d) $(p_2 \longrightarrow p_5) \overset{?}{\in} \Gamma$ $\qquad\qquad$ $\{p_0, \neg(p_1 \longrightarrow p_2), p_3 \vee p_2\} \subseteq \Gamma$

Probemos que

$\langle \exists \mathcal{D} : \mathcal{D} \in \mathscr{D} : \text{Hip}(\mathcal{D}) \subseteq \Gamma \;\&\; \text{concl}(\mathcal{D}) = p_2 \longrightarrow p_5 \rangle$

$$\mathcal{D} := \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\neg(p_1 \longrightarrow p_2) \qquad \dfrac{[p_2]_2}{p_1 \longrightarrow p_2} \to I_3}{\bot} \to E}{\neg p_2} \to I_2 \qquad [p_2]_1}{\bot} \to E}{p_5} \bot}{p_2 \longrightarrow p_5} \to I_1$$

Luego $\mathcal{D}$ atestigua $\Gamma \vdash (p_2 \longrightarrow p_5)$

Lema 32

$\Gamma \vdash (p_2 \longrightarrow p_5) \Longrightarrow (p_2 \longrightarrow p_5) \in \Gamma$