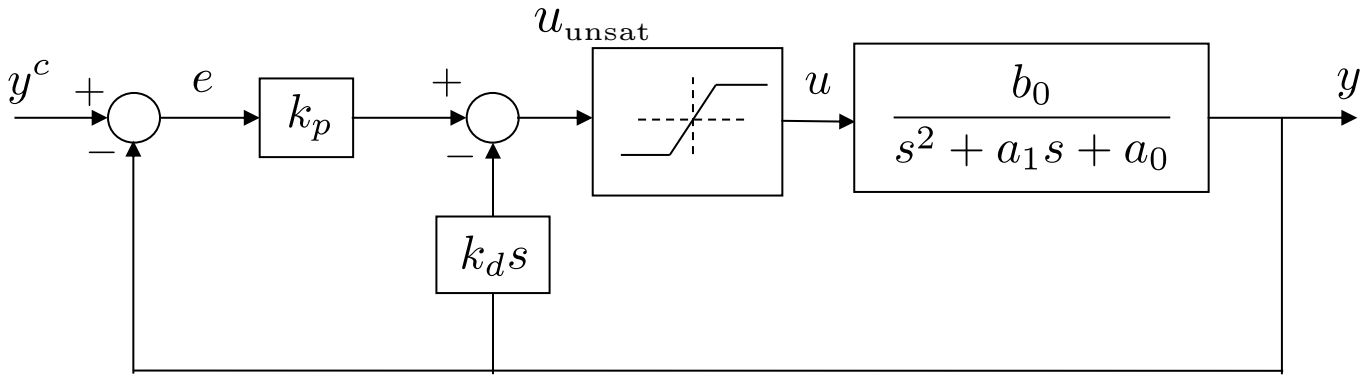
Pole Placement



The control signal u is largest immediately after a step on y_c, at which point the output of the differentiator is essentially zeros. Therefore $u \approx k_p e$. Let $u^{\max}$ be the input saturation limit, and $e^{\max}$, the largest expected step, then set

$$\boxed{k_p = \frac{u^{\max}}{e^{\max}}.}$$

The closed loop transfer function is

$$Y(s) = \frac{b_0 k_p}{s^2 + (a_1 + b_0 k_d)s + (a_0 + b_0 k_p)} Y^c(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} Y^c(s)$$

Equating terms gives

$$\omega_n = \sqrt{a_0 + b_0 k_p} \qquad \boxed{k_d = \frac{2\zeta\omega_n - a_1}{b_0}.}$$