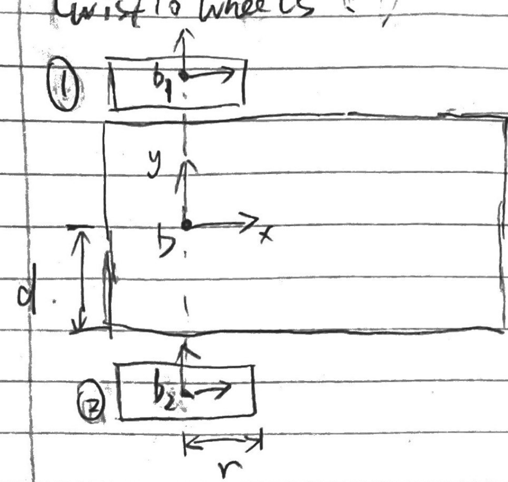
twistToWheels :



$$\begin{array}{c} \qquad\quad \theta \quad\; x \quad\; y \\ T_{1b} = (\;0\;,\;0\;,\;-d\;) \end{array}$$

$$T_{2b} = (\;0,\;0,\;d\;)$$

$$V_1 = \overset{\text{Adjoint}}{A_{1b}} V_b = \begin{bmatrix} 1 & 0 & 0 \\ -d & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} w_z \\ v_x \\ v_y \end{bmatrix}$$

$$= \begin{bmatrix} w_z \\ -dw_z + v_x \\ v_y \end{bmatrix} = \begin{bmatrix} w_z \\ r\dot{\phi}_1 \\ 0 \end{bmatrix}$$

$$\dot{\phi}_1 = \frac{-dw_z + v_x}{r}$$

$$\therefore \begin{bmatrix} \dot{\phi}_1 \\ 0 \end{bmatrix} = \frac{1}{r} \begin{bmatrix} -d & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} w_z \\ v_x \\ v_y \end{bmatrix}$$

$$\Rightarrow \dot{\phi}_1 = \frac{1}{r} \begin{bmatrix} -d & 1 & 0 \end{bmatrix} \begin{bmatrix} w_z \\ v_x \\ v_y \end{bmatrix}$$

Similarly,

$$\Rightarrow \dot{\phi}_2 = \frac{1}{r} \begin{bmatrix} d & 1 & 0 \end{bmatrix} \begin{bmatrix} w_z \\ v_x \\ v_y \end{bmatrix}$$

Eq ①

WheelsToTwist :

$$\dot{\phi} = H V_b \Rightarrow \begin{bmatrix} v_{left} \\ v_{right} \end{bmatrix} = \begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \end{bmatrix} = \frac{1}{r} \begin{bmatrix} -d & 1 & 0 \\ d & 1 & 0 \end{bmatrix} \begin{bmatrix} w_z \\ v_x \\ v_y \end{bmatrix}$$

$$\Rightarrow V_b = \overset{\text{Pseudo-inverse}}{H^{+}} v$$

col-based : $H^{+} = (H^T H)^{-1} H^T$ ✗ → sigular matrix

row-based : $H^{+} = H^T (H H^T)^{-1}$ ✓

$$H = \frac{1}{r}\begin{bmatrix} -d & 1 & 0 \\ d & 1 & 0 \end{bmatrix}$$

$$\therefore H^{+} = H^{T}(HH^{T})^{-1} = r\begin{bmatrix} -\frac{1}{2d} & \frac{1}{2d} \\ \frac{1}{2} & \frac{1}{2} \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} w_z \\ v_x \\ v_y \end{bmatrix} = r\begin{bmatrix} -\frac{1}{2d} & \frac{1}{2d} \\ \frac{1}{2} & \frac{1}{2} \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \end{bmatrix} \longrightarrow \text{Eq } 2$$