d) $(p_2 \longrightarrow p_5) \overset{?}{\in} \Gamma$ $\qquad\qquad$ $\{p_0, \neg(p_1 \longrightarrow p_2), p_3 \vee p_2\} \subseteq \Gamma$

Probemos que

$\langle \exists D : D \in \mathscr{D} : Hip(D) \subseteq \Gamma \ \& \ concl(D) = p_2 \longrightarrow p_5 \rangle$

Antes probemos que

$\langle \exists D' : D' \in \mathscr{D} : Hip(D') \subseteq \{\neg(p_1 \longrightarrow p_2)\} \ \& \ concl(D') = p_1 \wedge \neg p_2 \rangle$

$$D' := \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[\neg p_1]_1 \quad [p_1]_2}{\perp}\rightarrow E}{p_2}\perp}{p_1 \longrightarrow p_2}\rightarrow I_2 \quad \neg(p_1 \longrightarrow p_2)}{\perp}\rightarrow E}{p_1}RRA_1 \qquad \dfrac{\dfrac{\neg(p_1 \longrightarrow p_2) \quad \dfrac{[p_2]_3}{p_1 \longrightarrow p_2}\rightarrow I_4}{\perp}\rightarrow E}{\neg p_2}\rightarrow I_3}{p_1 \wedge \neg p_2}\wedge I$$

Luego D' atestigua $\{\neg(p_1 \longrightarrow p_2)\} \vdash p_1 \wedge \neg p_2$

Por otro lado

D atestigua $\Gamma \vdash (p_2 \longrightarrow p_5)$

$$D := \dfrac{\dfrac{\dfrac{\dfrac{\begin{matrix} \vdots D' \\ p_1 \wedge \neg p_2 \end{matrix}}{\neg p_2}\wedge E \quad [p_2]_1}{\perp}\rightarrow E}{p_5}\perp}{p_2 \longrightarrow p_5}\rightarrow I_1$$

Lema 32

$\Gamma \vdash (p_2 \longrightarrow p_5) \Longrightarrow (p_2 \longrightarrow p_5) \in \Gamma$