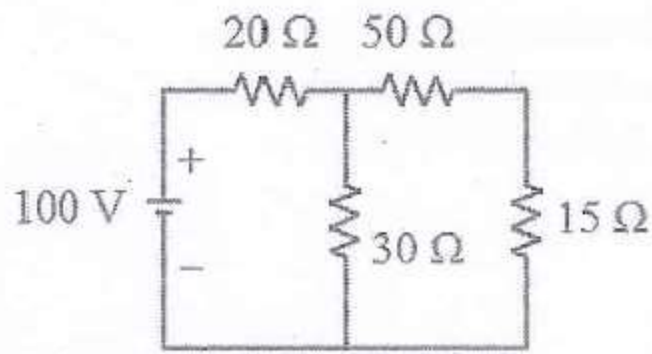
An algebraic sum of all the element voltages in a mesh is equal to

(A) The total of the voltage drops
(B) The source voltage
(C) Zero
(D) The total of the source voltage and the voltage drops.

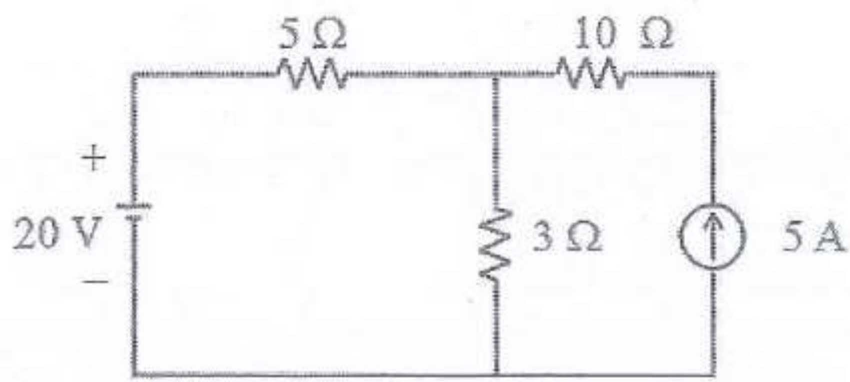
A 24V DC supply is applied across a voltage divider consisting of two 68kΩ resistors. The unknown output voltage is

(A) 12V
(B) 24V
(C) 0V
(D) 6V

Using KVL, determine total current drawn from the source and also current across 15 Ω resistor in the following circuit.



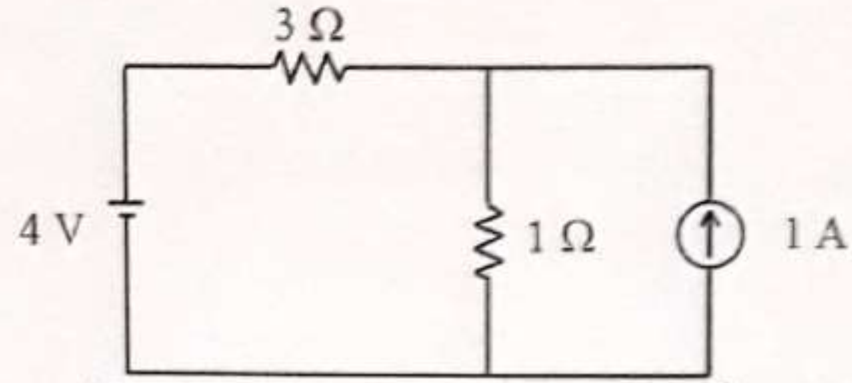
Identify the current through 3Ω resistor in the circuit shown below, using superposition theorem.



A resistance of 100 Ω is connected in series with a 50μF capacitor to a supply at 200 V, 50 Hz. Determine the

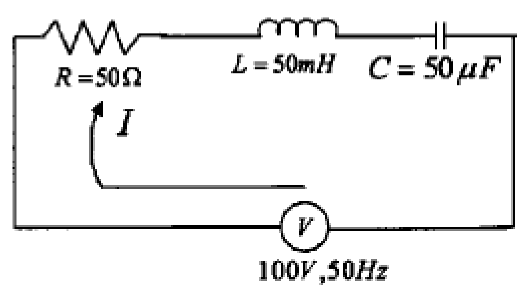
(i) Current
(ii) Power factor
(iii) Voltage across resistor and capacitor

For the circuit shown, find the voltage across the 1 ohm resistors.

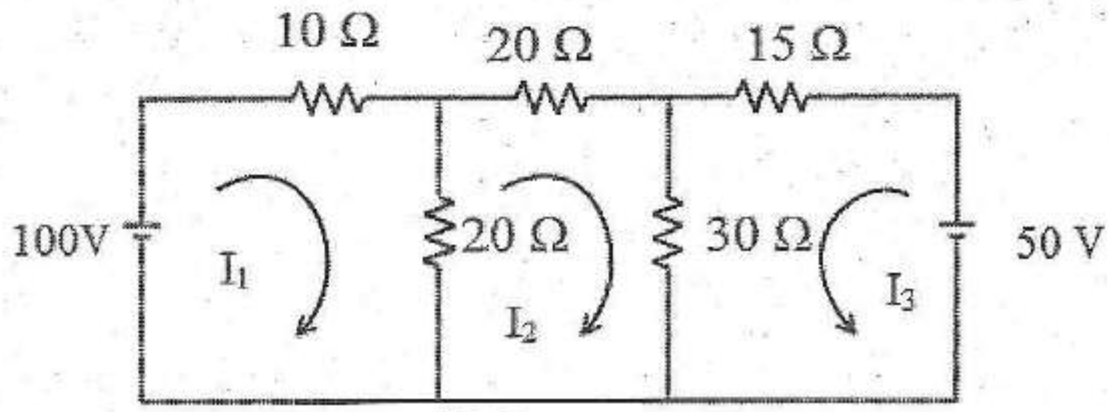


Two coils when connected in series have a resistance of 18Ω and when connected in parallel have a resistance of 4Ω. Find the resistance of each coil.

Find the current I in the circuit shown.



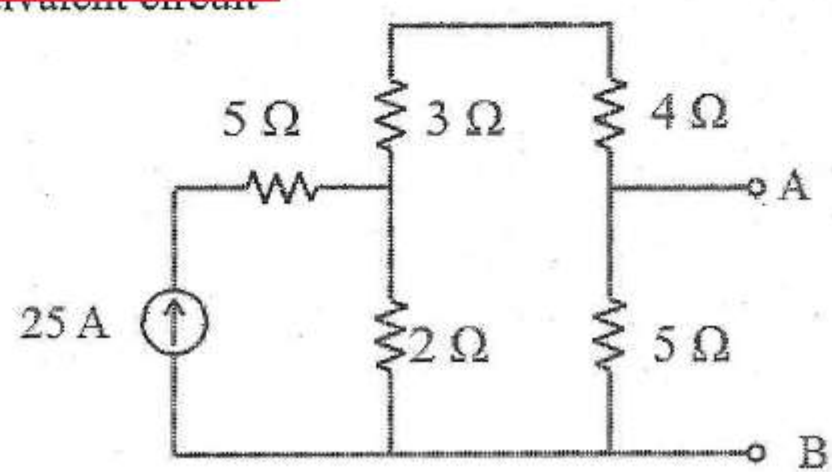
Using mesh analysis, determine mesh current in each loop given in the below circuit.



Simplify the given circuit into

(i) Thevenin's equivalent circuit and

~~(ii) Norton's equivalent circuit~~



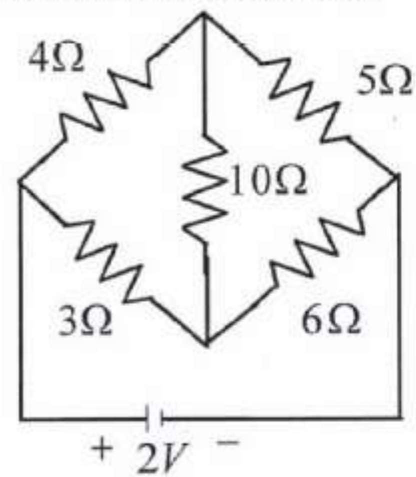
Analyze that the RMS and Average values of sinusoidal alternating current are $\frac{I_m}{\sqrt{2}}$ and $\frac{2I_m}{\pi}$.

Derive the average value, RMS value, ~~peak factor and form factor~~ for a full-wave rectified sinusoidal waveform. (8 Marks)

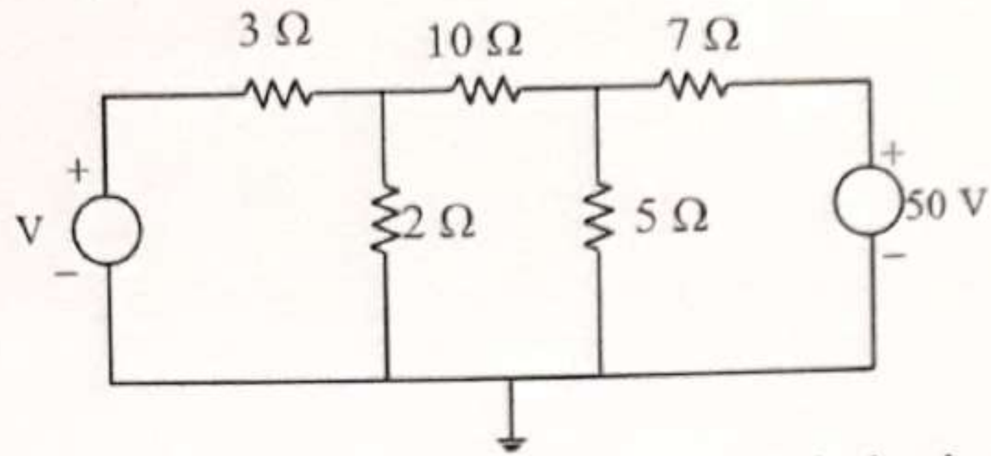
Derive the average value, RMS value, ~~form factor and peak factor~~ for the half-wave and full-wave rectified sine wave.

Derive the average and RMS value of the full wave rectified sine wave voltage.

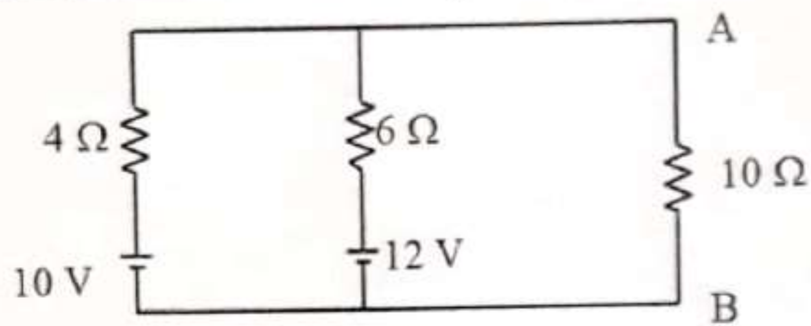
Determine the current through all the branches in circuit



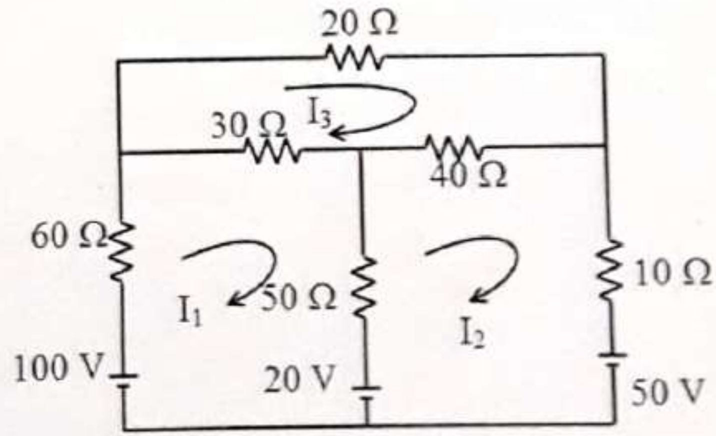
a.i. Find the voltage 'V' in the circuit shown below which makes the current in the 10 Ω resistor zero by using nodal analysis.



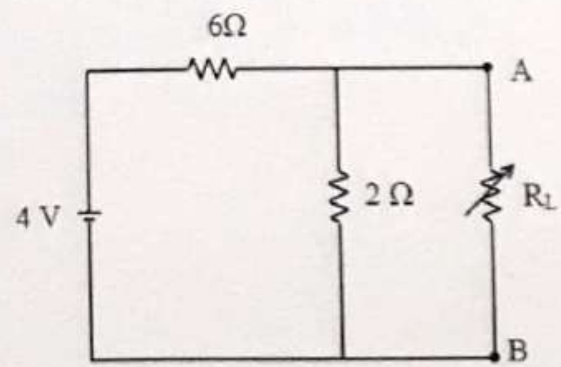
ii. Using Thevenin's theorem, find the current through 10Ω resistor in the circuit shown below.



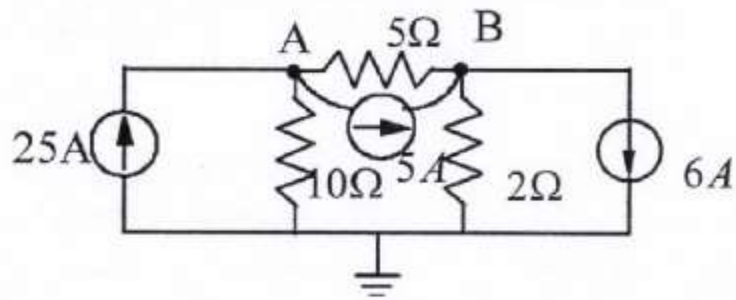
b.i. Find the current that flows through the 50 Ω resistor for the circuit shown below using mesh analysis.



ii. Determine the value of load resistance R_L when it is dissipating maximum power. Also find the maximum power dissipated in the load resistance for the circuit given below.



Compute the voltages at node A and B for the circuit shown below. (8 Marks)



Find the impedance, current and power factor of the following series circuit and draw the corresponding phasor diagrams

(i) R and L
(ii) R and C.

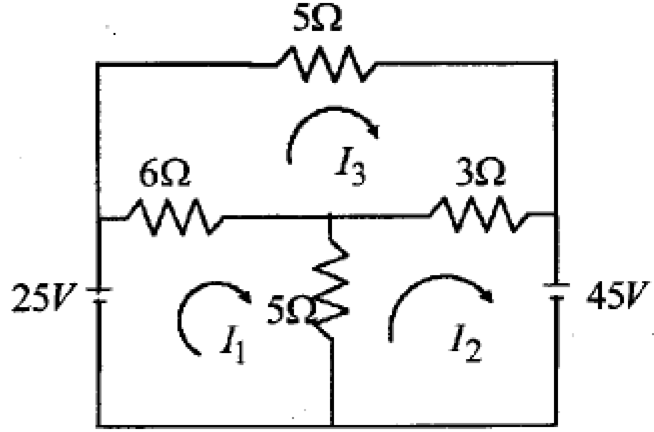
In each case the applied voltage is 200 volts and the frequency is 50Hz. Further $R = 10\Omega,\ L = 50mH\ \ and\ C = 100\mu F$

In an ac circuit, resistor R and inductor L are connected in series. Voltage and current equations are given as $e(t) = 200 \sin 314t$ and $i(t) = 20 \sin(314t - 30°)$ calculate (i) Rms value of the voltage and current. (ii) Frequency (iii) Power factor (iv) Power (v)Values of R and L.

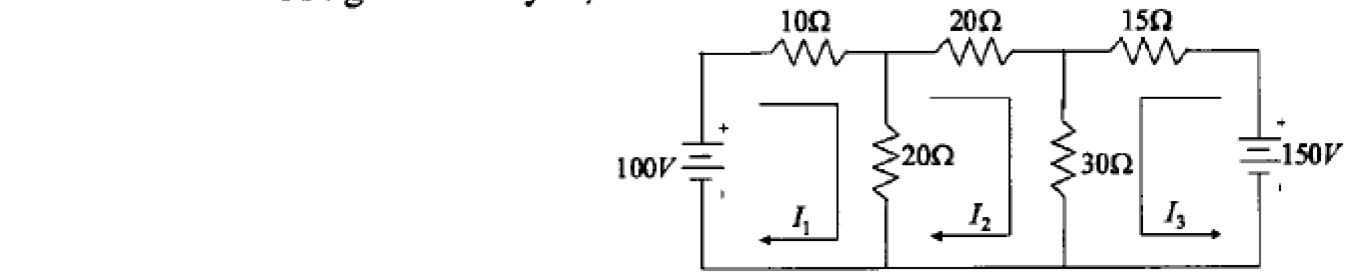
A coil of resistance 10Ω and inductance 0.1H is connected in series with a 150μF capacitor across 200V, 50Hz supply. Calculate

(i) Inductive reactance, capacitive reactance, impedance, current and power factor
(ii) Voltage across the coil and capacitor

Find the current in 5Ω resistor using Mesh analysis in the circuit shown below.



Using mesh analysis, find mesh currents in the circuit shown below.



Using nodal analysis, find all node voltages in the circuit shown below.