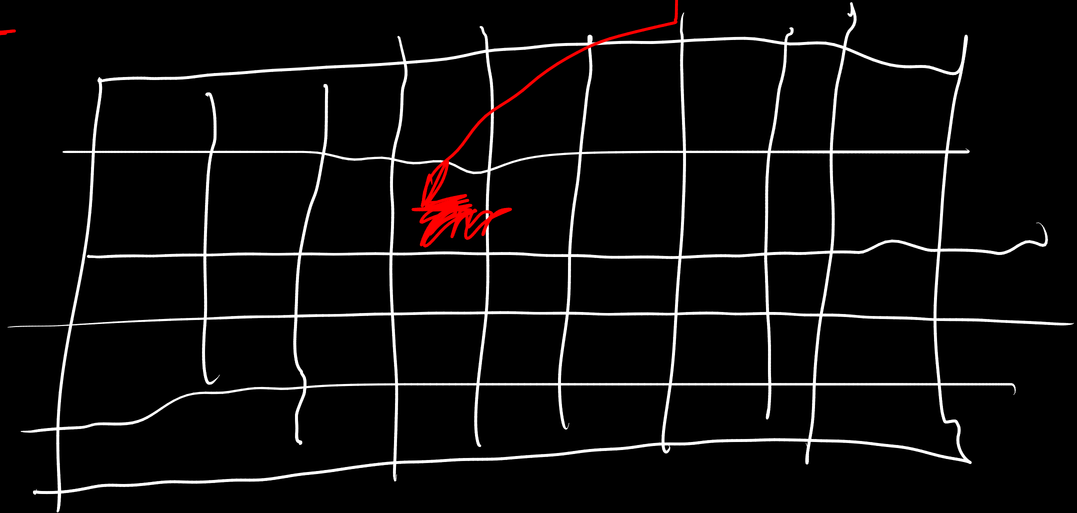
Electromagnetic Field

class →



Numeric Values of Potentials

Crashcourse on Potentials (2+1)D

2 Spatial
1 time

$\vec{E} \sim (E_x, E_y)$ $D-1$

$\vec{B} \sim B_z$ $\frac{(D-1)(D-2)}{2}$

Potentials $\rightarrow D$ ✓

4D (Real World)

$(\underline{\phi}, \vec{A})$

Scalar Potential

$$\vec{E} = -\vec{\nabla}\phi - \frac{\partial \vec{A}}{\partial t}$$

Arbitrary dimensions

$$\vec{B} = \vec{\nabla} \times \vec{A}$$ → 3D

$$B^i = \underbrace{\epsilon^{ijk}}_{3D} \partial_j A_k$$

$$\underline{E} = -\underline{\nabla\phi} - \frac{\partial A}{\partial t}$$

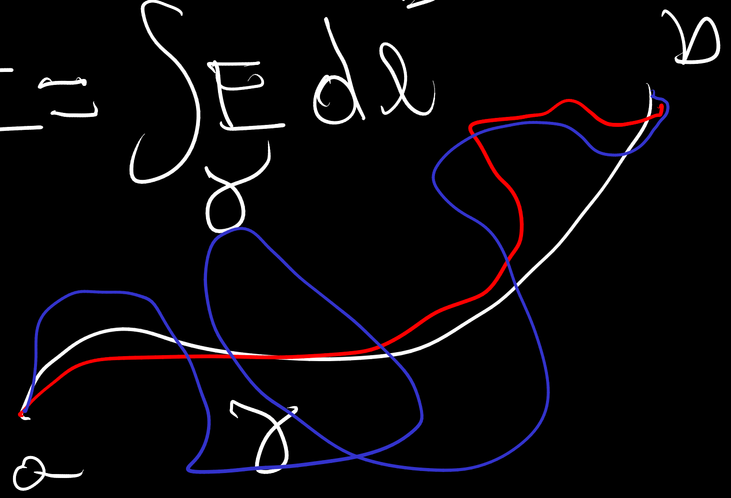
Under the assumption that A doesn't change with time

A $\overset{E}{\checkmark}$ field is conservative

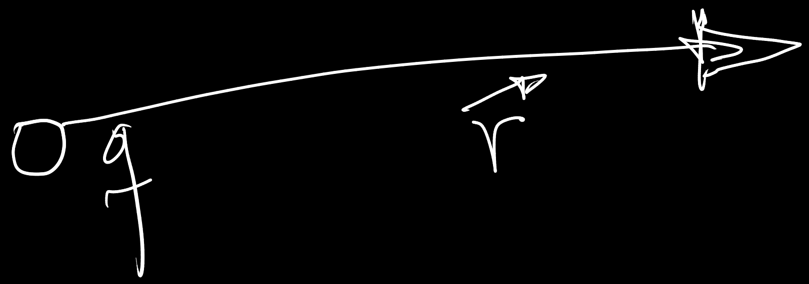
iff $E = \nabla V \rightarrow$

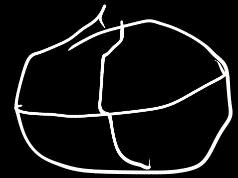
$\int_\gamma E\, dl$ only depends on the endpoints

$$I = \int_\gamma E\, d\vec{l}$$



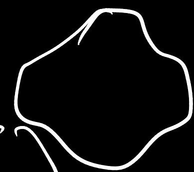
$$\oint E\, dl = 0$$



 $\sim 4\pi$

D-dimensions

$$\phi = \frac{q}{S_{D-2}\epsilon_0} \begin{cases} r^{3-D} & D \neq 3 \\ \ln(r) & D = 3 \end{cases}$$



4D $$\boxed{\phi = \frac{q}{4\pi r \epsilon_0}}$$

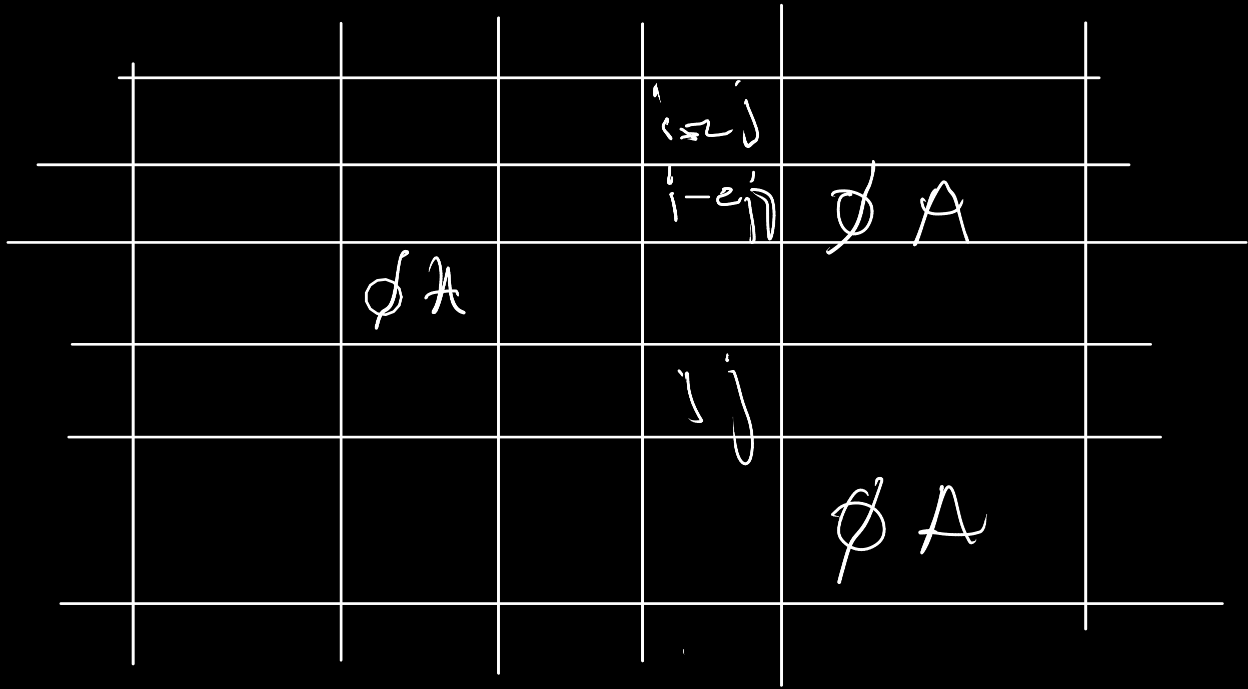
3D $$\phi = \frac{q \ln(r)}{2\pi \epsilon_0}$$

$$A = \frac{\mu_0 q \vec{v}}{4\pi r}$$

$$\boxed{\vec{A} \simeq q\vec{v}\,\phi\,\epsilon_0 \mu_0}$$

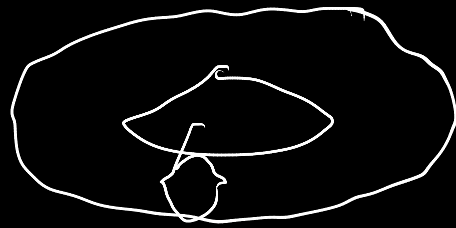
Approximation $v^{2} << \mu_0 \epsilon_0$

$\mu_0 \epsilon_0 \rightarrow 1/c^2$ c is the speed of light



Array

Toro



Maxwell's Equations

$$E = -\nabla\phi - \frac{\partial A}{\partial t} \qquad B = \nabla \times A$$

$$\nabla \times B = \mu_0 \epsilon_0 \frac{\partial E}{\partial t} \qquad \nabla E = 0$$

$$A^\mu = (\phi, \vec{A}) \qquad \mu = 0, 1, 2, 3$$

$$\frac{\partial^2 A^\mu}{\partial t^2} = \frac{\mu_0 \epsilon_0 \nabla^2 A^\mu}{1} \qquad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \cdots$$

Wave Equation

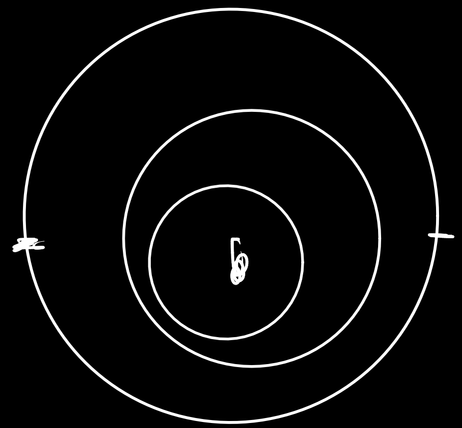
A behaves like a wave

Taylor DPotential

$$\frac{\partial A}{\partial t}(t+e) \simeq \frac{\partial A}{\partial t}\Big|t) + \frac{\partial^2 A}{\partial t^2} e + \dots$$

$$D\,t = \nabla^2 A \quad A$$

→ Energy stored



$$U = \int dV \left(E^2 + B^2\right)$$

$$\partial^{\mu} A^{\nu} = \begin{cases} \mu = 0 \longrightarrow \partial_t \ \mathrm{DPot} \\ \mu \neq 0 \sim \nabla \end{cases}$$

rval = DPotential[ν][x][y]; $\mu = 0$

rval = Potential[ν][ ][ ] $\mu = 1$

$\mu = 2$

Logic :

switch(ll) :

case 0 :

rval = DPot...

break;

case 1 :

nal = $\frac{1}{2}$ (Potential[v][i+1][v] + Poten [v][i-1]

All fields can be stored in an object $F_{\mu\nu}$

$$\boxed{F_{\mu\nu}} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$F_{\mu\nu} \overset{4D}{=} \begin{pmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{pmatrix}$$

$$F_{\mu\nu} = \begin{pmatrix} 0 & E_x & E_y \\ -E_x & 0 & B \\ -E_y & -B & 0 \end{pmatrix} \quad U = \int_{Grid} \left(E_x^2 + E_y^2 + B^2\right)/2$$

4

$$E_x = F_{01} \qquad E_y = F_{02}$$

$$B = F_{12}$$