PS10

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1. $\mathcal{L}\{\dot{y} + y = x\} = sY(s) + Y(s) = X(s)$

$Y(s)(s+1) = X(s)$ $\qquad \mathcal{L}\{X(s)\} = \frac{1}{s}$

$$Y(s) = \frac{X(s)}{s+1} = \frac{\frac{1}{s}}{s+1} = \frac{1}{s^2+s} = \frac{(s+1)-s}{s^2+s} = \frac{(s+1)-s}{s(s+1)} =$$

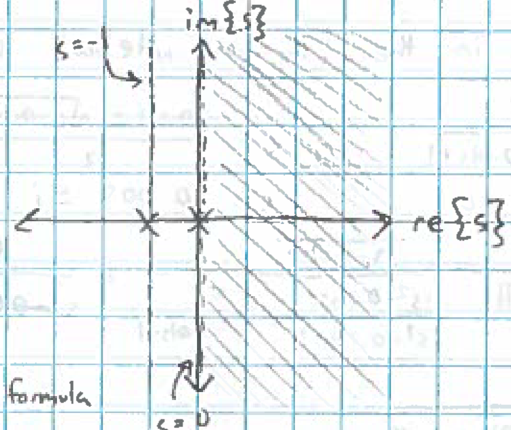
$$\frac{\cancel{s+1}}{s(\cancel{s+1})} + \frac{-\cancel{s}}{\cancel{s}(s+1)} = \frac{1}{s} + \frac{-1}{s+1}$$

$$\mathcal{L}^{-1}\left\{Y(s) = \frac{1}{s} + \frac{-1}{s+1}\right\} = 1e^{0t}u(t) + -1e^{-t}u(t) = y(t)$$

$$y(t) = u(t) - e^{-t}u(t) = (1-e^{-t})u(t)$$

ROC: $Re\{s\} > 0$

We only care about the pole at $s=0$ since the rightmost pole in the s-plane dominates.



Black's formula

2. a. $\frac{Y}{Y_{sp}} = \frac{KH}{1+KH} = \frac{\frac{K_I}{s}H}{1+\frac{K_I}{s}H}$

DC gain: $\lim_{s\to 0} \frac{\frac{K_I}{s}H}{1+\frac{K_I}{s}H} = 1$

The DC gain does not depend on the value of K_I

b. $$\frac{Y}{Y_{sp}} = \frac{\frac{K_I}{s}H}{\frac{s}{s}\left(1+\frac{K_I}{s}H\right)} = \frac{\frac{K_I}{s}H}{\frac{s+K_IH}{s}} = \frac{K_IH}{\cancel{s}}\cdot\frac{\cancel{s}}{s+K_IH} = \frac{K_IH}{s+K_IH}$$

$$\frac{Y}{Y_{sp}} = \frac{K_I\left(\frac{1/\tau}{s+1/\tau}\right)}{\left(\frac{s+1/\tau}{s+1/\tau}\right)s + K_I\left(\frac{1/\tau}{s+1/\tau}\right)} = \frac{\left(\frac{K_I/\tau}{s+1/\tau}\right)}{\frac{s^2+s/\tau+K_I/\tau}{s+1/\tau}} =$$

$$\frac{K_1/\tau}{\cancel{s+1/\tau}}\cdot\frac{\cancel{s+1/\tau}}{s^2+s/\tau+K_1/\tau} = \frac{K_1/\tau}{s^2+s/\tau+K_1/\tau}$$

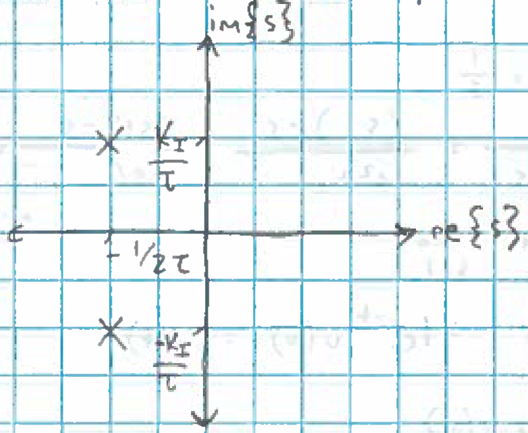
$$s^2 + s/\tau + K_1/\tau = 0$$

$$s^2 + \frac{1}{\tau}s + \frac{K_1}{\tau} = 0 \qquad s = \frac{-\frac{1}{\tau} \pm \sqrt{\left(\frac{1}{\tau}\right)^2 - 4(1)\left(\frac{K_I}{\tau}\right)}}{2(1)}$$

Since $K \gg \frac{1}{\tau}$ we can say our $\left(\frac{1}{\tau}\right)^2$ term is negligible. Thus we have:

$$s = \frac{-\frac{1}{\tau} \pm \sqrt{-4K_I/\tau}}{2} = -\frac{1}{2\tau} \pm \frac{K_I}{\tau} i$$

There are no zeroes since there is no value for which the numerator in Y/Y_{sp} is 0 for s. There are two poles.



Part 3 and 4 are in the iPython notebook submitted w/ this sheet

4. a. $H(s) = \frac{1}{s^2 - 0.01s + 1}$ $\qquad s = \frac{0.01 \pm \sqrt{(-0.01)^2 - 4}}{2}$ assume $(-0.01)^2$ is 0

$s^2 - 0.01s + 1 = 0 \qquad s = 0.005 \pm i$

b. $$\frac{Y}{Y_{sp}} = \frac{KH}{1+KH} = \frac{\frac{K}{s^2-0.01s+1}}{\frac{s^2-0.01s+1}{s^2-0.01s+1} + \frac{K}{s^2-0.01s+1}} = \frac{K}{\cancel{s^2-0.01s+1}} \cdot \frac{\cancel{s^2-0.01s+1}}{s^2-0.01s+1+K}$$

$$\frac{Y}{Y_{sp}} = \frac{K}{s^2-0.01s+1+K}$$

c. $$\frac{Y}{Y_{sp}} = \frac{\frac{K/s}{s^2-0.01s+1}}{1 + \frac{K/s}{s^2-0.01s+1}} = \frac{\frac{K}{s^3-0.01s^2+s}}{\frac{s^3-0.01s^2+s}{s^3-0.01s^2+s} + \frac{K}{s^3-0.01s^2+s}} = \frac{K}{\cancel{s^3-0.01s^2+s}} \cdot \frac{\cancel{s^3-0.01s^2+s}}{s^3-0.01s^2+s+K}$$

$$\frac{Y}{Y_{sp}} = \frac{K}{s^3-0.01s^2+s+K}$$

d. $$\frac{Y}{Y_{sp}} = \frac{\frac{sK}{s^2-0.01s+1}}{1 + \frac{sK}{s^2-0.01s+1}} = \frac{\frac{sK}{s^2-0.01s+1}}{\frac{s^2-0.01s+1}{s^2-0.01s+1} + \frac{sK}{s^2-0.01s+1}} = \frac{sK}{\cancel{s^2-0.01s+1}} \cdot \frac{\cancel{s^2-0.01s+1}}{s^2-0.01s+1+sK}$$

$$\frac{Y}{Y_{sp}} = \frac{sK}{s^2-0.01s+1+sK}$$

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Below is the link to my iPython notebook which is on NBViewer and contains my work for Problems 3 and 4.

http://nbviewer.ipython.org/github/dinopants174/ThinkDSP/blob/master/PS10_ZG.ipynb