Boer Mulders Functions

Notes

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Drell-Yan Process

- DY angular distribution;

$$\frac{d\sigma}{d\Omega} \propto 1 + \lambda cos^2\theta + \mu sin2\theta cos\phi + \frac{\nu}{2} sin^2\theta cos2\phi$$

- In "naive" DY model;
  - Transverse momentum of the quark is ignorec
  - No gluon emission is considered.
  - $\lambda = 1$, $\mu = \nu = 0$.
- QCD effects and non-zero intrinsic transverse momentum of the quarks;
  - $\lambda \neq 1$, $\nu, \mu \neq 0$.
  - Satisfy Lam-Tung relation;

$$1 - \lambda = 2\nu$$

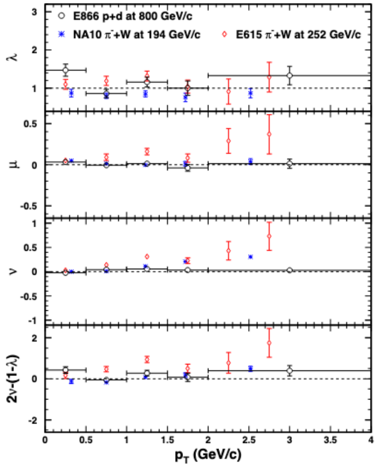

FIG. 1: Parameters λ, μ, ν and $2\nu - (1-\lambda)$ vs. p_T in the Collins-Soper frame. Solid circles are for E866 $p+d$ at 800 GeV/c, crosses are for NA10 $\pi^- + W$ at 194 GeV/c, and diamonds are E615 $\pi^- + W$ at 252 GeV/c. The error bars include the statistical uncertainties only.

Drell-Yan Process

- E866 and E615 results on DY angular distribution strongly suggest that new effects beyond conventional perturbative QCD are present.
- Boer-Mulders function can explain this results;
  - k_T dependent parton distribution function $h_1^\perp$.
  - Characterizes the correlation of a quark's transverse spin and its transverse momentum k_T in an unpolarized nucleon.
  - E866 experiment;
    - 1st report on the azimuthal angular distributions for proton-induced DY.
    - Proton-induced Drell-Yan data provide a stringent test of theoretical models.
    - $\cos 2\phi$ dependence is expected to be much reduced in proton-induced DY if the underlying mechanism involves the Boer-Mulders functions; due to the expectation that the BM functions are small for the sea-quark.
    - However, if the QCD vacuum effect is the origin of the $\cos 2\phi$ angular dependence, then the azimuthal behavior of proton-induced DY should be similar to that of pion-induced DY.
    - 1st measurement of the validity of the LT relation for proton-induced DY.

source : arXiv:hep-ex/0609005

Drell-Yan Process

- For NA10 data, Boer assumed that $h_1^\perp$ proportional to the spin-averaged parton distribution function f_1.

$$h_1^\perp(x, k_T^2) = C_H \frac{\alpha_T}{\pi} \frac{M_C M_H}{k_T^2 + M_C^2} e^{\alpha_T k_T^2} f_1(x)$$

- The $\cos 2\phi$ dependence then results from the convolution of the pion $h_1^\perp/f_1$ term with the nucleon $h_1^\perp/f_1$ term, and the parameter ν;

$$\nu = 16 k_1 \frac{p_T^2 M_C^2}{(p_T^2 + 4M_C^2)^2}$$

Predict large $\cos 2\phi$ dependence originating from QCD vacuum effects, suggest that $h_1^\perp$ for sea quarks are significantly smaller than those for valence quarks.

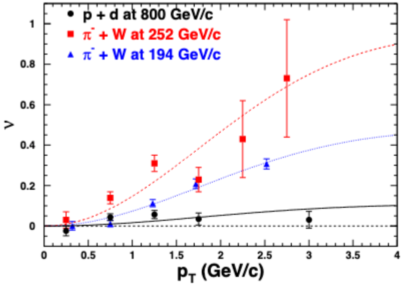

FIG. 2: Parameter ν vs. p_T in the Collins-Soper frame for three Drell-Yan measurements. Fits to the data using Eq. 3 and $M_C = 2.4$ GeV/c^2 are also shown.

Boer-Mulders Function (pD)

- Definition;

$$h_1^{\perp}(x,k_T^2)\epsilon_T^{ij}k_{T_j} = \frac{M}{2} F.T.\, \langle P|\, \bar{\psi}(0)\mathcal{L}_C(0,\epsilon)\gamma^i\gamma^+\gamma_5\psi(\epsilon)\,|P\rangle\,|_{\epsilon^+=0}$$

- Describes the net polarization of quarks inside an unpolarized proton.
- If this function is nonzero, then it reflects the presence of a handedness inside the proton $\vec{P}\cdot(\vec{k}_T\times\vec{s}_T)$.
- For DY process;

$$\nu_{pD} = \frac{2\mathcal{F}[\chi(e_u^2 h_1^{\perp,u} + e_d^2 h_1^{\perp,d})(h_1^{\perp,\bar{u}} + h_1^{\perp,\bar{d}})] + q \leftrightarrow \bar{q}}{\mathcal{F}[(e_u^2 f_1^u + e_d^2 f_1^d)(f_1^{\bar{u}} + f_1^{\bar{d}})] + q \leftrightarrow \bar{q}}$$

Boer-Mulders Function (pD)

- Parametrize $h_1^{\perp}(x, \vec{p}_T^2)$ in the factorized form as;

$$h_1^{\perp,q}(x, \vec{p}_T) = h_1^{\perp,q}(x) \frac{exp(-\vec{p}_T^2)/p_{bm}^2}{\pi p_{bm}^2}$$

- $h_1^{\perp,q}$ can be written as;

$$h_1^{\perp,q}(x) = H_q x^c (1-x) f_1^q(x)$$

- TMD unpolarized distribution function

$$f_1^q(x, \vec{p}_T^2) = f_1^q(x) \frac{exp(-\vec{p}_T^2/p_{unp}^2)}{\pi p_{unp}^2}$$

Boer-Mulders Function (pD)

- Integrating over x_1 and x_2;

$$\nu_{pD}(Q_T) = \frac{p^2_{unp}}{2M^2} \frac{\int dx_1 \int dx_2 [\alpha] Q^2_T exp(-Q^2_T/2p^2_{bm})}{p^2_{bm} \int dx_1 \int dx_2 [\beta] exp(-Q^2_T/2p^2_{unp})}$$

where $x_1 x_2 s = Q^2 + Q^2_T$.

- This expression is fitted to the E866 data.

TABLE I. Best fit values of the Boer-Mulders functions.

H_u	3.99
H_d	3.83
$H_{\bar{u}}$	0.91
$H_{\bar{d}}$	-0.96
p^2_{bm}	0.161
c	0.45
χ^2/d.o.f.	0.79

Boer-Mulders Function (pD)

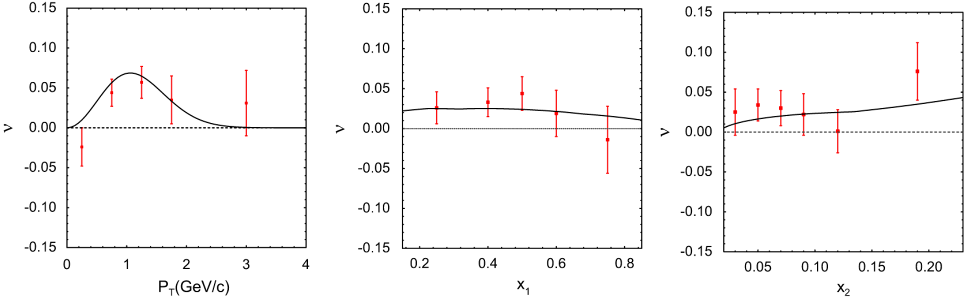

FIG. 1 (color online). Fits to the p_T, x_1, x_2-dependent cos2ϕ asymmetries ν_{pD} for Drell-Yan processes. Data are from the FNAL E866/NuSea collaboration.

Boer-Mulders Function (pD)

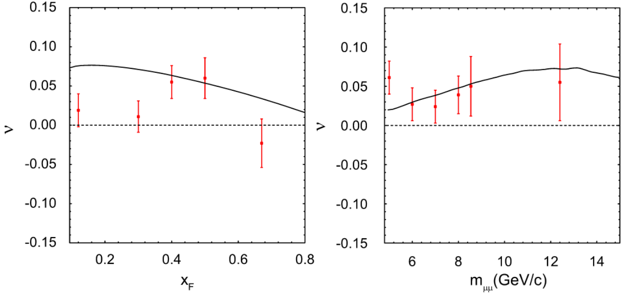

FIG. 3 (color online). The Q-dependent $\cos 2\phi$ asymmetry ν_{pD} for Drell-Yan processes at FNAL E866/NuSea, presenting both the experimental data and the results we estimate with the best fit values of the Boer-Mulders functions in Table I (line).

Boer-Mulders Function (pp)

» We can get similar experssion for pp DY;

$$\nu_{pp}(x_1, x_2, Q_T) = \frac{p_{\rm unp}^2[\kappa] Q_T^2 \exp(-\frac{Q_T^2}{2p_{\rm bm}^2})}{2M^2 p_{\rm bm}^2[\lambda]\exp(-\frac{Q_T^2}{2p_{\rm unp}^2})}, \quad (18)$$

where

$$[\kappa] = x_1^c(1-x_1)x_2^c(1-x_2)(4H_u f_1^u(x_1) H_{\bar{u}} f_1^{\bar{u}}(x_2) + H_d f_1^d(x_1) H_{\bar{d}} f_1^{\bar{d}}(x_2)) + (q \leftrightarrow \bar{q}),$$

$$[\lambda] = (4 f_1^u(x_1) f_1^{\bar{u}}(x_2) + f_1^d(x_1) f_1^{\bar{d}}(x_2)) + (q \leftrightarrow \bar{q}) \quad (19)$$

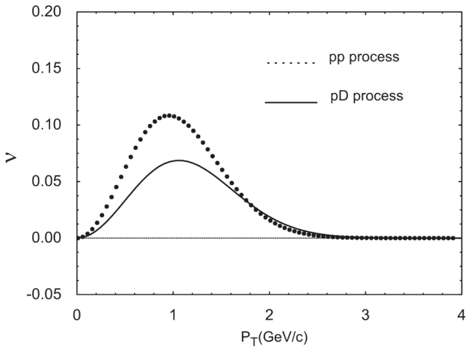

FIG. 4. The p_T-dependent $\cos 2\phi$ asymmetries ν in both pp (dotted curve) and pD (solid curve) Drell-Yan processes at FNAL E866/NuSea, calculated with the fitted Boer-Mulders functions presented in Table I.

Boer-Mulders Function ($p\vec{p}$)

» To seperate H_q (valance quarks) and $H_{\bar{q}}$ (sea quark) we can introduce free coef. ω;

$$h_1^{\perp,u}(x) = \omega H_u x^c(1-x) f_1^u(x), \quad (20)$$

$$h_1^{\perp,d}(x) = \omega H_d x^c(1-x) f_1^d(x), \quad (21)$$

$$h_1^{\perp,\bar{u}}(x) = \frac{1}{\omega} H_{\bar{u}} x^c(1-x) f_1^{\bar{u}}(x), \quad (22)$$

$$h_1^{\perp,\bar{d}}(x) = \frac{1}{\omega} H_{\bar{d}} x^c(1-x) f_1^{\bar{d}}(x). \quad (23)$$

$$\nu_{p\bar{p}}(x_1, x_2, Q_T) = \frac{p_{\text{unp}}^2[\mathrm{s}] Q_T^2 \exp(-\frac{Q_T^2}{2p_{\text{bm}}^2})}{2M^2 p_{\text{bm}}^2 [\tau] \exp(-\frac{Q_T^2}{2p_{\text{unp}}^2})},$$

where

$$[\mathrm{s}] = \omega^2 x_1^c(1-x_1) x_2^c(1-x_2)(4H_u f_1^u(x_1) H_u f_1^u(x_2) + H_d f_1^d(x_1) H_d f_1^d(x_2)) + (q \leftrightarrow \bar{q}),$$

$$[\tau] = (4 f_1^u(x_1) f_1^u(x_2) + f_1^d(x_1) f_1^d(x_2)) + (q \leftrightarrow \bar{q})$$

Boer-Mulders Function ($p\vec{p}$)

source: arXiv:0803.1692 [hep-ph], arXiv:hep-ph/9711485

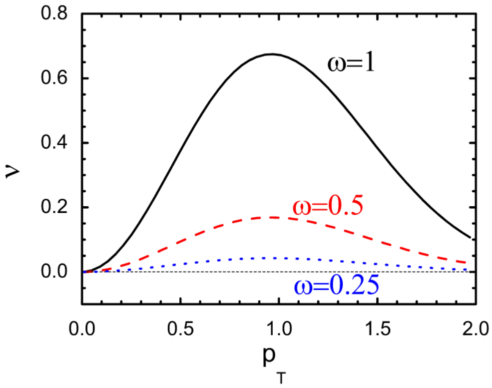

FIG. 6 (color online). The p_T-dependent $\cos 2\phi$ asymmetries ν for $p\bar{p}$ Drell-Yan process at GSI, with the kinematics: c.m. energy $s = 45\ \mathrm{GeV}^2$, and invariance mass square of the lepton pair $Q^2 = 2.5\ \mathrm{GeV}^2$. The solid, dashed, and dotted curves correspond to the free coefficient $\omega = 1, 0.5, 0.25$, respectively.

Pipeline

- Start with MC data;
  - Get ϕ vs. $\cos\theta$ distributions in p_T, x_1, x_2, x_F, $m_{\mu\mu}$ bins.
  - Use unfolding to make the corrections.
  - Fit the ϕ vs. $\cos\theta$ ditributions to extract λ, μ, ν.
  - Use ν values to extract BM function.
  - Cross-check (probably with Kei)
- Real data;
  - Understand the reco. cuts.
  - Background subtraction (Combinatorial Background, e906-root-ana)
  - Use the same steps as in MC data.