Solutions to Systems of Linear Equations

Lab 01

May 9, 2023

Problem

Consider the system of linear equations given by,

$$5x_1 + 10x_2 + 3x_3 + x_4 = 6.7$$

$$6x_1 + 7x_2 + 20x_3 - x_4 = 5.8$$

$$12x_1 + 2x_2 + 3x_3 - 30x_4 = 4.3$$

$$15x_1 - x_2 + x_3 + x_4 = 2.1$$

Construct a computer program to find the solution to the above system using,

- Jacobi method.
- Gauss-Seidel method.

and hence find the solution in above parts.
Note: You may use python/ Matlab to construct the computer program.

Task 01

We solve,

$$Ax = b \rightarrow x = A^{-1}b.$$

To obtain a unique solution, A should be invertible (non-singular). Check whether $det(A) \neq 0$.

Find

$$\begin{vmatrix} 5 & 10 & 3 & 1 \\ 6 & 7 & 20 & -1 \\ 12 & 2 & 3 & -30 \\ 15 & -1 & 1 & 1 \end{vmatrix}$$

Task 02

Solve

$$5x_1 + 10x_2 + 3x_3 + x_4 = 6.7$$

$$6x_1 + 7x_2 + 20x_3 - x_4 = 5.8$$

$$12x_1 + 2x_2 + 3x_3 - 30x_4 = 4.3$$

$$15x_1 - x_2 + x_3 + x_4 = 2.1$$

as it is, using the **Jacobi method**. Do the same using the **Gauss Seidel** method as well. Use 20 iterations as the stopping criteria.
i.e. for Jacobi method use:

$$x_1^{(k+1)} = \frac{6.7 - 10x_2^{(k)} - 3x_3^{(k)} - x_4^{(k)}}{5}$$

$$x_2^{(k+1)} = \frac{5.8 - 6x_1^{(k)} - 20x_3^{(k)} + x_4^{(k)}}{7}$$

$$x_3^{(k+1)} = \frac{4.3 - 12x_1^{(k)} - 2x_2^{(k)} + 30x_4^{(k)}}{3}$$

$$x_4^{(k+1)} = 2.1 - 15x_1^{(k)} + x_2^{(k)} - x_3^{(k)}$$

Task 03

Make the system diagonally dominant and find solutions using both **Jacobi** and **Gauss Seidel** methods. Use both the stopping criteria mentioned below. Choose a suitable initial guess.

Stopping Criteria

- Number of iterations
- $||x^{(k)} - x^{(k-1)}|| <$ Tolerance Value.

Definition

A matrix **A** of dimension $n \times n$ is said to be strictly diagonally dominant provided that,

$$|a_{ii}| > \sum_{j=1,\, j\neq i}^{n} |a_{ij}|,\ \textit{for all } i = 1, \ldots, n.$$

Task 04

For both the algorithms, change the initial guess and check how many iterations it take to reach a given tolerance value. Take the tolerance value 10^{-8}. Plot your results in the same graph.

Instructions on how to change the initial guess: Solve the system using $x = A^{-1}b$ and obtain the actual solutions $\{x_1^*, x_2^*, x_3^*, x_4^*\}$.

Change the initial guess, by scaling the actual solution in powers of 10.
i.e. Use the following as initial guesses

$$\{x_1^*, x_2^*, x_3^*, x_4^*\}$$

$$\{x_1^* \times 10, x_2^* \times 10, x_3^* \times 10, x_4^* \times 10\}$$

$$\{x_1^* \times 10^2, x_2^* \times 10^2, x_3^* \times 10^2, x_4^* \times 10^2\}$$

$$\vdots$$

$$\{x_1^* \times 10^{10}, x_2^* \times 10^{10}, x_3^* \times 10^{10}, x_4^* \times 10^{10}\}.$$

Task 05

Fix the number of iterations (say up to 20) and check for the error in both algorithms. Plot the **error** (e) **vs. number of iterations** (k) for both the algorithms on the same graph.

Calculate the error in each iteration (k) using,

$$e = \sqrt{e_1^{(k)} + e_2^{(k)} + e_3^{(k)} + e_4^{(k)}}$$

where,

$$e_i^{(k)} = (x_i^{(k)} - x_i^*)^2.$$