

Circuit Theory and Electronics Fundamentals

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RC circuit

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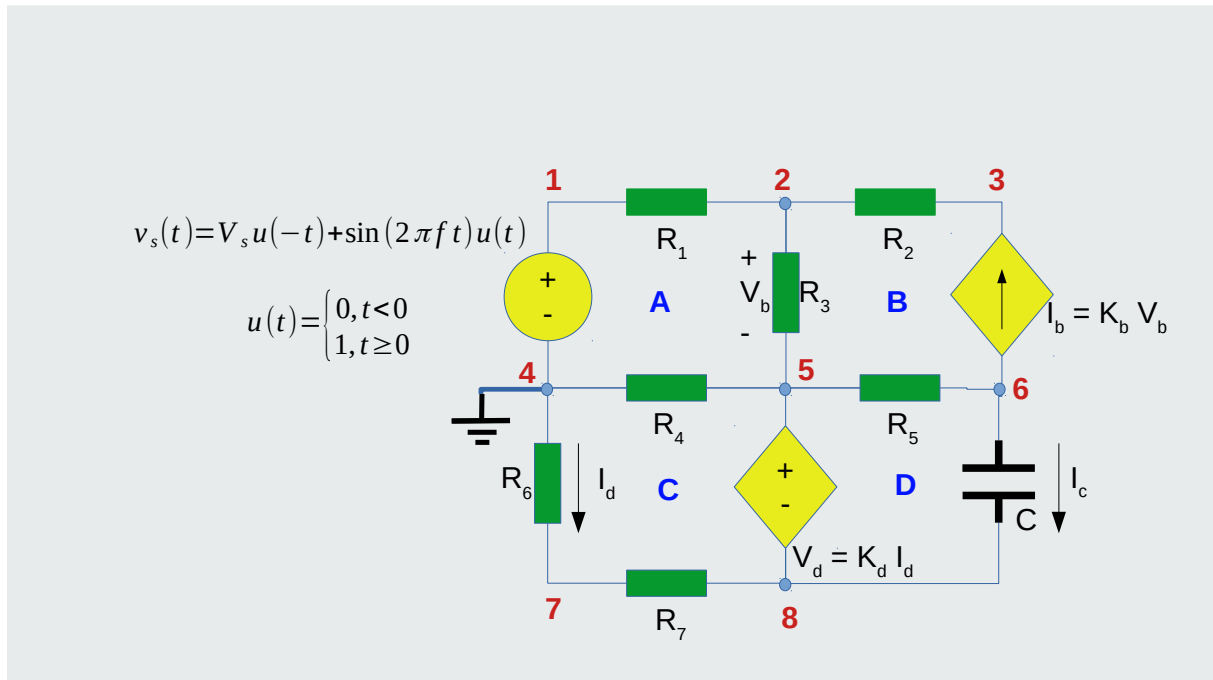
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1 Introduction

The objective of this laboratory assignment is to study a RC circuit containing a AC voltage source V_s , a capacitor C , a voltage controlled current source I_b , a current controlled voltage source V_d and resistors, R_1 , R_2 , R_3 , R_4 , R_5 , R_6 and R_7 . The circuit can be seen in Figure 1.

In Section 2, a theoretical analysis of the circuit is presented. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.



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TCFE: DEEC/Instituto Superior Técnico

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Figure 1: RC circuit to be analysed

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically, analysing the circuit for $t < 0$, calculating the equivalent resistance, determining the natural and forced solutions and superimposing them to find the total solution.

2.1 Nodal analysis

For $t < 0$, $v_s(t) = V_s(t)$, it is a DC circuit. We can determine the voltages in all nodes and currents in all branches using the nodal method. Since this is a linear circuit, we apply Ohm's Law, $V_i = R_i * I$ and the Kirchhoff Current Law (KCL), $\sum I_i = 0$.

We get the following equation, in matrix form:

$$\begin{bmatrix} -G_1 & G_1 + G_2 + G_3 & -G_2 & 0 & -G_3 & 0 & 0 & 0 \\ 0 & -G_2 - K_b & G_2 & 0 & K_b & 0 & 0 & 0 \\ 0 & K_b & 0 & 0 & -G_5 - K_b & G_5 & 0 & 0 \\ 0 & 0 & 0 & -G_6 & 0 & 0 & G_6 + G_7 & -G_7 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -K_c * G_6 & 1 & 0 & K_c * G_6 & -1 \\ 0 & -G_3 & 0 & -G_4 & G_4 + G_3 + G_5 & -G_5 & -G_7 & G_7 \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ V_s \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (1)$$

This equation solved using octave yields the following results:

Variable	Value [A or V]
V_1	5.13612248730V
V_2	4.88464690881V
V_3	4.36195145882V
V_4	-0.00000000000V
V_5	4.92000960269V
V_6	5.69027079572V
V_7	-1.96654083449V
V_8	-2.94453891610V
I_1	0.00024147744A
I_2	0.00025321233A
I_3	-0.00001173489A
I_4	-0.00120106056A
I_5	-0.00025321233A
I_6	0.00095958312A
I_7	0.00095958312A
I_S	-0.00024147744A
I_b	-0.00025321233A
I_c	-0.00000000000A
I_e	-0.00095958312A

Table 1: Node Analysis Results for $t < 0$

2.2 Equivalent resistance

Now, we have to determine the equivalent resistance R_{eq} as seen from the capacitor terminals. We take out all the independent voltage sources (make $v_s = 0$) and replace the capacitor with a voltage source $V_x = V(6) - V(8)$. The values of $V(6)$ and $V(8)$ were already obtained via nodal analysis in the previous subsection. To determine the current I_x supplied by V_x we run mesh analysis. This is necessary because the resistors are arranged in such a way that they cannot be simplified into an equivalent resistor by applying the usual equations for resistors in series and in parallel. The mesh method gives us these equations in matrix form:

$$\begin{bmatrix} R_1 + R_3 + R_4 & -R_3 & -R_4 & 0 \\ -K_b R_3 & K_b R_3 - 1 & 0 & 0 \\ -R_4 & 0 & R_4 + R_6 + R_7 - K_d & 0 \\ 0 & -R_5 & K_d & R_5 \end{bmatrix} \cdot \begin{bmatrix} I_A \\ I_B \\ I_C \\ I_D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ V_x \end{bmatrix} \quad (2)$$

This yields the following results:

Variable	Value [A or V]
V_x	8.63480971182V
I_x	0.00283856995A
R_{equiv}	3041.95770117000Ω

Table 2: Equivalent resistance

For the time constant:

$$\tau = R_{eq} \cdot C = 0.00313933181s$$

2.3 Natural solution

Using the capacitor voltage V_x for $t < 0$ as the initial condition, the natural solution of $v_{6n}(t)$ becomes:

$$v_{6n}(t) = V_x e^{\frac{-t}{R_{eq}C}} \quad (3)$$

This equation gives us the following plot in $[0,20]$ ms:

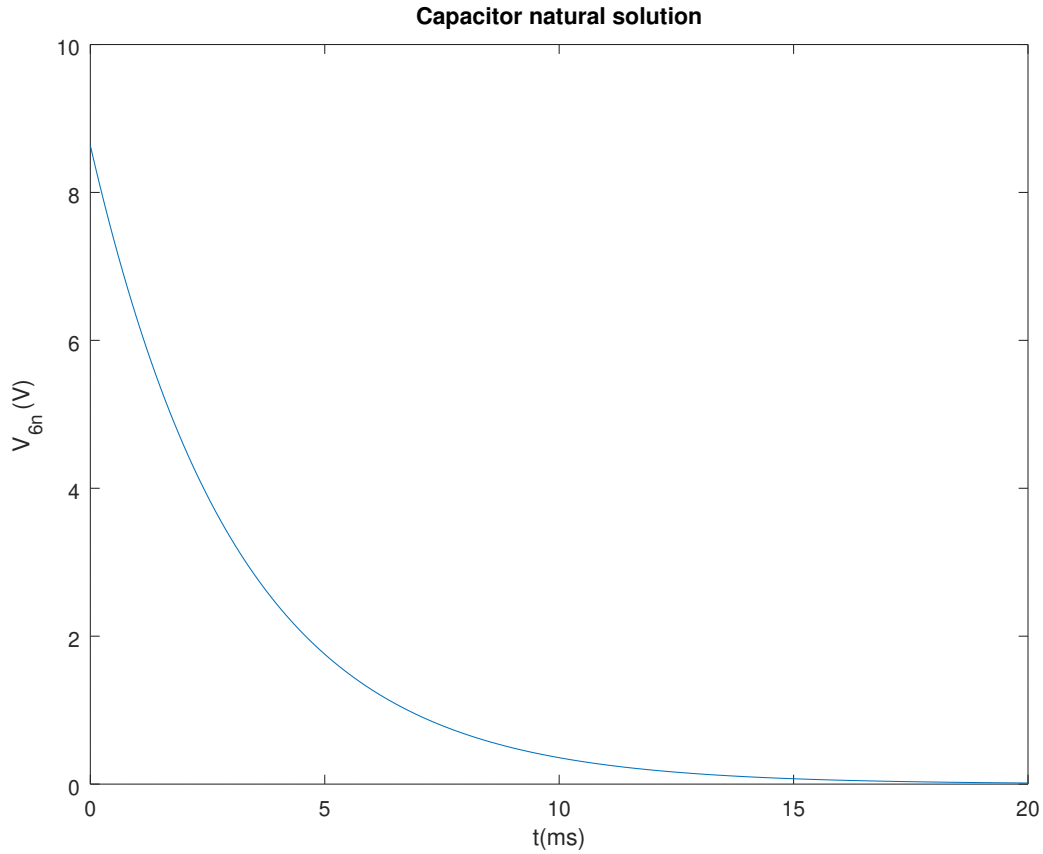


Figure 2: Natural solution $v_{6n}(t)$

2.4 Forced solution

To determine the forced solution in the same interval $[0, 20]$ ms we use a phasor voltage source V_s and replace C with its impedance Z_c .

We run nodal analysis to determine the phasor voltages in all nodes:

$$\omega = 2\pi f$$

$$\begin{bmatrix} -G_1 & G_1 + G_2 + G_3 & -G_2 & 0 & -G_3 & 0 & 0 & 0 \\ 0 & -G_2 - K_b & G_2 & 0 & K_b & 0 & 0 & 0 \\ 0 & K_b & 0 & 0 & -G_5 - K_b & G_5 + (j\omega C) & 0 & -(j\omega C) \\ 0 & 0 & 0 & -G_6 & 0 & 0 & G_6 + G_7 & -G_7 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -K_d G_6 & 1 & 0 & K_d G_6 & -1 \\ 0 & -G_3 & 0 & -G_4 & G_4 + G_3 + G_5 & -G_5 - (j\omega C) & -G_7 & G_7 + (j\omega C) \end{bmatrix} \cdot \begin{bmatrix} V_{1p} \\ V_{2p} \\ V_{3p} \\ V_{4p} \\ V_{5p} \\ V_{6p} \\ V_{7p} \\ V_{8p} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ i \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (4)$$

Solving this system of equation in octave yields the following results:

Variable	Value [A or V]
\tilde{V}_1	$(0.000000000000 + i \cdot 1.000000000000)V$
\tilde{V}_2	$(0.000000000000 + i \cdot 0.95103785412)V$
\tilde{V}_3	$(0.000000000000 + i \cdot 0.84926936022)V$
\tilde{V}_4	$(0.000000000000 + i \cdot 0.000000000000)V$
\tilde{V}_5	$(0.000000000000 + i \cdot 0.95792294963)V$
\tilde{V}_6	$(0.08501303490 + i \cdot -0.56899006711)V$
\tilde{V}_7	$(-0.000000000000 + i \cdot -0.38288433334)V$
\tilde{V}_8	$(-0.000000000000 + i \cdot -0.57329997939)V$

Table 3: Phasor voltages

2.5 Total solution

Converting the phasors to real time functions for $f=1\text{KHz}$, we can then superimpose the natural and forced solutions:

$$v_6(t) = V_x e^{\frac{-t}{R_{eq}C}} - \Re(\tilde{V}_6 e^{j\omega t})$$

This plot in the interval $[-5,20]\text{ms}$:

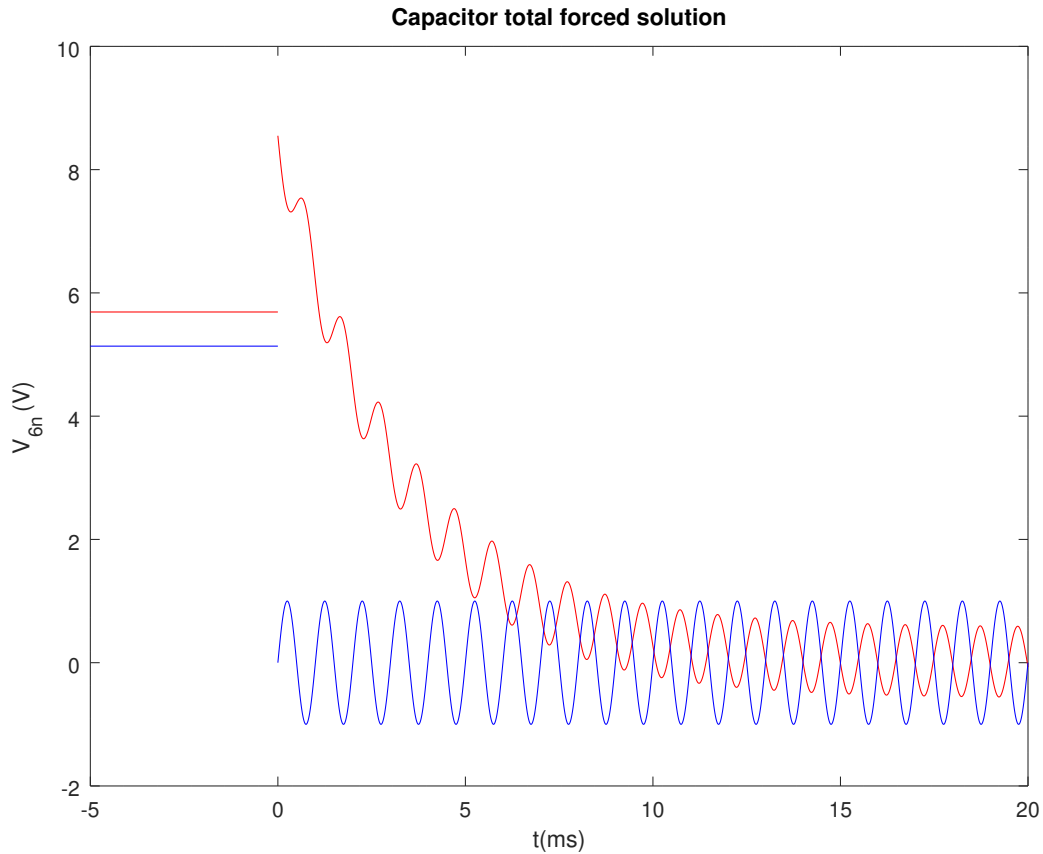


Figure 3: Final total solution $v_6(t)$

2.6 Frequency responses

The system 4 was solved for various values of f . The plots of the magnitude and phase of V_s , $V(6)$ and V_C , for values ranging from 0.1Hz to 1MHz are below:

For the frequency responses, we get the following plot:

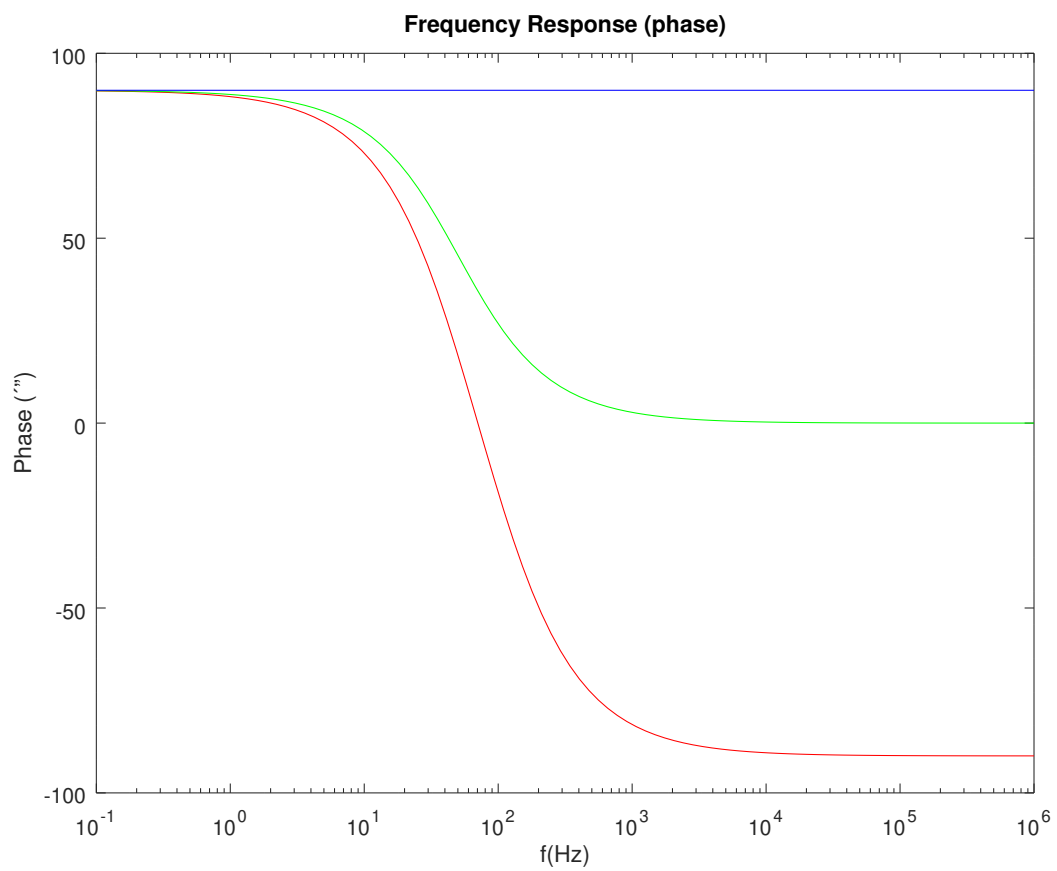


Figure 4: Frequency response-phase

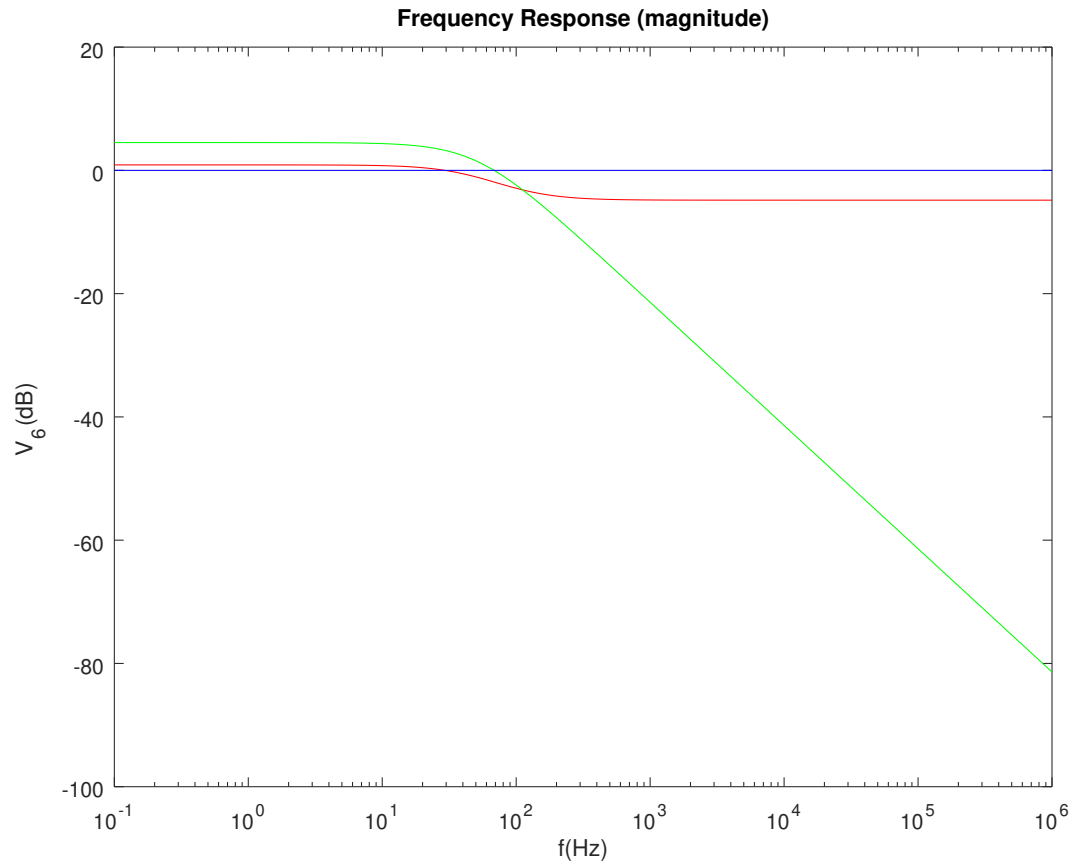


Figure 5: Frequency response-magnitude

3 Simulation Analysis

3.1 Operating Point Analysis

Table 4 shows the simulated operating point results for the circuit at times $t < 0$.

Name	Value [A or V]
@ca[i]	0.000000e+00
@gb[i]	-2.53212e-04
@r1[i]	2.414774e-04
@r2[i]	2.532123e-04
@r3[i]	-1.17349e-05
@r4[i]	-1.20106e-03
@r5[i]	-2.53212e-04
@r6[i]	9.595831e-04
@r7[i]	9.595831e-04
v(1)	5.136122e+00
v(2)	4.884647e+00
v(3)	4.361951e+00
v(5)	4.920010e+00
v(6)	5.690271e+00
v(8)	-2.94454e+00
v(71)	-1.96654e+00
v(72)	-1.96654e+00

Table 4: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

Table 5 shows the simulated operating point results for the circuit given that $V_s = 0$ and replacing the capacitor with a voltage source imposing the voltage on the terminals of said capacitor as calculated in the earlier analysis.

Name	Value [A or V]
@gb[i]	-6.24390e-18
@r1[i]	5.954528e-18
@r2[i]	6.243896e-18
@r3[i]	-2.89368e-19
@r4[i]	1.300919e-18
@r5[i]	-2.83857e-03
@r6[i]	-8.67362e-19
@r7[i]	1.165891e-21
v(1)	0.000000e+00
v(2)	-6.20107e-15
v(3)	-1.90901e-14
v(5)	-5.32907e-15
v(6)	8.634810e+00
v(8)	1.776357e-15
v(71)	1.777545e-15
v(72)	1.777545e-15

Table 5: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

This is necessary so as to provide initial conditions for the analyses made below given that V_s at time $t = 0$ is equal to 0 but the voltage difference in the terminals of the capacitor stays constant for very short time intervals.

3.2 Natural response

We will now use the values of $V(6)$ and $V(8)$ calculated above as initial conditions for a transient analysis of the natural response of the circuit when $V_s = 0$. This is represented in figure 1

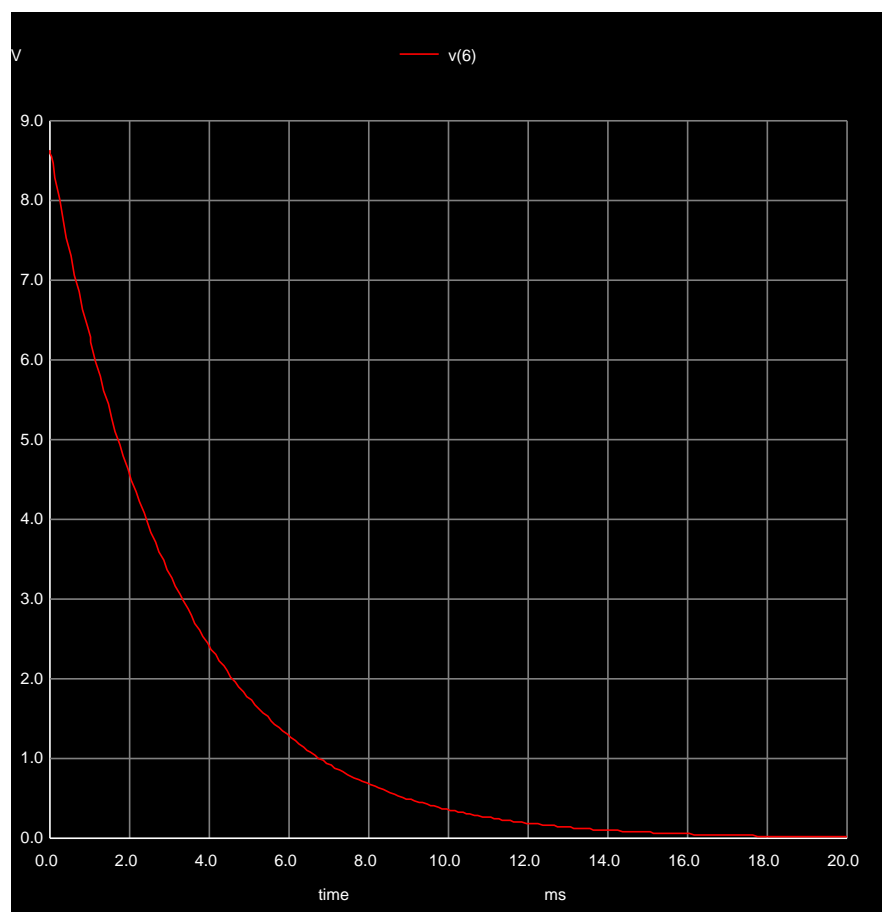


Figure 6: Natural Response

3.3 Forced response

Utilizing the same initial conditions and the value for V_s given for $t > 0$, an analysis of the forced response of the circuit over time was performed. This is represented in figure 1

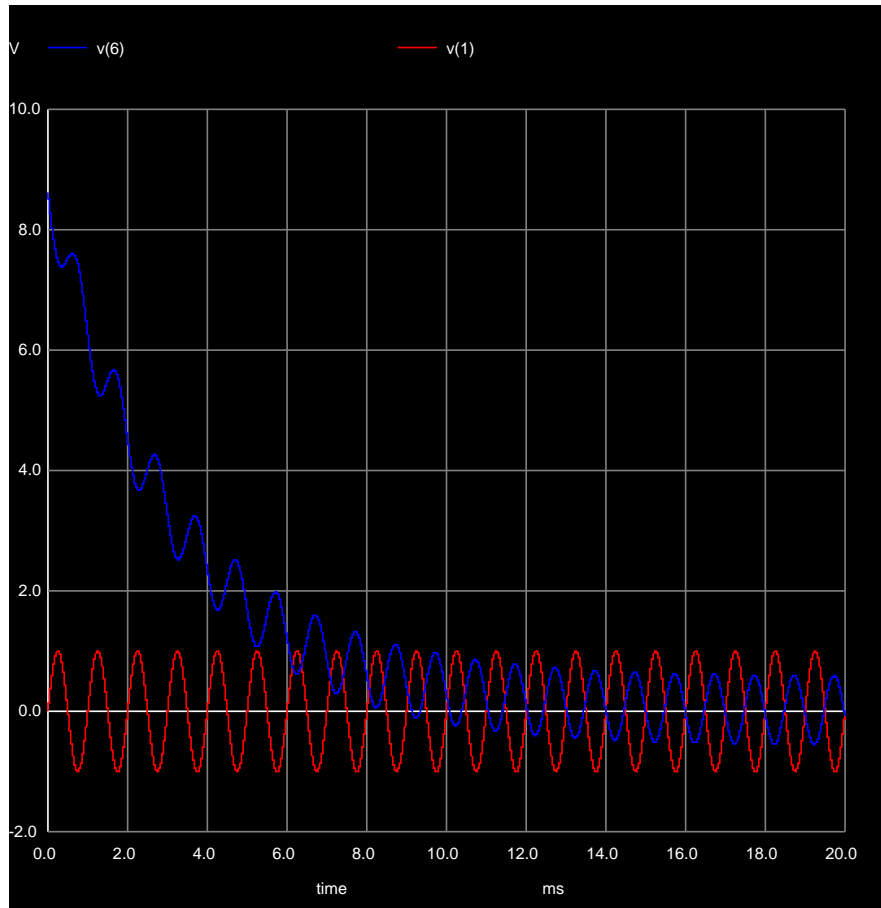


Figure 7: Forced Response

3.4 Frequency analysis

Finally, the frequency response of the circuit was studied and the magnitude and phase of both V_s and $V(6)$ was plotted for values of f from 0.1Hz to 1MHz .

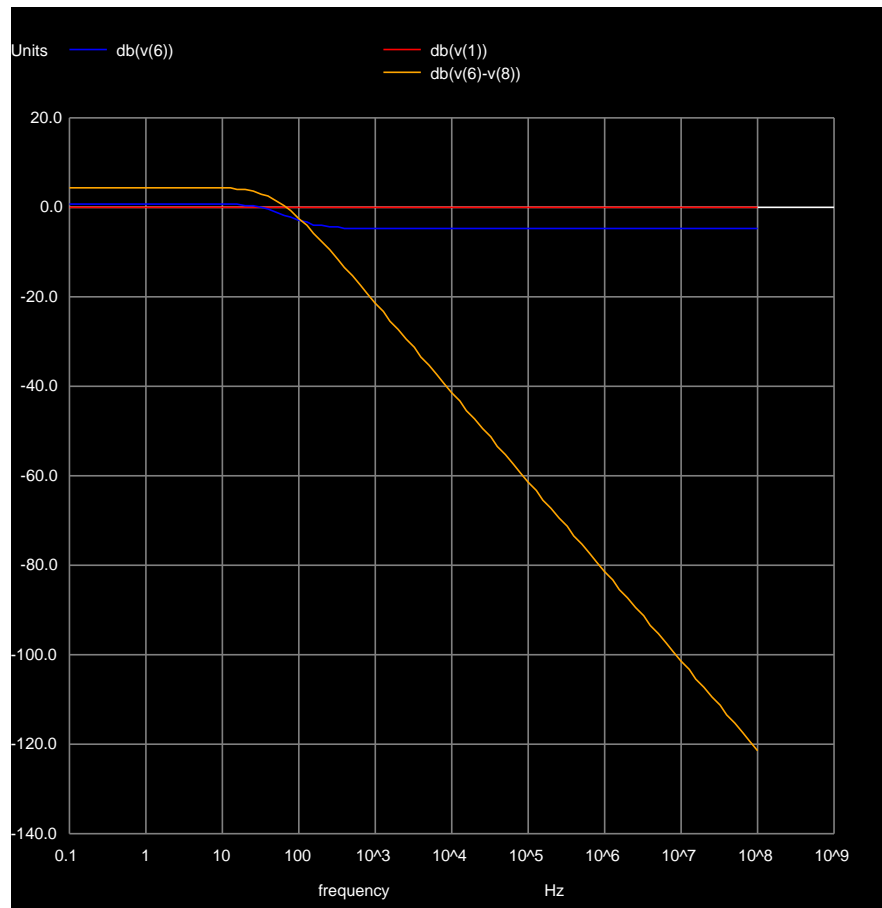


Figure 8: Frequency Response - Magnitude

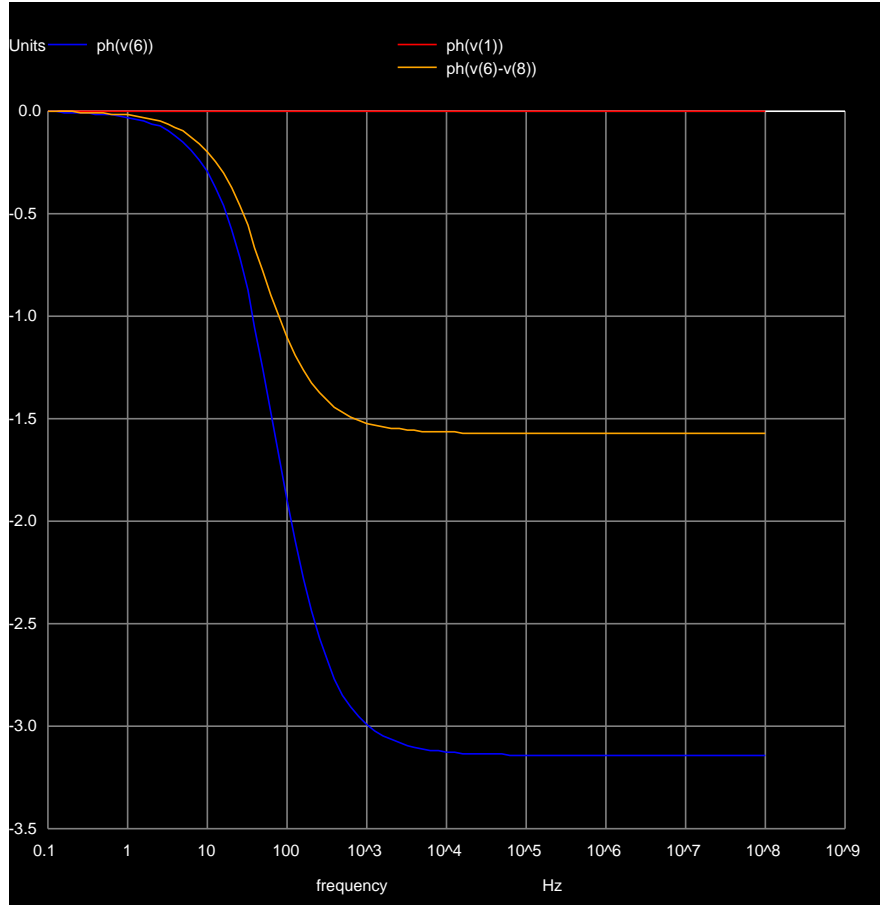


Figure 9: Frequency Response - Magnitude

The magnitude of the voltage V_s is always 1 (or 0 in dB), and its phase is always 0, by definition. As we can see, the magnitude of $V(6)$ drops sharply between the orders of magnitude of 10^1 to 10^3 , stabilizing at a value of around $-4.9dB$. The phase starts off being close to 0, but deviates to negative values, stabilizing at $-\pi$.

4 Conclusion

In this laboratory assignment the objective of analysing both the static solution of a circuit with fixed applied voltage and a capacitor, and the time-dependent solution of the same circuit has been successful. The results from both the theoretical analysis using octave and the circuit simulation using ngspice appear to match, for both the static analyses for $t < 0$ and for the calculation of boundary conditions, and for the natural, forced, and frequency responses.