Circuit Theory and Electronics Fundamentals

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RC circuit

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Contents

1 Introduction

The objective of this laboratory assignment is to study a RC circuit containing a AC voltage source V_s, a capacitor C, a voltage controlled current source I_b, a current controlled voltage source V_d and resistors, R_1, R_2, R_3, R_4, R_5, R_6 and R_7. The circuit can be seen in Figure 1.

In Section 2, a theoretical analysis of the circuit is presented. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

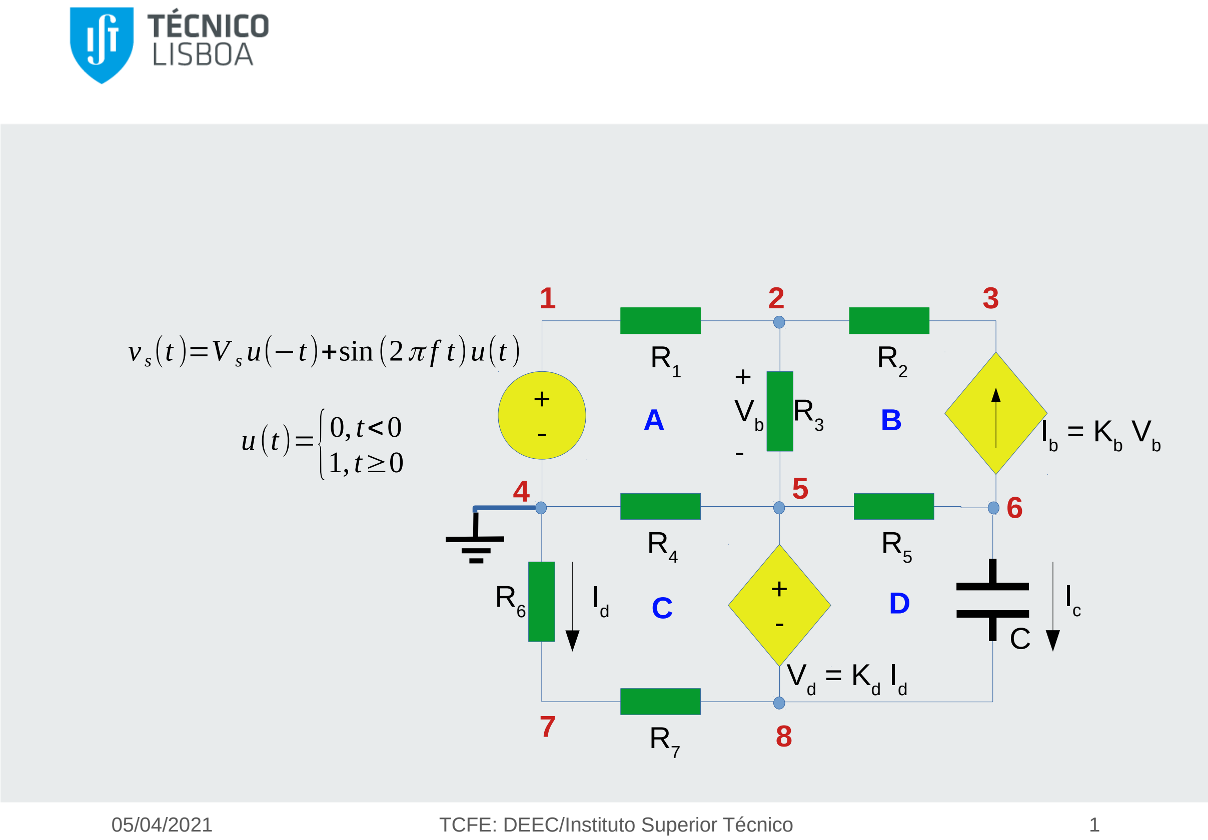

Figure 1: RC circuit to be analysed

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically, analysing the circuit for $t < 0$, calculating the equivalent resistance, determining the natural and forced solutions and superimposing them to find the total solution.

2.1 Nodal analysis

For t<0, $v_s(t) = V_s(t)$, it is a DC circuit. We can determine the voltges in all nodes and currents in all branches using the nodal method. Since this is a linear circuit, we apply Ohm's Law, $V_i = R_i * I$ and the Kirchoff Current Law (KCL), $\sum I_i = 0$.

We get the following equation, in matrix form:

$$\begin{bmatrix} -G_1 & G_1+G_2+G_3 & -G_2 & 0 & -G_3 & 0 & 0 & 0 \\ 0 & -G_2-K_b & G_2 & 0 & K_b & 0 & 0 & 0 \\ 0 & K_b & 0 & 0 & -G_5-K_b & G_5 & 0 & 0 \\ 0 & 0 & 0 & -G_6 & 0 & 0 & G_6+G_7 & -G_7 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -K_c*G_6 & 1 & 0 & K_c*G_6 & -1 \\ 0 & -G_3 & 0 & -G_4 & G_4+G_3+G_5 & -G_5 & -G_7 & G_7 \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ V_s \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

This equation solved using octave yields the following results:

Variable	Value [A or V]
V_1	$5.13612248730V$
V_2	$4.88464690881V$
V_3	$4.36195145882V$
V_4	$-0.00000000000V$
V_5	$4.92000960269V$
V_6	$5.69027079572V$
V_7	$-1.96654083449V$
V_8	$-2.94453891610V$
I_1	$0.00024147744A$
I_2	$0.00025321233A$
I_3	$-0.00001173489A$
I_4	$-0.00120106056A$
I_5	$-0.00025321233A$
I_6	$0.00095958312A$
I_7	$0.00095958312A$
I_S	$-0.00024147744A$
I_b	$-0.00025321233A$
I_c	$-0.00000000000A$
I_e	$-0.00095958312A$

Table 1: Node Analysis Results for t<0

2.2 Equivalent resistance

Now, we have to determine the equivalent resistance R_eq as seen from the capacitor terminals. We take out all the independent voltage sources (make $V_s = 0$) and replace the capacitor with a voltage source $Vx = V(6) - V(8)$. The values of$V(6)$ and $V(8)$ were already obtained via nodal analysis in the previous subsection. To determine the current I_x supplied by V_x we run mesh analysis:

$$\begin{bmatrix} R1+R3+R4 & -R3 & -R4 & 0 \\ -Kb*R3 & Kb*R3-1 & 0 & 0 \\ -R4 & 0 & R4+R6+R7-Kd & 0 \\ 0 & -R5 & Kd & R5 \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \\ I_D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ V_x \end{bmatrix} \tag{2}$$

This yields the following results:

Variable	Value [A or V]
V_x	$8.63480971182V$
I_x	$0.00283856995A$
R_{equiv}	3041.95770117000Ω

Table 2: Equivalent resistance

$I_D = I_x =$
$R_{eq} = \frac{V_x}{I_x} =$
This value is equal to R_5, which makes sense: since the current controled voltage source V_d has null internal resistance, all the current flows through mesh D (which only contains V_d andR_5).

For the time constant:

$\tau = R_{eq} \cdot C =$

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2.3 Natural solution

Using the capacitor voltage V_x for $t < 0$ as the initial condition, the natural solution of $v_{6n}(t)$ becomes:

$$v_{6n}(t) = V_x e^{\frac{-x}{1000 R_5 C}} \tag{3}$$

This equation gives us the following plot in [0,20]ms:

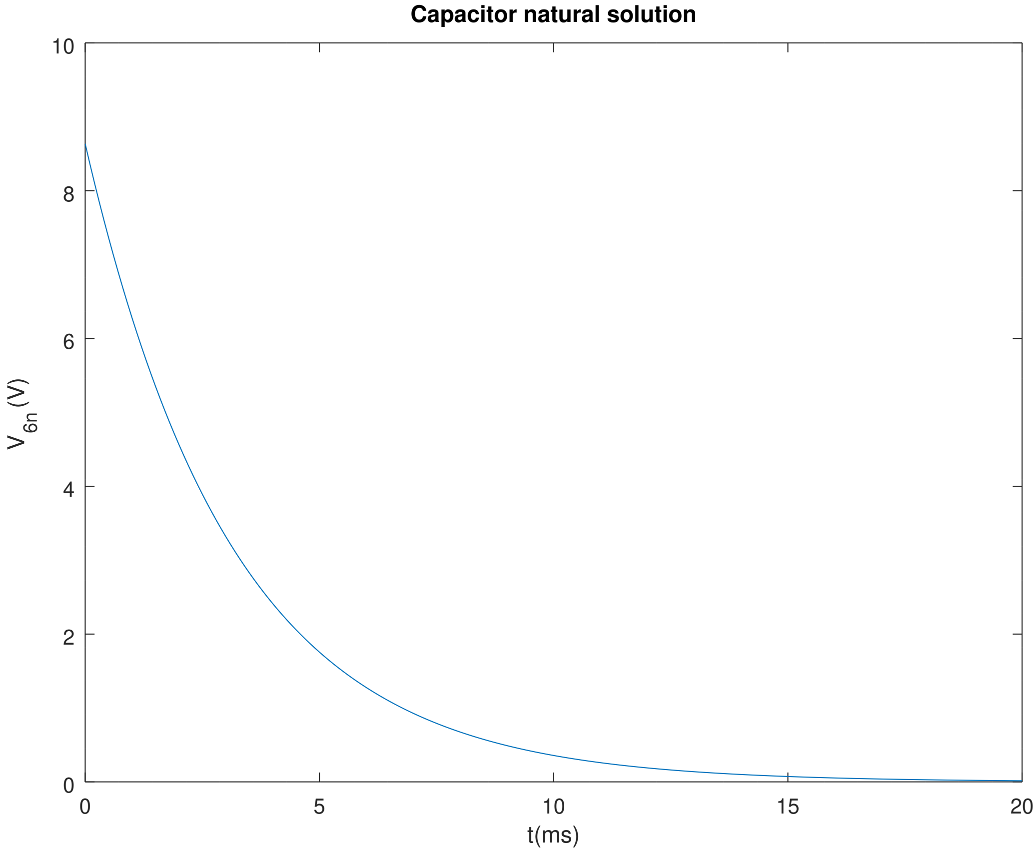

Figure 2: Natural solution $v_{6n}(t)$

2.4 Forced solution

To determine the forced solution in the same interval [0, 20]ms we use a phasor voltage source V_s and replace C with its impedance Z_c.

We run nodal analysis to determine the phasor voltages in all nodes:

$\omega = 2 * \pi * 1000$

$$\begin{bmatrix} -G1 & G1+G2+G3 & -G2 & 0 & -G3 & 0 & 0 & 0 \\ 0 & -G2-Kb & G2 & 0 & Kb & 0 & 0 & 0 \\ 0 & Kb & 0 & 0 & -G5-Kb & G5+(C*\omega*i) & 0 & -(C*\omega*i) \\ 0 & 0 & 0 & -G6 & 0 & 0 & G6+G7 & -G7 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -Kd*G6 & 1 & 0 & Kd*G6 & -1 \\ 0 & -G3 & 0 & -G4 & G4+G3+G5 & -G5-(C*\omega*i) & -G7 & G7+(C*\omega*i) \end{bmatrix} . \quad (4)$$

Solving this system of equation in octave yiels the following results:

Variable	Value [A or V]
$\tilde{V_1}$	$(0.00000000000 + i \cdot 1.00000000000)V$
$\tilde{V_2}$	$(0.00000000000 + i \cdot 0.95103785412)V$
$\tilde{V_3}$	$(0.00000000000 + i \cdot 0.84926936022)V$
$\tilde{V_4}$	$(0.00000000000 + i \cdot 0.00000000000)V$
$\tilde{V_5}$	$(0.00000000000 + i \cdot 0.95792294963)V$
$\tilde{V_6}$	$(0.08501303490 + i \cdot -0.56899006711)V$
$\tilde{V_7}$	$(-0.00000000000 + i \cdot -0.38288433334)V$
$\tilde{V_8}$	$(-0.00000000000 + i \cdot -0.57329997939)V$

Table 3: Phasor voltages

2.5 Total solution

Converting the phasors to real time functions for f=1KHz, we can then superimpose the natural and forced solutions:

$y_1 = \Re(V_f(6) * e^{x*\omega*i/1000)} + V_x * exp(-x/1000/R5/C)$

$y_2 = \sin(\omega * x2/1000)$

This plot in the interval [-5,20]ms:

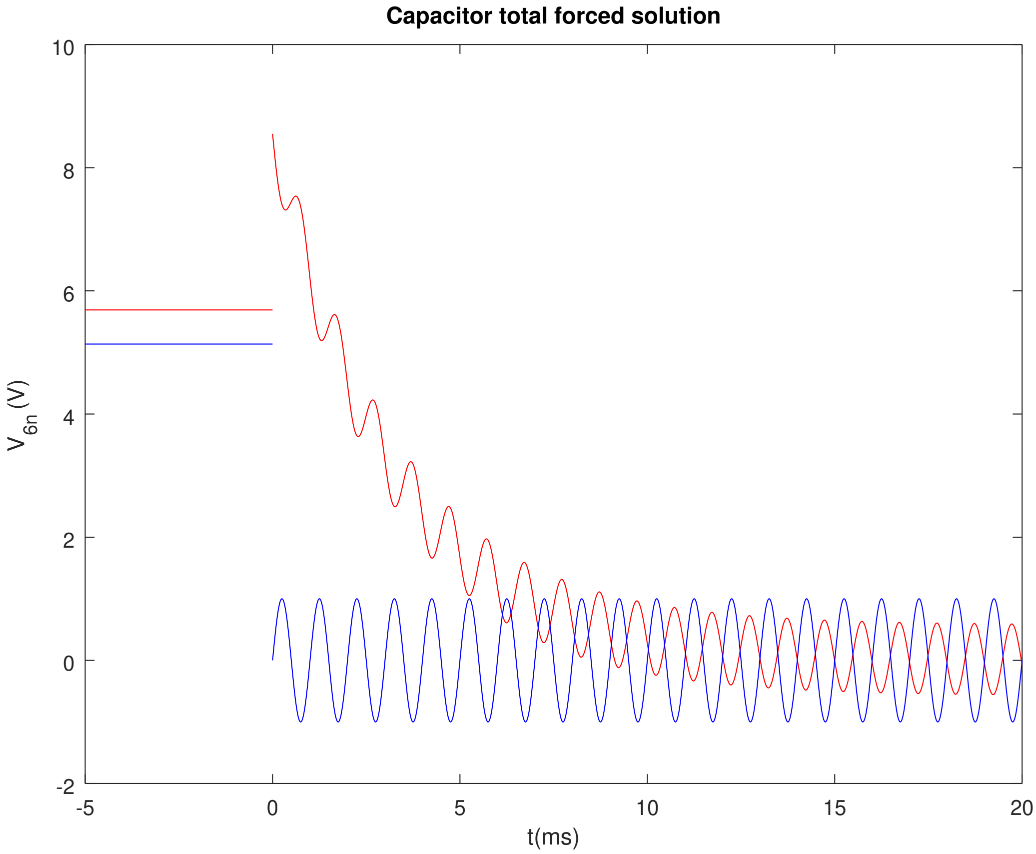

Figure 3: Final total solution $v_6(t)$

2.6 Frequency responses

For the frequency responses, we get the following plot:

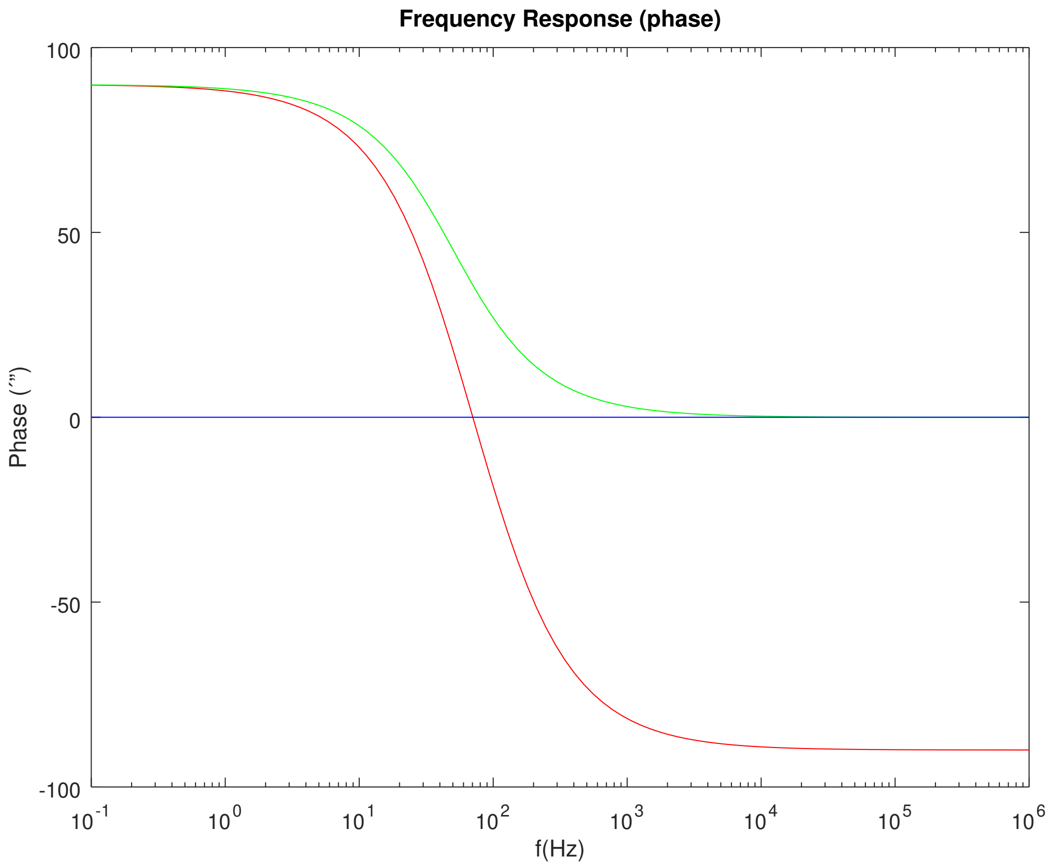

Figure 4: Frequency response-phase

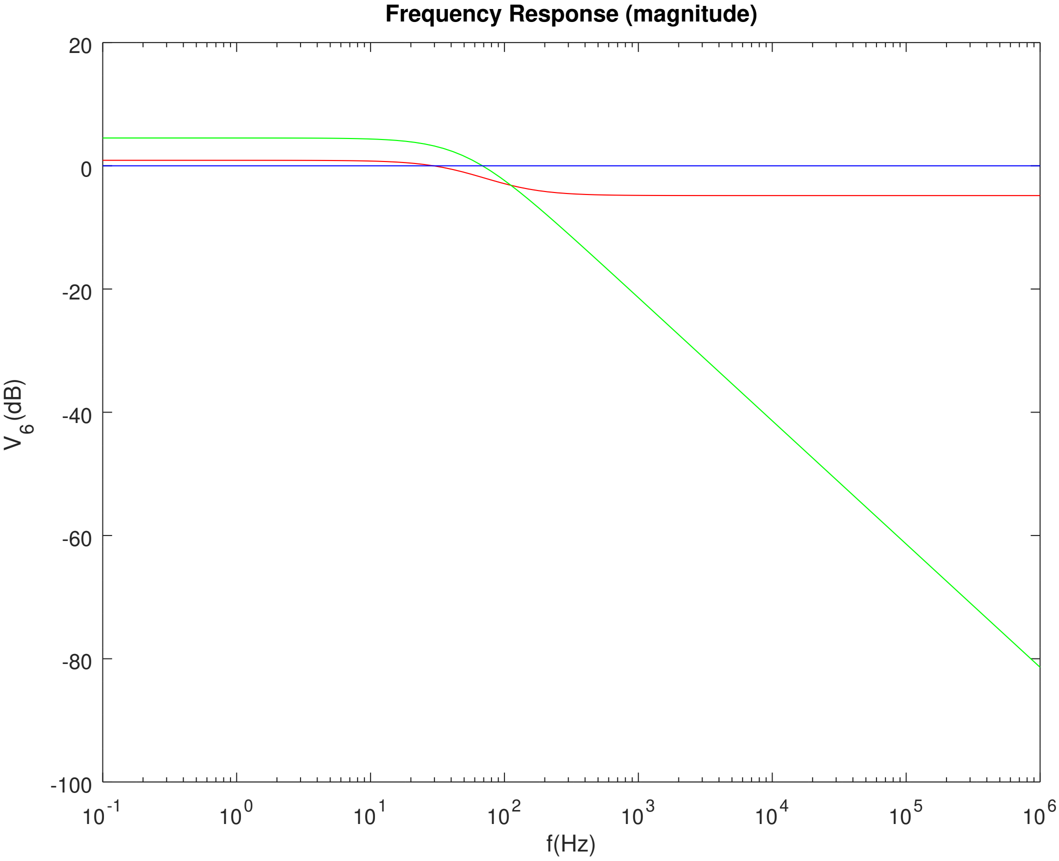

Figure 5: Frequency response-magnitude

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3 Simulation Analysis

3.1 Operating Point Analysis

Table 4 shows the simulated operating point results for the circuit at times $t < 0$.

Name	Value [A or V]
@ca[i]	0.000000e+00
@gb[i]	-2.53212e-04
@r1[i]	2.414774e-04
@r2[i]	2.532123e-04
@r3[i]	-1.17349e-05
@r4[i]	-1.20106e-03
@r5[i]	-2.53212e-04
@r6[i]	9.595831e-04
@r7[i]	9.595831e-04
v(1)	5.136122e+00
v(2)	4.884647e+00
v(3)	4.361951e+00
v(5)	4.920010e+00
v(6)	5.690271e+00
v(8)	-2.94454e+00
v(71)	-1.96654e+00
v(72)	-1.96654e+00

Table 4: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

Table 5 shows the simulated operating point results for the circuit given that $V_s = 0$ and replacing the capacitor with a voltage source imposing the voltage on the terminals of said capacitor as calculated in the earlier analysis.

Name	Value [A or V]
@gb[i]	-6.24390e-18
@r1[i]	5.954528e-18
@r2[i]	6.243896e-18
@r3[i]	-2.89368e-19
@r4[i]	1.300919e-18
@r5[i]	-2.83857e-03
@r6[i]	-8.67362e-19
@r7[i]	1.165891e-21
v(1)	0.000000e+00
v(2)	-6.20107e-15
v(3)	-1.90901e-14
v(5)	-5.32907e-15
v(6)	8.634810e+00
v(8)	1.776357e-15
v(71)	1.777545e-15
v(72)	1.777545e-15

Table 5: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

This is necessary so as to provide initial conditions for the analyses made below given that V_s at time $t = 0$ is equal to 0 but the voltage difference in the terminals of the capacitor stays constant for very short time intervals.

3.2 Natural response

We will now use the values of $V(6)$ and $V(8)$ calculated above as initial conditions for a transient analyisis of the natural response of the circuit when $V_s = 0$. This is represented in figure 1

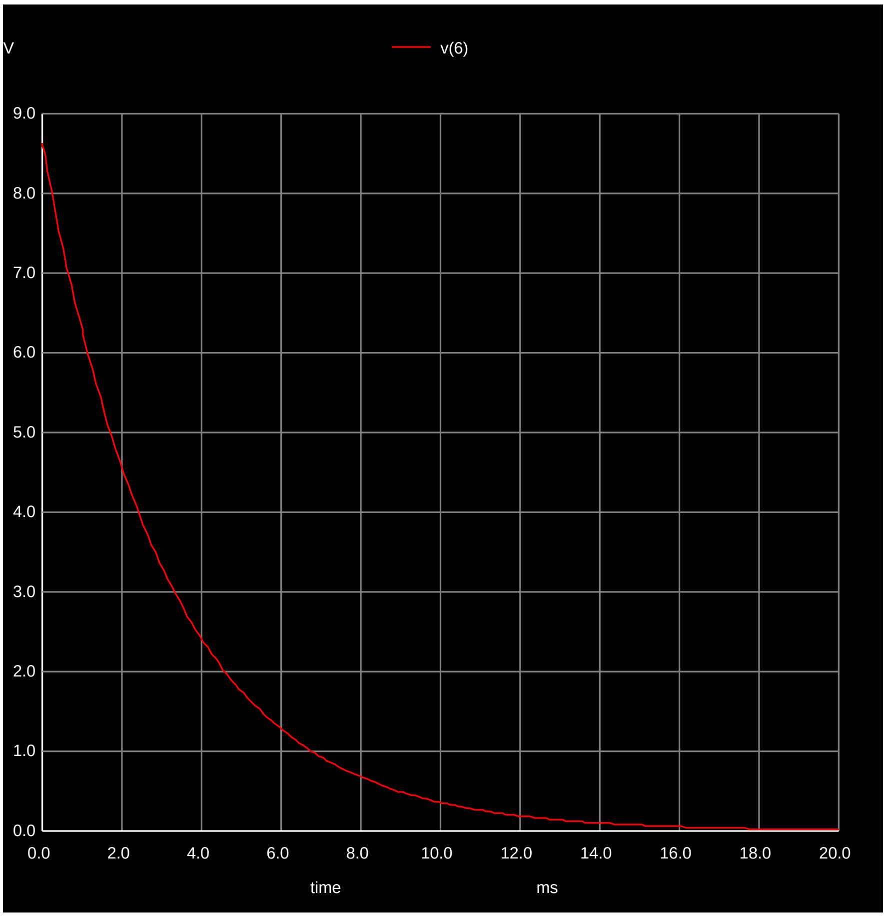

Figure 6: Natural Response

3.3 Forced response

Utilizing the same initial conditions and the value for V_s given for $t > 0$, an analysis of the forced response of the circuit over time was performed. This is represented in figure 1

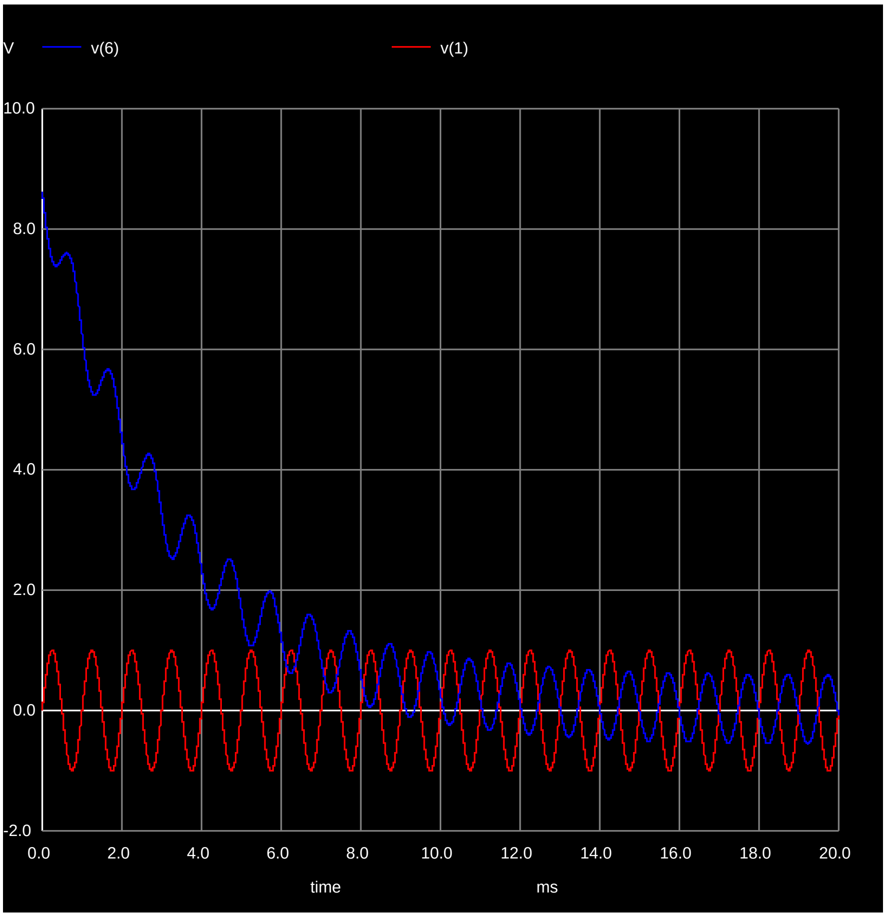

Figure 7: Forced Response

3.4 Frequency analysis

Finally, the frequency response of the circuit was studied and the magnitude and phase of both V_s and $V(6)$ was plotted for values of f from $0.1Hz$ to $1MHz$.

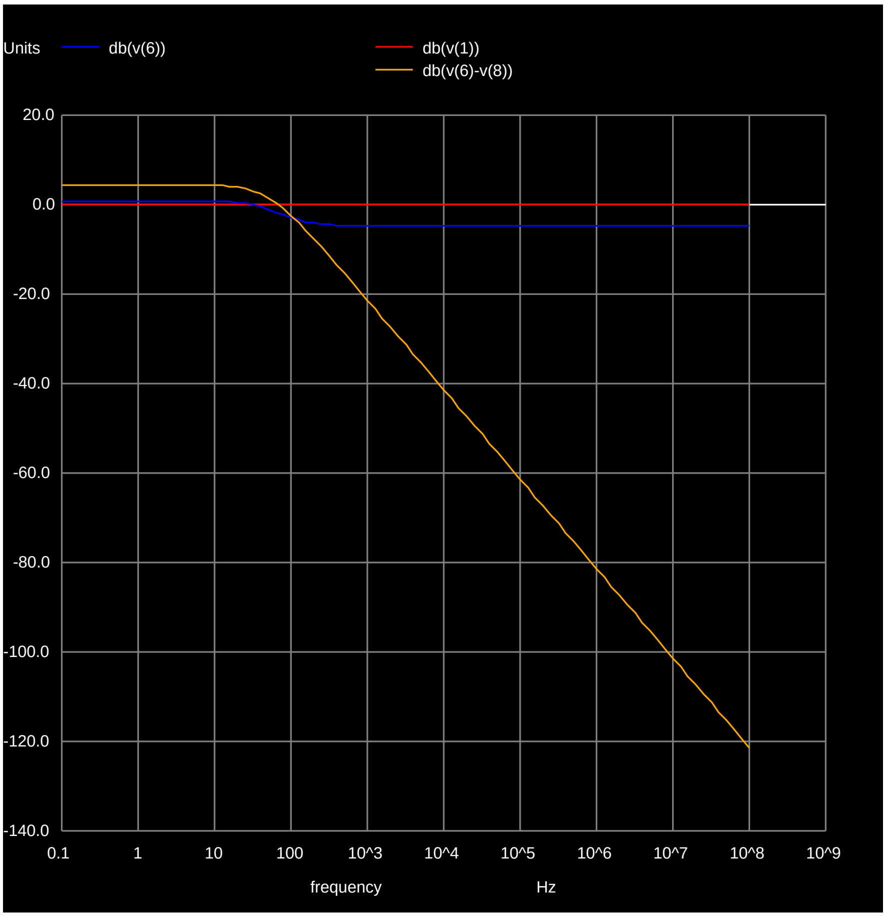

Figure 8: Frequency Response - Magnitude

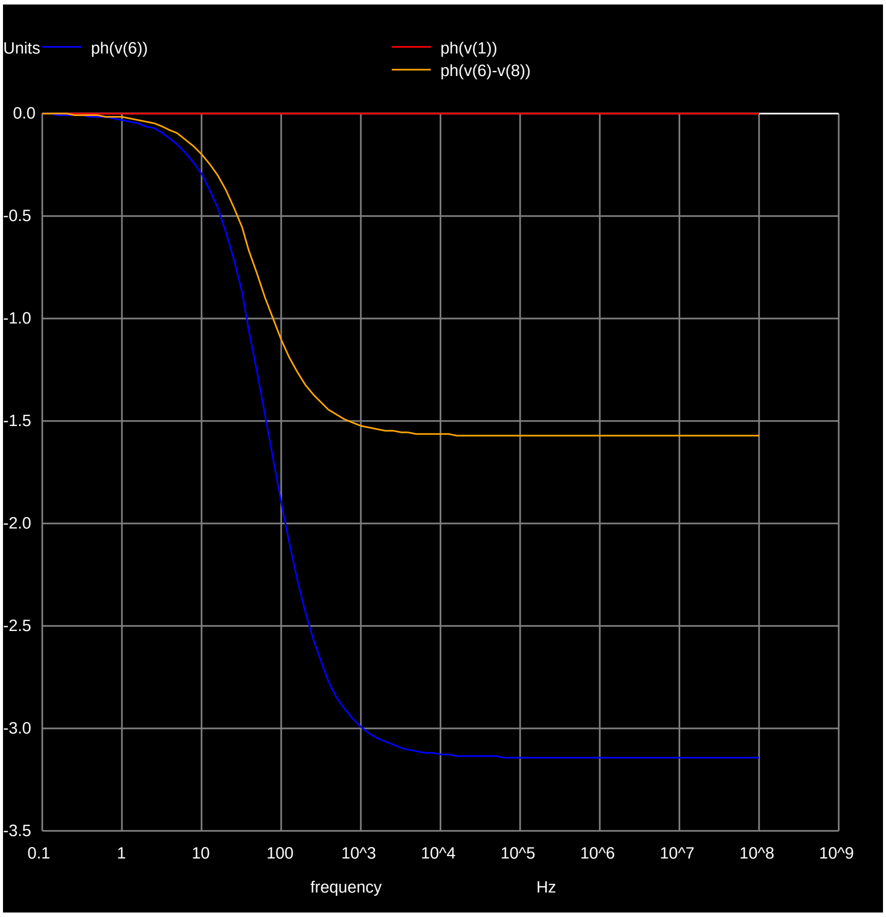

Figure 9: Frequency Response - Magnitude

The magnitude of the voltage V_s is always 1 (or 0 in dB), and its phase is always 0, by definition. As we can see, the magnitude of $V(6)$ drops sharply between the orders of magnitude of 10^1 to 10^3, stabilizing at a value of around $-4.9dB$. The phase starts off being close to 0, but deviates to negative values, stabilizing at $-\pi$.

4 Conclusion

In this laboratory assignment the objective of analysing a static DC circuit has been achieved. A static analysis has been performed on the circuit, through both the node analysis and mesh analysis methods, using the Octave software, and a simulation was run using ngspice. The three sets of results all match with all available decimal places of precision. The reason for this perfect match is the fact that although this circuit has multiple components and nodes, all of the components are linear, and no time dependence exists. The matching of results for the various methods also helps to confirm the accuracy of the equations used for the theoretical analysis.