Homework 1 Report

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Abstract— In this homework, I have simulated 3 different systems: a system of 3 metal spheres connected by beams falling in a viscous liquid, the same system but with N spheres, and a bending elastic beam subject to a single point load. The findings in each of these systems is discussed in the following sections.

I. 4.1 RIGID SPHERES AND ELASTIC BEAM FALLING IN VISCOUS FLOW

In this problem, we consider 3 falling metal spheres on an elastic beam that is falling under gravity. The constants are shown in the following table:

TABLE I

VARIABLES INVOLVED IN THE SYSTEM OF 3 METAL SPHERES

Variable	Description	Sample Value
μ	Viscosity of the fluid	1000 Pa-s
ρ_{fluid}	Density of the fluid	1000 kg/m^3
ρ_{metal}	Density of the spheres	7000 kg/m^3
R_1, R_2, R_3	Radius of each sphere	0.005 m, 0.025 m, 0.005 m
l	Length of the beam	0.10 m
r_0	Cross-sectional radius of the beam	0.001 m
E	Young's modulus of the beam	$1.0 \times *10^9$ Pa
EA	Stretching stiffness of the beam	$E\pi r_0^2$
EI	Bending stiffness of the beam	$E\pi r_0^4/4$
Δl	Half-length of the beam	0.05 m
x_1, x_2, x_3	x position of each sphere	initial at 0, Δl, $2\Delta l$
y_1, y_2, y_3	y position of each sphere	all initial at 0
m_1, m_2, m_3	Mass of each sphere	$\frac{4}{3}\pi R_i^3 \rho_{\text{metal}}$
C_i	Damping coefficient due to fluid	$6\pi\mu R_i$
g	Acceleration due to gravity	9.81 m/s^2
W_i	Weight of each sphere	$\frac{4}{3}\pi R_i^3(\rho_{\text{metal}} - \rho_{\text{fluid}}g)$

It should also be noted that the elastic energy of the beam is given by the following equation:

$$E^{\text{elastic}} = E_1^S + E_2^S + E^b$$

where E_1^S and E_2^S are the stretching energies of the beam on the left and right sides of the beam, respectively, and E^b is the bending energy of the beam. The stretching energy of the beam is given by the following equation:

$$E_1^S = \tfrac{1}{2}EA\left(1 - \frac{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}}{\Delta l}\right)^2 \Delta l$$

$$E_2^S = \tfrac{1}{2}EA\left(1 - \frac{\sqrt{(x_3-x_2)^2+(y_3-y_2)^2}}{\Delta l}\right)^2 \Delta l$$

$$E^b = \tfrac{1}{2}\frac{EI}{\Delta l}\left(2\tan\frac{\theta}{2}\right)^2$$

Using these variables, we may write our equations of motion for each sphere in the system:

(TODO: Write the 6 equations we used)

And the discretized equations of motion for the beam are given by the following equations:

(TODO: write the discretized version of the equations we used)

In the explicit method, I replace the velocity term used in the damping force with the old velocity at t_k. This makes the integration method less coupled and easier to work with.

(TODO: just copy the equations)

In the implicit method, I use the following Jacobian matrix to input into the Newton-Raphson method:

(TODO: insert how we calculate the Jacobian matrix here)

Now we may answer the questions posed in the assignment:

A. Q1: Shape of the structure at t = 0, 0.01, 0.05, 0.10, 1.0, 10.0

B. Q1: Position and Velocity Plot of R_2 as a function of time(TODO: t)

C. Q2: Terminal velocity of the system

(TODO: need to examine the velocity plots of R_1 and R_3)

D. Q3: What happens to the turning angle if all radii are the same?

(TODO: calculate turning angle and look at the shape of the structure)

E. Q4: What happens when we change the time step, specifically for the explicit simulation?

II. 4.2 GENERALIZED CASE OF ELASTIC BEAM FALLING IN VISCOUS FLOW

(TODO Opt: make a explicit version)

In the general case of the elastic beam falling, we have $N = 21$ spheres. We use the same variables as in the previous case, but some variables are calculated

A. Q1: Plot of position and velocity of middle node as a function of time

B. Q2: Final shape of deformed beam

C. Q3: What is the significance of spatial discretization and temporal discretization?

(TODO: Include plots of terminal velocity vs number of nodes and terminal velocity vs time step size)

III. 4.3 ELASTIC BEAM BENDING

A. Q1: Plot maximum vertical displacement of beam as a function of time. Compare it with the theoretical prediction from the Euler beam theory.

(TODO: Plot the max vertical displacement and check if y_{max} is a steady value. Determine if the value is accurate or not.)

B. Q2: What is the benefit of the simulation over the predictions of the Euler Beam Theory?

(TODO: Use simulation on $P = 20000$. Note that the Euler Beam Theory is only accurate for small deflections.)

(TODO opt: make a plot of P vs y_{max} using data from the simulation and the beam theory and find the P where the solutions diverge.)

We suggest that you use a text box to insert a graphic (which is ideally a 300 dpi TIFF or EPS file, with all fonts embedded) because, in an document, this method is somewhat more stable than directly inserting a picture.

Fig. 1.   Inductance of oscillation winding on amorphous magnetic core versus DC bias magnetic field

IV. CONCLUSIONS

A conclusion section is not required. Although a conclusion may review the main points of the paper, do not replicate the abstract as the conclusion. A conclusion might elaborate on the importance of the work or suggest applications and extensions.