# Circuit Theory and Electronics Fundamentals

Department of Physical Engineering, Técnico, University of Lisbon

RC circuit

April 7, 2021

Diogo Simões, Júlia Mestre, Rafael Dias

## Contents



## 1 Introduction

The objective of this laboratory assignment is to study a RC circuit containing a AC voltage source $V_s$, a capacitor $C$, a voltage controlled current source $I_b$, a current controlled voltage source $V_d$ and resistors, $R_1$, $R_2$, $R_3$, $R_4$, $R_5$, $R_6$ and $R_7$. The circuit can be seen in Figure 1.

In Section 2, a theoretical analysis of the circuit is presented. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

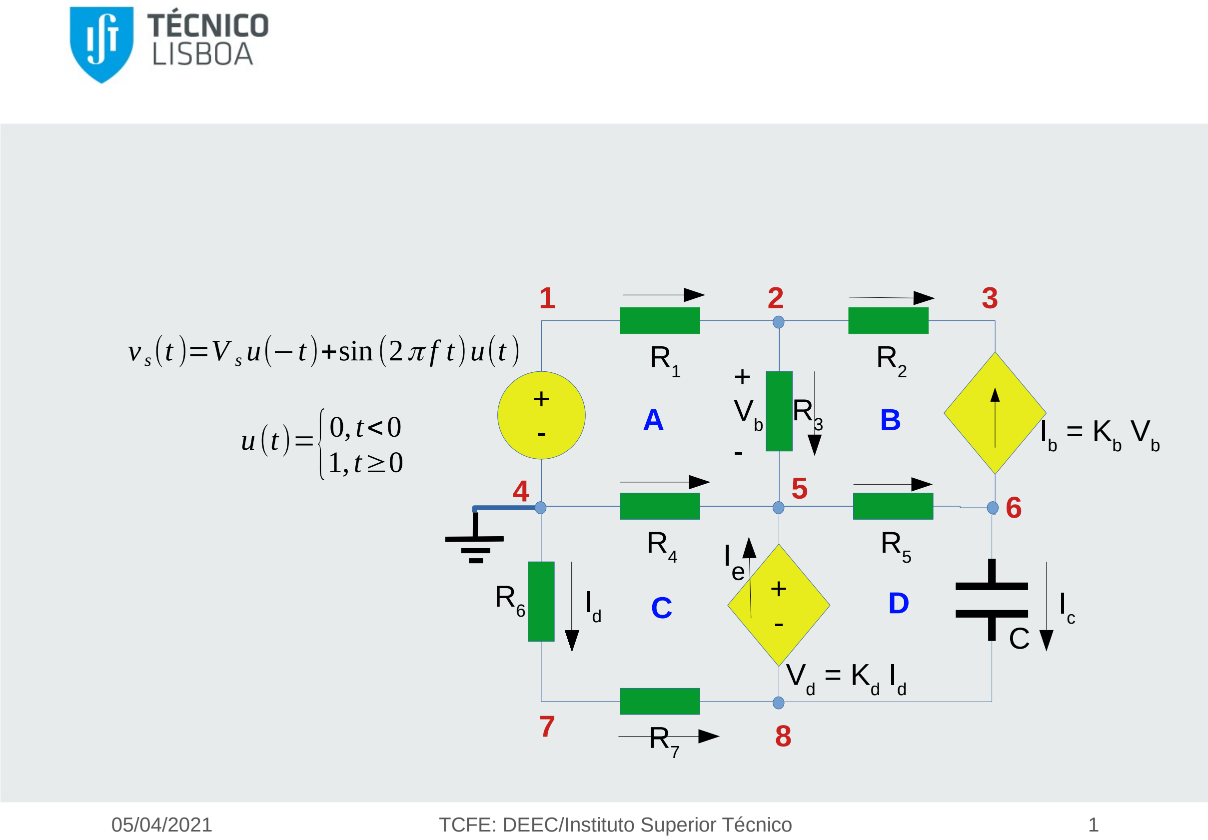


Figure 1: RC circuit to be analysed

## 2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically, analysing the circuit for $t < 0$, calculating the equivalent resistance, determining the natural and forced solutions and superimposing them to find the total solution.

### 2.1 Nodal analysis

For t<0, $v_s(t) = V_s(t)$, it is a DC circuit. We can determine the voltges in all nodes and currents in all branches using the nodal method. Since this is a linear circuit, we apply Ohm's Law, $V_i = R_i * I$ and the Kirchoff Current Law (KCL), $\sum I_i = 0$.

We get the following equation, in matrix form:

$$\begin{bmatrix} -G_1 & G_1+G_2+G_3 & -G_2 & 0 & -G_3 & 0 & 0 & 0 \\ 0 & -G_2-K_b & G_2 & 0 & K_b & 0 & 0 & 0 \\ 0 & K_b & 0 & 0 & -G_5-K_b & G_5 & 0 & 0 \\ 0 & 0 & 0 & -G_6 & 0 & 0 & G_6+G_7 & -G_7 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -K_c*G_6 & 1 & 0 & K_c*G_6 & -1 \\ 0 & -G_3 & 0 & -G_4 & G_4+G_3+G_5 & -G_5 & -G_7 & G_7 \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ V_s \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

This equation solved using octave yields the following results:

| Variable | Value [A or V] |
|---|---|
| $V_1$ | $5.13612248730V$ |
| $V_2$ | $4.88464690881V$ |
| $V_3$ | $4.36195145882V$ |
| $V_4$ | $-0.00000000000V$ |
| $V_5$ | $4.92000960269V$ |
| $V_6$ | $5.69027079572V$ |
| $V_7$ | $-1.96654083449V$ |
| $V_8$ | $-2.94453891610V$ |
| $I_1$ | $0.00024147744A$ |
| $I_2$ | $0.00025321233A$ |
| $I_3$ | $-0.00001173489A$ |
| $I_4$ | $-0.00120106056A$ |
| $I_5$ | $-0.00025321233A$ |
| $I_6$ | $0.00095958312A$ |
| $I_7$ | $0.00095958312A$ |
| $I_S$ | $-0.00024147744A$ |
| $I_b$ | $-0.00025321233A$ |
| $I_c$ | $-0.00000000000A$ |
| $I_e$ | $-0.00095958312A$ |

Table 1: Node Analysis Results for t<0

## 2.2 Equivalent resistance

Now, we have to determine the equivalent resistance $R_{eq}$ as seen from the capacitor terminals. We take out all the independent voltage sources (make $v_s = 0$) and replace the capacitor with a voltage source $Vx = V(6) - V(8)$. The values of$V(6)$ and $V(8)$ were already obtained via nodal analysis in the previous subsection. To determine the current $I_x$ supplied by $V_x$ we run mesh analysis. This is necessary because the resistors are arranged in such a way that they cannot be simplified into an equivalent resistor by applying the usual equations for resistors in series and in parallel. The mesh method gives us these equations in matriz form:

$$\begin{bmatrix} R_1+R_3+R_4 & -R_3 & -R_4 & 0 \\ -K_bR_3 & K_bR_3-1 & 0 & 0 \\ -R_4 & 0 & R_4+R_6+R_7-K_d & 0 \\ 0 & -R_5 & K_d & R_5 \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \\ I_D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ V_x \end{bmatrix} \tag{2}$$

This yields the following results:

| Variable | Value [A or V] |
|---|---|
| $V_x$ | $8.63480971182V$ |
| $I_x$ | $0.00283856995A$ |
| $R_{equiv}$ | $3041.95770117000\Omega$ |

Table 2: Equivalent resistance

For the time constant:
$\tau = R_{eq} \cdot C = 0.00313933181s$

## 2.3 Natural solution

Using the capacitor voltage $V_x$ for $t < 0$ as the initial condition, the natural solution of $v_{6n}(t)$ becomes:

$$v_{6n}(t) = V_x e^{\frac{-t}{R_{eq}C}} \tag{3}$$

This equation gives us the following plot in [0,20]ms:

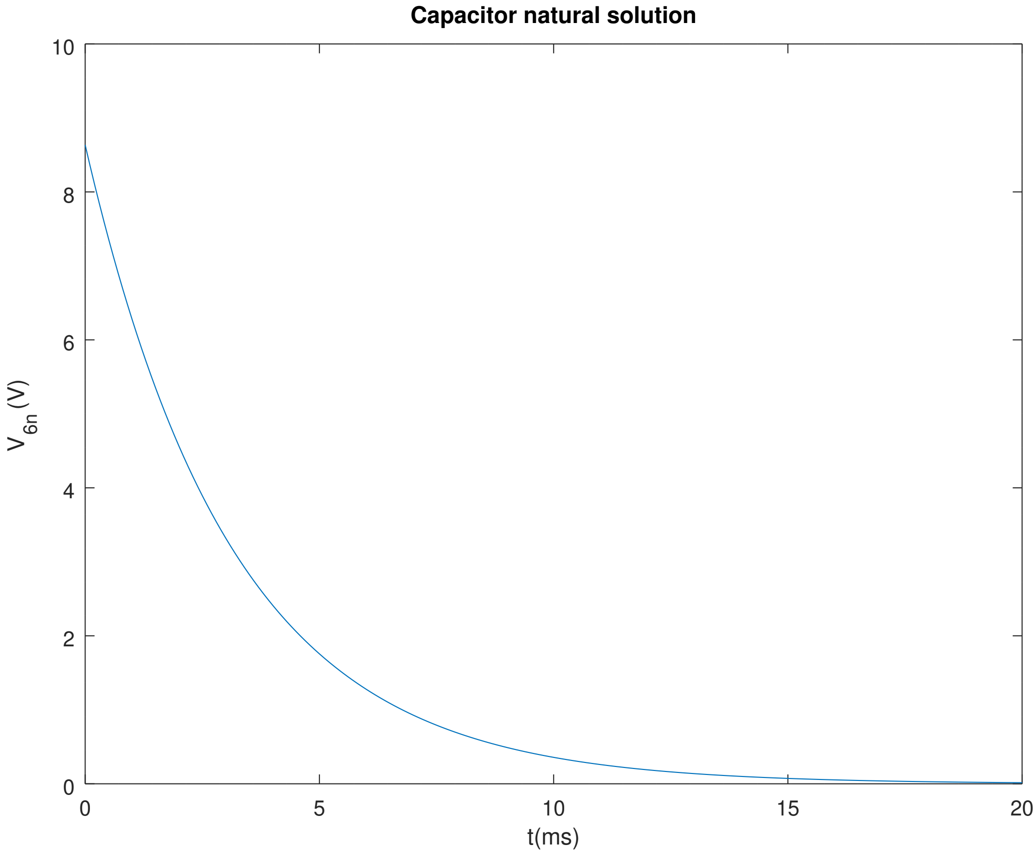


Figure 2: Natural solution $v_{6n}(t)$

## 2.4 Forced solution

To determine the forced solution in the same interval [0, 20]ms we use a phasor voltage source $V_s$ and replace $C$ with its impedance $Z_c$.

We run nodal analysis to determine the phasor voltages in all nodes:

$\omega = 2\pi f$

$$\begin{bmatrix} -G_1 & G_1+G_2+G_3 & -G_2 & 0 & -G_3 & 0 & 0 & 0 \\ 0 & -G_2-K_b & G_2 & 0 & K_b & 0 & 0 & 0 \\ 0 & K_b & 0 & 0 & -G_5-K_b & G_5+(j\omega C) & 0 & -(j\omega C) \\ 0 & 0 & 0 & -G_6 & 0 & 0 & G_6+G_7 & -G_7 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -K_dG_6 & 1 & 0 & K_dG_6 & -1 \\ 0 & -G_3 & 0 & -G_4 & G_4+G_3+G_5 & -G_5-(j\omega C) & -G_7 & G_7+(j\omega C) \end{bmatrix} \cdot \begin{bmatrix} V_{1p} \\ V_{2p} \\ V_{3p} \\ V_{4p} \\ V_{5p} \\ V_{6p} \\ V_{7p} \\ V_{8p} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ i \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{4}$$

Solving this system of equation in octave yields the following results:

| Variable | Value [A or V] |
|---|---|
| $\tilde{V_1}$ | $(0.00000000000 + i \cdot 1.00000000000)V$ |
| $\tilde{V_2}$ | $(0.00000000000 + i \cdot 0.95103785412)V$ |
| $\tilde{V_3}$ | $(0.00000000000 + i \cdot 0.84926936022)V$ |
| $\tilde{V_4}$ | $(0.00000000000 + i \cdot 0.00000000000)V$ |
| $\tilde{V_5}$ | $(0.00000000000 + i \cdot 0.95792294963)V$ |
| $\tilde{V_6}$ | $(0.08501303490 + i \cdot -0.56899006711)V$ |
| $\tilde{V_7}$ | $(-0.00000000000 + i \cdot -0.38288433334)V$ |
| $\tilde{V_8}$ | $(-0.00000000000 + i \cdot -0.57329997939)V$ |

Table 3: Phasor voltages

## 2.5 Total solution

Converting the phasors to real time functions for f=1KHz, we can then superimpose the natural and forced solutions:

$v_6(t) = V_x e^{\frac{-t}{R_{eq}C}} - \Re(\tilde{V_6} e^{j\omega t})$

This plot in the interval [-5,20]ms:

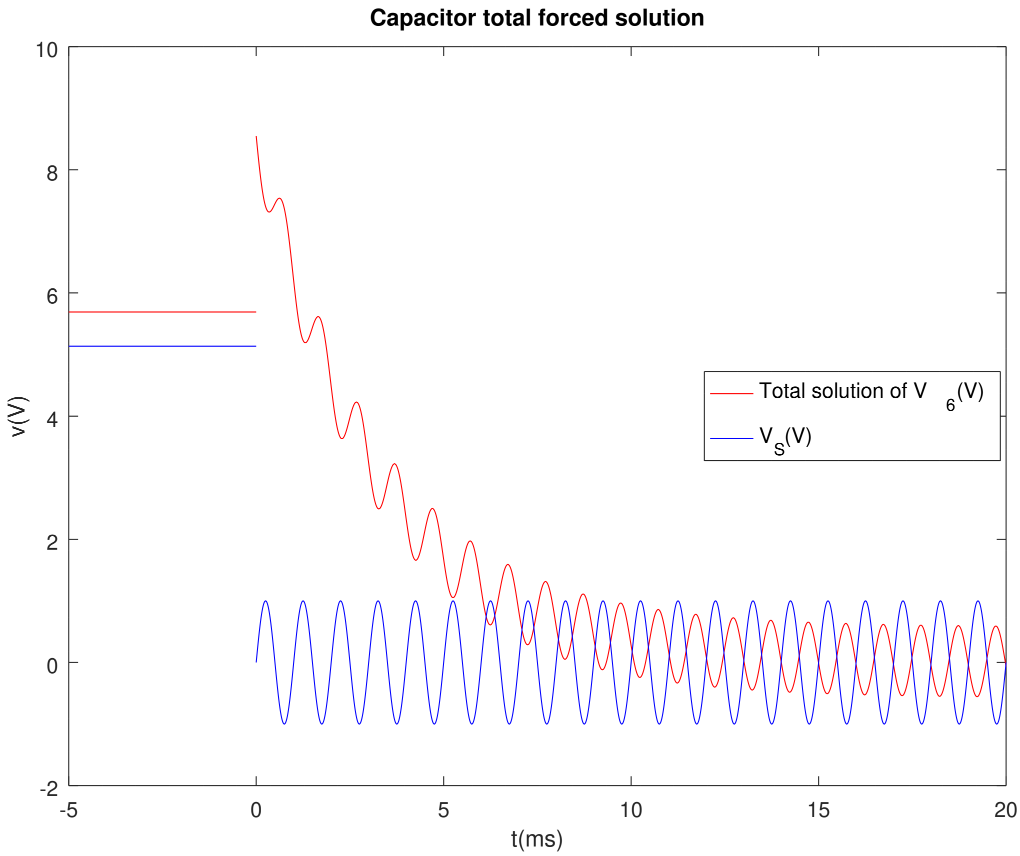


Figure 3: Final total solution $v_6(t)$

## 2.6 Frequency responses

The system 4 was solved for various values of $f$. The plots of the magnitude and phase of $V_s$, $V(6)$ and $V_C$, for values ranging from $0.1Hz$ to $1MHz$ are below:

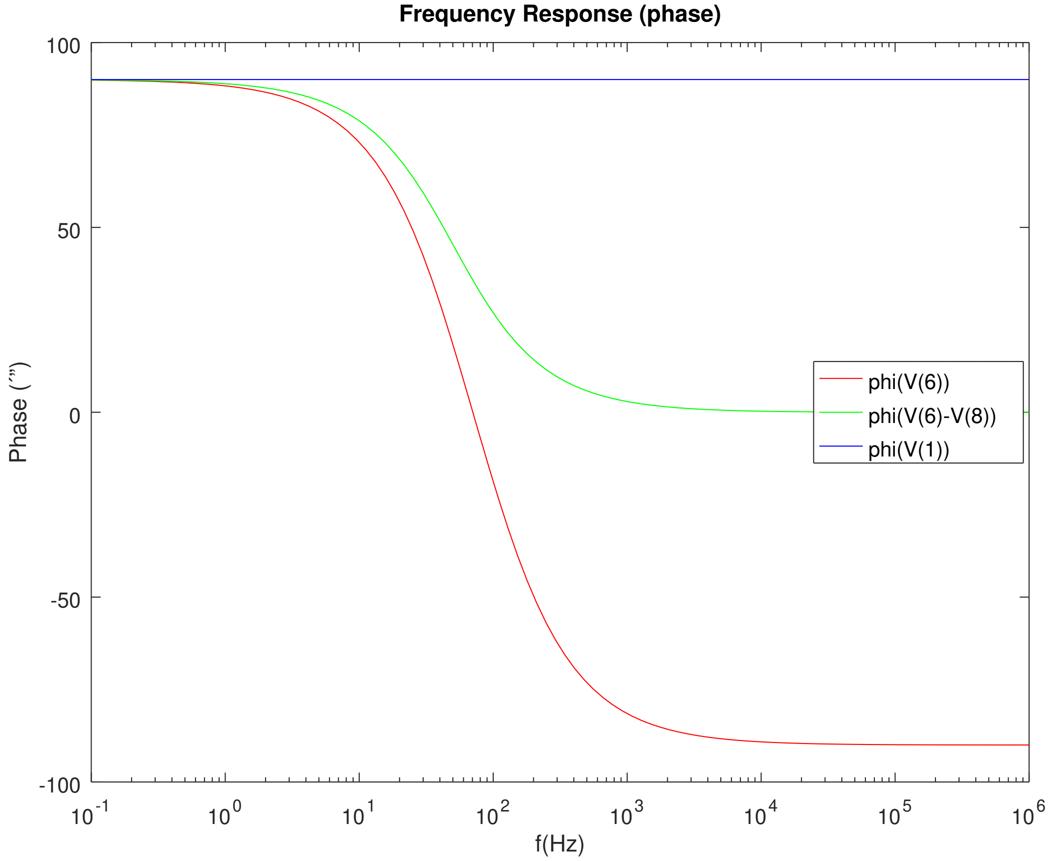


Figure 4: Frequency response-phase

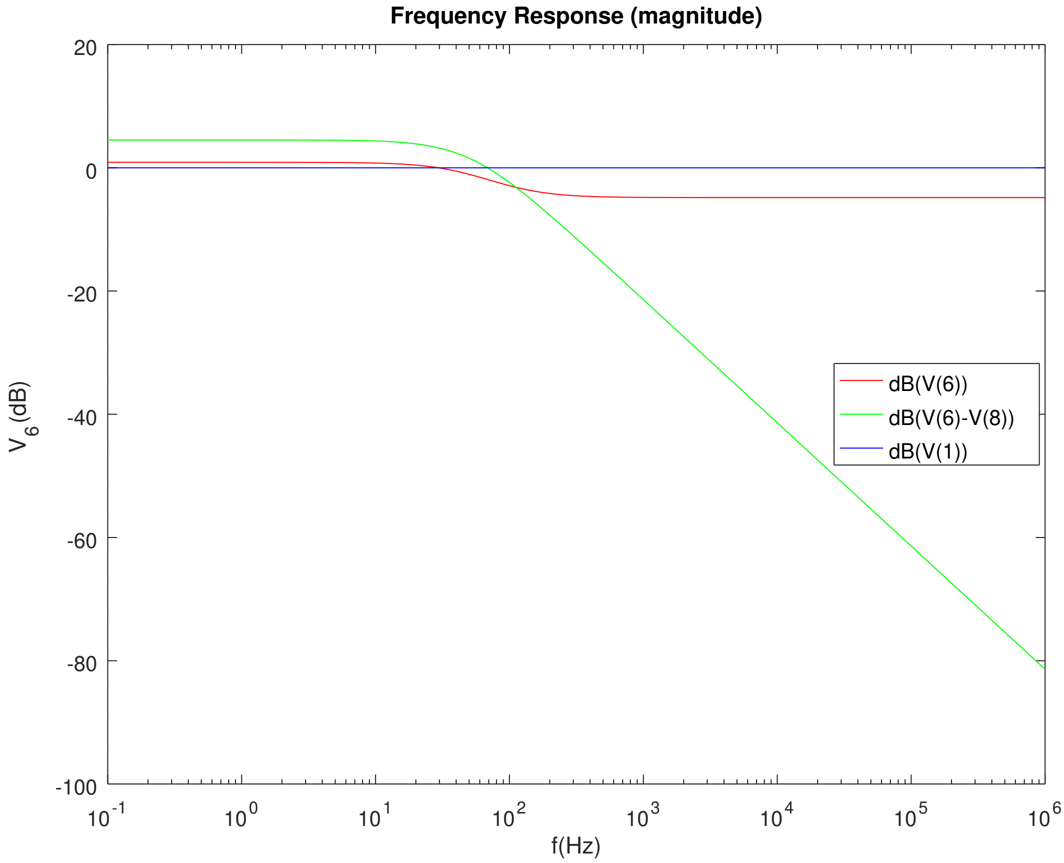


Figure 5: Frequency response-magnitude

# 3 Simulation Analysis

## 3.1 Operating Point Analysis

Table 4 shows the simulated operating point results for the circuit at times $t < 0$.

| Name | Value [A or V] |
|---|---|
| @ca[i] | 0.000000e+00 |
| @gb[i] | -2.53212e-04 |
| @r1[i] | 2.414774e-04 |
| @r2[i] | 2.532123e-04 |
| @r3[i] | -1.17349e-05 |
| @r4[i] | -1.20106e-03 |
| @r5[i] | -2.53212e-04 |
| @r6[i] | 9.595831e-04 |
| @r7[i] | 9.595831e-04 |
| v(1) | 5.136122e+00 |
| v(2) | 4.884647e+00 |
| v(3) | 4.361951e+00 |
| v(5) | 4.920010e+00 |
| v(6) | 5.690271e+00 |
| v(8) | -2.94454e+00 |
| v(71) | -1.96654e+00 |
| v(72) | -1.96654e+00 |

Table 4: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

Table 5 shows the simulated operating point results for the circuit given that $V_s = 0$ and replacing the capacitor with a voltage source imposing the voltage on the terminals of said capacitor as calculated in the earlier analysis.

| Name | Value [A or V] |
|---|---|
| @gb[i] | -6.24390e-18 |
| @r1[i] | 5.954528e-18 |
| @r2[i] | 6.243896e-18 |
| @r3[i] | -2.89368e-19 |
| @r4[i] | 1.300919e-18 |
| @r5[i] | -2.83857e-03 |
| @r6[i] | -8.67362e-19 |
| @r7[i] | 1.165891e-21 |
| v(1) | 0.000000e+00 |
| v(2) | -6.20107e-15 |
| v(3) | -1.90901e-14 |
| v(5) | -5.32907e-15 |
| v(6) | 8.634810e+00 |
| v(8) | 1.776357e-15 |
| v(71) | 1.777545e-15 |
| v(72) | 1.777545e-15 |

Table 5: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

This is necessary so as to provide initial conditions for the analyses made below given that $V_s$ at time $t = 0$ is equal to 0 but the voltage difference in the terminals of the capacitor stays constant for very short time intervals.

### 3.2 Natural response

We will now use the values of $V(6)$ and $V(8)$ calculated above as initial conditions for a transient analyisis of the natural response of the circuit when $V_s = 0$. This is represented in figure 1

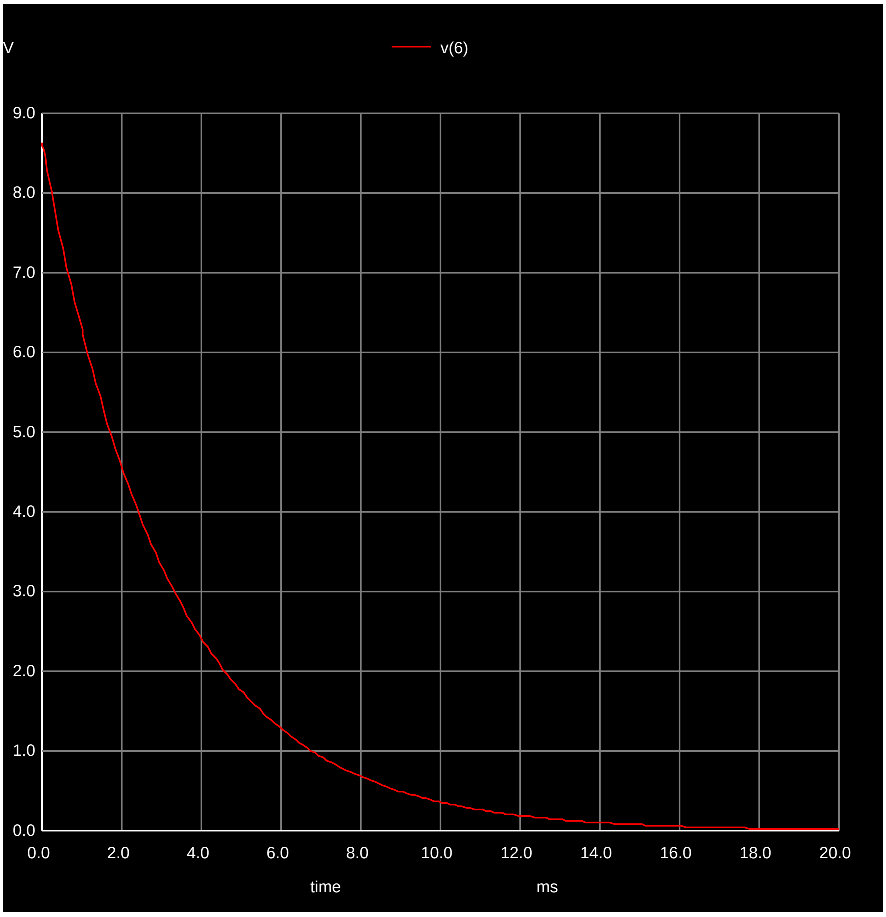


Figure 6: Natural Response

### 3.3 Forced response

Utilizing the same initial conditions and the value for $V_s$ given for $t > 0$, an analysis of the forced response of the circuit over time was performed. This is represented in figure 1

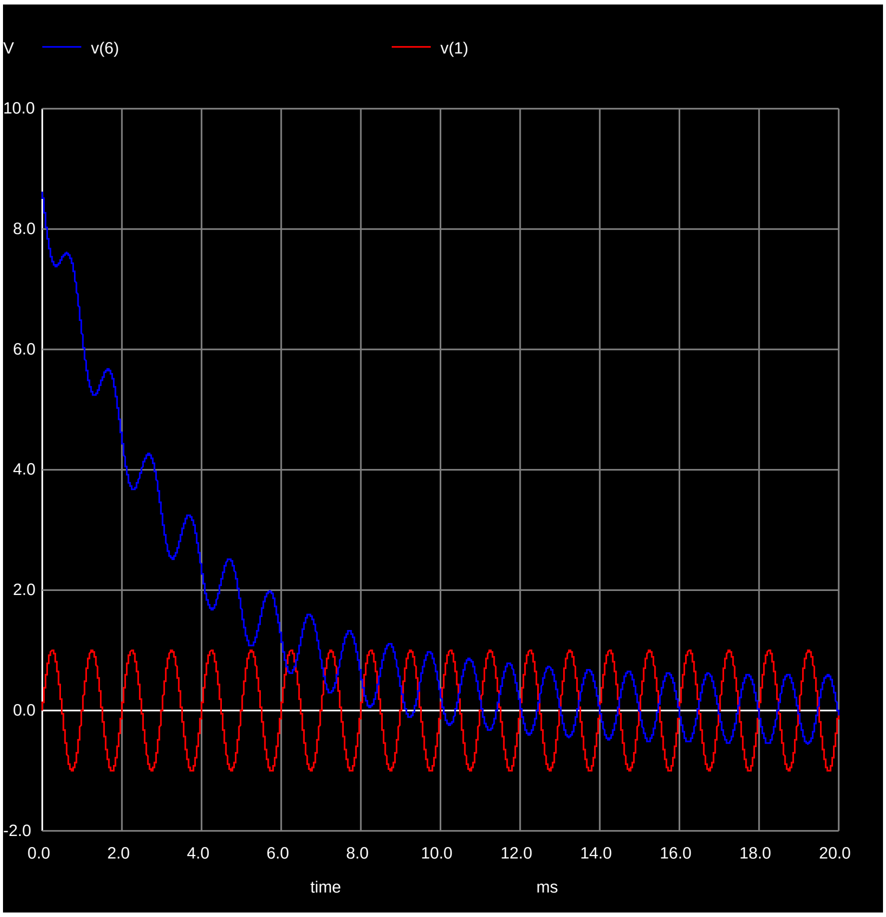


Figure 7: Forced Response

### 3.4 Frequency analysis

Finally, the frequency response of the circuit was studied and the magnitude and phase of both $V_s$ and $V(6)$ was plotted for values of $f$ from $0.1Hz$ to $1MHz$.

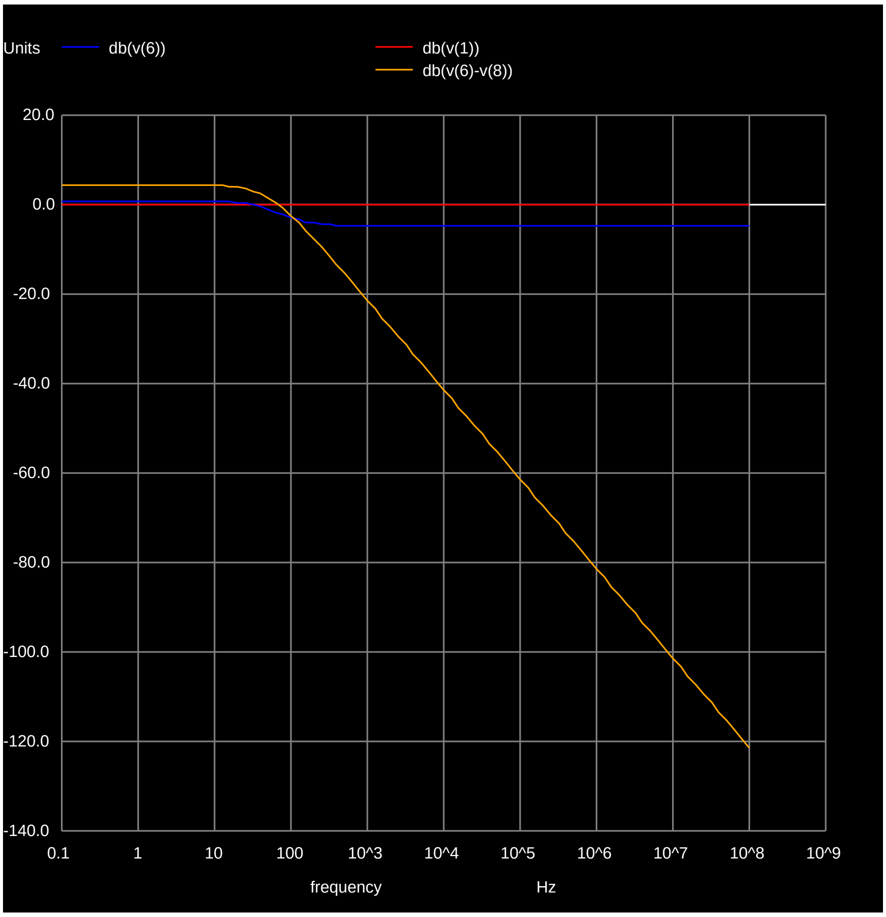


Figure 8: Frequency Response - Magnitude

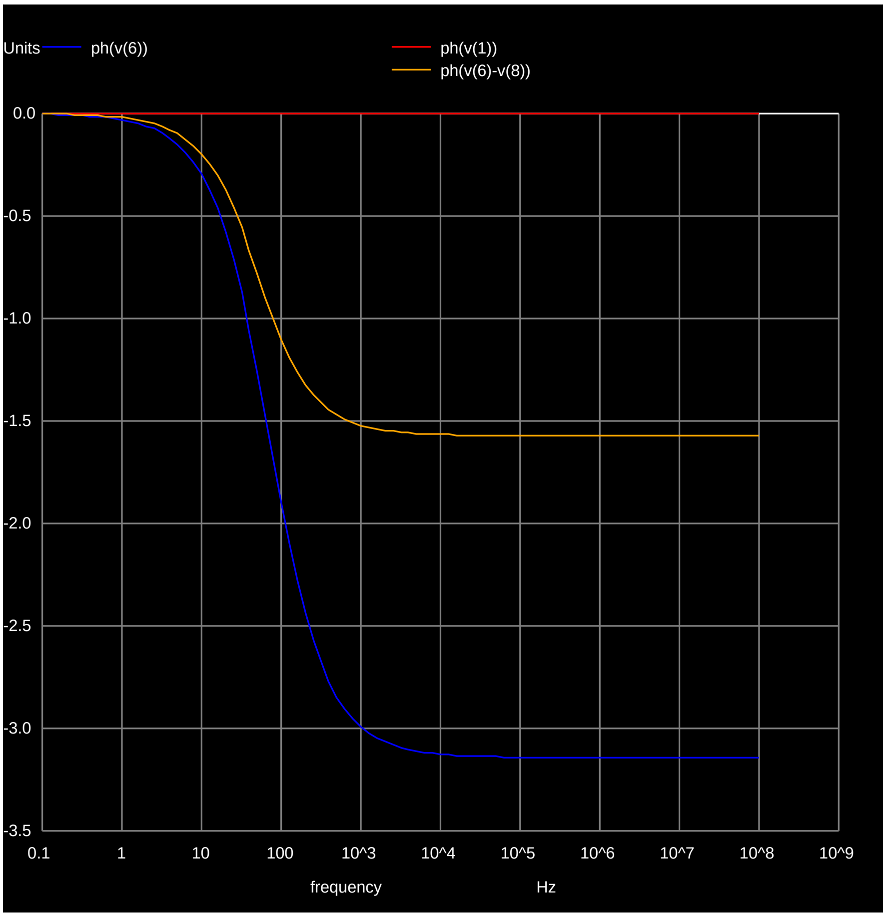


Figure 9: Frequency Response - Magnitude

The magnitude of the voltage $V_s$ is always 1 (or 0 in dB), and its phase is always 0, by definition. As we can see, the magnitude of $V(6)$ drops sharply between the orders of magnitude of $10^1$ to $10^3$, stabilizing at a value of around $-4.9dB$. The phase starts off being close to 0, but deviates to negative values, stabilizing at $-\pi$.

# 4 Conclusion

In this laboratory assignment the objective of analysing both the static solution of a circuit with fixed applied voltage and a capacitor, and the time-dependent solution of the same circuit has been successful. The results from both the theoretical analysis using octave and the circuit simulation using ngspice appear to match, for both the static analyses for $t < 0$ and for the calculation of boundary conditions, and for the natural, forced, and frequency responses.