Correlations and the covariance matrix

1. Export the data into Matlab using *csvread* and/or *readtable*. This question investigates the correlations of the return rates of the 29 stocks. When analysing return rate data one has several choices. A commonly used variable is the logarithmic change of price or the so called *log return rate*: Let S_{kt} be the price of k-th stock at time t, then consider $Y_{kt} = \log S_{kt} - \log S_{k(t-1)}$ (wrt the natural base).

(i) ~~Calculate the maximal correlation between the $\mathbf{Y}_k$, name and plot the two stock prices associated with the highest correlation as a function of time. On the graph present normalised time series so that they start from the same value 100 on 2/01/2013.~~

(ii) ~~Calculate the minimal correlation between the $\mathbf{Y}_k$, name and plot the two stock prices associated with the smallest correlation as a function of time. On the graph present normalised time series so that they start from the same value 100 on 2/01/2013.~~

(iii) Visualise the correlation matrices for two subperiods: 1/12/2014–1/09/2016 and 1/09/2016–1/02/2018 (you may use Matlab's command *imagesc*). Can you spot differences?
Plot the price of Dow Jones Industrial Average in the whole period. Can you relate it to your observations about the correlation matrices?

(iv) Plot the histogram of the correlation coefficients ρ_{ij} for the two periods from the previous point. Comment on your result.

(v) Find the maximal risk (expressed via standard deviation) of the 29 stocks and the most risky stock corresponding to this value. Calculate the maximal correlation between the most risky stock and one of the remaining stocks, name and plot these two stock prices as a function of time. On the graph present normalised time series so that they start from the same value 100 on 2/01/2013. (Here work with the entire time series.)

(vi) Find the minimal risk (expressed via standard deviation) of the 29 stocks and the least risky stock corresponding to this value. Calculate the maximal correlation between the least risky stock and one of the remaining stocks, name and plot these two stock prices as a function of time. On the graph present normalised time series so that they start from the same value 100 on 2/01/2013. (Here work with the entire time series.)

Portfolio Theory

2. In this section we consider simple return rates, that is $R_{kt} = \frac{S_{kt} - S_{k(t-1)}}{S_{k(t-1)}}$, where S_{kt} is the price of the k-th stock at time t. Carry out the following computational tasks for an unrestricted optimal portfolio P^* consisting of the 29 stocks included in the Dow Jones for an agent who wants to invest \$200,000 and has a risk aversion parameter $t = 0.2$.

(a) Compute the dollar investment in each of the stocks and the corresponding expected return and risk of P^*.

(b) Illustrate the problem graphically and plot on the same graph in the $\mu\sigma$-plane :

(i) The 29 stocks of the Dow Jones.

(ii) The minimum variance and efficient frontiers. Use a t-range $|t| \leq 0.35$ for your display.

(iii) A plot of 1000 random feasible portfolios satisfying $|x_i| \leq 20$ (for each of the 29 stocks) and $\sigma_i \leq 0.05$ for $i = 1, \ldots, 1000$.
You might notice that the random points occupy some region well-separated from the minimum variance frontier (MVF) - comment on this and explain why (This is a/the major part of the question).

(iv) The indifference curve of an investor with $t = 0.2$ and their optimal portfolio P^*.

3. Determine which investors shortsell in the market consisting of the 29 stocks, and which stocks they shortsell. Are there any stocks which no-one will shortsell or which everyone will shortsell?

4. **Three funds with different risk profiles:** In this question you will divide 29 stocks with respect to their risk profile into 3 funds. Sort stocks from highest to lowest risk (expressed via variance or standard deviation). Assuming that each stock has the same contribution to a given fund, form high-risk fund from the 9 most risky stocks, low-risk fund from the 10 least risky stocks and mid-risk fund from the rest.

   (a) Compute expected returns and covariance matrix of the 3 funds.

   (b) Let $\hat{P}$ be an unrestricted optimal portfolio consisting of the 3 funds for an agent who wants to invest \$200,000 and has a risk aversion parameter $t = 0.2$.

      (i) Compute the dollar investment in each of the funds and the corresponding expected return and risk of $\hat{P}$.

      (ii) Plot on the second $\mu\sigma$-plane graph :

      - The 3 funds.
      - The minimum variance and efficient frontiers. Use a t-range $|t| \leq 0.35$ for your display.
      - The indifference curve of an investor with $t = 0.2$ and their optimal portfolio $\hat{P}$.
      - The minimum variance frontier and optimal portfolio P^* from Question 2. Compare solutions P^* and $\hat{P}$ to the two problems based on computations and graphs.

Capital Asset Pricing Model

5. Assume that the daily risk free rate in the studied period was 0.002906%. Suppose that Standard & Poor's 500 Index is the market portfolio (S&P 500 Index prices are included in the data file). Make a new $\mu\beta$-plane graph showing the risk free asset, market portfolio, and the Security Market Line. Compute the β's of all relevant assets in this project (29 stocks, 3 funds, and two optimal portfolio P^* and $\hat{P}$ from Questions 2 and 4). Plot these assets on the same graph. Identify assets with β's greater than 1 and lower than 1. Comment on the result and describe what Portfolio Theory would recommend an investor to do.