

Functional and Logic Programming

Bachelor in Informatics and Computing Engineering
2021/2022 - 1st Semester

Introduction to Prolog

Agenda

- Prolog
- Facts and Rules
- Queries
- How Prolog works
- Arithmetic
- Recursion
 - Recursion
 - Recursion
- Lists

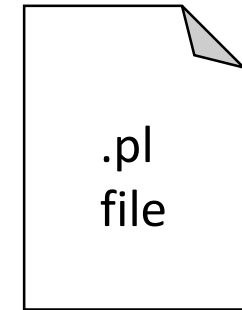
Prolog

- Prolog is the most widely used logic programming language
 - There are some language dialects, such as Edinburgh Prolog, and also a standardization - ISO Prolog
- There are several Prolog systems, both free and commercial
 - Some of the most popular are SICStus Prolog and SWI-Prolog
 - Another notable system is YAP, developed at DCC, FCUP

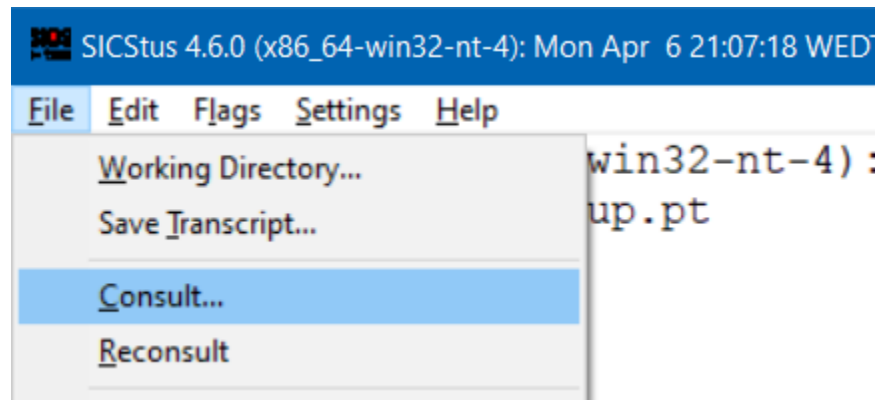
In this course, we'll be using SICStus Prolog v 4.7
(link to installer and keys are available in Moodle)

Prolog

- Write your code in a text file with a .pl extension
 - Use the text editor of your choice
- In SICStus, load it using the *File -> Consult...* Menu
 - Or call directly on SICStus:



```
| ?- consult('path/to/file.pl').
```



Alternatively, you can use SPIDER
(SICStus Prolog IDE, based on Eclipse)

Prolog Programs

- A Prolog program is a finite set of predicates
 - Predicates use facts and rules to express knowledge as relations
 - Relations are generalizations of functions
 - Usually more versatile, usable in multiple directions
- A computation is a proof of a goal from a program
 - Using [a form of] SLD resolution with a unification algorithm
- A **correct** program does not allow the deduction of unwanted facts
- A **complete** program allows the deduction of everything intended

Terms

- Everything in Prolog is a *term*, which can be a *constant*, a *variable* or a *compound term*
- **Constants** represent elementary objects
 - **Numbers**
 - **Integers** (eg. 4, -8) (bases other than decimal can also be used, eg. 8'755)
 - **Floats** (eg. 1.5, -1.6) (also supports exponent, eg. 23.4E-2)
 - **Atoms**
 - Start with lower-case letter (eg. john_doe, johnSmith42)
 - String within single quotes (eg. 'John Doe', 'John Smith 42')

Terms

- **Variables** act as placeholders for arbitrary terms
 - Start with a capital letter (eg. Variable1)
 - Start with an underscore (eg. _Var2)
 - Single underscore (_) (anonymous variable)
- **Compound terms** are comprised of a *functor* and *arguments* (which are terms)
 - The functor is characterized by its *name* (an atom) and *arity* (the number of arguments), usually represented as *name/arity*
 - Eg. point/2 represents a functor named *point* with two arguments
 - point(4, 2) is a possible instance of point/2, and so is point(foo, point(3, bar))

Agenda

- Prolog
- **Facts and Rules**
- Queries
- How Prolog works
- Arithmetic
- Recursion
 - Recursion
 - Recursion
- Lists

Facts

- Facts express a relation that is true
 - You can (kind of) interpret them as lines in a database table

Statements end with a period

<code>male(homer) .</code>	<code>% homer is a male</code>
<code>female(marge) .</code>	<code>% marge is a female</code>
<code>father(homer, bart) .</code>	<code>% homer is the father of bart</code>
<code>mother(marge, bart) .</code>	<code>% marge is the mother of bart</code>

Arguments between parentheses and separated by commas

Predicate (relation) names start with lowercase letter

Semantics

- The semantics (interpretation) needs to be defined and shared

```
father(homer, bart).      % homer is the father of bart
father(homer, bart).      % the father of homer is bart
```

- This inherent ambiguity only highlights the importance of using appropriate and descriptive names as well as code comments

```
% single-line comment
```


```
/* multi-line
   comment */
```

Naming conventions and code comments represent a part of the evaluation of the practical assignment

Rules

- Rules allow for the deduction of new knowledge from existing knowledge (facts and other rules)
 - Rules are expressed in the form of Horn Clauses:
 - Head :- Body

```
grandfather(X, Y) :- father(X, Z), parent(Z, Y).    % X is the grandfather of Y
                                                    % if X is the father of Z
                                                    % and Z is a parent of Y
```



%multiple definitions of a rule with the same head: rule one **or** rule two **or**...

```
parent(X, Y) :- father(X, Y).    % X is a parent of Y if X is the father of Y
parent(X, Y) :- mother(X, Y).   % X is a parent of Y if X is the mother of Y
```

Disjunction

- Disjunction can also be expressed with the ; operator

```
parent(X, Y):- father(X, Y).      % X is a parent of Y if X is the father of Y
parent(X, Y):- mother(X, Y).      % X is a parent of Y if X is the mother of Y
```

% is equivalent to

```
parent(X, Y):- father(X, Y) ; mother(X, Y)
```

- The disjunction operator (;) should be used sparingly
 - Always use parentheses to clarify

Rules

- Rules have both a declarative and a procedural interpretation
 - Declarative interpretation

<code>grandfather(X, Y) :-</code>	<code>% X is the grandfather of Y</code>
<code> father(X, Z),</code>	<code>% if X is the father of Z</code>
<code> parent(Z, Y) .</code>	<code>% and Z is a parent of Y</code>

- Procedural interpretation

<code>grandfather(X, Y) :-</code>	<code>% to solve grandfather(X,Y)</code>
<code> father(X, Z),</code>	<code>% first solve father(X, Z)</code>
<code> parent(Z, Y) .</code>	<code>% and then parent(Z, Y)</code>
	<code>% (solve = execute)</code>

Rules

- The head of a rule can have 0 or more arguments

```
parent(X, Y):- father(X, Y).      % X is a parent of Y if X is the father of Y

fathers:- father(X, Y).           % fathers is true if there is a(t least one)
                                  % father/child relation
```

A rule with no arguments is a good entry point to a program

Variables in Programs

- Variables are universally instantiated in logic programs

```
plus(0, S, S).           % 0 is the neutral element of addition
mult(1, V, V).           % 1 is the neutral element of multiplication
```

```
human(Homer).            % everything is human
father(homer, Bart)       % homer is the father of everything
```

```
grandfather(X, Y) :- father(X, Z), parent(Z, Y).
```

Variables occurring only in the body of a rule can be seen as existentially quantified

We need to be careful when using variables with facts

Agenda

- Prolog
- Facts and Rules
- **Queries**
- How Prolog works
- Arithmetic
- Recursion
 - Recursion
 - Recursion
- Lists

Queries

- Computations in Prolog start with a question, which has two possible answers:
 - Yes (possible with answer substitution - variable binding)
 - No
- The attempt to prove the question right/wrong (is it a consequence of the program?) produces the computations

```
| ?- male(homer) .  
yes
```

```
| ?- father(homer, bart) .  
yes
```

```
| ?- female(marge) .  
yes
```

```
| ?- father(marge, bart) .  
no
```

Variables in Queries

- Queries can include variables
 - Variables are existentially quantified in queries
- A variable starting with an underscore is a '*don't care*'

```
| ?- father(X, bart).  
X = homer ?  
yes
```

```
| ?- father(_X, bart).  
yes
```

```
| ?- male(_).  
yes
```

```
| ?- male(X).  
X = homer ?  
yes
```

```
| ?- male(X).  
X = homer ? ;  
X = bart ? n  
no
```

If satisfied with the answer, just hit enter
If you want another answer, type 'n', 'no' or ';'.

Variables and Compound Queries

- Queries can be more complex, combining goals
- Variables are used to glue together the different goals
 - Underscore alone (`_`) is the exception

```
| ?- male(X), parent(X, bart).  
X = homer ? ;  
no
```

```
| ?- male(_X), parent(_X, bart).  
yes
```

```
| ?- male(_X), parent(Y, bart).  
Y = homer ? ;  
Y = marge ? ;  
Y = homer ? ;  
Y = marge ? ;  
no
```

Why the duplicates?
Just wait a few slides!

Closed World Assumption

- Assumption that everything that is true is known to be true (ie, is represented as a clause in the program)
- Therefore, everything that cannot be deduced from the clauses in the program is assumed to be false

```
| ?- male(donald) .  
no
```

- Requires attention to make sure everything we want to deduce can be deduced from the program clauses

Coding Efficiency Considerations

- Use implicit unification instead of additional variables

```
change_player(X, Y) :- X = 1, Y = 2.  
change_player(X, Y) :- X = 2, Y = 1.
```

Should instead be written as

```
change_player(1, 2).  
change_player(2, 1).
```

- Always place input arguments before output arguments
 - SICStus indexes predicates by their first argument

= is the unification operator (kind of '[possibly] equal');
\= (not unifiable) can be interpreted as 'can't be equal'

Coding Style Considerations

- Although white space and code indentation are meaningless, there are some coding style guidelines you should consider following, to increase code readability:
 - Indent the code consistently
 - Put each sub-goal on a separate, indented line
 - Use human-readable names for predicates and variables
 - Try to limit the length of code lines and number of lines per clause
 - ...

See Covington et al. (2012). Coding Guidelines for Prolog. Theory and Practice of Logic Programming, 12(6): 889-927

Horn Clauses

- Everything in Prolog is expressed as a Horn Clause

- Rules are complete horn clauses (head :- body)

```
parent(X, Y) :- father(X, Y) .
```

- Fact are horn clauses where the body is always true (just the head)

```
male(homer) :- true           ⇔           male(homer) .
```

- Queries are horn clauses without a head (just the body)

```
| ?- father(X, bart) .
```

Predicates

- A **predicate** is a set of clauses for the same functor
 - **Clauses** are either facts or rules
- Functors with the same name but different arity are different predicates

```
father(X) :- father(X, Y).    % X is a father
                               % if X is the father of some Y
```


Documentation

- Documentation should include a **mode declaration** for each argument:
 - + (input): the argument is instantiated when the predicate is called
 - - (output): the argument is not instantiated in the predicate call
 - ? (in/out): the argument can be instantiated or not

```
% square(+number, -square)
```

```
% calculates the square  
% of a given number
```

```
% parent(?parent, ?child)
```

```
% parent/child relation
```

- One of the most powerful properties of Prolog is its versatility

Prolog Versatility

- The versatility of Prolog can be seen in most predicates:
 - For instance, parent/2 allows:
 - Confirming that two given people are parent/child
 - Obtaining the children of a given person
 - Obtaining the parents of a given person
 - Obtaining all parent/child pairs
- In most other languages, we would need to implement four different functions to achieve this, or include extra logic to test instantiation

Agenda

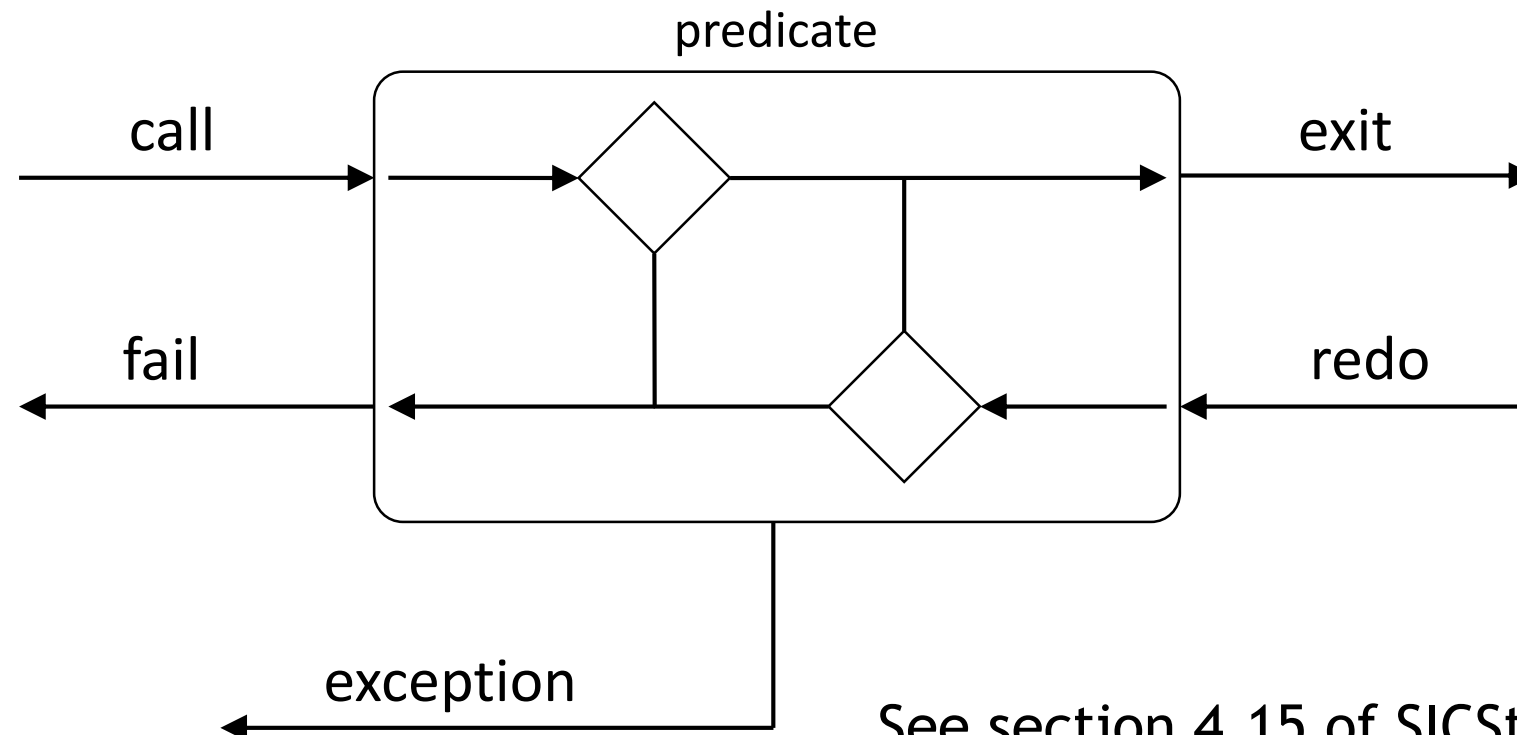
- Prolog
- Facts and Rules
- Queries
- **How Prolog works**
- Arithmetic
- Recursion
 - Recursion
 - Recursion
- Lists

How Prolog Works

- Prolog's mechanics work
 - Top to bottom
 - The order of clauses is important
 - Left to right
 - In rules, prove sub-goals in left-to-right order
 - With backtracking
 - If a sub-goal fails, go back to previous decision point

The Prolog Box Model

- Each call to a goal can be modelled as a four-gate box model



See section 4.15 of SICStus's Manual
for more information on exceptions

Tracing

- Trace mode allows us to follow the computations step by step
 - Can be activated from the menu Flags -> Debugging -> trace
 - Or in the code, by calling *trace*
 - Disable it by calling *notrace*

```
foo(bar) :- fubar(bar, baz),  
            trace,                % activate trace mode  
            qux(baz),             % the call to qux will be traced  
            notrace,             % deactivate trace mode  
            quux(bar) .
```

See section 5 of SICStus's Manual for
more information on Trace and Debugging

Tracing

- Trace message format:

N S InvID Depth Port: Goal ?

- N (only visible at Exit ports) indicates that the goal call may backtrack to find alternative solutions
- S indicates the existence of a spypoint
- InvID (Invocation ID) is a unique identifier for each goal (can be used to match messages from the various ports)
- Depth is an indication of the general call depth
- Port is one of Call, Exit, Redo, Fail or Exception
- Goal is the current goal of the computation

Agenda

- Prolog
- Facts and Rules
- Queries
- How Prolog works
- **Arithmetic**
- Recursion
 - Recursion
 - Recursion
- Lists

Arithmetic

- Arithmetic expressions are not evaluated immediately
 - Example: $A = 4+2$ unifies A with the term $+(4, 2)$, not the value 6
- The *is* predicate can be used to evaluate an arithmetic expression
- The right-side of *is* needs to be instantiated

```
| ?- A = 4+2.
A = 4+2 ?
yes
| ?- B is 4+2.
B = 6 ?
yes
| ?- 6 is 4+2.
yes
| ?- 4+2 is 4+2.
no
```

```
| ?- C is 4+B.
! Instantiation error in argument 2 of (is)/2
! goal:  _419 is 4+_427
```

See section 4.7 of the SICStus Manual for more information on Arithmetic

Arithmetic

- Arithmetic expressions can be compared for (in)equality
 - $\text{Expr1} ::= \text{Expr2}$ evaluates both expressions and if they are equal
 - $\text{Expr1} \neq \text{Expr2}$ evaluates both expressions and if they are different
 - Comparison

$E1 < E2$ $E1 > E2$ $E1 \leq E2$ $E1 \geq E2$

- Prolog can also compare and order terms

$T1 @< T2$ $T1 @> T2$ $T1 @\leq T2$ $T1 @\geq T2$

- $\text{Term1} == \text{Term2}$ verifies whether the two terms are literally identical
- $\text{Term1} \neq \text{Term2}$ checks if the two terms are not literally identical

Arithmetic

- There are several functions available
 - $X+Y$, $X-Y$, $X*Y$, X/Y (float quotient)
 - $X//Y$ is the integer quotient, truncated towards 0
 - $X \text{ div } Y$ is the integer quotient (rounded down)
 - $X \text{ rem } Y$ is integer remainder: $X-Y*(X//Y)$
 - $X \text{ mod } Y$ is integer remainder: $X-Y*(X \text{ div } Y)$
 - Many other functions
 - $\text{round}(X)$, $\text{truncate}(X)$, $\text{floor}(X)$, $\text{ceiling}(X)$
 - $\text{abs}(X)$, $\text{sign}(X)$, $\text{min}(X, Y)$, $\text{max}(X, Y)$
 - $\text{sqrt}(X)$, $\text{log}(X)$, $\text{exp}(X)$, $X ** Y$, $X ^ Y$
 - $\text{sin}(X)$, $\text{cos}(X)$, $\text{tan}(X)$, ...

```
| ?- A is 5 // 2.
A = 2 ?
yes
| ?- A is -5 // 2.
A = -2 ?
yes
| ?- A is 5 div 2.
A = 2 ?
yes
| ?- A is -5 div 2.
A = -3 ?
yes
| ?- A is 5 rem 2.
A = 1 ?
yes
| ?- A is -5 rem 2.
A = -1 ?
yes
| ?- A is 5 mod 2.
A = 1 ?
yes
| ?- A is -5 mod 2.
A = 1 ?
yes
```

Natural Numbers

- Arithmetic in Prolog deviates from pure Logic Programming
 - It is, however, necessary for efficiency
- A more ‘*logical*’ representation of (natural) numbers
 - 0 is natural
 - The successor of X - $s(X)$ - is natural if X is natural
 - 0, $s(0)$, $s(s(0))$, $s(s(s(0)))$, ...

```
natural_number(0) .  
natural_number(s(X)) :- natural_number(X) .
```

Adding Natural Numbers

- Addition can then be seen as a ternary relation

```
% plus(X, Y, Z) : X+Y = Z
```

```
plus(0, X, X) :-  
    natural_number(X).
```

```
plus(s(X), Y, s(Z)) :-  
    plus(X, Y, Z).
```

```
| ?- plus( s(s(0)), s(0), Z) .  
Z = s(s(s(0))) ?  
yes  
| ?- plus( s(s(0)), Y, s(s(s(0)))) .  
Y = s(0) ?  
yes  
| ?- plus( X, s(0), s(s(s(0)))) .  
X = s(s(0)) ?  
yes  
| ?- plus( X, Y, s(s(0))) .  
X = 0,  
Y = s(s(0)) ? ;  
X = s(0),  
Y = s(0) ? ;  
X = s(s(0)),  
Y = 0 ? ;  
no
```

Agenda

- Prolog
- Facts and Rules
- Queries
- How Prolog works
- Arithmetic
- **Recursion**
 - Recursion
 - Recursion
- Lists

Recursion

- Some relations are recursive

```
ancestor(X, Y) :-                % X is an ancestor of Y
    parent(X, Y) .               % if X is a parent of Y

ancestor(X, Y) :-                % X is an ancestor of Y
    parent(X, Z) ,               % if X is a parent of Z
    ancestor(Z, Y) .             % and Z is an ancestor of Y
```

- Recursion is based on the inductive proof

- One or more base clauses
- One or more recursion clauses

The order of clauses and goals may influence performance, or even cause infinite computations

Recursion

- Example: sum all numbers between 1 and N

```
sumN(0, 0).                                     % Base clause

sumN(N, Sum) :- N > 0,                          % Guard - make sure we don't
                                                    % have infinite recursion
    N1 is N-1,
    sumN(N1, Sum1),
    Sum is Sum1 + N.                            % Recursive call
```


Recursion

- Example: sum all numbers between 1 and N

```
sumN(0, 0).
```

```
sumN(N, Sum) :- N > 0,
```

```
    N1 is N-1,
    sumN(N1, Sum1),
    Sum is Sum1 + N.
```

```
?- sumN(2, Sum).
1      1 Call: sumN(2,_925) ?
2      2 Call: 2>0 ?
2      2 Exit: 2>0 ?
3      2 Call: _1935 is 2-1 ?
3      2 Exit: 1 is 2-1 ?
4      2 Call: sumN(1,_1955) ?
5      3 Call: 1>0 ?
5      3 Exit: 1>0 ?
6      3 Call: _6589 is 1-1 ?
6      3 Exit: 0 is 1-1 ?
7      3 Call: sumN(0,_6609) ?
?      7      3 Exit: sumN(0,0) ?
8      3 Call: _1955 is 0+1 ?
8      3 Exit: 1 is 0+1 ?
?      4      2 Exit: sumN(1,1) ?
9      2 Call: _925 is 1+2 ?
9      2 Exit: 3 is 1+2 ?
?      1      1 Exit: sumN(2,3) ?
Sum = 3 ?
```

Tail Recursion

- Tail Recursion can increase efficiency
 - Add a new argument to the predicate: the accumulator
 - Make the recursive call the last call

```
sumN(N, Sum) :- sumN(N, Sum, 0).           % Encapsulate
sumN(0, Sum, Sum).                         % Base case - the result is
                                           %      in the accumulator

sumN(N, Sum, Acc) :- N > 0,
                    N1 is N-1,
                    Acc1 is Acc + N,
                    sumN(N1, Sum, Acc1).    % Recursive call is now
                                           %      the last sub-goal
```

To increase efficiency, we actually need to add a *cut* in the base clause - we'll see this operator 'next' week

Tail Recursion

```
| ?- trace, sumN(2, S), notrace.
% The debugger will first creep -- showing everything
1      1 Call: sumN(2,_941) ?
2      2 Call: 2>0 ?
2      2 Exit: 2>0 ?
3      2 Call: _2067 is 2-1 ?
3      2 Exit: 1 is 2-1 ?
4      2 Call: sumN(1,_2087) ?
5      3 Call: 1>0 ?
5      3 Exit: 1>0 ?
6      3 Call: _6721 is 1-1 ?
6      3 Exit: 0 is 1-1 ?
7      3 Call: sumN(0,_6741) ?
?      7      3 Exit: sumN(0,0) ?
8      3 Call: _2087 is 0+1 ?
8      3 Exit: 1 is 0+1 ?
?      4      2 Exit: sumN(1,1) ?
9      2 Call: _941 is 1+2 ?
9      2 Exit: 3 is 1+2 ?
?      1      1 Exit: sumN(2,3) ?
10     1 Call: notrace ?
% The debugger is switched off
S = 3 ?
yes
```

```
| ?- trace, sumN(2, S, 0), notrace.
% The debugger will first creep -- showing
1      1 Call: sumN(2,_941,0) ?
2      2 Call: 2>0 ?
2      2 Exit: 2>0 ?
3      2 Call: _2111 is 2-1 ?
3      2 Exit: 1 is 2-1 ?
4      2 Call: _2129 is 0+2 ?
4      2 Exit: 2 is 0+2 ?
5      2 Call: sumN(1,_941,2) ?
6      3 Call: 1>0 ?
6      3 Exit: 1>0 ?
7      3 Call: _8679 is 1-1 ?
7      3 Exit: 0 is 1-1 ?
8      3 Call: _8697 is 2+1 ?
8      3 Exit: 3 is 2+1 ?
9      3 Call: sumN(0,_941,3) ?
9      3 Exit: sumN(0,3,3) ?
5      2 Exit: sumN(1,3,2) ?
1      1 Exit: sumN(2,3,0) ?
10     1 Call: notrace ?
% The debugger is switched off
S = 3 ?
yes
```

Agenda

- Prolog
- Facts and Rules
- Queries
- How Prolog works
- Arithmetic
- Recursion
 - Recursion
 - Recursion
- **Lists**

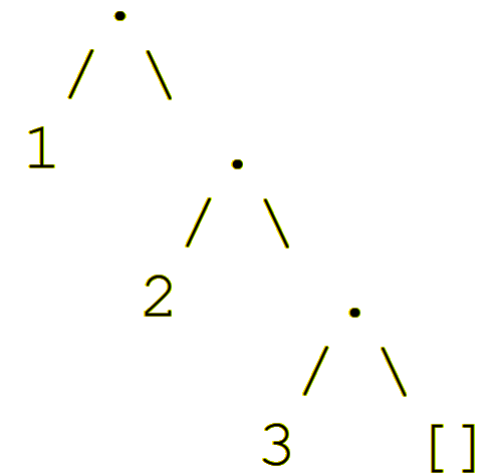
Lists

- Lists are the quintessential data structure in Prolog
- Empty list represented as []
- Elements separated by commas within square brackets
 - [a, b, c]
 - [4, 8, 15, 16, 23, 42]
- Lists elements can be anything, including other lists
 - [2, [a, b, c], [3, [x, y], 4], 5]

Lists

- The internal representation uses the `.` functor and two arguments - the head and tail of the list
 - Ex.: `[1, 2, 3] = .(1, .(2, .(3, [])))`

```
| ?- A = .(1, .(2, .(3, [])) ).
A = [1,2,3] ?
yes
```



- Strings are a representation of lists of character ASCII codes

```
| ?- A = "Hello".
A = [72,101,108,108,111] ?
yes
```

Lists

- Easily separate the head of the list from the rest of the list
 - The head of the list can separate more than one element

```
[ H | T ] % where T is a list with the remaining elements of the list
[ 4 ] = [ 4 | [ ] ] % tail of list with one element is empty list
[4, 8, 15, 16, 23, 42] = [4 | [8, 15, 16, 23, 42] ]
[4, 8, 15, 16, 23, 42] = [ 4, 8 | [ 15, 16, 23, 42] ]
```

- Definition of what is a list

- An empty list
- A list construct where tail is a list

```
is_list( [ ] ).
```

```
is_list( [H|T] ):- is_list(T).
```

List Length

- There are several useful built-in predicates to work with lists
 - ***length(?List, ?Size)***
 - Size of a list (very flexible)

Can also be easily implemented recursively

```
length( [], 0 ).  
length( [_|T], L ):-  
    length(T, L1),  
    L is L1+1.
```

Actually, there's a small caveat with this solution. Can you find it? (homework)

```
| ?- length([1,2,3], 3).  
yes  
| ?- length([1,2,3], L).  
L = 3 ?  
yes  
| ?- length(L, 3).  
L = [_A,_B,_C] ?  
yes  
| ?- length(L, S).  
L = [],  
S = 0 ? ;  
L = [_A],  
S = 1 ? ;  
L = [_A,_B],  
S = 2 ? ;  
L = [_A,_B,_C],  
S = 3 ?  
yes
```


List Membership

- ***member(?Elem, ?List)***
 - List member (very flexible)
- ***memberchk(?Elem, ?List)***
 - Member verification (determinate)

Can also be easily implemented recursively

```
member( X, [X|_] ).  
member( X, [_|T] ) :-  
    member(X, T) .
```

```
memberchk( X, [X|_] ) .  
memberchk( X, [Y|T] ) :-  
    X \= Y,  
    memberchk(X, T) .
```

```
| ?- member(2, [1,2,3]) .  
yes  
| ?- member(2, L) .  
L = [2|_A] ? ;  
L = [_A,2|_B] ?  
yes  
| ?- member(M, [1,2]) .  
M = 1 ? ;  
M = 2 ? ;  
no  
| ?- member(M, L) .  
L = [M|_A] ? ;  
L = [_A,M|_B] ?  
yes
```

Appending Lists

- ***append(?L1, ?L2, ?L3)***
 - Appends two lists into a third (very flexible)

Can also be easily implemented recursively

```
append( [ ], L2, L2 ).  
append( [H|T], L2, [H|T3] ):-  
    append( T, L2, T3 ).
```

```
| ?- append([1,2], [3,4], [1,2,3,4]).  
yes  
| ?- append([1,2], [3,4], L).  
L = [1,2,3,4] ?  
yes  
| ?- append([1,2], L, [1,2,3,4]).  
L = [3,4] ?  
yes  
| ?- append(L, [3,4], [1,2,3,4]).  
L = [1,2] ?  
yes  
| ?- append(L1, L2, [1,2,3]).  
L1 = [],  
L2 = [1,2,3] ? ;  
L1 = [1],  
L2 = [2,3] ? ;  
L1 = [1,2],  
L2 = [3] ? ;  
L1 = [1,2,3],  
L2 = [] ? ;  
no
```

Sorting Lists

- ***sort(+List, -SortedList)***
 - Sorts a (proper) list
- ***keysort(+PairList, -SortedList)***
 - Sorts a (proper) key-value pair list
 - If a key appears more than once, elements retain original order

```
| ?- sort([4,2,3,1], [1,2,3,4]).  
yes  
| ?- sort([4,2,3,1], SL).  
SL = [1,2,3,4] ?  
yes  
| ?- keysort([2-1, 1-2, 4-3, 3-4], SL).  
SL = [1-2,2-1,3-4,4-3] ?  
yes  
| ?- keysort([2-1, 1-2, 4-3, 3-4, 1-1], SL).  
SL = [1-2,1-1,2-1,3-4,4-3] ?  
yes
```

Can also be implemented recursively
Homework!

Lists Library

- The Lists library has numerous predicates to work with lists
- Libraries can be imported using the *use_module* directive:

```
:-use_module(library(lists)).
```

See section 10.24 of the SICStus Manual for a complete description of available predicates

Lists Library

- Some useful predicates from the lists library
 - nth0(?Pos, ?List, ?Elem) / nth1(?Pos, ?List, ?Elem)
 - nth0(?Pos, ?List, ?Elem, ?Rest) / nth1(?Pos, ?List, ?Elem, ?Rest)
 - select(?X, ?XList, ?Y, ?YList)
 - delete(+List, +ToDel, -Rest) / delete(+List, +ToDel, + Count, -Rest)
 - last(?Init, ?Last, ?List)
 - segment(?List, ?Segment)
 - sublist(+List, ?Part, ?Before, ?Length, ?After)

Lists Library

- `append(+ListOfLists, -List)`
- `reverse(?List, ?Reversed)`
- `rotate_list(+Amount, ?List, ?Rotated)`
- `transpose(?Matrix, ?Transposed)`
- `remove_dups(+List, ?PrunedList)`
- `permutation(?List, ?Permutation)`
- `sumlist(+ListOfNumbers, ?Sum)`
- `max_member(?Max, +List) / min_member(?Min, +List)`
- `max_member(:Comp, ?Max, +List) / min_member(:Comp, ?Min, +L)`

Lists Library

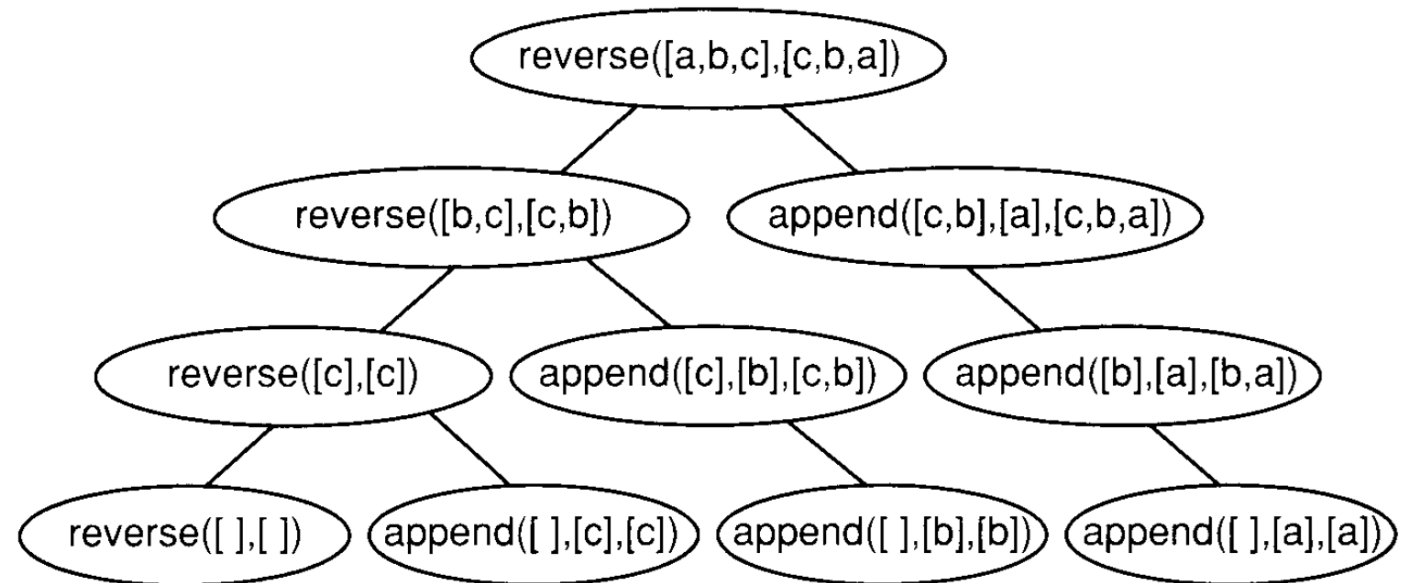
- `maplist(:Pred, +L) / maplist(:Pr, +L1, ?L2) / maplist(:Pr, +L1, ?L2, ?L3)`
- `map_product(:Pred, +Xs, +Ys, ?List)`
- `scanlist(:Pred, +Xs, ?Start, ?Final)`
- `cumlist(:Pred, +Xs, ?Start, ?List)`
- `some(:Pred, +List) / some(:Pred, +Xs, ?Ys) / some(:Pr, +Xs, ?Ys, ?Zs)`
- `include(:P, +X, ?L) / include(:P, +X, +Y, ?L) / include(:P, +X, +Y, +Z, ?L)`
- `exclude(:P, +X, ?L) / exclude(:P, +X, +Y, ?L) / exclude(:P, +X, +Y, +Z, ?L)`
- `group(:Pred, +List, ?Front, ?Back)`

Lists

- Several of these predicates can be implemented using append
 - However, sometimes we can find more efficient versions
- Example: list reverse

```
reverse([], []).
reverse([X|Xs], Rev) :-
    reverse(Xs, Ys),
    append(Ys, [X], Rev).
```

- Size of proof tree is quadratic to the number of elements in the list

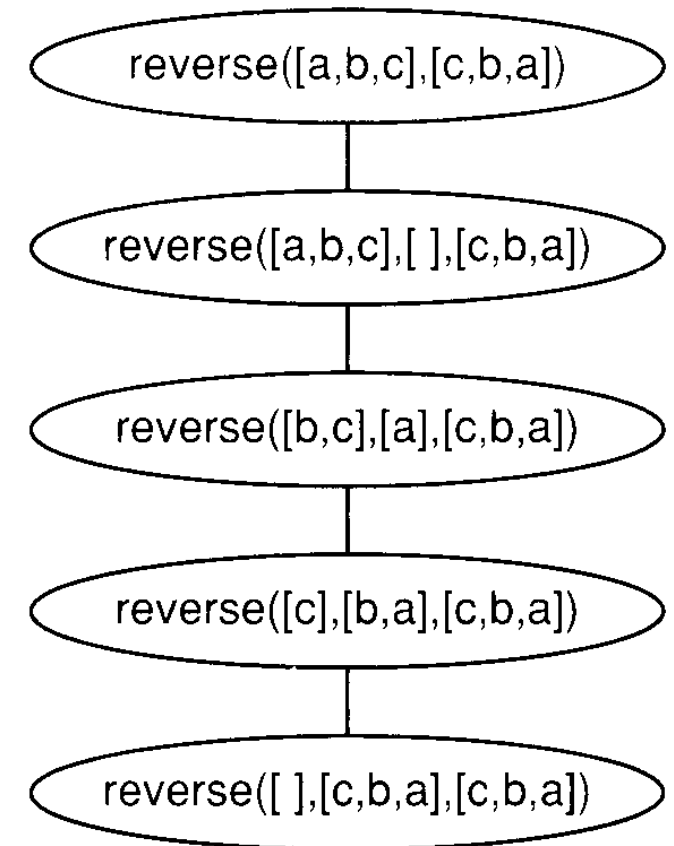


Lists

- We can use an accumulator (tail recursion) to reverse the list

```
reverse(Xs, Rev) :- reverse(Xs, [], Rev).  
reverse([X|Xs], Acc, Rev) :-  
    reverse(Xs, [X|Acc], Rev).  
reverse([], Rev, Rev).
```

- The accumulator holds the reversed list in the last step of the recursion
- Now the process is linear to the number of elements in the list



Additional Readings

- Prolog
 - Leon Sterling and Ehud Shapiro (1994). The Art of Prolog. The MIT Press (2nd ed). ISBN: 978-0262691635
 - Krzysztof R. Apt (1996). From Logic Programming to Prolog. Prentice Hall. ISBN: 978-0132303682
 - Patrick Blackburn, Johan Bos and Kristina Striegnitz (2006). Learn Prolog Now! College Publications. ISBN: 978-1904987178
 - Max Bramer (2013). Logic Programming with Prolog. Springer (2nd ed). ISBN: 978-1447154860

Q & A

