Circuit Theory and Electronics Fundamentals

Department of Physical Engineering, Técnico, University of Lisbon

First Laboratory Report

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Contents

1 Introduction

The objective of this laboratory assignment is to study a circuit containing a DC voltage source V_a, a current source, I_d, a voltage controlled current source I_b, a current controlled voltage source V_c and resistors, R_1, R_2, R_3, R_4, R_5, R_6 and R_7. The circuit can be seen in Figure 1.

In Section 2, a theoretical analysis of the circuit is presented. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

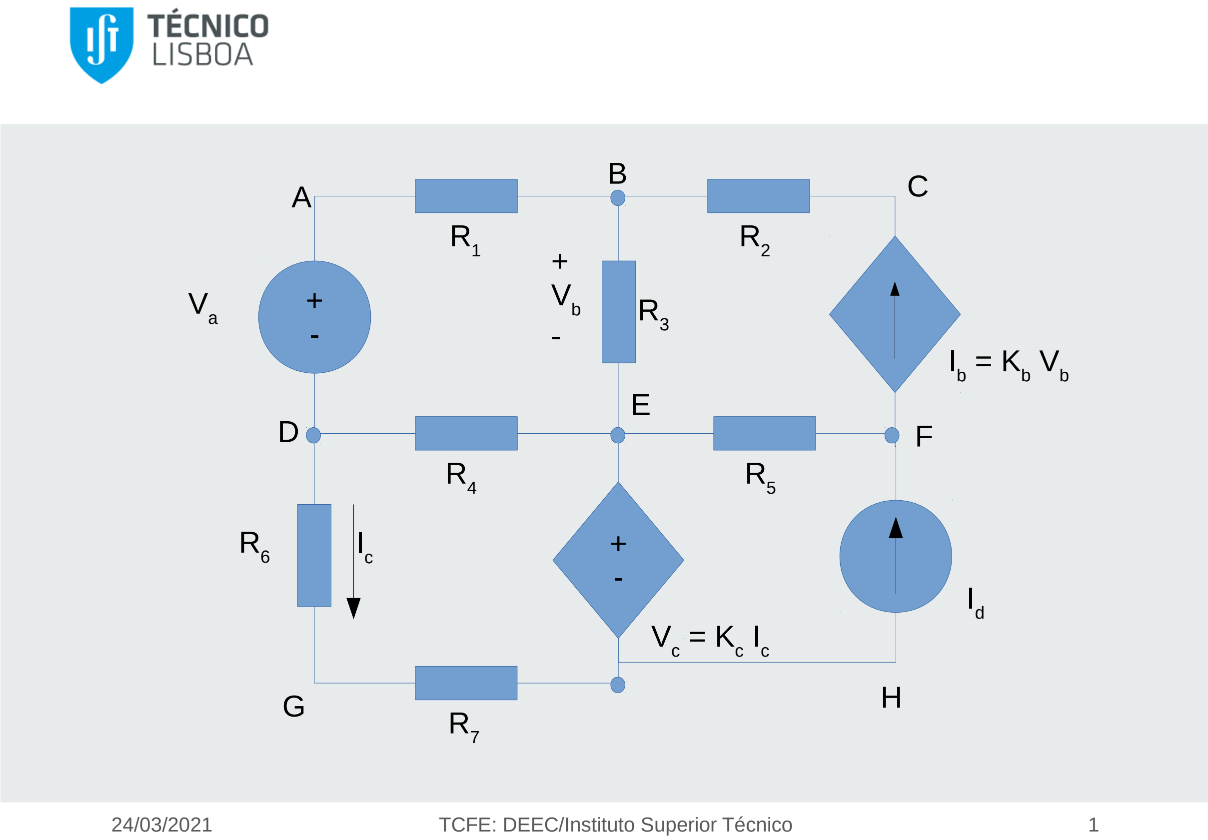

Figure 1: DC circuit to be analysed

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically.

2.1 Mesh analysis

We considered 4 meshes delimited by the nodes ABDE, BCEF, DEGH and EFH and named them MA,MB,MC and MD respectively (it should be noted that all all currents $I_{MA}, I_{MB}, I_{MC}, I_{MD}$ run clockwise in their respective meshes). Since this is a linear circuit, we can apply to each one of these meshes the Kirchhoff Voltage Law (KVL):

$$\sum V_i = 0 \tag{1}$$

Applying Ohm's Law:

$$V_i = R_i * I \tag{2}$$

We get the following equations:

V$_a = (R_1 + R_2 + R_3)I_{MA} - R_3 I_{MB} - R_4 I_{MC}$

-K$_b R_3 I_{MA} + I_b(K_b R_3 - 1) = 0$

-I$_{MA} R_4 + I_{MC}(R_4 - K_c + R_6 + R_7) = 0$

$I_{MD} = -I_d$
(3)

In matrix form:

$$\begin{bmatrix} R_1+R_2+R_3 & -R_3 & -R_4 & 0 \\ -K_b*R_3 & K_b*R_3-1 & 0 & 0 \\ -R_4 & 0 & R_4-K_c+R_6+R_7 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_{MA} \\ I_{MB} \\ I_{MC} \\ I_{MD} \end{bmatrix} = \begin{bmatrix} V_a \\ 0 \\ 0 \\ -I_d \end{bmatrix} \tag{4}$$

This equation solved using octave yields the following results:

Variable	**Value [A or V]**
VA	8.08066140340V
VB	7.82918582491V
VC	7.30649037492V
VD	2.94453891610V
VE	7.86454851879V
VF	11.77414152690V
VG	0.97799808161V
VH	0.00000000000V
I1	0.00024147744A
I2	0.00025321233A
I3	-0.00001173489A
I4	-0.00120106056A
I5	-0.00128522267A
I6	0.00095958312A
I7	0.00095958312A
Ia	0.00024147744A
Ib	-0.00025321233A
Ic	-0.00007242723A
Id	0.00103201034A

Table 1: Mesh Analysis Results

2.2 Node analysis

The Kirchhoff Current Law (KCL) states that,for each node, the current from every branch conected must sum to 0:

$$\sum I_i = 0 \tag{5}$$

Using KCL and Ohm's law (2) we can obtain an equation for each node that isn't connected to a voltage source.

$(V_B - V_A)G_1 + (V_B - V_C)G_2 + (V_B - V_E)G_3 = 0$

$(V_C - V_B)G_2 + (V_E - V_B)K_b - I_d = 0$

$(V_F - V_E)G_5 + (V_B - V_E)K_b = 0$

$(V_G - V_D)G_6 + (V_G - V_H)G_6 = 0$
(6)

In nodes that are conected directly to voltage sources we cannot obtain equations as the ondes above, however, by using supernodes, we can obtain pairs of equations for each pair of nodes connected to a source.

The supernodes we'll be using are couplings of two nodes conected by a voltage source. The first two equations come from the fact that we know the voltage of each source, so we obtain

$$V_B - V_E = V_a$$

$V_E - V_H = K_c(V_D - V_G)G_6$(7)

We can obtain another equation by fixing one node as groung.

$$V_H = 0V \tag{8}$$

We also know that equation (5) aplies to supernodes, and from this we can derive the last linearly independent equation

$$I_d + (V_H - V_G)G_7 + (V_E - V_F)G_5 + (V_E - V_B)G_3 + (V_E - V_D)G_4 = 0 \tag{9}$$

All the previous equations can be compounded into a matrix

$$\begin{bmatrix} -G_1 & G_1+G_2+G_3 & -G_2 & 0 & -G_3 & 0 & 0 & 0 \\ 0 & -G_2-K_b & G_2 & 0 & K_b & 0 & 0 & 0 \\ 0 & K_b & 0 & 0 & -G_5-K_b & G_5 & 0 & 0 \\ 0 & 0 & 0 & -G_6 & 0 & 0 & G_6+G_7 & -G_7 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -K_c*G_6 & 1 & 0 & K_c*G_6 & -1 \\ 0 & -G_3 & 0 & -G_4 & G_4+G_3+G_5 & -G_5 & -G_7 & G_7 \end{bmatrix} \cdot \begin{bmatrix} V_A \\ V_B \\ V_C \\ V_D \\ V_E \\ V_F \\ V_G \\ V_H \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ I_d \\ 0 \\ V_a \\ 0 \\ 0 \\ -I_d \end{bmatrix} \tag{10}$$

This equation solved using octave yields the following results:

Variable	Value [A or V]
VA	8.08066140340V
VB	7.82918582491V
VC	7.30649037492V
VD	2.94453891610V
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Ic	-0.00007242723A
Id	0.00103201034A

Table 2: Node Analysis Results

3 Simulation Analysis

3.1 Operating Point Analysis

Table 3 shows the simulated operating point results for the circuit under analysis. When compared to the theoretical analysis results, we see the same values up to the 5 decimal places provided by ngspice.

Name	Value [A or V]
@gb[i]	-2.53212e-04
@id[current]	1.032010e-03
@r1[i]	2.414774e-04
@r2[i]	2.532123e-04
@r3[i]	-1.17349e-05
@r4[i]	-1.20106e-03
@r5[i]	-1.28522e-03
@r6[i]	9.595831e-04
@r7[i]	9.595831e-04
a	8.080661e+00
b	7.829186e+00
c	7.306490e+00
d	2.944539e+00
e	7.864549e+00
f	1.177414e+01
g1	9.779981e-01
g2	9.779981e-01

Table 3: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

It should be noted that nodes G1 and G2 represent the same node G, and exist seperatly so as to allow the measuring of current I_c in ngspice, for the purpose of defining the dependent voltage source V_c.

4 Conclusion

In this laboratory assignment the objective of analysing a static DC circuit has been achieved. A static analysis has been performed on the circuit, through both the node analysis and mesh analysis methods, using the Octave software, and a simulation was run using ngspice. The three sets of results all match with all available decimal places of precision. The reason for this perfect match is the fact that although this circuit has multiple components and nodes, all of the components are linear, and no time dependence exists. The matching of results for the various methods also helps to confirm the accuracy of the equations used for the theoretical analysis.