Circuit Theory and Electronics Fundamentals

Department of Physical Engineering, Técnico, University of Lisbon

RC circuit

April 7, 2021
Diogo Simões, Júlia Mestre, Rafael Dias

Contents

1 Introduction

The objective of this laboratory assignment is to study a RC circuit containing a AC voltage source V_s, a capacitor C, a voltage controlled current source I_b, a current controlled voltage source V_d and resistors, R_1, R_2, R_3, R_4, R_5, R_6 and R_7. The circuit can be seen in Figure 1.

In Section 2, a theoretical analysis of the circuit is presented. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

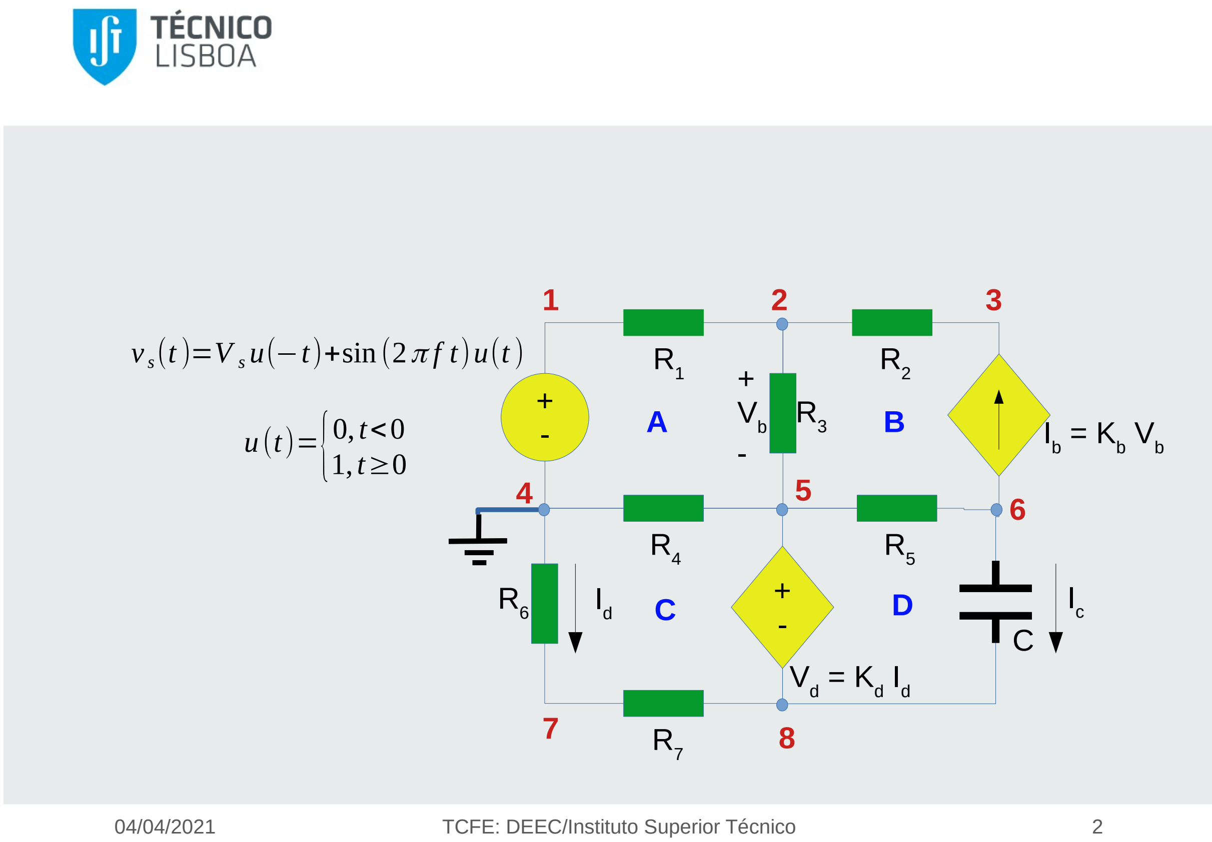

Figure 1: RC circuit to be analysed

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically, using both the mesh and node methods.

2.1 t¡0

For t¡0, vs(t)= Vs(t), it is a DC circuit. We can determine the voltges in all nodes and currents in all branches using the nodal method. Since this is a linear circuit, we apply Ohm's Law, $V_i = R_i * I$ and the Kirchoff Current Law (KCL), $\sum I_i = 0$.

We get the following equation, in matrix form:

$$\begin{bmatrix} -G_1 & G_1+G_2+G_3 & -G_2 & 0 & -G_3 & 0 & 0 & 0 \\ 0 & -G_2-K_b & G_2 & 0 & K_b & 0 & 0 & 0 \\ 0 & K_b & 0 & 0 & -G_5-K_b & G_5 & 0 & 0 \\ 0 & 0 & 0 & -G_6 & 0 & 0 & G_6+G_7 & -G_7 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -K_c*G_6 & 1 & 0 & K_c*G_6 & -1 \\ 0 & -G_3 & 0 & -G_4 & G_4+G_3+G_5 & -G_5 & -G_7 & G_7 \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ V_s \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (1)$$

This equation solved using octave yields the following results:

Variable	Value [A or V]

Table 1: Node Analysis Results for t¡0

3 Simulation Analysis

3.1 Operating Point Analysis

Table 2 shows the simulated operating point results for the circuit at times $t < 0$.

Name	Value [A or V]
@ca[i]	0.000000e+00
@gb[i]	-2.53212e-04
@r1[i]	2.414774e-04
@r2[i]	2.532123e-04
@r3[i]	-1.17349e-05
@r4[i]	-1.20106e-03
@r5[i]	-2.53212e-04
@r6[i]	9.595831e-04
@r7[i]	9.595831e-04
v(1)	5.136122e+00
v(2)	4.884647e+00
v(3)	4.361951e+00
v(5)	4.920010e+00
v(6)	5.690271e+00
v(8)	-2.94454e+00
v(71)	-1.96654e+00
v(72)	-1.96654e+00

Table 2: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

Table 3 shows the simulated operating point results for the circuit given that $V_s = 0$ and replacing the capacitor with a voltage source imposing the voltage on the terminals of said capacitor as calculated in the earlier analysis.

Name	Value [A or V]
@gb[i]	-6.24390e-18
@r1[i]	5.954528e-18
@r2[i]	6.243896e-18
@r3[i]	-2.89368e-19
@r4[i]	1.300919e-18
@r5[i]	-2.83857e-03
@r6[i]	-8.67362e-19
@r7[i]	1.165891e-21
v(1)	0.000000e+00
v(2)	-6.20107e-15
v(3)	-1.90901e-14
v(5)	-5.32907e-15
v(6)	8.634810e+00
v(8)	1.776357e-15
v(71)	1.777545e-15
v(72)	1.777545e-15

Table 3: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

3.2 Natural response

We will now use the values of $V(6)$ and $V(8)$ calculated above as initial conditions for a transient analyisis of the natural response of the circuit when $V_s = 0$. This is represented in figure 1

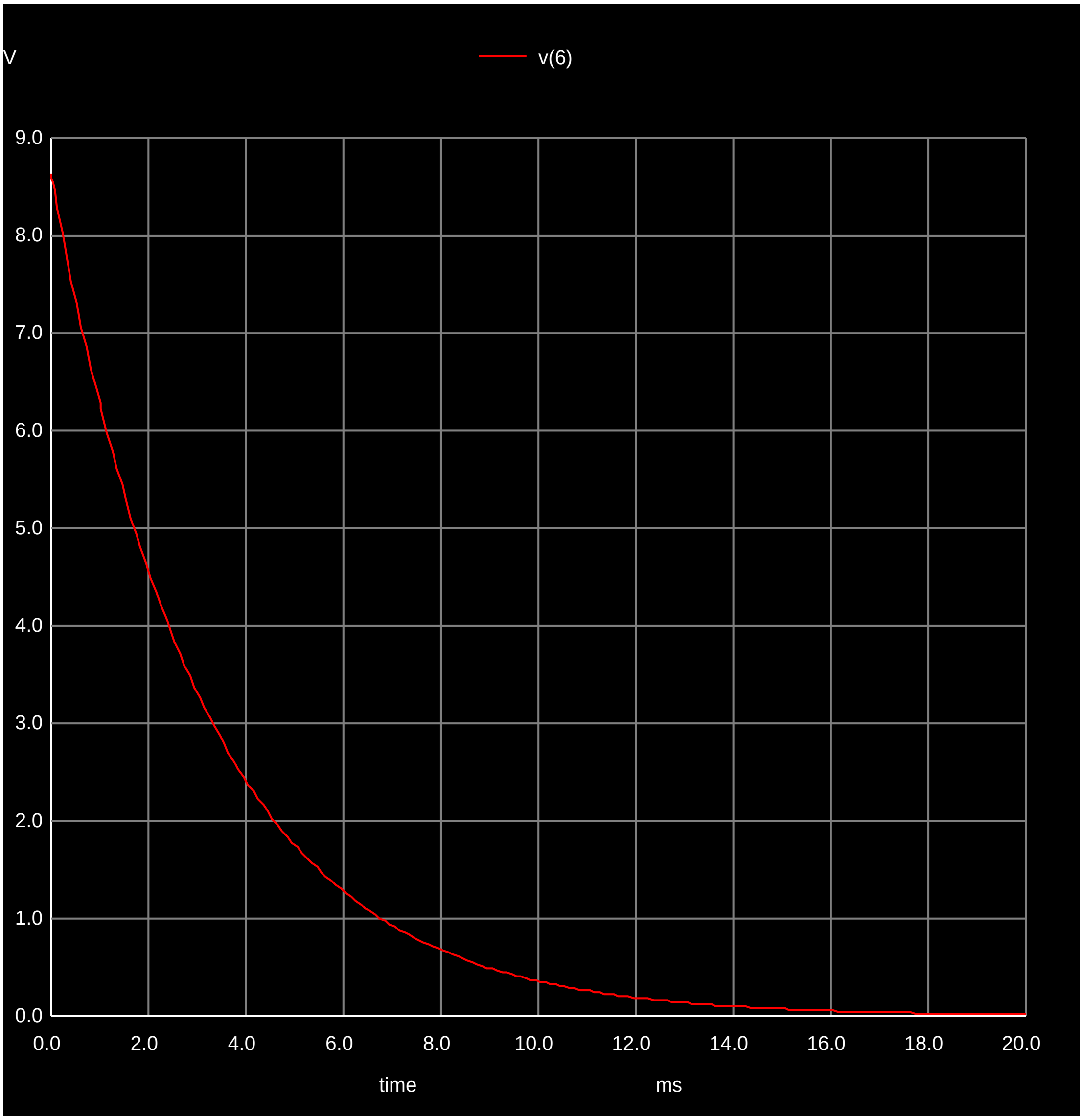

Figure 2: RC circuit to be analysed

3.3 Forced response

Utilizing the same initial conditions and the value for V_s given for $t > 0$, an analysis of the forced response of the circuit over time was performed. This is represented in figure 1

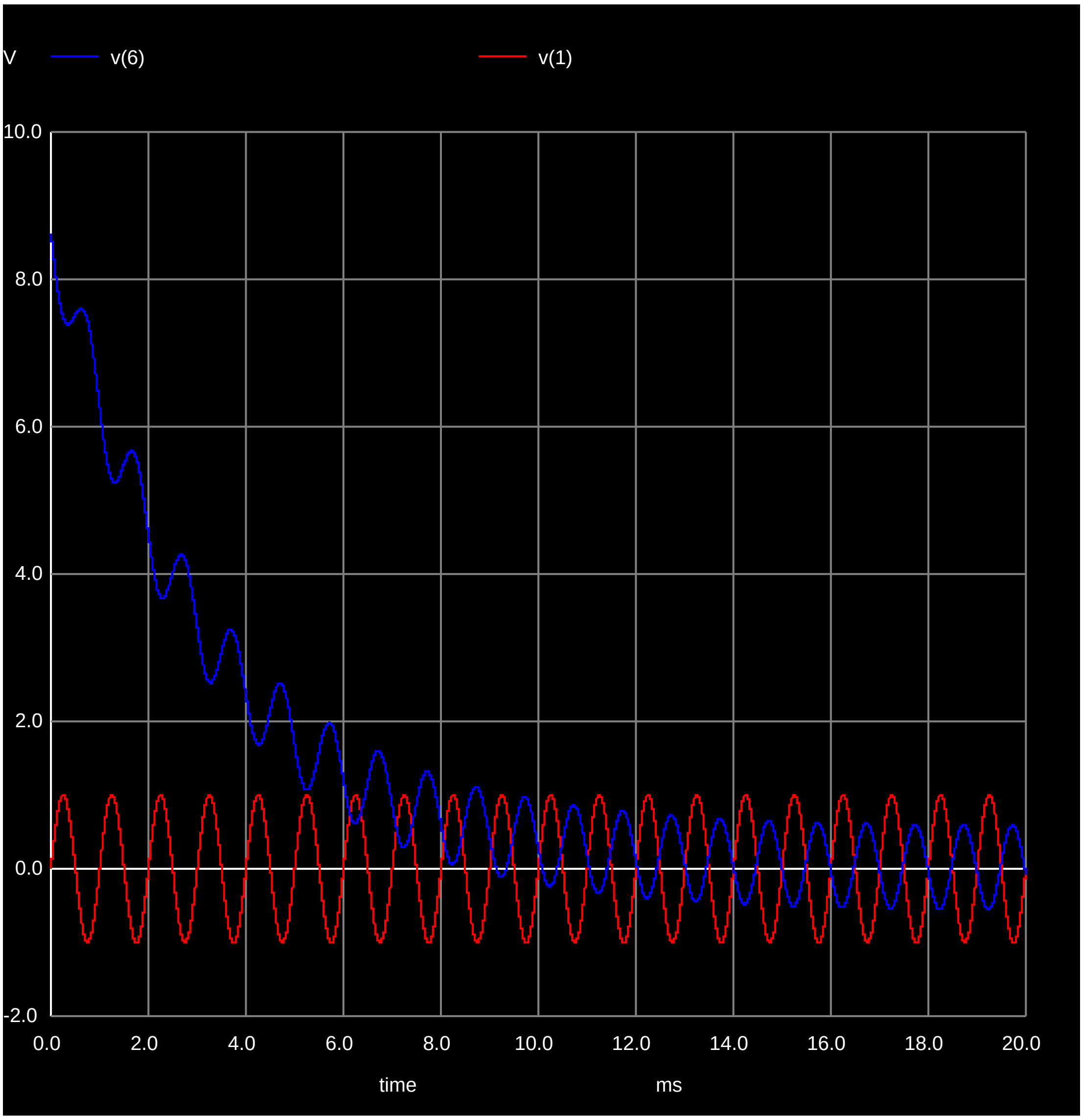

Figure 3: RC circuit to be analysed

3.4 Frequency analysis

Finally, the frequency response of the circuit was studied and the magnitude and phase of both V_s and $V(6)$ was plotted for values of f from $0.1Hz$ to $1MHz$.

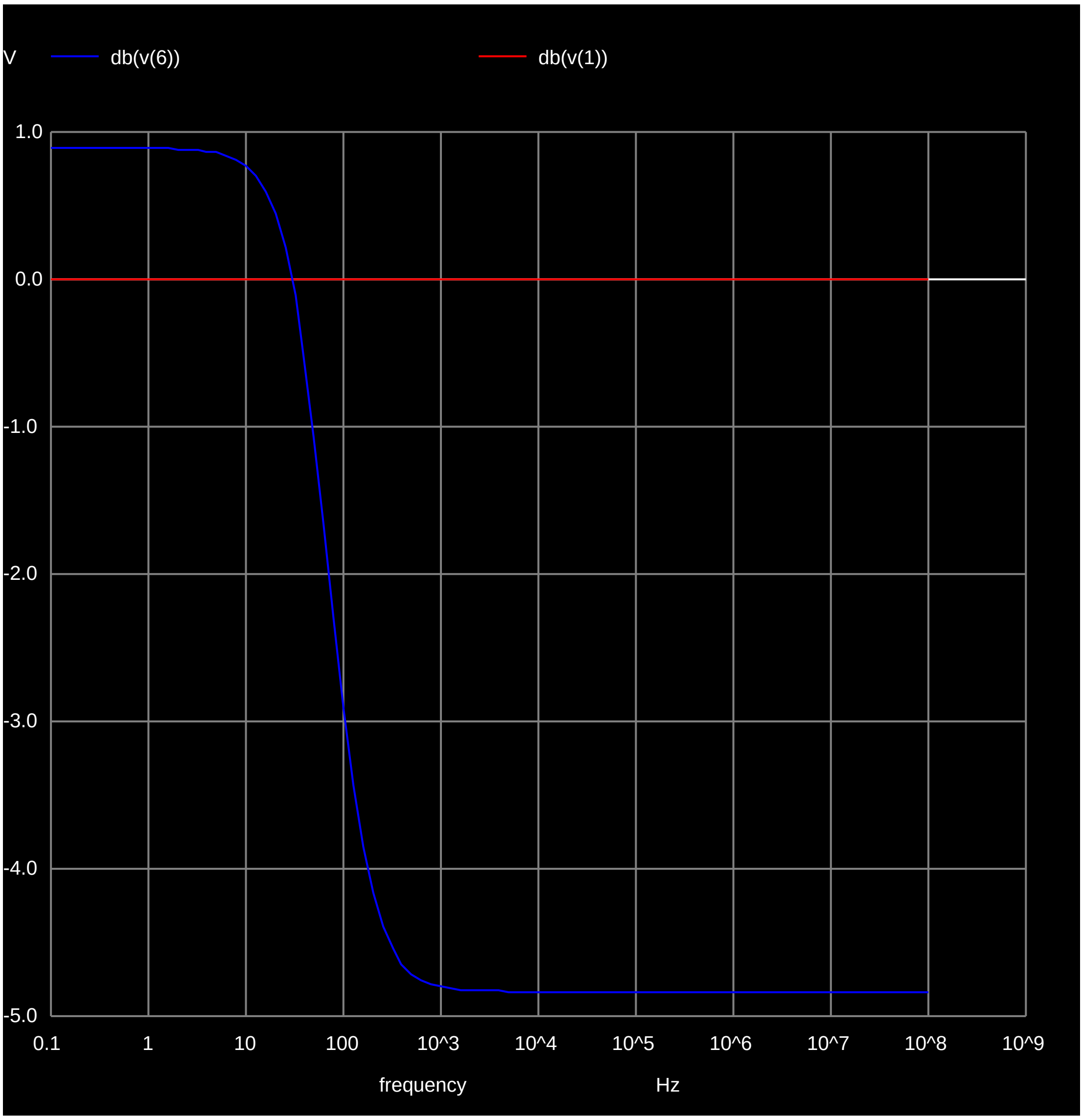

Figure 4: RC circuit to be analysed

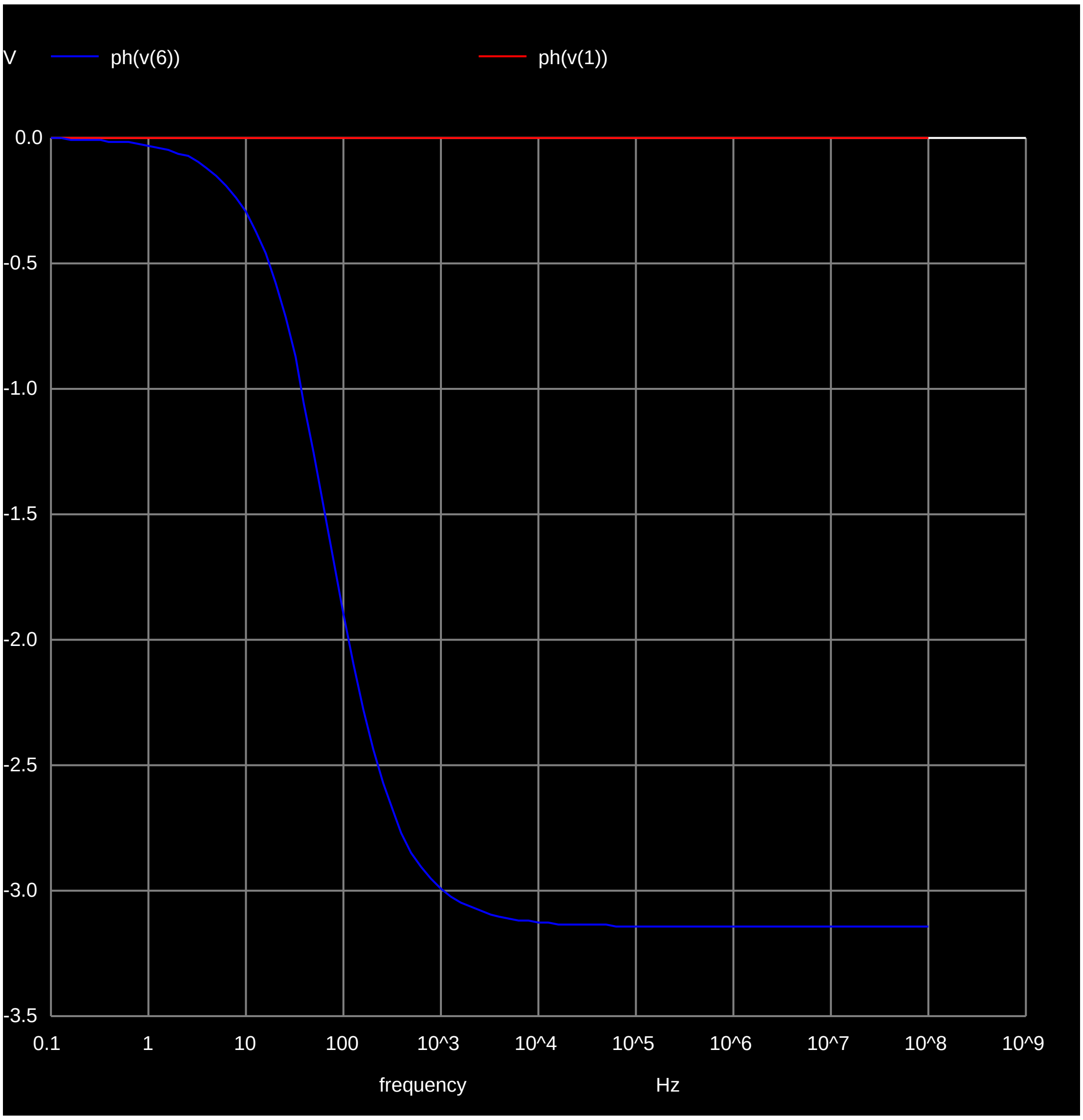

Figure 5: RC circuit to be analysed

The magnitude of the voltage V_s is always 1 (or 0 in dB), and its phase is always 0, by definition. As we can see, the magnitude of $V(6)$ drops sharply between the orders of magnitude of 10^1 to 10^3, stabilizing at a value of around $-4.9dB$. The phase starts off being close to 0, but deviates to negative values, stabilizing at $-\pi$.

4 Conclusion

In this laboratory assignment the objective of analysing a static DC circuit has been achieved. A static analysis has been performed on the circuit, through both the node analysis and mesh analysis methods, using the Octave software, and a simulation was run using ngspice. The three sets of results all match with all available decimal places of precision. The reason for this perfect match is the fact that although this circuit has multiple components and nodes, all of the components are linear, and no time dependence exists. The matching of results for the various methods also helps to confirm the accuracy of the equations used for the theoretical analysis.