b) Si $\varphi \vdash \psi$ & $\neg\varphi \vdash \psi \implies \vdash \psi$

$\varphi \vdash \psi$ & $\neg\varphi \vdash \psi$

$\equiv$ {Def def de $\vdash$}

$\langle \exists \mathcal{D}' \in \mathbb{D} :: Hip(\mathcal{D}') \subseteq \{\varphi\}$ & $concl(\mathcal{D}') = \psi \rangle$

& $\langle \exists \mathcal{D}'' \in \mathbb{D} :: Hip(\mathcal{D}'') \subseteq \{\neg\varphi\}$ & $concl(\mathcal{D}'') = \psi \rangle$

Sea $\mathcal{D} \in \mathbb{D}$ tal que

$$\mathcal{D} := \dfrac{\begin{pmatrix} \vdots\ \mathcal{D}''' \\ \varphi \vee \neg\varphi \end{pmatrix} \quad \begin{pmatrix} [\varphi]_3 \\ \vdots\ \mathcal{D}' \\ \psi \end{pmatrix} \quad \begin{pmatrix} [\neg\varphi]_4 \\ \vdots\ \mathcal{D}'' \\ \psi \end{pmatrix}}{\psi}\ \vee E_{3,4}$$

Donde

$Hip(\mathcal{D}''') = \emptyset$

$$\mathcal{D}''' := \dfrac{\dfrac{\dfrac{\dfrac{[\varphi]_2}{\varphi \vee \neg\varphi}\ \vee I}{[\neg(\varphi \vee \neg\varphi)]_1 \quad \neg\varphi \vee [\varphi]_2}\ \to E}{\bot}\ \to I_2}{\dfrac{\dfrac{\neg\varphi}{\varphi \vee \neg\varphi}\ \vee I}{[\neg(\varphi \vee \neg\varphi)]_1 \quad \varphi \vee \neg\varphi}\ \to E}{\bot}\ RAA_1$$

Luego $concl(\mathcal{D}) = \psi$

$Hip(\mathcal{D})$

= {Def 22 con respecto a $(\vee E)$}

$Hip(\mathcal{D}''') \cup (Hip(\mathcal{D}') \setminus \{\varphi\}) \cup (Hip(\mathcal{D}'') \setminus \{\neg\varphi\})$

= {Def de $Hip(\mathcal{D}''')$; Por Hipotesis}

$\emptyset \cup (\emptyset \setminus \{\varphi\}) \cup (\emptyset \setminus \{\neg\varphi\})$

= {Def de unión y diferencia entre conjuntos}

$\emptyset$

= {Def $\vdash$}

$\vdash \psi$ Por lo tanto $\varphi \vdash \psi$ & $\neg\varphi \vdash \psi \implies \vdash \psi$