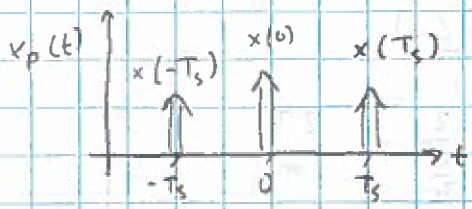
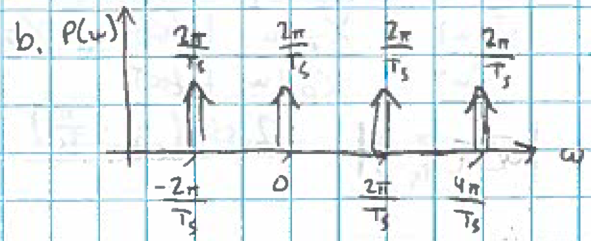
PS08

Zoher Ghadyali
Sig Sys 2015
03/25/2015

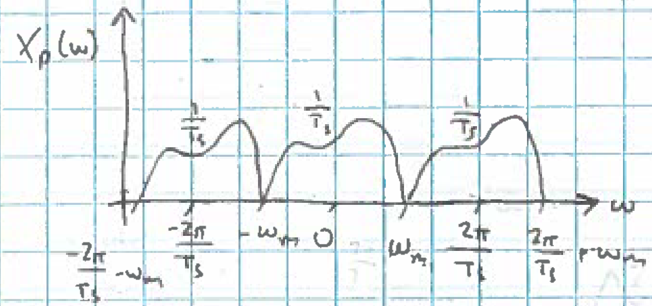
1. a.



b.



c.

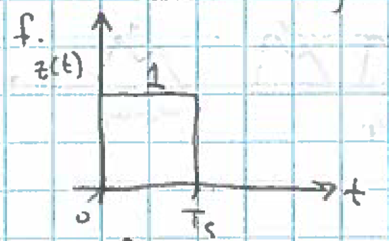


d. $2\omega_m \le \frac{2\pi}{T_s}$

$T_s \le \frac{\pi}{\omega_m}$

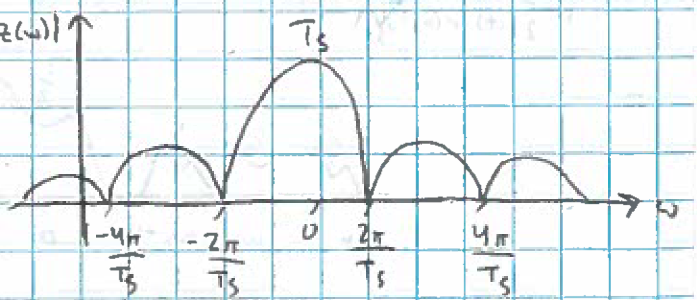
e. You can recover $x(t)$ from $x_p(t)$ exactly by multiplying $X_p(\omega)$ with a low-pass filter with cut-off frequency ω_m. In the time domain, this would be convolving with a sinc function which is the equivalent of interpolating with sincs, recreating $x(t)$ exactly.

f.

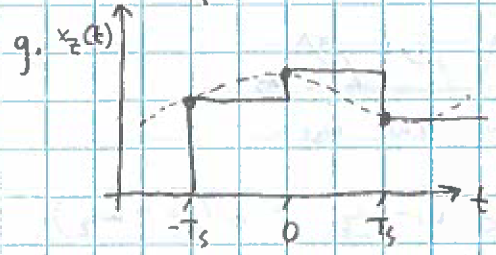


h. $x(t-\frac{T_s}{2}) = e^{-j\omega\frac{T_s}{2}} X(\omega)$

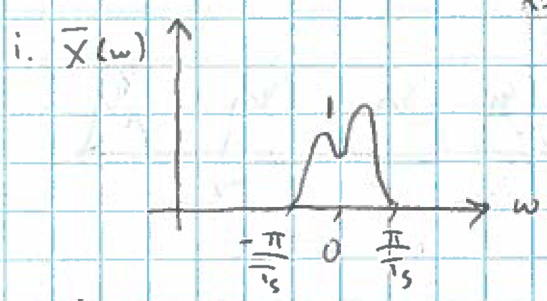
$Z(\omega) = e^{-j\omega\frac{T_s}{2}} \frac{2\sin(\frac{T_s}{2}\omega)}{\omega}$

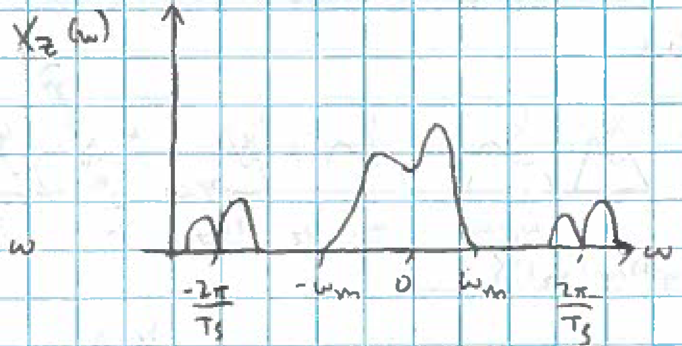


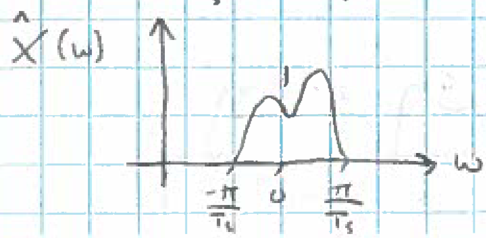
g.



i.







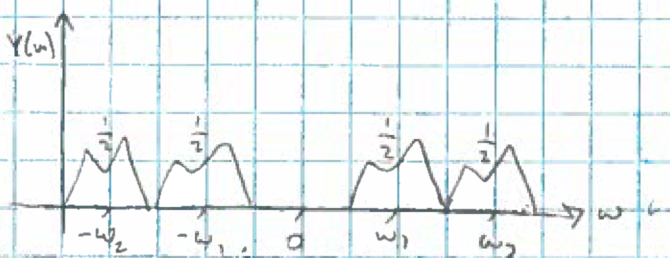
j. $\bar{X}(\omega)$ and $\hat{X}(\omega)$ are very similar. However, the edges of $\bar{X}(\omega)$ are attenuated a little due to the envelope of the sinc function which decays at a rate of $1/\omega$. Other than that, $\bar{X}(\omega)$ and $\hat{X}(\omega)$ are very similar to $X(\omega)$.

k. $$\frac{\bar{X}(\omega)}{\hat{X}(\omega)} = \frac{X_z(\omega)\cancel{H(\omega)}}{X_p(\omega)\cancel{H(\omega)}} = \frac{\cancel{X_p(\omega)}\,Z(\omega)}{\cancel{X_p(\omega)}}$$

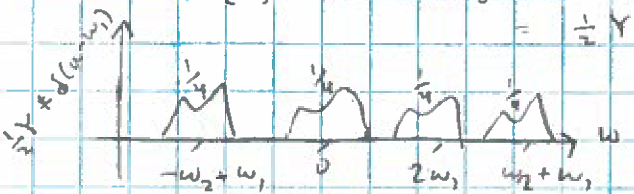
$$\left|Z\left(\frac{\pi}{T_s}\right)\right| = \frac{2\sin\left(\frac{T_s}{2}\cdot\frac{\pi}{T_s}\right)}{\frac{\pi}{T_s}} = \frac{2}{\frac{\pi}{T_s}} = \frac{2T_s}{\pi}$$

$$\frac{\bar{X}(\omega)}{\hat{X}(\omega)} = \frac{2T_s}{\pi}$$

2. a.



b. $$FT\{y(t)\cos(\omega_1 t)\} = \frac{1}{2\pi} Y * \pi\delta(\omega-\omega_1) + \pi\delta(\omega+\omega_1)$$
$$= \frac{1}{2} Y * \delta(\omega-\omega_1) + \delta(\omega+\omega_2)$$



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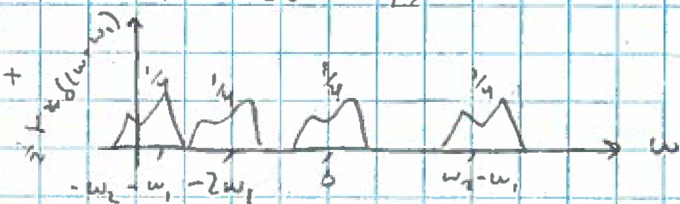

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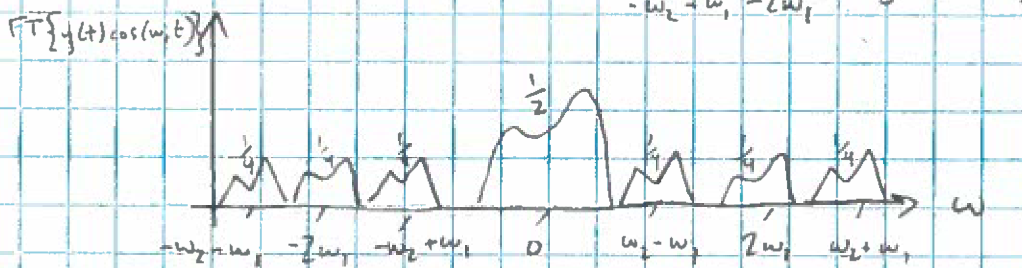
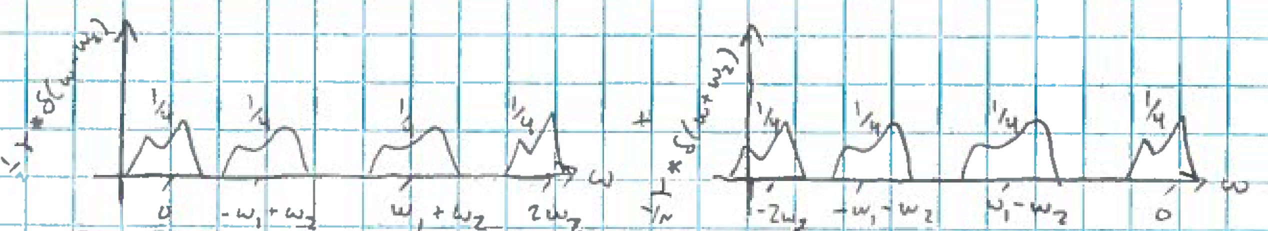
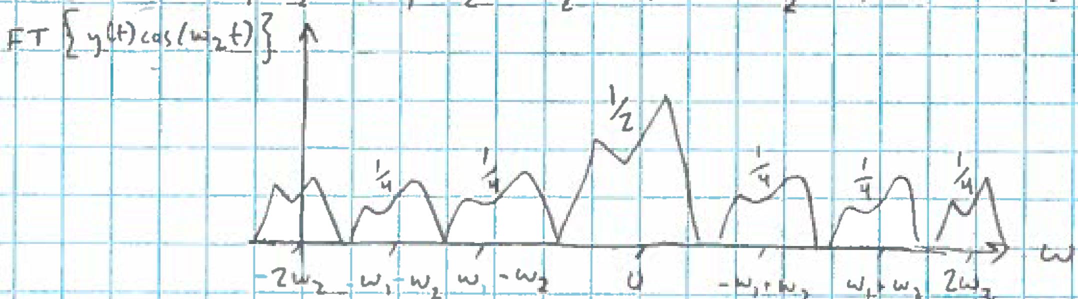


$$FT\{y(t)\cos(\omega_2 t)\} = \frac{1}{2} Y * \delta(\omega-\omega_2) + \delta(\omega+\omega_2)$$





c. $y(t)$ is a recieved AM radio signal from two different AM transmitters. To recover $x_1(t)$, you multiply $y(t)$ by a $\cos(\omega_1 t)$ which demodulates the frequency components at ω_1. Applying a low-pass filter in the frequency domain with a cut-off of ω_m will recover a scaled version of $x_1(t)$.

To recover $x_2(t)$, you do the same thing except $y(t)$ is multiplied by a $\cos(\omega_2 t)$.

3. a. $V_{in}(t) = V_R(t) + V_L(t) + V_{out}(t) = Ri(t) + V_L(t) + V_{out}(t)$

$$= RC\frac{d}{dt}V_{out}(t) + L\frac{d}{dt}C\frac{d}{dt}V_{out}(t) + V_{out}(t)$$

$$V_{in}(t) = LC\frac{d^2}{dt^2}V_{out}(t) + RC\frac{d}{dt}V_{out}(t) + V_{out}(t)$$

b. $V_{in}(\omega) = LC(j\omega)^2 V_{out}(\omega) + RCj\omega V_{out}(\omega) + V_{out}(\omega)$

$$= V_{out}(\omega)(-LC\omega^2 + RCj\omega + 1)$$

$$\frac{V_{in}(\omega)}{V_{out}(\omega)} = -LC\omega^2 + 1 + RC\omega j$$

$$\frac{V_{out}(\omega)}{V_{in}(\omega)} = H(\omega) = \frac{1}{-LC\omega^2 + 1 + RC\omega j}$$

c. $$|H(\omega)| = \frac{1}{\sqrt{(1-LC\omega^2)^2 + (RC\omega)^2}}$$

d. Since we are trying to maximize $|H(\omega)|$, we are trying to minimize the denominator so we are looking for where the derivative of the denominator is 0.

$$0 = \frac{d}{d\omega}\sqrt{(1-LC\omega^2)^2 + (RC\omega)^2}$$

$$0 = \frac{d}{d\omega}(1-LC\omega^2)^2 + (RC\omega)^2$$

$$= 2(1-LC\omega^2)\frac{d}{d\omega}(1-LC\omega^2) + 2RC\omega\frac{d}{d\omega}RC\omega$$

$$= 2(1-LC\omega^2)(-LC2\omega) + 2RC\omega(RC)$$

$$= -4LC\omega(1-LC\omega^2) + 2R^2C^2\omega$$

$$= -4LC\omega + 4L^2C^2\omega^3 + 2R^2C^2\omega$$

$$= 2C\omega(-2L + 2L^2C\omega^2 + R^2C) \Rightarrow \omega = 0 \text{ is a possible solution}$$

$$0 = -2L + 2L^2C\omega^2 + R^2C$$

$$2L - R^2C = 2L^2C\omega^2$$

$$\omega^2 = \frac{2L - R^2C}{2L^2C} \Rightarrow \omega = \pm\sqrt{\frac{2L-R^2C}{2L^2C}} \text{ are also possible solutions}$$

To check which of these solutions are minimums, we need to check the second derivative to see if it is positive for these solutions.

$$H'(\omega) = -4LC\omega + 4L^2C^2\omega^3 + 2R^2C^2\omega$$

$$H''(\omega) = -4LC + 12L^2C^2\omega^2 + 2R^2C^2$$

$$= -4LC + 2R^2C^2 + 12L^2C^2\omega^2$$

$$= 2C(-2L + R^2C) + 6C(2L^2C\omega^2)$$

If you substitute in $\omega = 0$ to find the sign of the second derivative, you need to know if $-2L + R^2C > 0$. From our other solutions for ω, we know $2L - R^2C \geq 0$ which means $2L \geq R^2C$. As a result, $-2L + R^2C < 0$ so the second derivative is negative which means $\omega = 0$ is a maximum and not a minimum. Thus,

$$\omega = \pm\sqrt{\frac{2L - R^2C}{2L^2C}} \quad \text{maximize } |H(\omega)|$$

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