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Circuit Theory and Electronics Fundamentals

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First Laboratory Report

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1 Introduction

The objective of this laboratory assignment is to study a circuit containing a DC voltage source V_a, a current source, I_d, a voltage controlled current source I_b, a current controlled voltage source V_c and resistors, R_1, R_2, R_3, R_4, R_5, R_6 and R_7. The circuit can be seen in Figure 1.

In Section 2, a theoretical analysis of the circuit is presented. In Section 5, the circuit is analysed by simulation, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 6.

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically.

3 Mesh analysis

We considered 4 meshes delimited by the nodes ABDE, BCEF, DEGH and EFH and named them MA,MB,MC,MD, respectively. Since this is a linear circuit, we can apply to each one of these meshes the Kirchhoff Voltage Law (KVL):

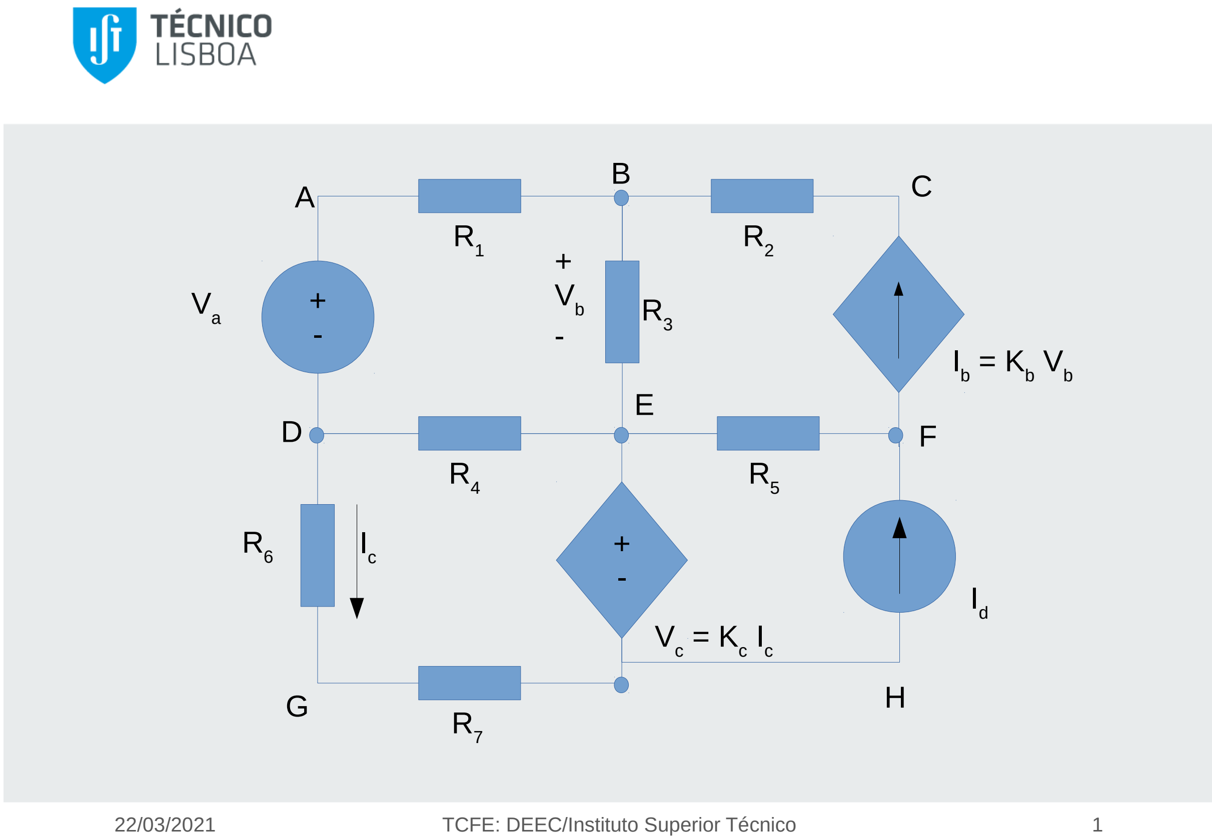

Figure 1: Voltage driven serial RC circuit.

$$\sum V_i = 0 \tag{1}$$

Applying Ohm's Law:

$$V_i = R_i * I \tag{2}$$

We get the following equations:

V$_a = (R_1 + R_2 + R_3)I_{MA} - R_3 I_{MB} - R_4 I_{MC}$

-K$_b R_3 I_{MA} + I_b(K_b R_3 - 1) = 0$

-I$_{MA} R_4 + I_{MC}(R_4 - K_c + R_6 + R_7) = 0$

I$_{MD} = -I_d$

(3)

In matrix form:

$$\begin{bmatrix} R_1 + R_2 + R_3 & -R_3 & -R_4 & 0 \\ -K_b * R_3 & K_b * R_3 - 1 & 0 & 0 \\ -R_4 & 0 & R_4 - K_c + R_6 + R_7 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} .$$

$$\cdot \begin{bmatrix} I_M A \\ I_M B \\ I_M C \\ I_M D \end{bmatrix} = \begin{bmatrix} V_a \\ 0 \\ 0 \\ -I_d \end{bmatrix} \quad (4)$$

4 Node analysis

The Kirchhoff Current Law (KCL) states that,for each node, the current from every branch conected must sum to 0:

$$\sum I_i = 0 \tag{5}$$

Using KCL and Ohm's law (2) we can obtain an equation for each node that isn't connected to a voltage source.

$$(V_B - V_A)G_1 + (V_B - V_C)G_2 + (V_B - V_E)G_3 = 0$$

$$(V_C - V_B)G_2 + (V_E - V_B)K_b - I_d = 0$$

$$(V_F - V_E)G_5 + (V_B - V_E)K_b = 0$$

$$(V_G - V_D)G_6 + (V_G - V_H)G_6 = 0$$

(6)

In nodes that are conected directly to voltage sources we cannot obtain equations as the ondes above, however, by using supernodes, we can obtain pairs of equations for each pair of nodes connected to a source.

The supernodes we'll be using are couplings of two nodes conected by a voltage source. The first two equations come from the fact that we know the voltage of each source, so we obtain

$$V_B - V_E = V_a$$

$$V_E - V_H = K_c(V_D - V_G)G_6$$

(7)

We can obtain another equation by fixing one node as groung.

$$V_H = 0V$$

(8)

We also know that equation (5) aplies to supernodes, and from this we can derive the last linearly independent equation

$$I_d + (V_H - V_G)G_7 + (V_E - V_F)G_5 + (V_E - V_B)G_3 + (V_E - V_D)G_4 = 0$$

(9)

All the previous equations can be compounded into a matrix

$$\begin{bmatrix} -G_1 & G_1+G_2+G_3 & -G_2 & 0 & -G_3 & 0 & 0 & 0 \\ 0 & -G_2-K_b & G_2 & 0 & K_b & 0 & 0 & 0 \\ 0 & K_b & 0 & 0 & -G_5-K_b & G_5 & 0 & 0 \\ 0 & 0 & 0 & -G_6 & 0 & 0 & G_6+G_7 & -G_7 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -K_c*G_6 & 1 & 0 & K_c*G_6 & -1 \\ 0 & -G_3 & 0 & -G_4 & G_4+G_3+G_5 & -G_5 & -G_7 & G_7 \end{bmatrix} \cdot$$

$$\cdot \begin{bmatrix} I_M A \\ I_M B \\ I_M C \\ I_M D \end{bmatrix} = \begin{bmatrix} V_a \\ 0 \\ 0 \\ -I_d \end{bmatrix} \text{(10)}$$

5 Simulation Analysis

5.1 Operating Point Analysis

Table 1 shows the simulated operating point results for the circuit under analysis. When compared to the theoretical analysis results, we see the same values up to 11 decimal places.

It should be noted that nodes G1 and G2 represent the same node G, and exist seperatly so as to allow the measuring of current I_c in ngspice, for the purpose of defining the dependent voltage source V_c.

6 Conclusion

In this laboratory assignment the objective of analysing a static DC circuit has been achieved. A static analysis has been performed on the circuit, through both the node analysis and mesh analysis methods, using the Octave software, and a simulation was run using ngspice. The three sets of results all match to 11 decimal places of precision. The reason for this perfect match is the fact that although this circuit has multiple components and nodes, all of the components are linear, and no time dependence exists. The matching of results for the various methods also helps to confirm the accuracy of the equations used for the theoretical analysis.

Name	**Value [A or V]**
@gb[i]	-2.53212e-04
@id[current]	1.032010e-03
@r1[i]	2.414774e-04
@r2[i]	2.532123e-04
@r3[i]	-1.17349e-05
@r4[i]	-1.20106e-03
@r5[i]	-1.28522e-03
@r6[i]	9.595831e-04
@r7[i]	9.595831e-04
a	8.080661e+00
b	7.829186e+00
c	7.306490e+00
d	2.944539e+00
e	7.864549e+00
f	1.177414e+01
g1	9.779981e-01
g2	9.779981e-01

Table 1: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.