c) $p_3 \overset{?}{\in} \Gamma$ $\qquad \{p_0, \neg(p_1 \longrightarrow p_2), p_3 \vee p_2\} \subseteq \Gamma$

Probemos que

$\langle \exists \mathcal{D} : \mathcal{D} \in \mathscr{D} : \mathrm{Hip}(\mathcal{D}) \subseteq \Gamma \ \& \ \mathrm{concl}(\mathcal{D}) = p_3 \rangle$ sii $\mathcal{D}$ atestigua $\Gamma \vdash p_3$

Antes probemos que

$\langle \exists \mathcal{D}' : \mathcal{D}' \in \mathscr{D} : \mathrm{Hip}(\mathcal{D}') \subseteq \{\neg(p_1 \longrightarrow p_2)\} \ \& \ \mathrm{concl}(\mathcal{D}') = p_1 \wedge \neg p_2 \rangle$

$$\mathcal{D}' := \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[\neg p_1]_1 \quad [p_1]_2}{\bot}\,{\to E}}{p_2}\,\bot}{p_1 \longrightarrow p_2}\,{\to I_2} \quad \neg(p_1 \longrightarrow p_2)}{\bot}\,{\to E}}{p_1}\,RRA_1 \qquad \dfrac{\dfrac{\neg(p_1 \longrightarrow p_2) \quad \dfrac{[p_2]_3}{p_1 \longrightarrow p_2}\,{\to I_4}}{\bot}\,{\to E}}{\neg p_2}\,{\to I_3}}{p_1 \wedge \neg p_2}\,\wedge_I$$

Luego $\mathcal{D}'$ atestigua $\{\neg(p_1 \longrightarrow p_2)\} \vdash p_1 \wedge \neg p_2$

Por otro lado

$\mathcal{D}$ atestigua $\Gamma \vdash p_3$

$$\mathcal{D} := \dfrac{\dfrac{p_3 \vee p_2 \quad \dfrac{[\neg p_3]_1 \quad [p_3]_2}{\bot}\,{\to E} \quad \dfrac{\dfrac{\begin{matrix}\vdots\,\mathcal{D}' \\ p_1 \wedge \neg p_2\end{matrix}}{\neg p_2}\,\wedge E \quad [p_2]_3}{\bot}\,{\to E}}{\bot}\,\vee E_{2,3}}{p_3}\,RRA_1$$

Lema 32

$\Gamma \vdash p_3 \implies p_3 \in \Gamma$