DIOGO ALVES

LISTA 3

EXERCÍCIO 2.16: CALCULE (A MÃO) A DFT bidimensional DA MATRIZ:

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$$

EM SEGUIDA, CALCULE A TRANSFORMADA INVERSA DO RESULTADO.

RESPOSTA

A PARTIR DA DEFINIÇÃO

$$DFT(A) = \hat{A}_{K,L} = (A, \mathcal{E}_{K,L}) = \sum_{R=0}^{M-1} \sum_{S=0}^{N-1} A_{R,S}\, e^{-i.2\pi.\left(\frac{KR}{M} + \frac{LS}{N}\right)}$$

$$\hat{A}_{K,L} = A_{0,0}\, e^{-i2\pi\left(K.\frac{0}{M} + L.\frac{0}{N}\right)} + A_{0,1}\, e^{-i2\pi\left(K.\frac{0}{M} + L.\frac{1}{N}\right)} + A_{1,0}\, e^{-i2\pi\left(K.\frac{1}{M} + L.\frac{0}{N}\right)} + A_{1,1}\, e^{-i2\pi\left(\frac{K.1}{M} + \frac{L.1}{N}\right)}$$

$$\hat{A}_{K,L} = A_{0,0}\, e^{0} + A_{0,1}\, e^{-i\pi L} + A_{1,0}\, e^{-i\pi.K} + A_{1,1}\, e^{-i\pi(K+L)}$$

$$\hat{A}_{K,L} = 1.1 + (-1)(-1)^L + 2.(-1)^K + 0$$

$$\hat{A}_{K,L} = 1 - (-1)^L + 2.(-1)^K$$

$\boxed{K=0, L=0}$ $\hat{A}_{0,0} = 1 - 1 + 2 = 2$

$\boxed{K=0, L=1}$ $\hat{A}_{0,1} = 1 + 1 + 2 = 4$

$\boxed{K=1, L=0}$ $\hat{A}_{1,0} = 1 - 1 - 2 = -2$

$\boxed{K=1, L=1}$ $\hat{A}_{1,1} = 1 + 1 - 2 = 0$

$$\hat{A} = \begin{bmatrix} 2 & 4 \\ -2 & 0 \end{bmatrix}$$

$$\boxed{e^{-i\pi} = \cos\pi - i\,\mathrm{sen}\,\pi = -1}$$

A PARTIR DA DEFINIÇÃO

$$IDFT(\hat{A}) = A_{R,S} = \frac{1}{MN}\langle \hat{A}, \overline{\mathcal{E}_{K,L}} \rangle = \left( \sum_{K=0}^{M-1} \sum_{S=0}^{N-1} \hat{A}_{K,L} \cdot e^{i \cdot 2\pi \cdot \left( \frac{K.R}{M} + \frac{L.S}{N} \right)} \right) \cdot \frac{1}{MN}$$

$$A_{R,S} = \frac{1}{M.N}\left( \hat{A}_{0,0} \cdot e^{i2\pi\left(\frac{0.R}{M} + \frac{0.S}{N}\right)} + \hat{A}_{0,1}\, e^{i2\pi\left(\frac{0.R}{M} + \frac{S.1}{M}\right)} + \hat{A}_{1,0}\, e^{i2\pi\left(\frac{1.R}{M} + \frac{0.S}{N}\right)} + \hat{A}_{1,1}\, e^{i2\pi\left(\frac{R}{M} \frac{S}{N}\right)} \right)$$

$$A_{R,S} = \frac{1}{4}\left( \hat{A}_{0,0} \cdot e^{0} + \hat{A}_{0,1}\, e^{i\pi S} + \hat{A}_{1,0}\, e^{i2\pi R} + \hat{A}_{1,1}\, e^{i2\pi(R+S)} \right)$$

$$A_{R,S} = \frac{1}{4}\left( 2 \cdot 1 + 4 \cdot (-1)^{S} + (-2) \cdot (-1)^{R} + 0 \right)$$

$$A_{R,S} = \frac{1}{4}\left( 2 + 4 \cdot (-1)^{S} + (-2)(-1)^{R} \right)$$

$e^{i\pi} = \cos\pi + i \operatorname{sen}\pi = -1$

$\boxed{R=0,\ S=0}$ $A_{0,0} = \frac{1}{4}(2+4-2) = \frac{4}{4} = 1$

$\boxed{R=0,\ S=1}$ $A_{0,1} = \frac{1}{4}(2+4(-1) - 2.(1)) = \frac{-4}{4} = -1$

$\boxed{R=1,\ S=0}$ $A_{1,0} = \frac{1}{4}(2+4(+1) - 2(-1)) = \frac{8}{4} = 2$

$\boxed{R=1,\ S=1}$ $A_{1,1} = \frac{1}{4}(2+4(-1) - 2(-1)) = \frac{0}{4} = 0$

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$$