PS07

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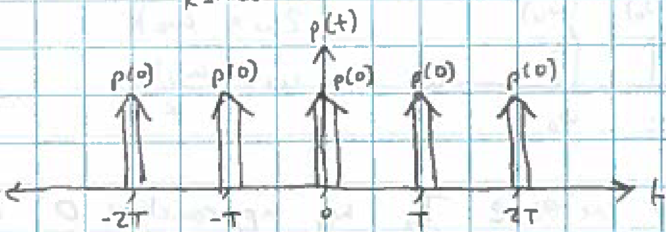
Sig Sys 2015
03/11/2015

1. a. $$p(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT)$$




b. Fourier series representation of $p(t)$: $\tilde{p}(t)$

$$\tilde{p}(t) = \sum_{k=-\infty}^{\infty} C_k e^{j\frac{2\pi}{T}kt}$$

$$C_k = \frac{1}{T}\int_{-T/2}^{T/2} p(t)\, e^{-j\frac{2\pi}{T}kt}\,dt = \frac{1}{T}\int_{-T/2}^{T/2} \sum_{k=-\infty}^{\infty} \delta(t-kT)\, e^{-j\frac{2\pi}{T}kt}\,dt$$

$$= \sum_{k=-\infty}^{\infty} \frac{1}{T}\int_{-T/2}^{T/2} \delta(t-kT)\, e^{-j\frac{2\pi}{T}kt}\,dt$$

From the plot above, we know that $p(t)$ is only non-zero at a point so from $T/2$ to $-T/2$, the integral is only evaluated at a single point, $t=0$.

$$C_k = \sum_{k=-\infty}^{\infty} \frac{1}{T} e^{-j\frac{2\pi}{T}k(0)} = \sum_{k=-\infty}^{\infty} \frac{1}{T} = \frac{1}{T}$$

$$\tilde{p}(t) = \sum_{k=-\infty}^{\infty} \frac{1}{T} e^{j\frac{2\pi}{T}kt}$$

c. $$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{j\frac{2\pi}{T}kt}$$

$$X(\omega) = \int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} C_k e^{j\frac{2\pi}{T}kt} e^{-j\omega t}\,dt = \sum_{k=-\infty}^{\infty} C_k \int_{-\infty}^{\infty} e^{j\frac{2\pi}{T}kt} e^{-j\omega t}\,dt$$

$$\int_{-\infty}^{\infty} e^{j\frac{2\pi}{T}kt - j\omega t}\,dt = \int_{-\infty}^{\infty} e^{jt(\frac{2\pi}{T}k - \omega)}$$ define $\omega_0 = \frac{2\pi}{T}k - \omega$

$$\int_{-\infty}^{\infty} e^{jt\omega_0} = 2\pi\,\delta(\omega - \omega_0) = 2\pi\,\delta(\omega - \tfrac{2\pi}{T}k + \omega) = 2\pi\,\delta(2\omega - \tfrac{2\pi}{T}k)$$

Substituting back in: $X(\omega) = \sum_{k=-\infty}^{\infty} C_k\, 2\pi\,\delta(2\omega - \frac{2\pi}{T}k)$

Now define $\omega_0 = \frac{2\pi}{T}$

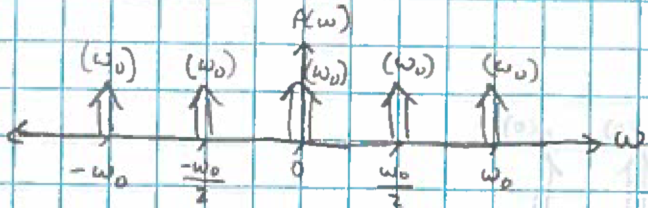
$$X(\omega) = \sum_{k=-\infty}^{\infty} C_k\, 2\pi\,\delta(2\omega - \omega_0 k)$$

d. $C_k = \frac{1}{T}$

$$P(\omega) = \sum_{k=-\infty}^{\infty} \frac{1}{T} 2\pi\, \delta(2\omega - \omega_0 k) = \sum_{k=-\infty}^{\infty} \omega_0\, \delta(2\omega - \omega_0 k)$$

$$= \omega_0 \sum_{k=-\infty}^{\infty} \delta(2\omega - \omega_0 k)$$

e.




$$2\omega - \omega_0 k = 0$$
$$2\omega = \omega_0 k$$
$$\omega = \frac{\omega_0 k}{2}$$

$\omega_0$ is $\frac{2\pi}{T}$. So if we increase $T$, $\omega_0$ approaches 0 which makes $P(\omega)$ approach 0 at all points. Decreasing $T$ increases the spacing between the impulses and scales up the impulses in $P(\omega)$.

Increasing $T$ for $p(t)$ increases the spacing between impulses. Decreasing $T$ makes $p(t)$ approach a single impulse at $t=0$.

This behavior is kind of intuitive when transition from the time domain to the frequency domain as $T$ varies.

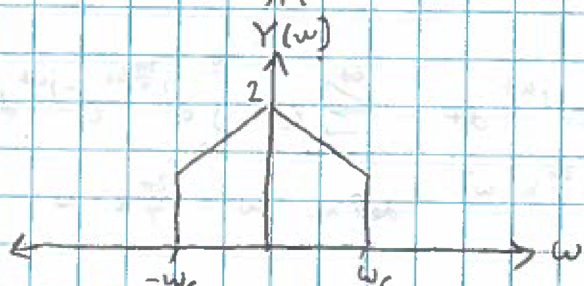
2. a. $h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega) e^{j\omega t}\, d\omega \qquad \int_{-\infty}^{-\omega_c} H(\omega)\, d\omega = 0 \qquad \int_{\omega_c}^{\infty} H(\omega)\, d\omega = 0$

$$\int_{-\omega_c}^{\omega_c} e^{j\omega t}\, d\omega = \frac{1}{jt} e^{j\omega t}\Big|_{-\omega_c}^{\omega_c} = \frac{1}{jt} e^{j\omega_c t} - \frac{1}{jt} e^{-j\omega_c t}$$

$$h(t) = \frac{1}{2\pi}\left(\frac{1}{jt} e^{j\omega_c t} - \frac{1}{jt} e^{-j\omega_c t}\right) = \frac{1}{\pi t}\left(\frac{1}{2j} e^{j\omega_c t} - \frac{1}{2j} e^{-j\omega_c t}\right)$$

$$h(t) = \frac{\sin(\omega_c t)}{\pi t}$$

b.




c. This LTI system is known as an ideal low-pass filter will cut-off frequency $\omega_c$ because it attenuates to 0 all frequencies above $\omega_c$ and below $-\omega_c$. This is because if you multiply $H(\omega)$ by frequencies above $\omega_c$, they become 0.

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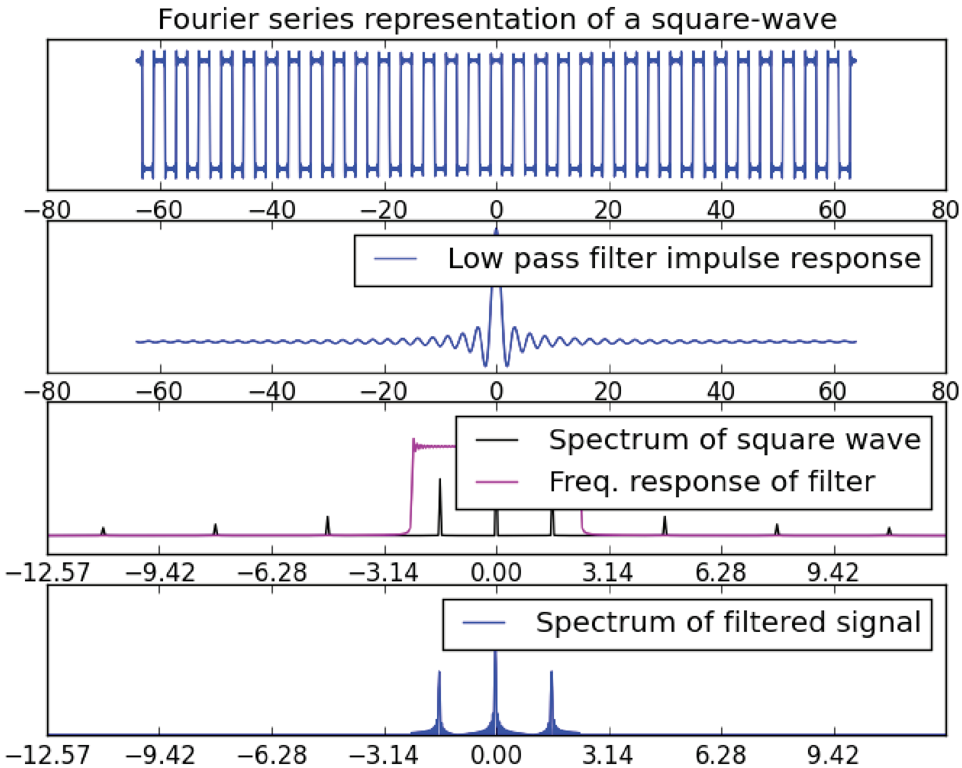
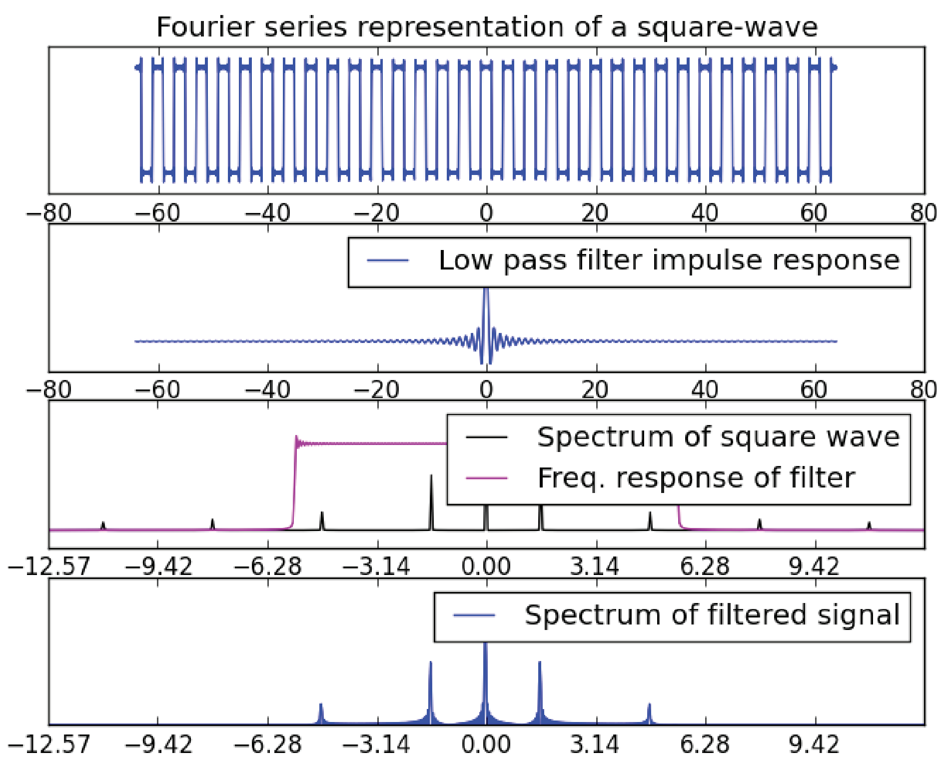
3d

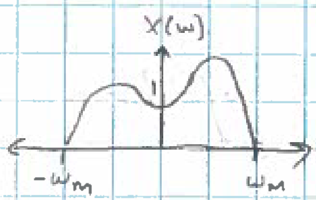
$\omega_c = 0.75\pi$




$\omega_c = 1.75\pi$



3.




$y(t) = x(t)\cos(w_c t)$

$y(t) = x(t)\,h(t)$ where $h(t) = \cos(w_c t)$

$H(w) = \pi\,\delta(w - w_c) + \pi\,\delta(w + w_c)$

$Y(w) = \frac{1}{2\pi} X * H(w)$

$Y(w) = \frac{1}{2\pi} X * \pi\,\delta(w - w_c) + \pi\,\delta(w + w_c)$

$Y(w) = \frac{1}{2} X * \delta(w - w_c) + \frac{1}{2} X * \delta(w + w_c)$