Task 1)

a) N-target associated with m of M-measurements

$m \leq \min(N, M)$

Total number of association events:

$$\sum_{m=0}^{\min(N,M)} \frac{N!\,M!}{m!\,(N-m)!\,(M-m)!}$$

In figure 8.1 $N = 3$, $M = 4$ $\min(N,M) = 3$, so

$$\sum_{m=0}^{3} \frac{3!\,4!}{m!\,(3-m)!\,(4-m)!}$$

$\downarrow$ using matlab

$= \underline{\underline{73}}$ ←

b)

x_1	x_2	x_3
0	0	0
0	0	2
0	1	0
0	1	2
0	2	0
~~0~~	~~2~~	~~2~~
0	4	0
0	4	2

x_1	x_2	x_3
1	0	0
1	0	2
~~1~~	~~1~~	~~0~~
~~1~~	~~1~~	~~2~~
1	2	0
~~1~~	~~2~~	~~2~~
1	4	0
1	4	2

x_1	x_2	x_3
3	0	0
3	0	2
3	1	0
3	1	2
3	2	0
~~3~~	~~2~~	~~2~~
3	4	0
3	4	2

$\underline{\underline{19}} \leftarrow (3 \times 2 \times 4 - 5 = 19)$

$\frac{19}{73} \approx 26\%$ this is very helpful as # of data associations are reduced to about $\frac{1}{5}$ from previously.

c) No misdetection: $\underline{\underline{3}}$ (counted from the table in b))

d)
$$\sum_{n=0}^{3} \frac{2^{4-m}\,3!\,4!}{m!\,(3-m)!\,(4-m)!}$$

$\downarrow$ using matlab

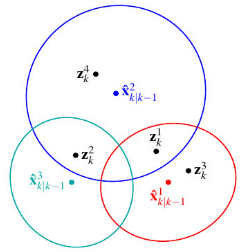
$16 + 96 + 144 + 48 = \underline{\underline{304}}$

$m = \{0 \quad 1 \quad 2 \quad 3\}$

Oh yes! There is [illegible] a lot to save by not considering the "new targets" with no validation gates. 304 is more than 4 times higher than 73 from a) and exponentially higher than the number from b) which is 19.

Task 2)

a)



$$A = \begin{bmatrix} \ln P_D^1 + \ln l^{1,1}/\lambda & \ln P_D^2 + \ln l^{2,1}/\lambda & -\infty \\ -\infty & \ln P_D^2 + \ln l^{2,2}/\lambda & \ln P_D^3 + \ln l^{3,2}/\lambda \\ \ln P_D^1 + \ln l^{1,3}/\lambda & -\infty & -\infty \\ -\infty & \ln P_D^2 + \ln l^{2,4}/\lambda & -\infty \\ \ln(1-P_D^1) & -\infty & -\infty \\ -\infty & \ln(1-P_D^2) & -\infty \\ -\infty & -\infty & \ln(1-P_D^3) \end{bmatrix} = \begin{array}{ccc} x^1 & x^2 & x^3 \\ \left[\begin{array}{ccc} -5.69 & 5.37 & -\infty \\ -\infty & -3.80 & 6.58 \\ 4.78 & -\infty & -\infty \\ -\infty & 5.36 & -\infty \end{array}\right. & & \left.\right\} \text{measurement } 1 \to 4 \\ \left.\begin{array}{ccc} -0.46 & -\infty & -\infty \\ -\infty & -0.52 & -\infty \\ -\infty & -\infty & -0.60 \end{array}\right] & & \left.\right\} \text{dummy measurements} \end{array}$$

Init:

unassigned queue $= \{1, 2, 3\}$, $\varepsilon = 0.01$

prices $= \{1, 2, 3, 4\}$

Loop:

Iteration 1: $t^* = 1$, (queue $= \{2, 3\}$)

best items per customer $= \{3, 1, 2\}$

$i^* \leftarrow 3$

$a(1) \leftarrow 3$

"no customer had $\{3\}$" ?

$y \leftarrow 4.78 - (-0.46) = 5{,}24$

price$\{3\} = 0 + y + \varepsilon = 5.25$

Iteration 2: $t^* \leftarrow 2$, (queue $= \{3\}$)

best item per customer $= \{3, 1, 2\}$

$i^* \leftarrow 1$

$a(2) \leftarrow 1$

"no customer had $\{1\}$"

$y \leftarrow 5.37 - 5.36 = 0.01$

price$\{1\} = 0 + y + \varepsilon = 0.02$

Iteration 3: $t^* \leftarrow 3$, (queue $= \{-\}$)

best item per customer $= \{3, 1, 2\}$

$i^* \leftarrow 2$

$a(3) \leftarrow 2$

"no customer had $\{2\}$"

$y \leftarrow 6.58 - (-0.6) = 6.64$

price$\{2\} = 0 + y + \varepsilon = 6.65.$

No more unassigned customers, terminate!

$\underline{\underline{a(1) = 3,\ a(2) = 1,\ a(3) = 2}}$

b) $A_s = \{3, 1, 2\}$

$\mathcal{L} = A_s$

$R = \emptyset$

Task 3

$\begin{bmatrix}\eta\\ \epsilon\end{bmatrix} \triangleq \eta + \epsilon$ imaginary product $\epsilon_1\epsilon_2 = -\epsilon_1\epsilon_2 + \epsilon_1 \times \epsilon_2$ using this

Commutative

a) $$(\alpha v)^{2n+1} = \underbrace{\alpha^{2n+1}}_{\text{scalar}} (v)^{2n+1} = \alpha^{2n+1}\left(\underbrace{v \cdot v \cdot \ldots v}_{2n+1}\right) = \alpha^{2n+1}\left(\left(-v^Tv + \underbrace{v \times v}_{0}\right) \cdot \underbrace{v \cdot \ldots \cdot v}_{2n-1}\right) = \alpha^{2n+1}\left(\underbrace{(-v^Tv) \cdot \ldots \cdot (-v^Tv)}_{n}\, v\right)$$

$$= \alpha^{2n+1}\left((-v^Tv)^n v\right) = \alpha^{2n+1}(-1)^n v = \underline{\underline{\alpha^{2n+1}(-1)^n v}}$$

$$v^Tv = [a \;\; b \;\; c]\begin{bmatrix}a\\b\\c\end{bmatrix} = a^2 + b^2 + c^2 = 1, \text{ since } |v| = 1$$

$$(\alpha v)^{2n} = \alpha^{2n}(v)^{2n} = \alpha^{2n}\left((-v^Tv)^n\right) \overset{\text{same as above}}{=} \underline{\underline{\alpha^{2n}(-1)^n}}$$

b) $$e^{\alpha v} = \sum_{n=0}^{\infty} \frac{(\alpha v)^n}{n!} = \left[\sum_{n=0}^{\infty} \frac{(\alpha v)^{2n}}{2n!} + \sum_{n=0}^{\infty} \frac{(\alpha v)^{2n+1}}{(2n+1)!}\right] = \left[\sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n}}{(2n)!} + \sum_{n=0}^{\infty} \frac{(-1)^n \alpha^{2n+1}}{(2n+1)!}\right] \overset{\text{definition}}{=} \underline{\underline{\cos(\alpha) + \sin(\alpha)\, v}}$$

proven from previous task 3a)

Theorem 10.1.2:

$$\begin{bmatrix}0\\v'\end{bmatrix} = q \otimes \begin{bmatrix}0\\v\end{bmatrix} \otimes q^* \overset{\text{using (10.21)}}{=} \begin{bmatrix}\cos\frac{\alpha}{2}\\ n\sin\frac{\alpha}{2}\end{bmatrix} \otimes \begin{bmatrix}0\\v\end{bmatrix} \otimes \begin{bmatrix}\cos\frac{\alpha}{2}\\ -n\sin\frac{\alpha}{2}\end{bmatrix}$$

$$= \begin{bmatrix}0 - \sin\frac{\alpha}{2} n^T v\\ 0 + \cos\frac{\alpha}{2} v + n \sin\frac{\alpha}{2} \times v\end{bmatrix} \otimes \begin{bmatrix}\cos\frac{\alpha}{2}\\ -n\sin\frac{\alpha}{2}\end{bmatrix}$$

$$= \begin{bmatrix}\sin\frac{\alpha}{2}\cos\frac{\alpha}{2} n^T v + \left(\cos\frac{\alpha}{2} v + \sin\frac{\alpha}{2} n \times v\right)^T \times n \sin\frac{\alpha}{2}\\ \left(\cos\frac{\alpha}{2}\right)^2 v + \underbrace{\cos\frac{\alpha}{2}\sin\frac{\alpha}{2}}_{\frac{1}{2}\sin\alpha} n \times v + \left(\sin\frac{\alpha}{2}\right)^2 \underbrace{n^T v\, n}_{=v} - \underbrace{\left(\cos\frac{\alpha}{2} v + \sin\frac{\alpha}{2} n \times v\right) \times n \sin\frac{\alpha}{2}}_{\frac{1}{2}\sin\alpha}\end{bmatrix}$$

only interested imaginary part:

$$v' = \underbrace{\left(\left(\cos\frac{\alpha}{2}\right)^2 + \left(\sin\frac{\alpha}{2}\right)^2\right)}_{=1} v + \frac{1}{2}\sin\alpha\, n \times v - \frac{1}{2}\sin\alpha\, n \times v \times n \sin\frac{\alpha}{2}$$

(10.33) $$v' = (1 - \cos\alpha)(v^T n) n + \cos\alpha\, v + \sin\alpha\,(n \times v)$$

$$= (v^T n) n - (v^T n) n \cos\alpha + v\cos\alpha + (n \times v)\sin\alpha$$

$$= (v^T n) n - (v^T n) n \cos\alpha + v\cos\alpha + \underbrace{|n||v|}_{1 \cdot 1} \sin\alpha\, \angle(n, v)$$

$1 - \cos(\alpha) = 2\sin^2\left(\frac{\alpha}{2}\right)$

$$v = e^{\alpha v} = \cos(\alpha) + \sin(\alpha) v$$

Task 4 Quaternion kinematics

$\dot{q} = \frac{1}{2} q \omega$

$\uparrow$ half rotation

$$q(t+\Delta t) = q(t)\, \Delta q(t, t+\Delta t)$$

$$\Delta q(t, t+\Delta t) = e^{\frac{\Delta\theta(t, t+\Delta t)}{2}}$$

$$\Delta\theta(t, t+\Delta t) = \Delta\alpha(t, t+\Delta t)\, \Delta v(t, t+\Delta t)$$

$\alpha(t, t+\Delta t)$: angle of rotation from $t \to t+\Delta t$

$v(t, t+\Delta t)$: axis of rotation

$$\lim_{\Delta t \to 0} \frac{\Delta\theta(t, t+\Delta t)}{\Delta t} = \omega(t) \,?$$

a) $\omega(t)$: angular velocity given in body frame

b)
$$\dot{q}(t) = \lim_{\Delta t \to 0} \frac{q(t+\Delta t) - q(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{q(t)\, q(t, t+\Delta t) - q(t)}{\Delta t}$$

$$= \lim_{\Delta t \to 0} \frac{q(t)\left(q(t, t+\Delta t) - 1\right)}{\Delta t}$$

$$= q(t) \lim_{\Delta t \to 0} \frac{q(t, t+\Delta t) - 1}{\Delta t}$$

fra hint

$$= q(t) \lim_{\Delta t \to 0} \frac{e^{\frac{\Delta\theta(t, t+\Delta t)}{2}} - 1}{\Delta t}$$

$$= q(t) \lim_{\Delta t \to 0} \frac{\cancel{1} + \frac{\Delta\theta(t+\Delta t)}{2}\left[1 + O(\Delta\theta(t, t+\Delta t))\right] \cancel{-1}}{\Delta t}$$

$\Delta t \to 0$

$$\approx q(t) \lim_{\Delta t \to 0} \frac{\Delta\theta(t+\Delta t)}{\Delta t\, 2}$$

$$= q(t) \frac{1}{2}\omega = \frac{1}{2} q \otimes \begin{bmatrix} 0 \\ \omega \end{bmatrix} = \dot{q} \quad (10.43)$$

Task 5: The error state dynamics

a) $\delta\dot{p} = \dot{p}_t - \dot{p} = \delta v$

Corresponding differential equations are linear for true & nominal state.

Diff. equation for position is $\dot{p} = v$ (10.47), which is linear, $\dot{p}_t = v_t$

Difference between two linear equations linear as well.

b)

$$\dot{v}_t = a_t = R_t(a_m - a_{bt} - a_n) + g_t = R_t(\underbrace{a_m - a_b}_{a_B} \underbrace{- \delta a_b}_{\delta a_B} - a_n) + g$$

$$\dot{v} = R(a_m - a_n) + g = R a_B + g$$

$R_t \approx R(I + S(\delta\theta))$ $\leftarrow$ Approximation 1) from 10.45 , δ-quantities are very small

$\downarrow$

$$\dot{v}_t \approx R(I + S(\delta\theta))(a_B + \delta a_{a_B}) + g = R a_B + R\delta a_B + R S(\delta\theta) a_B + \cancel{R S(\delta\theta)\delta a_B} + g$$

approximation: assume negligible

$$\delta\dot{v} = \dot{v}_t - \dot{v} \approx R(\delta a_B + S(\delta\theta) a_B)$$

$$= R(-\delta a_b - a_n - S(a_B)\,\delta\theta)$$

$$= -R S(a_m - a_b)\delta\theta - R\,\delta a_b - R a_n$$

c)
$$2\delta\dot{q} = \left(\begin{bmatrix} 0 & -\omega_t^T \\ \omega_t & -S(\omega_t) \end{bmatrix} - \begin{bmatrix} 0 & -\omega^T \\ \omega & S(\omega) \end{bmatrix}\right)\delta q \approx \left(\begin{bmatrix} 0 & -(\omega_t - \omega)^T \\ (\omega_t - \omega) & -(S(\omega_t) - S(\omega)) \end{bmatrix}\right)\begin{bmatrix} 1 \\ \frac{\delta\theta}{2} \end{bmatrix} = \left(\begin{bmatrix} 0 & -\delta\omega^T \\ \delta\omega & -S(2\omega + \delta\omega) \end{bmatrix}\right)\begin{bmatrix} 1 \\ \delta\theta/2 \end{bmatrix}$$

$\uparrow$

$\delta q \approx \begin{bmatrix} 1 \\ \frac{\delta\theta}{2} \end{bmatrix}$ (10.41) : approximation