NOVA IMS Information Management School

Research Seminar II

Introduction to Structural Equation Modeling

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Summary

1. Introduction and motivation for SEM
2. Representation of a structural equation model
3. Confirmatory factor analysis (CFA)
4. Structural equation modeling (path analysis with latent variables)
5. PLS

1. Introduction and motivation to SEM

Constructs

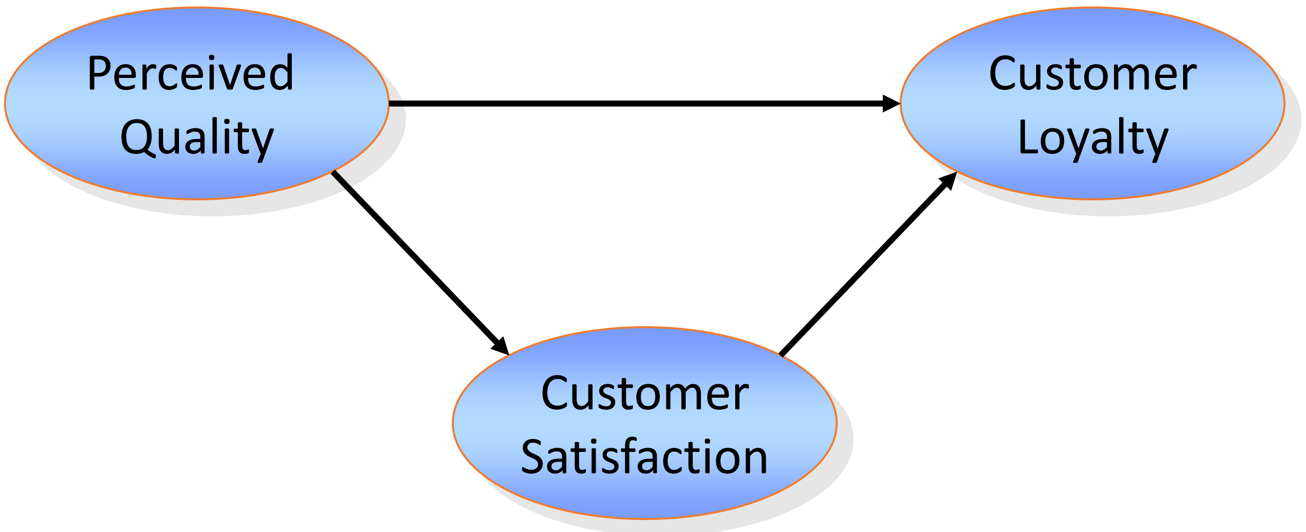
- **Theoretical propositions** consist of relationships between abstract constructs.
- **Testing theories** (i.e., theoretical propositions) require measuring these constructs accurately, before the strength of their relationships can be tested.
- **Measurement** refers to careful, deliberate observations of the real world and is the essence of empirical research.
- While some constructs in social science research, such as a person's age, weight, or a firm's size, may be easy to measure, other constructs, such as creativity, prejudice, or alienation, may be considerably harder to measure.

Variables

- **Latent Variables**: variables are not directly observable, the only object of observation are the manifestations or causes of these variables, eg. Intelligence, perceptions of quality of an organization, satisfaction, etc.

- **Measurement Variables**: variables used to indirectly "measure" the latent variables. Typically are expressions of latent variables.

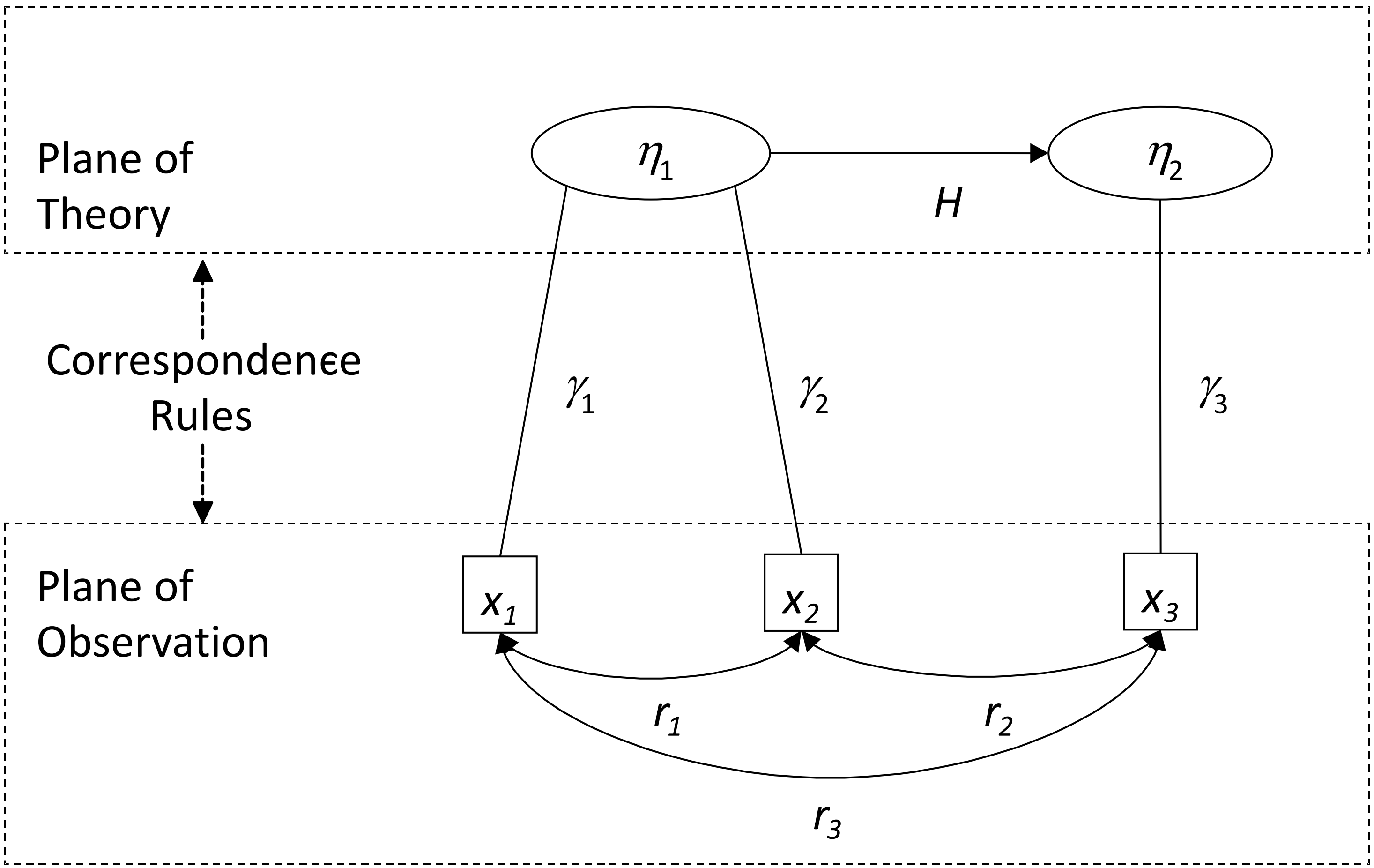
Marketing Example: Final Part of the ECSI Model



One exogenous variable: Perceived Quality(ξ_1)

Two endogenous variables: Customer Satisfaction (η_1)
Customer Loyalty (η_2)

Linking Theory and Observation



Bagozzi (1984)

Theory Testing

- Core theory
  - Background of the theoretical context: What latent constructs are involved and how are they connected?
  - Formulation of the theoretical hypotheses
- Measurement theory
  - Operationalization of the latent constructs: How are the latent constructs measured?
  - Formulation of correspondent hypotheses

Procedure for Developing Better Measures

1. *Domain identification and conceptualization*
2. *Item generation*
3. *Item refinement*
4. *Data colection*
5. *Item reduction and refinement*
6. *Scale evaluation*

Scale development

7. *Data colection*
8. *Confirmatory factor analysis*
9. *Final reliability and validity assessment*

CFA

SEM
1.Specification 2.Identification 3.Estimation 4.Fit 5.Reespecification

SEM

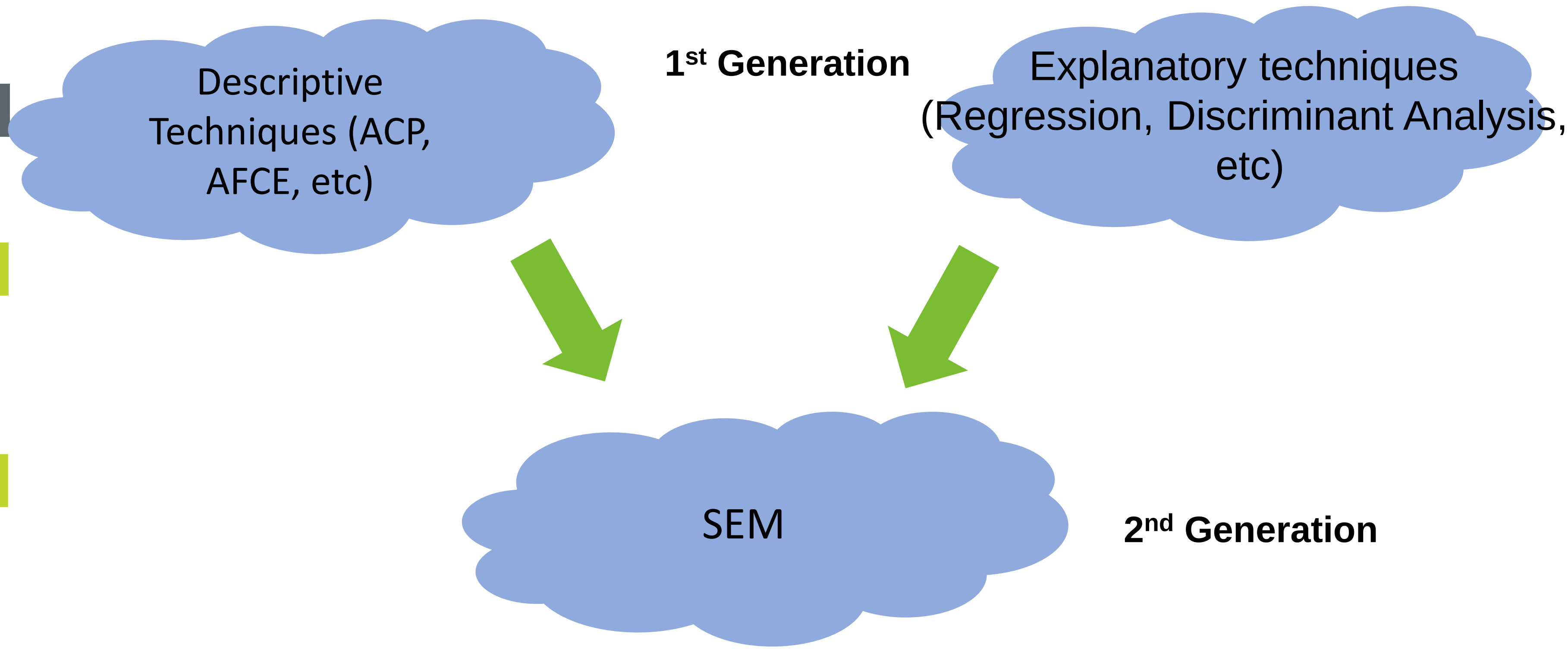
Structural Equation Modeling (SEM) represents an approach which integrates various portions of the research process in an holistic fashion. It involves:

- development of a theoretical frame where each concept draws its meaning partly through the nomological network of concepts it is embedded,
- specification of the auxiliary theory which relates empirical measures and methods for measurement to theoretical concepts
- constant interplay between theory and data based on interpretation of data via ones objectives, data properties, and level of theoretical knowledge and measurement.

Statistically - SEM represents a second generation analytical technique which:

- Combines an **econometric** perspective focusing on **prediction** and
- a **psychometric** perspective modeling **latent** (unobserved) variables inferred from observed - measured variables.
- Resulting in greater flexibility in modeling theory with data compared to first generation techniques

SEM Represents a second generation of statistical techniques:



SEM modeling flexibility include:

- Modeling multiple predictors and criterion variables
- Construct latent (unobservable) variables
- Model errors in measurement for observed variables due to noise and other unique factors
- Confirmatory analysis - Statistically test prior substantive/theoretical and measurement assumptions against empirical data

SEM advantages over path analysis

- Path analysis uses a single measure for each construct
- Forces the researcher to choose among alternative measures
- The indicators supporting the analysis are subject to measurement error
- Using multiple measures for each construct controls and reduces the effect of measurement error

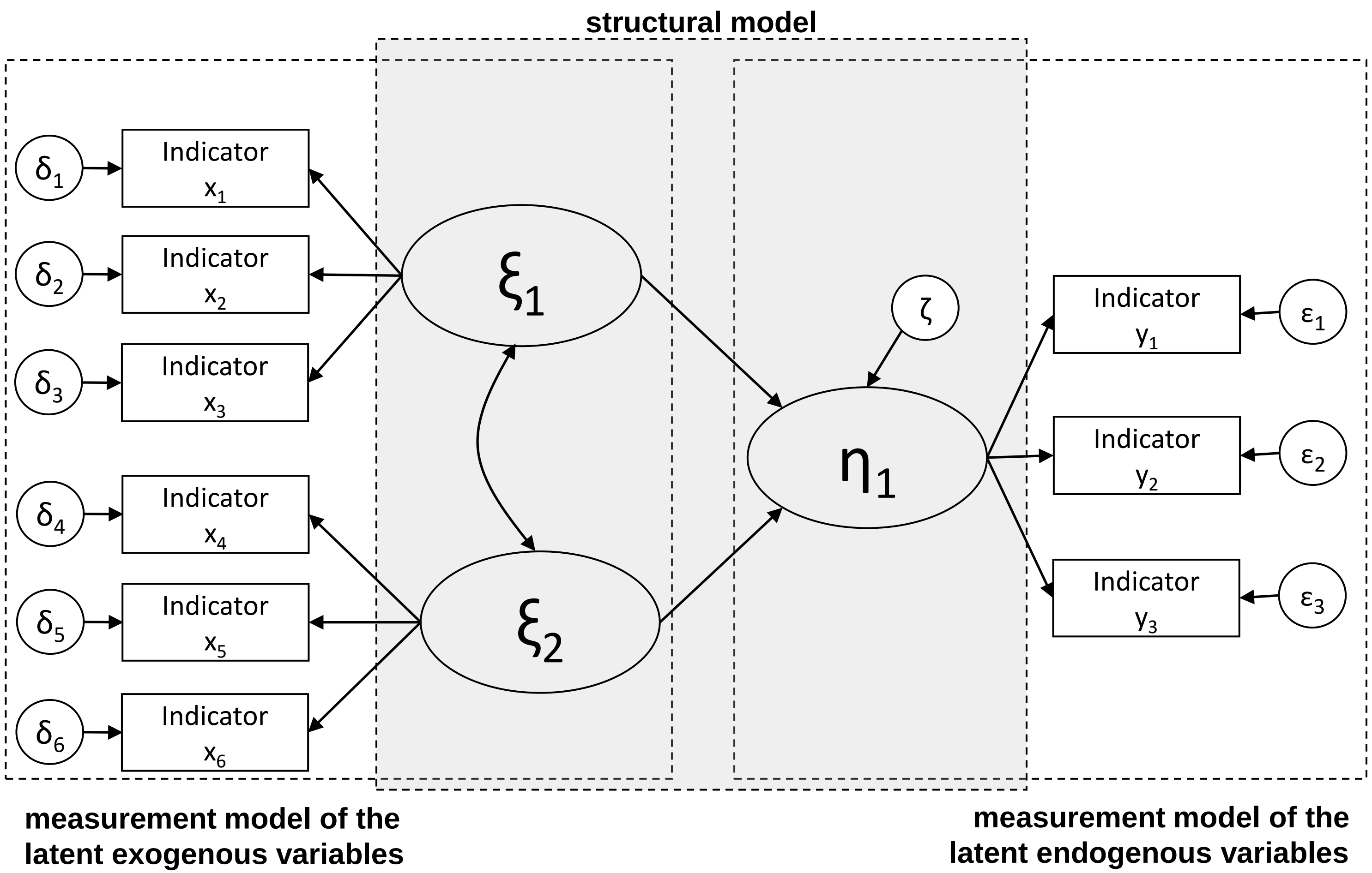
Viewed as an extension or generalization of first generation techniques - SEM can be used to perform the following analyses:

- Factor or component based analysis
- Discriminant analysis
- Multiple regression
- Canonical correlation
- MAN(C)OVA

Constructs

- **Latent constructs are theoretical**; they cannot be observed directly and therefore cannot be measured directly.
- To measure a latent construct, **researchers capture indicators** that represent the underlying construct.
- The indicators are directly observable and are believed by the researcher to **represent the variable that cannot be observed**.
- The researcher must operationally define the latent variable of interest in terms of behavior believed to represent it. As such, the **unobserved variable is linked to one or more that are observable**, thereby making its measurement possible.

Schematic Representation of a Structural Equation Model



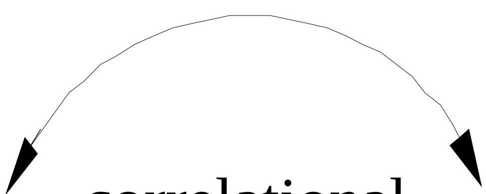
Y_{11} Y_{12} Y_{13}

indicators are normally represented as squares. For questionnaire based research, each indicator would represent a particular question.

η_1

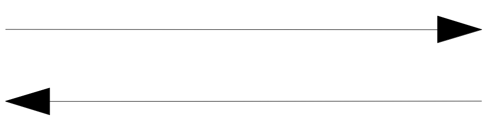
ε_{11}

Latent variables are normally drawn as circles. In the case of error terms, for simplicity, the circle is left off. Latent variables are used to represent phenomena that cannot be measured directly. Examples would be beliefs, intention, motivation.



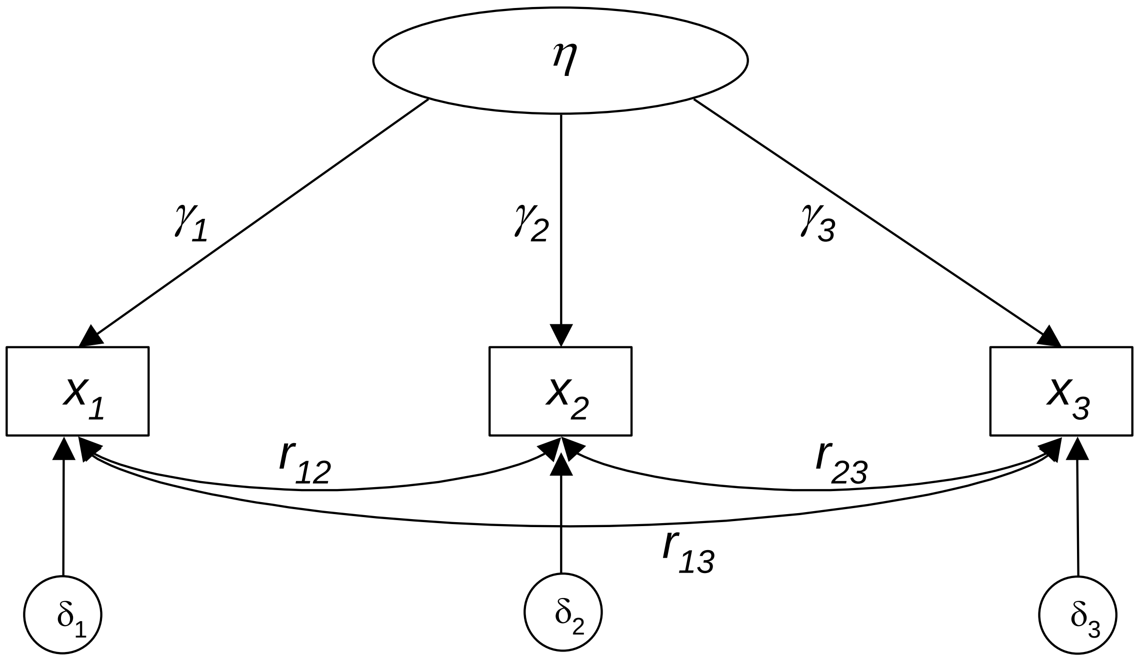
correlational relationship

recursive relationship

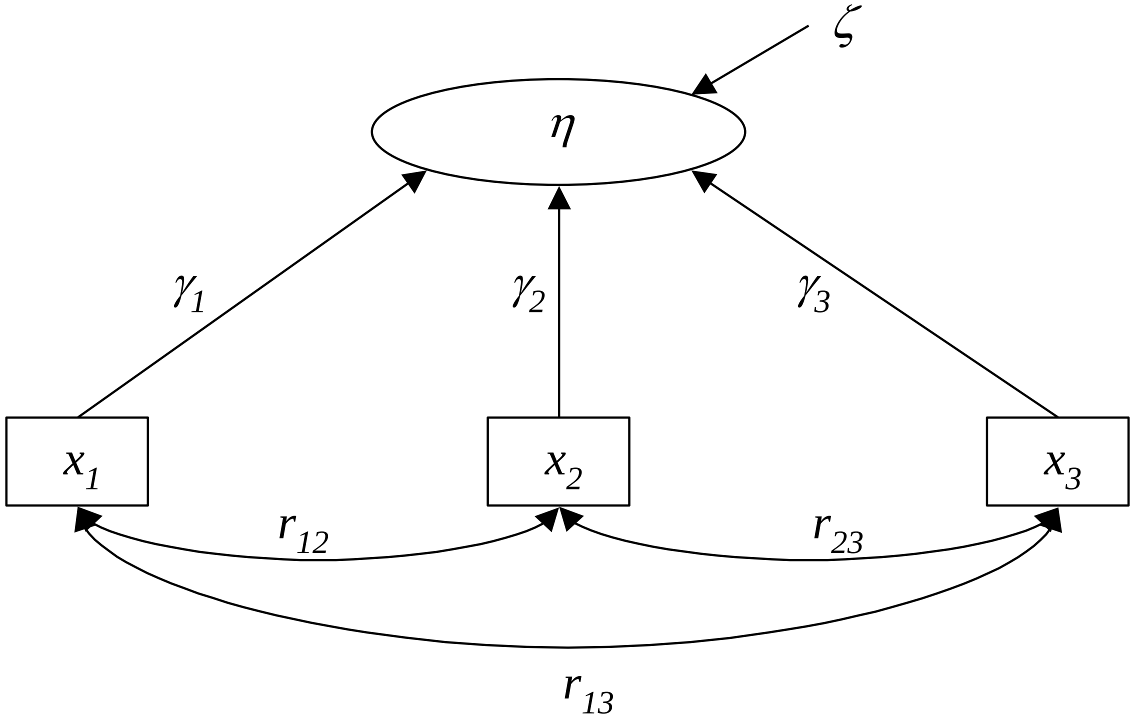


non-recursive relationship

A Reflective Measurement Model with Three Indicators



A Formative Measurement Model with Three Indicators



Selecting a Reflective or a Formative Measurement Model

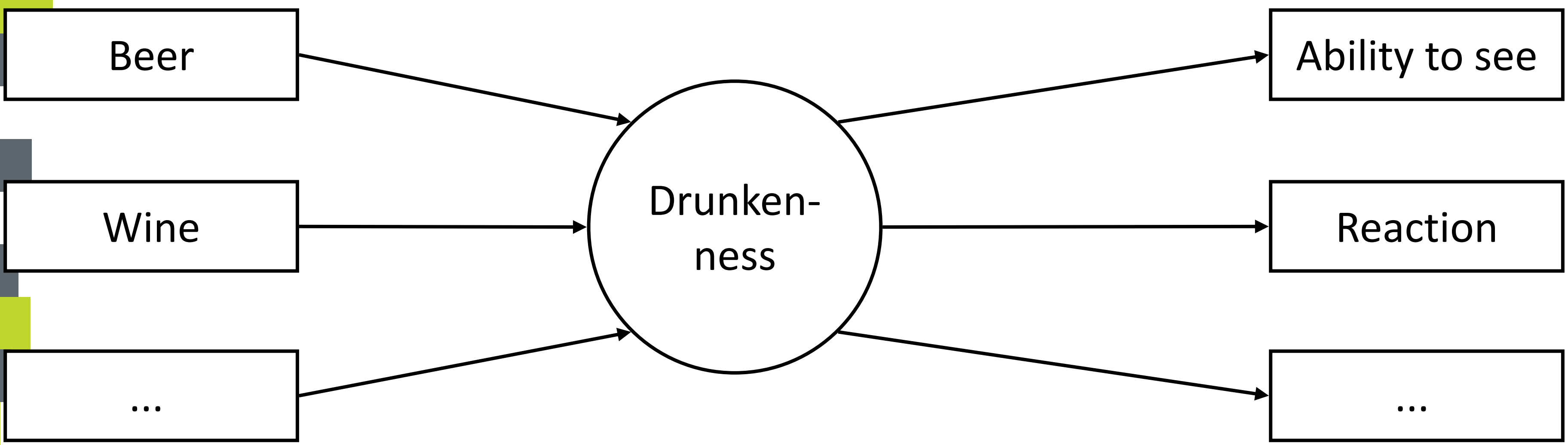
Reflective Indicators

- Construct occurrence
- Interchangeable
- Highly correlated
- Explicit consideration of measurement errors
- Several criteria for goodness
- Connected to construct by factor loadings

Formative Indicators

- Determine the construct values
- Eliminating indicators means changing the construct meaning
- Not necessarily correlated
- No measurement errors
- Validity not testable
- Connected to construct by regression coefficients

Formative and Reflective Measurement Models



Chin (1998), Diamantopoulos/Siguaw (2002)

Estimation Methods

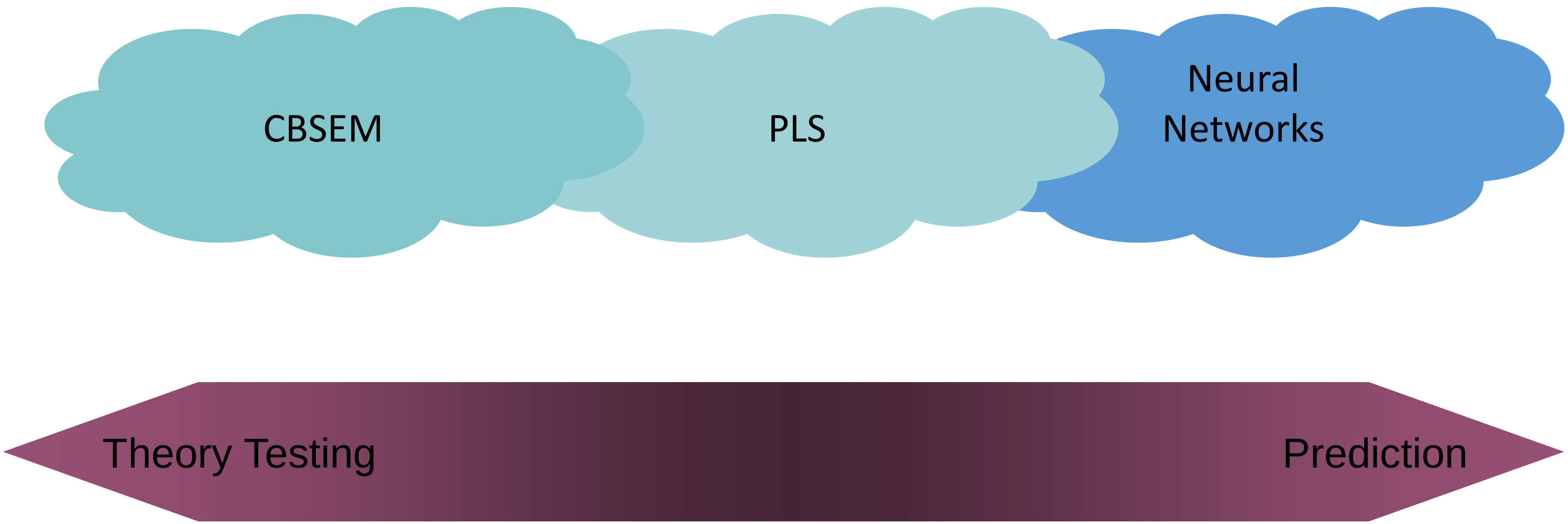
- There are two major types of estimation methods:
  - Methods based on the covariance, or more specifically in minimizing the difference between the matrix of covariance (or correlation) of the sample and the corresponding matrix of the theoretical model. The most known method is the maximum likelihood (ML).
  - Methods based on Minimizing the variance of the residuals of dependent variables (whether they are latent or observed variables). The most known method is the Partial Least Squares (PLS).

Methods to Estimate Structural Equation Models

Criterion	Structural Equation Modeling with Partial Least Squares	Covariance-Based Structural Equation Modeling
Objective:	Prediction oriented	Parameter oriented
Approach:	Variance based	Covariance based
Assumptions:	Predictor specification (nonparametric)	Typically multivariate normal distribution and independent observations (parametric)
Parameter estimates:	Consistent as indicators and sample size increase (i.e., consistency at large)	Consistent
Latent variable scores:	Explicitly estimated	Indeterminate
Epistemic relationship between a latent variable and its measures:	Can be modeled in either formative or reflective mode	Typically only with reflective indicators
Implications:	Optimal for prediction accuracy	Optimal for parameter accuracy
Model complexity:	Large complexity (e.g., 100 constructs and 1000 indicators)	Small to moderate complexity (e.g., less than 100 indicators)
Sample size:	Power analysis based on the portion of the model with the largest number of predictors. Minimal recommendation range from 30 to 100 cases.	Ideally based on power analysis of specific model – minimal recommendations range from 100 to 800.

Chin/Newsted (1999)

Prediction vs. Theory Testing

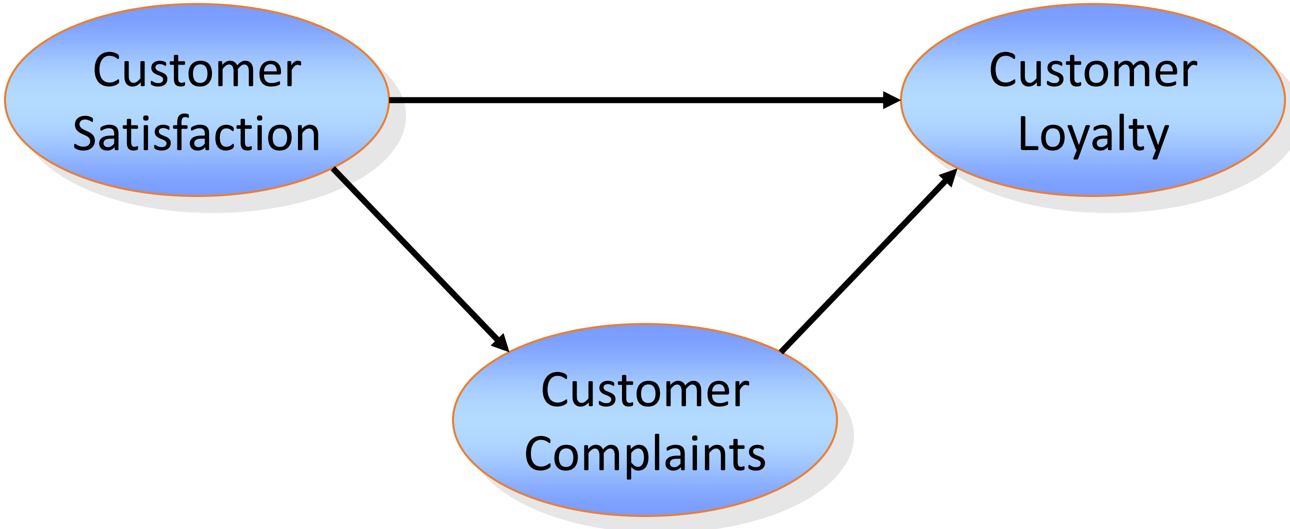


The Selection of SEM Methods within the Research Process

- Early, exploratory phase: PLS
- Theory testing and theory comparison: CBSEM
- Prediction: PLS

2. Representation of a structural equation model

Marketing Example: Final Part of the ECSI Model



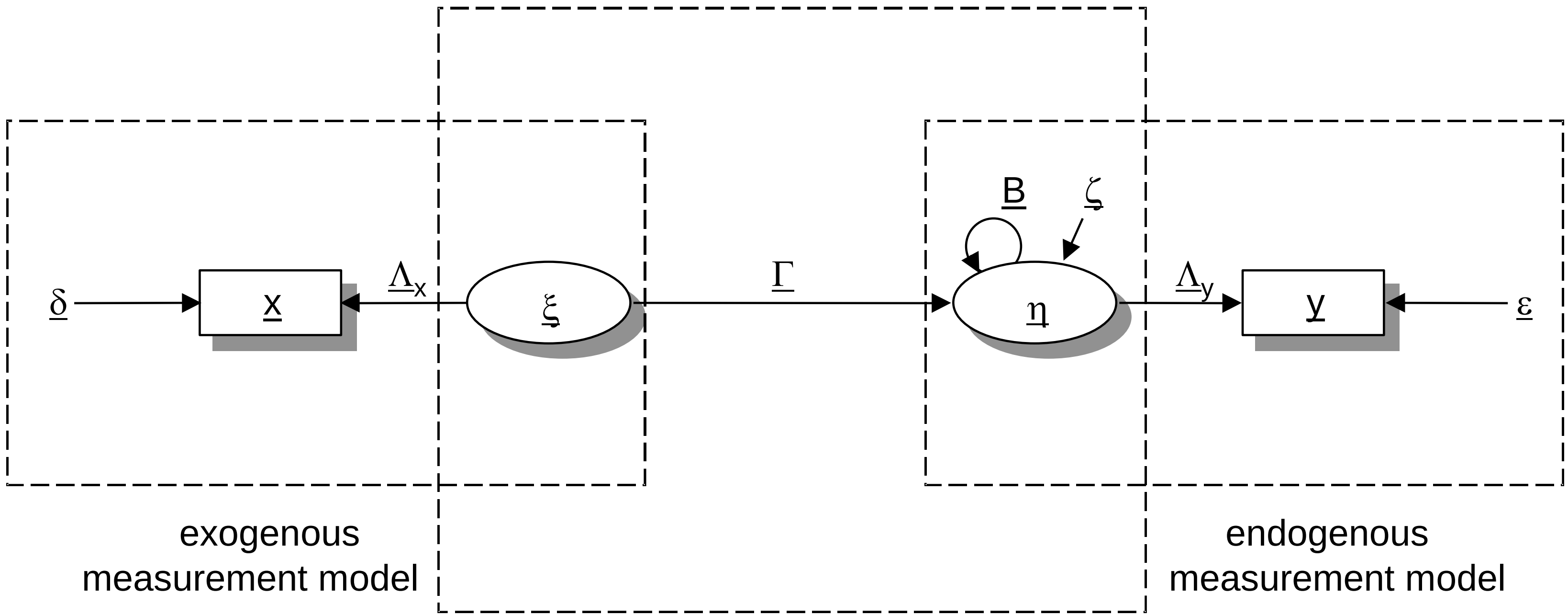
One exogenous variable: Customer Satisfaction (ξ_1)

Two endogenous variables: Customer Complaints (η_1)
Customer Loyalty (η_2)

Example for Measurement Model: Final Part of the ECSI Model

Latent Variable	Indicators
Customer Satisfaction (ξ_1)	Overall satisfaction (x_1)
	Fulfilment of expectations (x_2)
	Outcome of comparison with ideal provider (x_3)
Customer Complaints (η_1)	Perceived complaint handling (y_1)
Customer Loyalty (η_2)	Likelihood of repurchase with same provider (y_2)
	Willingness to pay more for current provider (y_3)
	Likelihood of recommendation (y_4)

The Complete Model



exogenous measurement model

$\underline{x} = \underline{\Lambda}_x \, \underline{\xi} + \underline{\delta}$

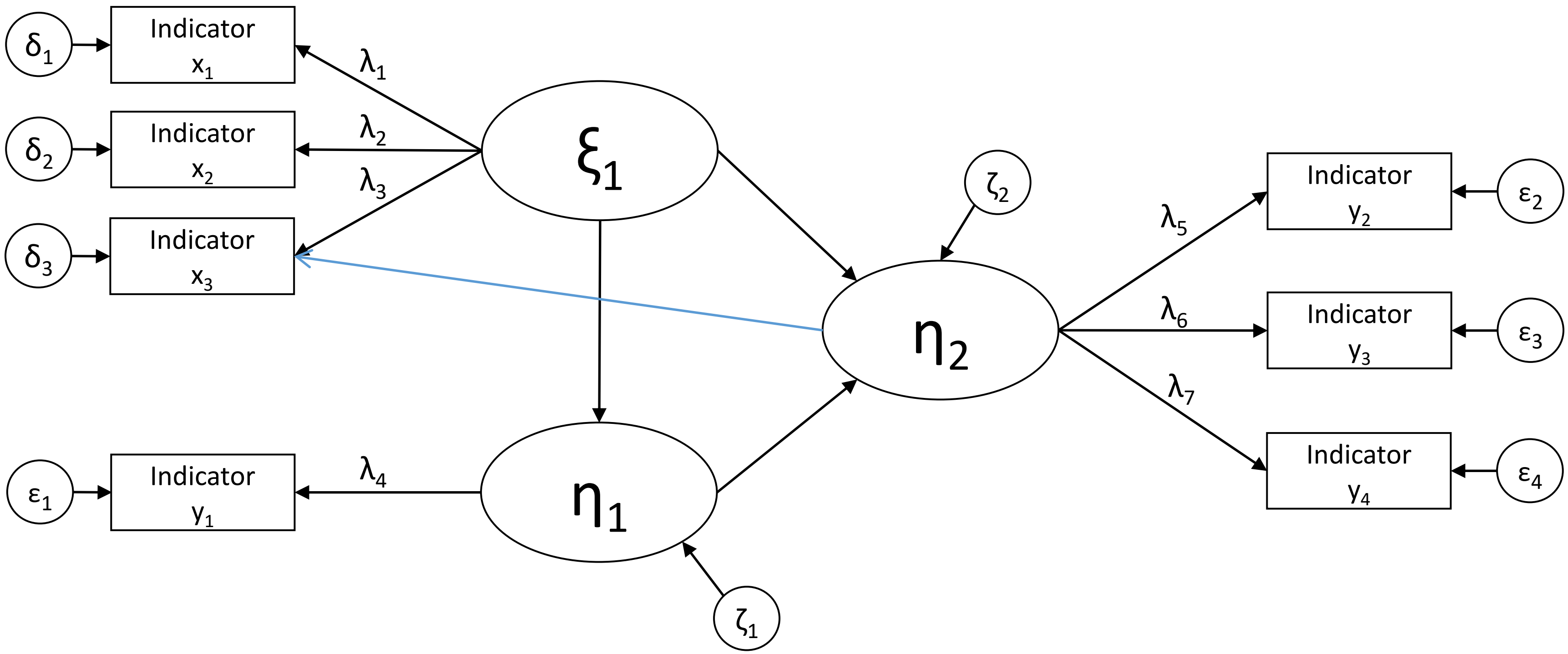
endogenous measurement model

$\underline{y} = \underline{\Lambda}_y \, \underline{\eta} + \underline{\varepsilon}$

structural model

$\underline{\eta} = \underline{\Gamma} \, \underline{\xi} + \underline{B} \, \underline{\eta} + \underline{\zeta}$

Schematic Representation of the Example Model

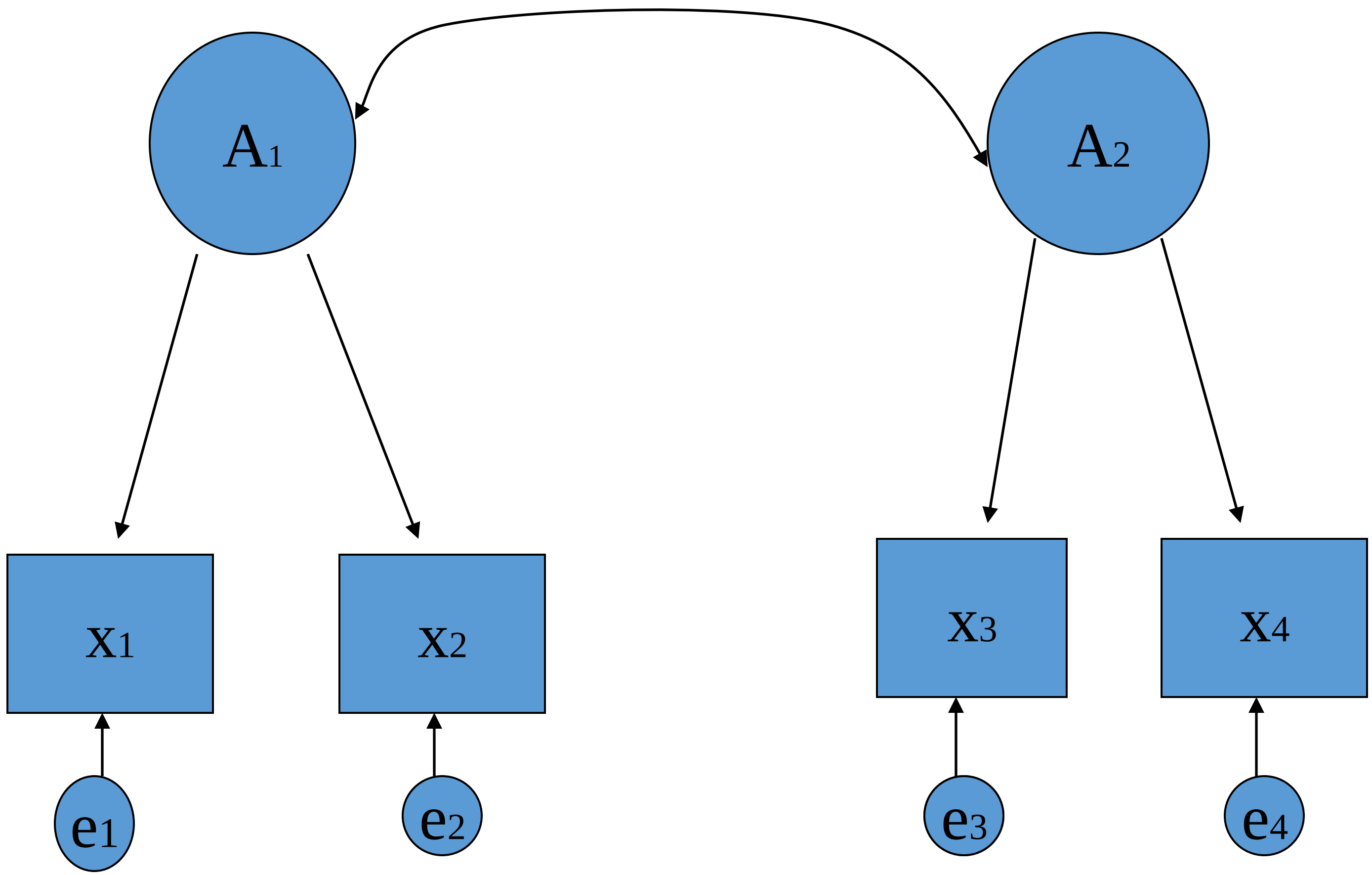


Three Main Applications of CBSEM

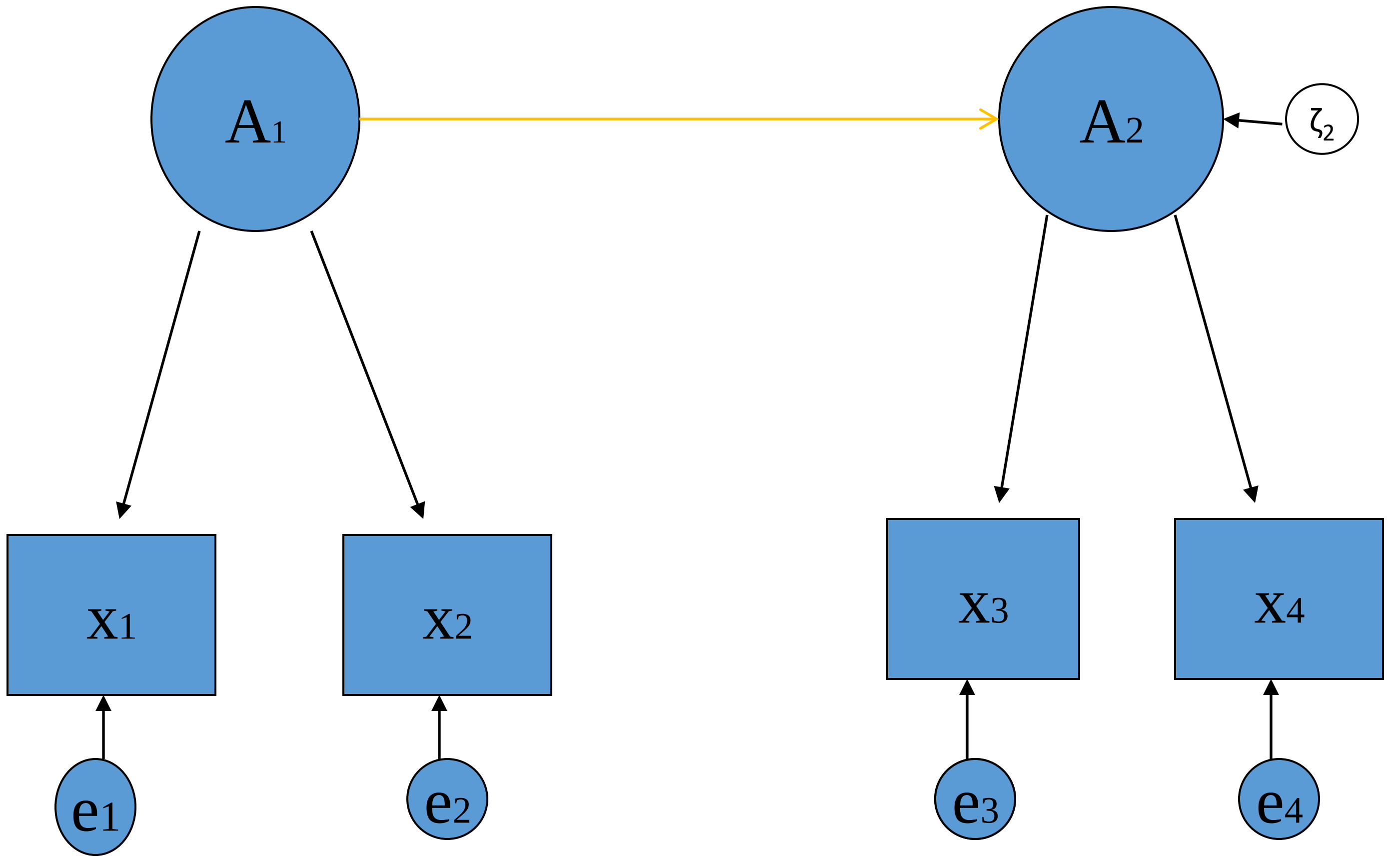
- Path models with observed variables
- Confirmatory factor analysis
- Structural equation models with latent variables
- But as a previous step we also may have exploratory factor analysis (still at the scale development phase)

Path models with observable variables

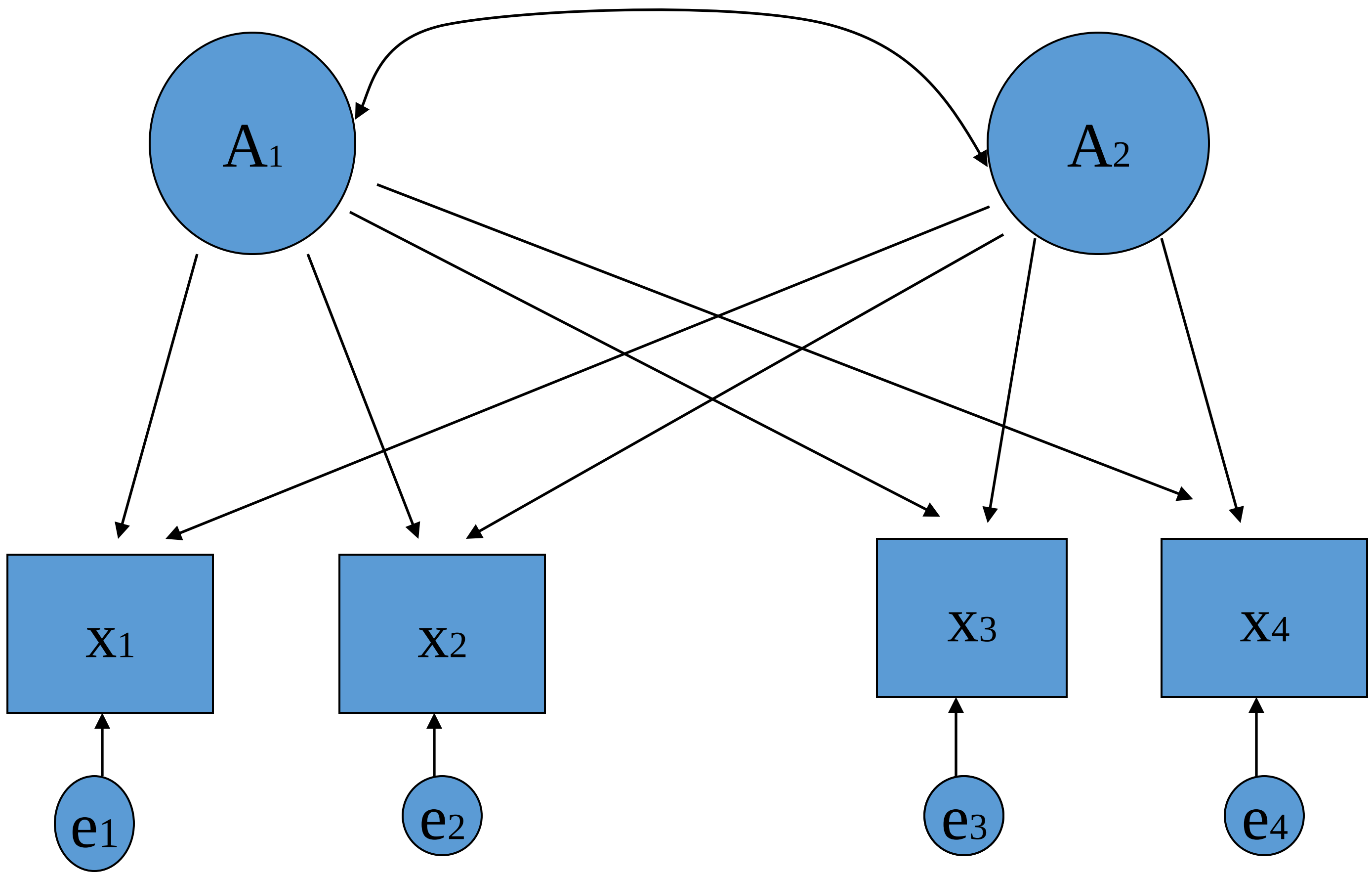
Confirmatory Factor Analysis



Structural equation models with latent variables



Exploratory Factor Analysis



Strictly Confirmatory (SC):

- In a strictly confirmatory situation the researcher has formulated one single model, and has obtained empirical data to test it. The model should be either accepted or rejected.

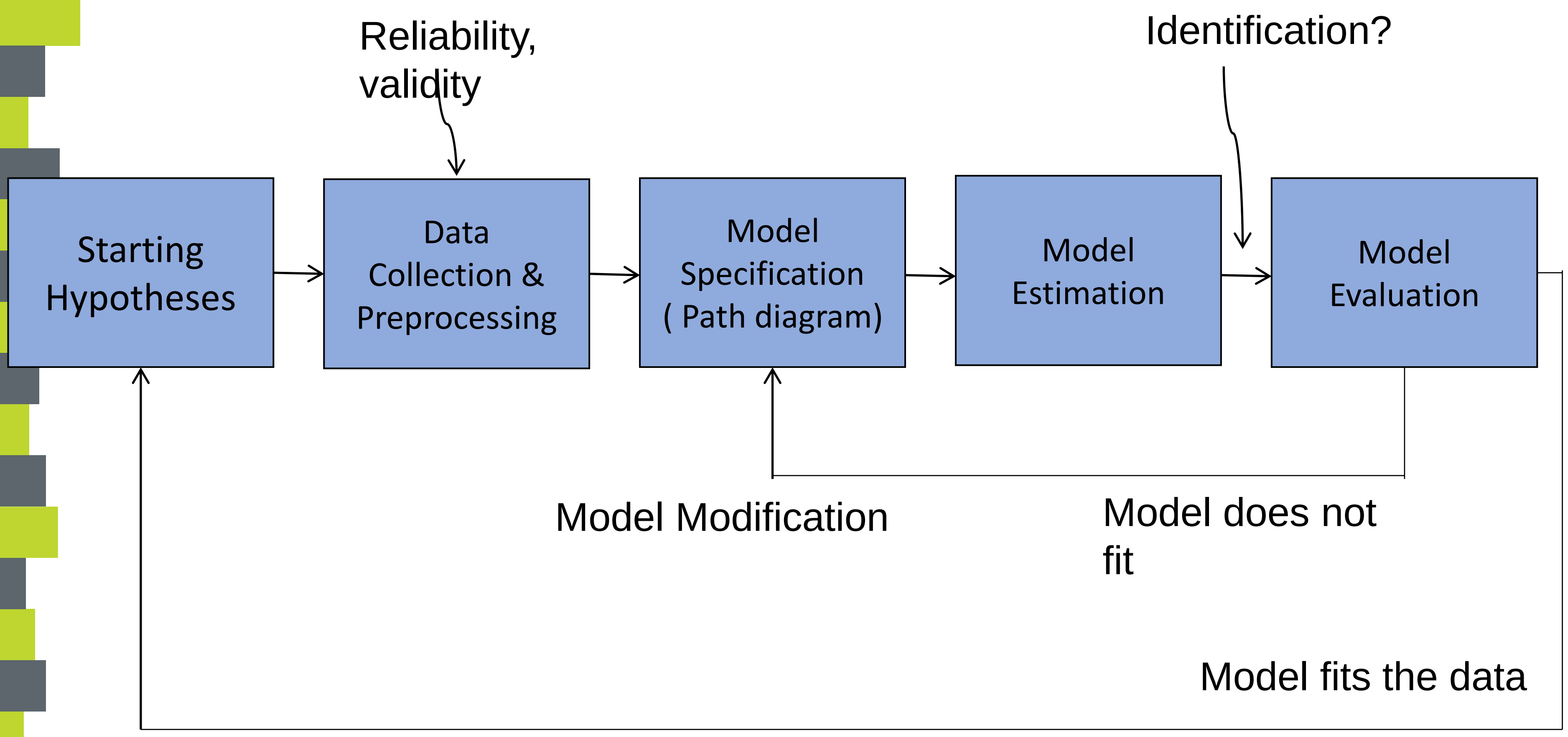
Alternative Models (AM):

- The researcher has specified several alternative models (or competing models) and, on the basis of an analysis of a single set of empirical data, one of the models should be selected.

Model Generating (MG):

- The researcher has specified a tentative initial model
- If the initial model does not fit the given data, the model should be modified and tested again using the same data
- Several models can be tested
- Finding of a model that fits the data well from a statistical point of view and additionally every parameter of the model can be given a substantively meaningful interpretation
- The re-specification of each model should be both theory driven and data driven
- Although a model may be tested in each round, the whole approach is model generating, rather than model testing

The Process of Linear Causal Modeling



3. Confirmatory factor analysis

The two step approach

- Anderson and Gerbing (1988) proposed a two-step approach to SEM
- The first step uses confirmatory factor analysis to develop an acceptable measurement model
  - We look for evidence that the indicators really are measuring the underlying constructs and that the measurement model fits the data
  - The measurement model does not specify any casual relationships between latent constructs and each latent variable is allowed to correlate freely with every other latent variable

The two step approach

- Anderson and Gerbing (1988) proposed a two-step approach to SEM
- The second step consists in using SEM to test the structural model, i.e. cause-effect relationships between latent constructs. Among other things this analysis allows to test hypotheses that certain latent constructs have causal effects on other latent constructs

Confirmatory factor analyis

- Focus on testing the measurement model
- It is based on accessing:
  - Goodness of fit
  - Reliability
  - Validity
  - How to modify the measurement model to achieve a better fit

Types of measurement error

1. Random measurement error: we can control for it and estimate it if we have at least two indicators. Alternatively, we can control for it, if we have an estimate of it from other studies.
2. Non-random measurement errors: we can control for them and estimate them if we have at least three indicators, and we can partly control for them and estimate them when we have two indicators (see Brown 2006, 65).

What About Measurement Error?

$$X_O = X_T + \varepsilon$$

$$X_O = X_T + \varepsilon_R + \varepsilon_S$$

X_O : Observed value

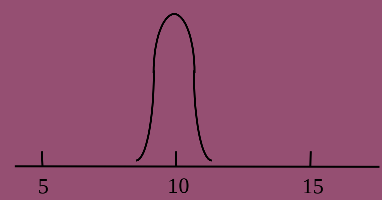
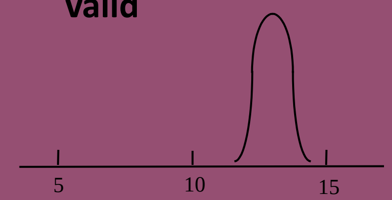
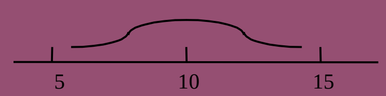
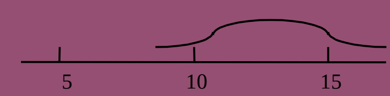
X_T : True value

ε : Measurement error

ε_R : Random measurement error

ε_S : Systematic measurement error

Disentangling Measurement Error

		Systematic Error: None	Systematic Error: High
Random Error	Low	a) **Reliable and valid**  True value = 10 Expected value = 10	b) **Reliable and not valid**  True value = 10 Expected value = 13
Random Error	High	c) **Neither reliable nor valid**  True value = 10 Expected value = 10	d) **Neither reliable nor valid**  True value = 10 Expected value = 13

Reliability

- Reliability refers to the ***accuracy***, precision of what is observed.
- It is concerned with the extent to which the obtained value of a variable differs from the true value as a result of random error.
- This means that reliability is the ***relative absence of error variance*** in a measurement instrument.
- Reliability is the degree to which the measure of a construct is consistent or dependable. In other words, if we use this scale to measure the same construct multiple times, do we get pretty much the same result every time, assuming the underlying phenomenon is not changing?

Reliability

- Sources of unreliability:
  - primary sources is the observer's (or researcher's) subjectivity
  - a second source of unreliable observation is asking imprecise or ambiguous questions
  - a third source of unreliability is asking questions about issues that respondents are not very familiar about or care about

Reliability

- A reliable instrument is one that performs in consistent, predictable ways.
- For a scale to be reliable, the scores it yields must represent some true state of the variable being assessed.
- In practice, this implies that the score produced by the instrument should not change unless there has been an actual change in the variable the instrument is measuring and, thus, that any observed change in scores can be attributed to actual change in that variable.

Validity

- Whether what is observed is a representative of the hypothetical concept is a question of validity (if we measure what we want to measure).
- The emphasis here is on ***what is being measured.***
- A valid measurement instrument measures what it is supposed to measure (high formal validity means high factor loading/coefficient between the latent variable and the indicator).
- Validity is the degree to which systemic measurement errors are small and have negligible effect over the measurement.

Validity

- Convergent validity: all the indicators of a latent construct should be efectivly measuring that construct

- Discriminant validity: a group of indicators intended to measure one latent variable should not be at the same time measuring another latent variable.

Developing Better Measures

- Ideally, indicators should already have been shown reliable and valid in previous research
- They should be composite scales of known reliability and validity
- It is usually a better and safer approach to use previously defined instruments
- Sometimes you have to develop new questionaire items that will be used as indicator variables (possible lower level of reliability)

Developing Better Measures

- It is of most importance to develop a good measurement model before obtaining data from additional subjects for CFA
- If you forget this, you may arrive to a situation where you can not test the structural model because the measurement model provides poor fit to data
- In this situation researchers are tempted to use exploratory factor analysis on the same data to find the factor structure and apply CFA to the same data
- Performing both exploratory and confirmatory FA on the same data may lead to a final model that is not generalize to other samples and to the population.

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4. Structural equation modeling

Five Generic Steps in Structural Equations Modeling

- Model Specification
- Identification
- Estimation
- Testing Fit
- Eventually Respecification

Model Specification

- Determine which constructs enter the model
- Determine the type of relationship between them (theory)
- Determine the measurement model for each construct
- Specify other relationships (e.g., between disturbance terms)
- Set restrictions

Identification of a model: necessary conditions

- The model is **over-identified** if there are more variances and covariances (elements in the variance-covariance matrix) than unknown free parameters. It is not necessarily sufficient for identification.

- The *degree of freedom **(df)*** is the difference between the number 'information points' in the variance-covariance input matrix (variances and covariances) and the number of unknown parameters.

Identification of the model: Degrees of Freedom…

- We should have: df ≥ 0
- df ≡ {k*(k+1)}/2 - t.
- Therefore:
- {k * (k+1)}/2 -t ≥ 0

k = number of indicators
t = number of free parameters to estimate
For example: K= 3, t = 4, (3*4)/2- 4 = 2

Estimation

- Maximum Likelihood (ML)
- Unweighted Least Squares (ULS)
- Generalized Least Squares (GLS)
- Weighted least squares for arbitrary distributions or asymptotic distribution free (WLS or ADF)

Global Fit Measures

- Types of fit measures:
  - Descriptive Measures
  - Statistical Inference
  - Measurement of approximate fit
  - Information criteria

5. Partial Least Squares (PLS)

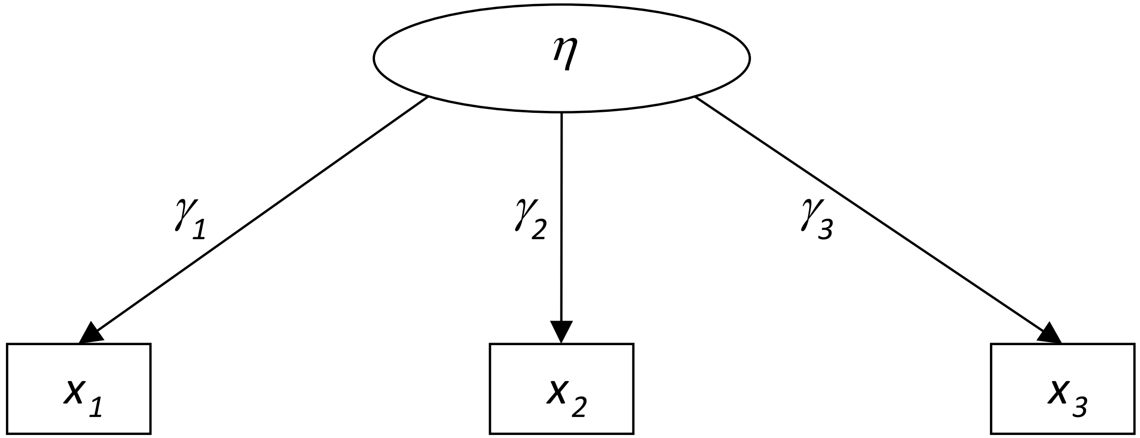
PLS Path Model Specification

- Which latent variables form part of the model?
- How are these latent variables connected among each other?
- Assign the indicators to the respective latent variables.
- Determine for each latent variable: Is it reflective or formative?

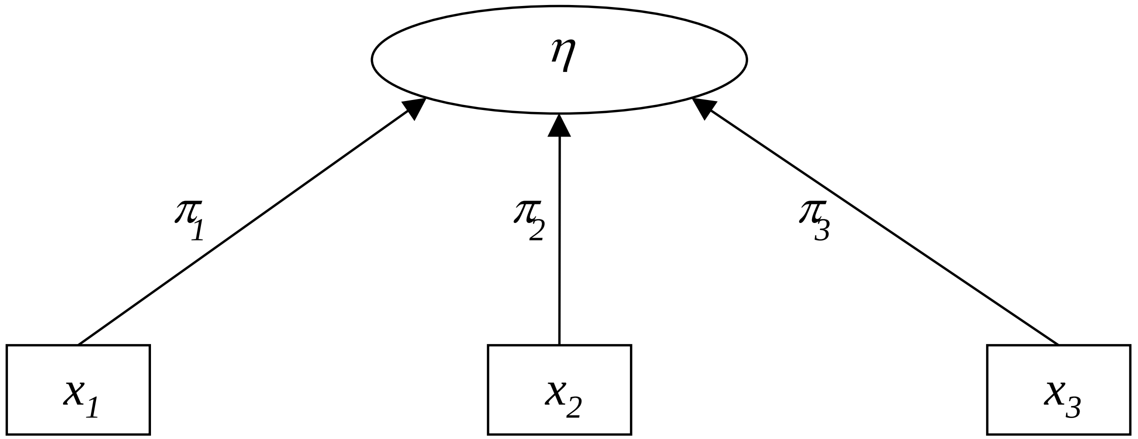
Specification Guidelines

- Each latent variable must be connected by arrows to at least one other latent variable.
- The model must be recursive (there must be no loop between latent variables)
  - the inner design matrix has or can be transformed to a diagonal form

Reflective Measurement Models Are Specified as Mode A



Formative Measurement Models Are (Usually) Specified as Mode B



The PLS Algorithm

Two phases:

1. In an iterative process, (standardized) scores for each latent variable are estimated.
2. For each endogenous latent variable, a (multiple) OLS regression is conducted.

Model assessment
Overview

- Assessing the measurement model
  - Reliability
  - Validity
- Assessing the structural model
  - Determination coefficients
  - Interpretation of path coeficients
  - Significance testing of path coeficients
  - Effect size
- Global assessment
  - Predictivity

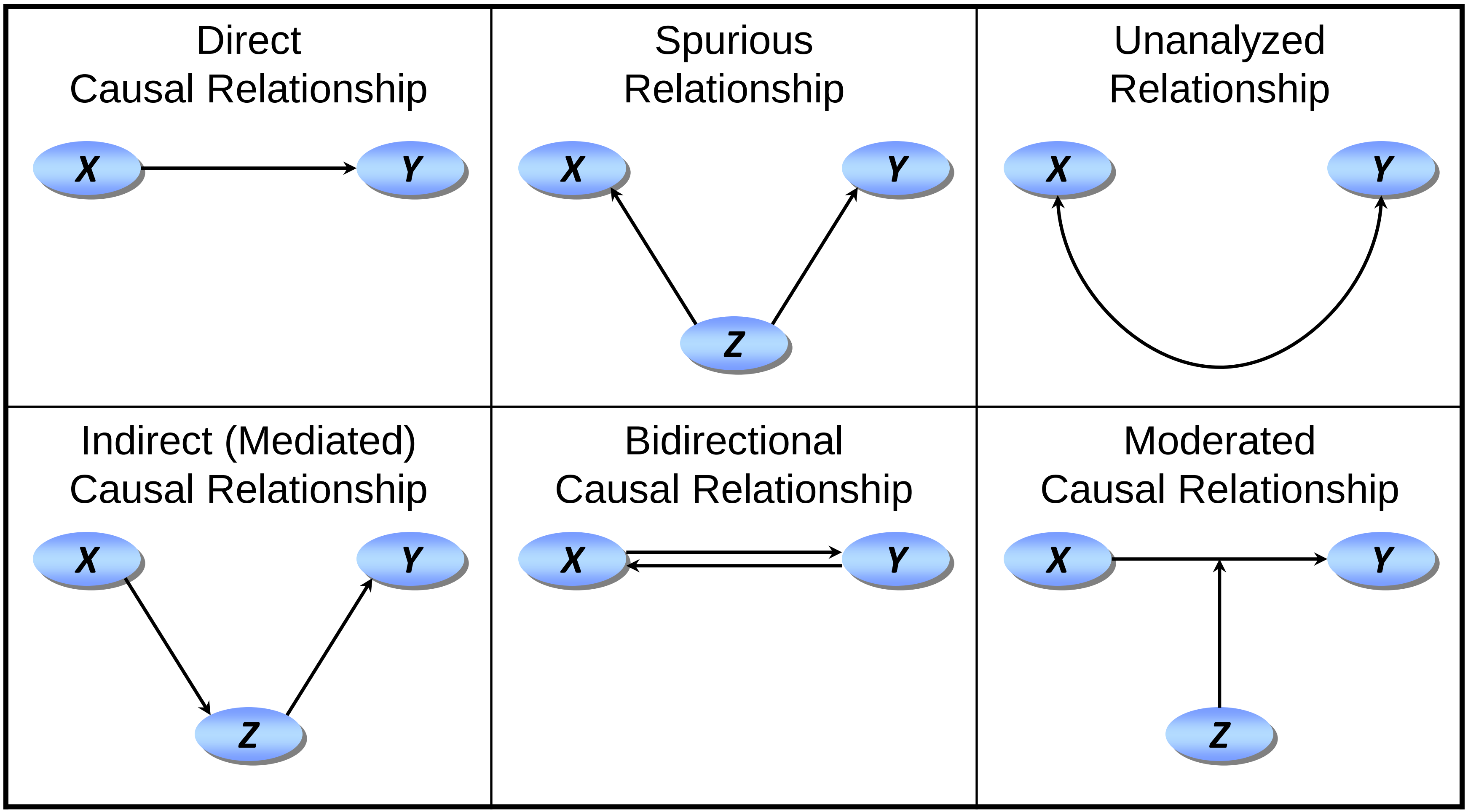
Assessing the measurement model

- There is not a formal CFA step
- The accessement of the measurement model is made over the estimation results of the SEM model
- There are not test statistics and fit indexes
- The analysis is concentrated on relibaility and validity

Assessing the measurement model

- As in covariance based SEM start analysing the mesurement model
- Only after obtaining a good measurement model you will access the strutural model

Examples of Causal Relationships between Latent Variables



Special topics

- Mediating and moderating effects
- Interactions
- Categorical indicators
- Multi-group analysis
- …

Literature

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Research Seminar II

Introduction to Structural Equation Modeling

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