Increased (Platform) Competition Reduces (Seller) Competition

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Abstract

Online platforms no longer simply operate the platforms on which third-party sellers sell their products to consumers. Platforms now also enter the sellers' product markets by introducing their private-label products, in direct competition with the third-party sellers. I build a model to explore how increased platform competition affects platforms' entry decisions and thereby platform-seller competition. I analyze the trade-offs that arise in the platforms' entry decisions and in the sellers' platform choices. I find that increased platform competition reduces platform-seller competition if the rival platform is a relatively weak seller. I also find that increased platform competition reduces consumer surplus and total welfare unless the rival platform is a sufficiently strong platform.

Key Words: online platforms, platform competition, platform-seller competition.

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1 Introduction

An increasing number of digital platforms are operating platforms on which third-party sellers sell to consumers and selling their private-label products on their platforms. For example, Amazon acts both as an online platform for third-party products and as a seller to sell its private-label products (e.g., Amazon Basics) on its platform. Online platforms commonly introduce their private-label products by imitating successful products and engage in platform-seller competition. This practice has raised antitrust concerns because product imitation might lead to unfair competition between the platform and the third-party seller. Relevant investigations and new proposed legislation arise. India has introduced new laws in 2019 to prevent Amazon from selling its private-label products. The United States has proposed the Ending Platform Monopolies Act in 2021 to stop platform-seller competition between "Big Tech" firms' private-label products and third-party products.

In this paper, I build a model in which two platforms compete to attract third-party sellers. Each platform endogenously chooses between engaging in product imitation and thereby platform-seller competition or committing not to do so. Each seller's platform choice depends on platforms' commitments. Each seller can only afford to sell on one platform and cannot afford to operate its direct channel (i.e., the seller's own website or store). I use the model to study how increased platform competition affects each platform's product imitation behavior and thereby platform-seller competition. I also explore the implications for consumer surplus and total welfare of increased platform competition.

A dominant platform is likely to imitate successful products that sell on its platform. This happens because the platform can obtain the third-party sellers' sales data via its platform and identify the successful products, which can be used for product imitation and thereby unfair platform-seller competition.

To model these practices, I study a benchmark setting in which a monopolistic platform P chooses whether to imitate successful products and then engage in platform-seller competition. Two third-party sellers (S1 and S2) sell independent products. If P imitates one seller's product, P incurs a fixed entry cost F, and then P and the seller compete on prices. In addition, P's imitated product can be more or less popular than the seller's product that P imitates. I find that each seller sells on P and P imitates each seller's product and thereby engages in platform-seller competition in equilibrium.

Increased platform competition arises in recent years. For example, Amazon is considered as a dominant online shopping platform for years. However, Walmart is a rising star in competing with Amazon because Walmart's online platform has been much improved lately including introducing its "W+ membership" with similar benefits with "Amazon Prime membership".

Platforms can differ in platform strength and selling strength in the product market. Platform strength characterizes the platform effect on the seller's sales including consumer loyalty and the platform boost to the seller's sales.[1] A platform's increased imitation cost reduces

[1]Consumers value platforms differently. Some consumers are loyal to one platform, while others have no innate preference. A platform's ability to boost a seller's sales is usually different from the other platform.

the platform's selling strength and increases the seller's selling strength in the platform-seller competition.

Each seller faces a trade-off when choosing a platform. On the one hand, the stronger platform provides the stronger boost to the seller's sales. On the other hand, the stronger the platform, the more intense platform-seller competition the seller faces because the stronger platform has less incentive to commit not to enter. I assume platforms cannot make discriminating commitments, then each platform also faces a trade-off. On the one hand, committing not to enter can enhance the likelihood of attracting sellers. On the other hand, if a seller always sells on the platform regardless of the platform's commitment, then the platform secures more profit by making no commitment (and subsequently entering the seller's market and imitating the seller's product) than by committing not to enter the seller's market.

I find three primary conclusions in equilibrium. First, if one platform's relative platform strength is sufficiently pronounced, both sellers sell on the strong platform that imitates each seller's product because the benefit from a strong platform boost exceeds the benefit from no platform-seller competition. Second, as the strong platform's relative platform strength declines, the weak platform commits not to enter to ensure that the weak seller will sell on the weak platform. The weak seller sells on the weak platform because the weak seller's benefit from no platform-seller competition exceeds the benefit from a strong platform boost. The strong seller sells on the strong platform because the strong seller's benefit from a strong platform boost exceeds the benefit from no platform-seller competition. The strong platform makes no commitment because its benefit from imitating the strong seller's product exceeds its benefit from attracting an additional seller (i.e., the weak seller). Third, if two platforms are sufficiently similar or symmetric, the relatively intense inter-platform competition compels both platforms to commit not to enter, in order to attract sellers.

I find that increased platform competition reduces platform-seller competition if P's rival platform is a relatively weak seller. Specifically, intense platform competition between similar or symmetric platforms deters product imitation and thereby leads to no platform-seller competition.

I also find that increased platform competition increases sellers' aggregate profits due to the reduced platform-seller competition. However, increased platform competition reduces consumer surplus and total welfare unless P's rival platform is a sufficiently strong platform. This is the case because increased platform competition reduces platform-seller competition and reduced platform-seller competition induces higher prices.

My paper contributes to a recent strand of literature that discusses the platform's choices: operating a platform, or being an active seller or both. Hagiu and Wright (2015) discuss the platform's strategic choice between running a platform and acting as a seller. Hagiu et al. (2022) study whether a platform should introduce its private-label products on its own platform. Jiang et al. (2011) show that independent sellers may strategically reduce their sales by increasing prices to prevent the platform from introducing its private-label products. Madsen and Vellodi (2022) discuss the platform's behavior of introducing its private-label products by using the third-party sellers' sales data. However, as far as I know, my research is the first to analytically study how increased platform competition affects platforms' choices of introducing their private-label products and thereby platform-seller competition.

My paper also relates to a literature exploring the welfare implications when the platform enters the product market and competes against independent sellers who sell their products on the platform. Etro (2020) addresses that entry by the monopoly platform can be socially efficient under competitive sellers and standard demand conditions due to reduced prices. Bedre-Defolie and Anderson (2021) show that the platform's entry hurts consumers and independent sellers. Lam and Liu (2021) show that the platform's entry with the usage of detailed information on independent sellers can benefit independent sellers by softening competition between sellers, but may hurt consumers because high-demand sellers strategically raise their prices to hide popularity to deter the platform's entry. However, these papers do not consider the possibility of platform competition.

My paper is also related to the literature on competing platforms. Haan et al. (2021) show that each platform faces a trade-off when choosing its competing strategies: intense competition reduces its profit on the consumer side, but also increases its market power over advertisers due to the increasing number of singlehoming consumers. Choi and Jeaon (2021) discuss how market power affects competing platforms' incentives to adopt technological innovations that create trade-offs between the consumer and the advertiser side. However, this literature does not consider the setting where the platform can enter the product market and compete against independent sellers.

The analysis proceeds as follows. Section 2 describes the key elements of my model. Section 3 provides the main analysis. Sections 3.1 and 3.2 characterize outcomes in the absence of platform-seller competition and outcomes in the presence of platform-seller competition. Section 3.3 characterizes equilibrium outcomes in a benchmark case with a monopolistic platform. Section 3.4 characterizes equilibrium outcomes in the case of primary interest with platform competition. Section 3.5 compares the outcomes in the monopolistic platform and platform competition settings. Section 4 provides concluding observations.

2 Model elements

I consider a setting in which two platforms (P1 and P2) compete to attract independent sellers (S1 and S2). Each independent seller sells one product. Each seller can only sell its product on a platform (i.e., the independent seller does not have its own store or website). Each seller can only sell its product on one platform.

I consider a game in which P1 and P2 simultaneously: (i) choose to either commit never to act as a seller or make no such commitment and; (ii) set their per-unit commissions. After the commitments and the commissions are specified, S1 and S2 choose which platform to sell on. Then, a platform that made no commitment will make its entry decision (i.e., whether to enter and which to enter). Then, each party in each product market chooses its own price.

Suppose S1 and S2 sell independent products. Suppose each consumer will purchase one unit of each product. Each seller's type is common knowledge.

If Sj sells on Pk and Pk enters Sj's product market ($k, j \in \{1, 2\}$), Pk imitates Sj's product and Pk and Sj sell differentiated products and compete on prices. Following Singh and Vives (1984), I assume the demands for Pk's product and Sj's product are given by

$$q_{kj}^P \;=\; \theta_j \, \alpha_j - \beta_j^P \, p_{kj}^P + \eta_j \, p_{kj}^S, \tag{1}$$

$$q_{kj}^S \;=\; \alpha_j - \beta_j^S \, p_{kj}^S + \eta_j \, p_{kj}^P. \tag{2}$$

where $\beta_j^P > 0$, $\beta_j^S > 0$, $\eta_j > 0$ are parameters, p_{kj}^S is the price of Sj's product, p_{kj}^P is the price of Pk's product, and α_j represents the popularity of Sj's product.[2] θ_j measures Pk's ability to replicate the popularity of Sj's product. Pk's imitated product can be more or less popular than Sj's product, i.e., $\theta_j \gtreqless 1$. Let $\Omega_j \equiv \frac{[\eta_j]^2}{\beta_j^P \beta_j^S} \in (0, 1)$ denote the extent to which Sj's product and Pk's product are homogeneous.[3] As Ω_j approaches 0, consumer demands for Sj's product and for Pk's product become nearly independent. As Ω_j approaches 1, consumers view Sj's product and Pk's product as nearly perfect substitutes.

(1) and (2) imply that if Sj sells on Pk and Pk does not enter Sj's product market, then $q_{kj}^P = 0$. Therefore, (1) implies

$$\theta_j \, \alpha_j - \beta_j^P \, p_{kj}^P + \eta_j \, p_{kj}^S \;=\; 0 \quad \Leftrightarrow \quad p_{kj}^P \;=\; \frac{\theta_j \, \alpha_j + \eta_j \, p_{kj}^S}{\beta_j^P}. \tag{3}$$

(2) and (3) imply that

$$q_{kj}^S \;=\; \alpha_j - \beta_j^S \, p_{kj}^S + \eta_j \, \frac{\theta_j \, \alpha_j + \eta_j \, p_{kj}^S}{\beta_j^P} \;=\; \alpha_j \left[1 + \frac{\theta_j \, \eta_j}{\beta_j^P} \right] - p_{kj}^S \left[\beta_j^S - \frac{(\eta_j)^2}{\beta_j^P} \right]. \tag{4}$$

(4) implies if Sj sells on Pk, then the demand for Sj's product in the absence of platform entry is:

$$q_{kj}^S \;=\; A_j - b_j^S \, p_{kj}^S, \tag{5}$$

where

$$A_j \;=\; \alpha_j \left[1 + \frac{\theta_j \, \eta_j}{\beta_j^P} \right] \;>\; 0 \;\; \text{and} \;\; b_j^S \;=\; \beta_j^S - \frac{[\eta_j]^2}{\beta_j^P} \;>\; 0. \tag{6}$$

(6) implies that:

$$A_j \;=\; \alpha_j \left[\frac{\beta_j^P + \theta_j \, \eta_j}{\beta_j^P} \right] \quad \Leftrightarrow \quad \alpha_j \left[\beta_j^P + \theta_j \, \eta_j \right] \;=\; \beta_j^P \, A_j; \tag{7}$$

[2] Pk's product and Sj's product are substitutes, so $\eta_j > 0$.

[3] See Singh and Vives (1984).

$$b_j^S \;=\; \frac{\beta_j^S\,\beta_j^P - \left[\eta_j\right]^2}{\beta_j^P} \quad\Leftrightarrow\quad \beta_j^S\,\beta_j^P - \left[\eta_j\right]^2 \;=\; \beta_j^P\,b_j^S. \tag{8}$$

(6) implies that:

$$\beta_j^S\,\beta_j^P - \left[\eta_j\right]^2 \;>\; 0. \tag{9}$$

(6) also implies that:

$$b_j^S \;=\; \beta_j^S \left[1 - \frac{\left(\eta_j\right)^2}{\beta_j^S\,\beta_j^P}\right] \;=\; \beta_j^S\left[\,1 - \Omega_j\,\right]. \tag{10}$$

(2) and (5) imply that the impact of platform entry on a seller's sales is ambiguous: (i) platform entry reduces the seller's popularity (i.e., $\alpha_j < A_j$ from (6)) because consumers have more choices; (ii) platform entry increases the sensitivity of the seller's sales to his price (i.e., $\beta_j^P > b_j^S$ from (6)) due to the relatively intense competition; (iii) platform entry reduces the seller's price.[4]

Consumers value P1 and P2 differently: a fraction f_1 of consumers are loyal to P1 (i.e., only purchase from P1); a fraction f_2 of consumers are loyal to P2 (i.e., only purchase from P2); and the remainder of the consumers $(1 - f_1 - f_2)$ have no innate preference for P1 and P2. If a seller sells its product at the same price on both platforms, a fraction $f_1 + \frac{1-f_1-f_2}{2}$ of consumers will buy it from P1, and a fraction $f_2 + \frac{1-f_1-f_2}{2}$ of consumers will buy it from P2. If a seller sells its product on P1 and P2 at different prices, a fraction f_k of consumers will buy the product from Pk, and a fraction $1 - f_k$ of consumers will buy the product from Pi, when the product is cheaper on Pi ($i, k \in \{1, 2\}$, $i \neq k$).

If Sj sells on Pk, consumers' demand for Sj's product is ($j, k, i \in \{1, 2\}$, $k \neq i$)

$$Q_{kj}^S \;=\; B_k\left[\,1 - f_i\,\right]\,q_{kj}^S. \tag{11}$$

where $B_k > 1$ is an exogenous "boost" provided by Pk to sales on Pk, and q_{kj}^S is specified in (2) in the presence of platform entry or (5) in the absence of platform entry.

Sj's unit production cost is $c_j^S \geq 0$. If Pk enters Sj's product market, then Pk can imitate Sj's product at cost $c_j^P \geq 0$ ($k, j \in \{1, 2\}$). I assume Pk has sunk fixed cost if Pk does not enter the product market. I assume that each independent seller has zero or sunk fixed cost, whereas Pk must incur a positive fixed cost ($F > 0$) to enter Sj' product market. Pk charges Sj a per-unit commission w_{kj} for each sale.

The ensuing discussion is facilitated by introducing measures of the competitive strengths of the sellers. To begin, let $\Delta_{kj} \;\equiv\; \alpha_j - \beta_j^S\,c_j^S + \eta_j\,c_{kj}^P$ denote Sj's "selling strength" and let

[4] A seller's popularity (e.g., A_j in (5)) refers to consumers' demand for the seller's product when the price of the product is zero (e.g., $p_j^S = 0$ in (5)). Therefore, some consumers, who would have purchased the product from the seller in the absence of platform entry, are attracted to the platform's imitation product in the presence of platform entry.

$\overline{\Delta}_{kj} \equiv \theta_j \, \alpha_j - \beta_j^P \, c_{kj}^P + \eta_j \, c_j^S$ denote Pk's "selling strength" in the presence of platform entry and price competition between Pk and Sj. (1) and (2) imply that Δ_{kj} and $\overline{\Delta}_{kj}$ represent consumers' demand for Sj's product and for Pk's product, respectively, in the presence of platform entry when Pk and Sj price their products at cost under price competition. Thus, Δ_{kj} measures Sj's ability to increase its sales through its product popularity and its relative cost advantage, holding prices constant at cost. $\overline{\Delta}_{kj}$ measures the corresponding ability of Pk as an active seller.

Similarly, I define $\widetilde{\Delta}_{kj} \equiv A_j - b_j^S \, c_j^S$ to be Sj's "selling strength" on Pk in the absence of platform entry, where A_j, and b_j^S are specified in (6). (5) implies that $\widetilde{\Delta}_{kj}$ represents consumers' demand for Sj's product in the absence of platform entry when Sj prices its product at cost. Thus, $\widetilde{\Delta}_{kj}$ measures Sj's ability to increase its sales as a monopolistic seller on Pk holding prices constant at cost.

Furthermore, I define $\Theta_k \equiv B_k \, [\, 1 - f_i \,]$ to be Pk's "platform strength" ($j, k, i \in \{1, 2\}$, $k \neq i$). Thus, a platform's (e.g., Pk) strength increases as it secures more loyal customers (i.e., f_k increases) or its boost (B_k) increases.

(6) implies that

$$\widetilde{\Delta}_{kj} = A_j - b_j^S \, c_j^S = \alpha_j \left[1 + \frac{\theta_j \, \eta_j}{\beta_j^P} \right] - \left[\beta_j^S - \frac{(\eta_j)^2}{\beta_j^P} \right] c_j^S$$

$$= \alpha_j - \beta_j^S \, c_j^S + \frac{\theta_j \, \eta_j}{\beta_j^P} \, \alpha_j + \frac{(\eta_j)^2}{\beta_j^P} \, c_j^S = \Delta_{kj} - \eta_j \, c_{kj}^P + \frac{\theta_j \, \eta_j}{\beta_j^P} \, \alpha_j + \frac{(\eta_j)^2}{\beta_j^P} \, c_j^S$$

$$= \Delta_{kj} + \eta_j \left[\frac{\theta_j}{\beta_j^P} \, \alpha_j + \frac{\eta_j}{\beta_j^P} \, c_j^S - c_{kj}^P \right] = \Delta_{kj} + \eta_j \left[\frac{\theta_j \, \alpha_j + \eta_j \, c_j^S - \beta_j^P \, c_{kj}^P}{\beta_j^P} \right] = \Delta_{kj} + \frac{\eta_j}{\beta_j^P} \, \overline{\Delta}_{kj}. \tag{12}$$

The ensuing analysis will focus on settings in which $\Delta_{kj} > 0$ and $\overline{\Delta}_{kj} > 0$ (and thus, $\widetilde{\Delta}_{kj} > 0$ from (12)) so that (i) consumers' demand for a seller's product is strictly positive in the absence of platform entry; and (ii) consumers' demands for a seller's product and for a platform's product are strictly positive in the presence of platform entry when the platform and the seller compete on prices.

The key notation is summarized in Table 1.

Table 1 **Key Notation**

Variables/Symbols	Description
α_j	Sj's popularity in the presence of platform entry
A_j	Sj's popularity in the absence of platform entry
c_j^S	Sj's unit production cost
c_{kj}^P	Pk's unit cost of imitating Sj's product
F	A platform's fixed cost to enter a seller' product market
w_{kj}	Pk's per-unit commission for Sj
p_{kj}^S	Sj's price if Sj sells on Pk
p_{kj}^P	Pk's price when competing against Sj
q_{kj}^S	Initial demand for Sj's product if Sj sells on Pk
q_{kj}^P	Initial demand for Pk's product when Pk enters Sj's market
Q_{kj}^S	Sj's sales if Sj sells on Pk
π_j	Sj's profit
Π_k	Pk's profit
Θ_k	Pk's platform strength
$\overline{\Delta}_{kj}$	Pk's selling strength as a duopolistic seller
$\widehat{\Delta}_{kj}$	Pk's selling strength as a monopolistic seller
Δ_{kj}	Sj's selling strength on Pk in the presence of platform entry
$\widetilde{\Delta}_{kj}$	Sj's selling strength on Pk in the absence of platform entry

3 Analysis

In Section 3.1, I derive the outcomes in the setting where a seller faces no competition from the platform on which it operates.[5] In Section 3.2, I derive the outcomes in the setting where a seller faces active competition from the platform on which it operates.[6] In Section 3.3, I analyze a benchmark case with a monopolistic platform. In Section 3.4, I study the case of primary interest with platform competition. In Section 3.5, I compare the outcomes in the Monopolistic Platform (MP) and Platform Competition (PC) settings.

To facilitate the statement and proof of key conclusions in the ensuing analysis, it is useful to define the following terms for $k, i, j \in \{1, 2\}$ and $k \neq i$:

$$\Phi_{1j} \equiv 2\,\beta_j^P\,\beta_j^S + \left[\,\eta_j\,\right]^2; \tag{13}$$

$$\Phi_{2j} \equiv 4\,\beta_j^P\,\beta_j^S - \left[\,\eta_j\,\right]^2; \tag{14}$$

[5]In other words, a seller sells on a platform and the platform does not enter the seller's product market.

[6]In other words, a seller sells on a platform and the platform enters the seller's product market.

$$\Omega_j \equiv \frac{[\eta_j]^2}{\beta_j^P \, \beta_j^S}; \tag{15}$$

$$M_{kj} \equiv \frac{1}{2\,[\,1-\Omega_j\,]} \left\{ \frac{\left[\overline{\Delta}_{kj}\right]^2}{2\,\beta_j^P} + \frac{\Omega_j\,[\,2+\Omega_j\,]\,\left[\,26-\Omega_j+2\,(\Omega_j)^2\,\right][\Delta_{kj}]^2}{2\,\beta_j^S\,[\,8+\Omega_j\,]^2} \right.$$
$$\left. + \frac{\Omega_j\left[\,6+2\,\Omega_j+(\Omega_j)^2\,\right]\,\overline{\Delta}_{kj}\,\Delta_{kj}}{\eta_j\,[\,8+\Omega_j\,]} \right\}; \tag{16}$$

$$\phi_{kj} \equiv \left[\left(\frac{8+\Omega_j}{4\sqrt{1-\Omega_j}\,(\,2+\Omega_j\,)} \right) \frac{\widetilde{\Delta}_{ij}}{\Delta_{kj}} \right]^2; \tag{17}$$

$$\xi_1\left(\frac{\overline{\Delta}_{kj}}{\Delta_{kj}}\right) \equiv \left[\frac{5\,\eta_j\,(\,2+\Omega_j\,)}{\beta_j^S\,(\,8+\Omega_j\,)} + \frac{\overline{\Delta}_{kj}}{\Delta_{kj}} \right] \left[\frac{\eta_j\,(\,2+\Omega_j\,)}{\beta_j^S\,(\,8+\Omega_j\,)} + \frac{\overline{\Delta}_{kj}}{\Delta_{kj}} \right] + \frac{4\,\beta_j^P\,[\,2+\Omega_j\,]^2}{\beta_j^S\,[\,8+\Omega_j\,]^2}; \tag{18}$$

$$\xi_2\left(\frac{\overline{\Delta}_{kj}}{\Delta_{kj}}\right) \equiv \left[\frac{8+\Omega_j}{4\sqrt{1-\Omega_j}\,(\,2+\Omega_j\,)} \right]^2 \left[1+\frac{\eta_j}{\beta_j^P}\,\frac{\overline{\Delta}_{kj}}{\Delta_{kj}} \right]^2. \tag{19}$$

3.1 Outcomes in the absence of platform entry

In this section, I study the setting in which a seller (e.g., Sj) faces no competition from the platform (e.g., Pk) on which it operates.

Lemma 1. *Suppose Sj faces no competition from Pk when Sj sells on Pk ($j, k \in \{1, 2\}$). Given w_{kj}, Sj's equilibrium output (i.e., sales) (Q_{kj}^S) is $\frac{\Theta_k \left[\widetilde{\Delta}_{kj} - b_j^S \, w_{kj}\right]}{2}$, and S$j$'s total profit is $\frac{\Theta_k}{b_j^S}\left[\,q_{kj}^S\,\right]^2$ where $q_{kj}^S = \frac{Q_{kj}^S}{\Theta_k}$.*

Lemma 1 reports that in the absence of competition from Pk, Sj's profit when it sells its product on Pk increases as: (i) Pk's platform strength (Θ_k) increases; (ii) Sj's output (q_{kj}^S) increases; and (iii) the sensitivity of the demand for Sj's product to its price (b_j^S) declines.[7]

Lemma 1 implies that in the absence of platform entry: (i) Sj will reduce his output (q_{kj}^S) when he faces a higher commission; and (ii) Sj's sales (Q_{kj}^S) increase as Pk's platform strength (Θ_k) increases and as Sj's selling strength ($\widetilde{\Delta}_{kj}$) increases.

[7](5) implies that b_j^S is the sensitivity of the demand for Sj's product to its price in the absence of platform entry.

Lemma 2. *Suppose Sj faces no competition from Pk when Sj sells on Pk ($j, k \in \{1,2\}$). Then Pk's profit-maximizing commission for Sj is* $w_{kj} = \frac{\widetilde{\Delta}_{kj}}{2\, b_j^S}$.

Lemma 2 reports that if Pk does not sell on its platform, then Pk's profit-maximizing commission for Sj increases as: (i) Sj's selling strength ($\widetilde{\Delta}_{kj}$) increases; and (ii) the sensitivity of the demand for Sj's product to its price (b_j^S) declines.[8]

Lemma 2 implies that in the absence of platform entry, a platform optimally charges the stronger seller a higher commission than it charges the weaker seller. Therefore, the profit-maximizing level of output is less sensitive to the commission for the strong seller.[9]

Lemma 3. *Suppose Sj faces no competition from Pk when Sj sells on Pk ($j, k \in \{1,2\}$). Then Sj's equilibrium output (Q_{kj}^S) is* $\frac{\Theta_k\, \widetilde{\Delta}_{kj}}{4}$, *S$j$'s profit is* $\frac{\Theta_k \left[\widetilde{\Delta}_{kj}\right]^2}{16\, b_j^S}$, *and P$k$'s profit from the commission it collects from Sj is* $\frac{\Theta_k \left[\widetilde{\Delta}_{kj}\right]^2}{8\, b_j^S}$.

Lemma 3 reports that in the absence of platform entry, Sj's profit increases as: (i) Pk's platform strength (Θ_k) increases; (ii) Sj's selling strength ($\widetilde{\Delta}_{kj}$) increases; (iii) and the sensitivity of the demand for Sj's product to its price (b_j^S) declines.

3.2 Outcomes in the presence of platform entry

This section considers the setting in which a seller (e.g., Sj) faces active competition from the platform (e.g., Pk) on which it operates ($j, k \in \{1,2\}$). In this setting, Pk first sets its commission w_{kj}, and then Pk and Sj choose their prices (p_{kj}^P and p_{kj}^S), simultaneously and noncooperatively.

Lemma 4. *Suppose Sj competes against Pk when Sj sells on Pk ($j, k \in \{1,2\}$). Given w_{kj}, Sj's equilibrium output (Q_j^S) is* $\frac{\Theta_k \left[\frac{\eta_j}{\beta_j^P}\, \overline{\Delta}_{kj} + 2\, \Delta_{kj} - 2\, \beta_j^S (1-\Omega_j)\, w_{kj} \right]}{4-\Omega_j}$, *P$k$'s equilibrium output ($Q_{kj}^P$) is* $\frac{\Theta_k \left[2\, \overline{\Delta}_{kj} + \frac{\eta_j}{\beta_j^S}\, \Delta_{kj} - \eta_j (1-\Omega_j)\, w_{kj} \right]}{4-\Omega_j}$, *and S$j$'s total profit is* $\frac{\Theta_k}{\beta_j^S} \left[q_{kj}^S\right]^2$ *where* $q_{kj}^S = \frac{Q_{kj}^S}{\Theta_k}$.

Lemma 4 reports that when Sj competes against Pk, Sj's profit increases as: (i) Pk's platform strength (Θ_k) increases; (ii) Sj's output (q_{kj}^S) increases; and (iii) the sensitivity of

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[8] (5) implies that b_j^S is the effect of product j's price on its demand in the absence of platform entry.

[9] A seller is considered to be a strong seller when his selling strength is sufficiently pronounced (i.e., the seller sells a highly attractive product or has a low marginal cost). DeGraba (1990) provides related conclusions.

the demand for Sj's product to its price (β_j^S) declines.[10]

Lemma 4 implies that if Pk enters Sj's product market, then Sj will increase his output as the commission he faces (i.e., $\frac{\partial q_{kj}^S}{\partial w_{kj}} < 0$) declines. Furthermore, Pk will sell more of its product in equilibrium when it charges Sj a lower commission (i.e., $\frac{\partial q_{kj}^P}{\partial w_{kj}} < 0$). This is the case because Sj charges a lower price for his product when his costs effectively decline due to a lower commission rate (i.e., $\frac{\partial p_{kj}^S}{\partial w_{kj}} > 0$ from (37)). In response, Pk reduces the price for its product (i.e., $\frac{\partial p_{kj}^P}{\partial w_{kj}} > 0$ and $\frac{\partial p_{kj}^P}{\partial p_{kj}^S} > 0$ from (34)), which causes its equilibrium output to increase.

Lemma 5. *Suppose Sj competes against Pk when Sj sells on Pk ($j, k \in \{1, 2\}$). Then Pk's profit-maximizing commission for Sj is*

$$\frac{1}{2[1-\Omega_j]}\left[\frac{\Omega_j\,\overline{\Delta}_{kj}}{\eta_j} + \frac{\left[8+(\Omega_j)^2\right]\Delta_{kj}}{\beta_j^S\,(8+\Omega_j)}\right].$$

Lemma 2 implies that the commission Sj faces under no entry increases as Sj's selling strength increases (i.e., $\frac{\partial\left(w_{kj}^{NE}\right)}{\partial \Delta_j} > 0$). This is the case because the demand of a stronger seller is less sensitive to the commission he faces.[11] Lemma 5 implies that this effect is weakened in the presence of platform entry (i.e., $0 < \frac{\partial\left(w_{kj}^{E}\right)}{\partial \Delta_j} < \frac{\partial\left(w_{kj}^{NE}\right)}{\partial \Delta_j}$).[12]

In the presence of competition between a platform (Pk) and a seller (Sj), the commission Sj faces: (i) increases in Sj's selling strength (i.e., $\frac{\partial\left(w_{kj}^{E}\right)}{\partial \Delta_j} > 0$), partially offsetting Sj's competition advantage ("Offsetting Effect"); and (ii) increases in Pk's selling strength (i.e., $\frac{\partial\left(w_{kj}^{E}\right)}{\partial \overline{\Delta}_{kj}} > 0$), partially amplifying Pk's competition advantage ("Amplifying Effect").[13] Both

[10] (2) implies that β_j^S is the sensitivity of the demand for Sj's product to its price in the presence of platform entry.

[11] (12) and Lemma 2 imply that $\frac{\partial\left(w_{kj}^{NE}\right)}{\partial \widetilde{\Delta}_j} = \frac{\partial\left(w_{kj}^{NE}\right)}{\partial \Delta_j} = \frac{1}{2\,b_j^S} > 0$.

[12] w_{kj}^{NE} denotes the commission Sj faces under no entry. w_{kj}^{E} denotes the commission Sj faces under entry.

(10), Lemmas 2 and 5 imply that

$$\frac{\partial\left(w_{kj}^{E}\right)}{\partial \Delta_j} < \frac{\partial\left(w_{kj}^{NE}\right)}{\partial \Delta_j} \;\Leftrightarrow\; \frac{8+[\Omega_{kj}]^2}{2\,\beta_j^S\,[\,1-\Omega_{kj}\,]\,[\,8+\Omega_{kj}\,]} < \frac{1}{2\,b_j^S}$$

$$\Leftrightarrow\; \frac{8+[\Omega_{kj}]^2}{\beta_j^S\,[\,1-\Omega_{kj}\,]\,[\,8+\Omega_{kj}\,]} < \frac{1}{\beta_j^S\,[\,1-\Omega_{kj}\,]}$$

$$\Leftrightarrow\; \frac{8+[\Omega_{kj}]^2}{8+\Omega_{kj}} < 1 \;\Leftrightarrow\; [\Omega_{kj}]^2 < \Omega_{kj} \;\Leftrightarrow\; \Omega_{kj} < 1.$$

[13] DeGraba (1990) provides related conclusions with respect to the "Offsetting Effect". Inderst and Shaffer (2009) provides related conclusions with respect to the "Amplifying Effect".

the Offsetting Effect and the Amplifying Effect favor Pk and harm Sj in the competition. The increased commission places Sj at a disadvantage by raising Sj's cost.[14] The fact that the commission is increasing in the selling strengths of both Sj and Pk renders Sj's sales and Pk's sales less sensitive to the commission (from Lemma 4). Therefore, it is relatively inexpensive for Pk to raise its rival's (i.e., Sj) costs.

Lemma 5 also implies that in the presence of competition between a platform and a seller, the commission the seller faces increases as the degree of product homogeneity increases (i.e., $\frac{\partial w_{kj}}{\partial \Omega_{kj}} > 0$), partially lessening competition between the platform and the seller.

Lemma 6. *Suppose Sj competes against Pk when Sj sells on Pk ($j, k \in \{1, 2\}$). Then Sj's equilibrium output is* $\frac{\Theta_k [2+\Omega_j] \Delta_{kj}}{8+\Omega_j}$*, Sj's profit is* $\frac{\Theta_k}{\beta_j^S}\left[\frac{(2+\Omega_j)\Delta_{kj}}{8+\Omega_j}\right]^2$*, Pk's equilibrium output is* $\frac{\Theta_k \overline{\Delta}_{kj}}{2} + \frac{\eta_j \Theta_k [2+\Omega_j] \Delta_{kj}}{2\beta_j^S [8+\Omega_j]}$*, and Pk's profit from the commission it collects from Sj is* $\Theta_k M_{kj} - F$*. Pk sells more than Sj if Pk is a stronger seller than Sj (i.e.,* $Q_{kj}^P > Q_{kj}^S$ *if* $\frac{\overline{\Delta}_{kj}}{\Delta_{kj}} > 1$*).*

Lemmas 4, 5, and 6 imply that when Pk and Sj compete, Pk optimally charges a commission to offset the effect of Pk's selling strength ($\overline{\Delta}_{kj}$) on Sj's sales and profit, i.e., Sj's equilibrium sales and profit are independent of the selling strength of the platform on which Sj sells.

3.3 Benchmark Case: Monopolistic Platform

In this section, I analyze the benchmark monopolistic platform setting in which one platform (P) interacts with two sellers (S1 and S2). P simultaneously: (i) chooses to either commit never to act as a seller or make no such commitment and; (ii) sets its per-unit commission. After the commitment and the commission are specified, S1 and S2 choose whether to sell on the platform.[15] If P makes no commitment, then P will make its entry decision (whether to enter and which market to enter). Next, each party in each product market chooses its own price. Without loss of generality, I suppose S1 is stronger than S2 ($c_2^S > c_1^S$, and therefore $\Delta_1 > \Delta_2$) in the ensuing discussion.

Condition FS $\Theta_2 M_{22} - \frac{\Theta_2 [\widetilde{\Delta}_{21}]^2}{8 b_1^S} - \frac{\Theta_2 [\widetilde{\Delta}_{22}]^2}{8 b_2^S} < F < \Theta_1 M_{11} - \frac{\Theta_1 [\widetilde{\Delta}_{11}]^2}{8 b_1^S} - \frac{\Theta_1 [\widetilde{\Delta}_{12}]^2}{8 b_2^S}$.

Condition FS ensures that $F < \Theta_k M_{k2} - \frac{\Theta_k [\widetilde{\Delta}_2]^2}{8 b_2^S} < \Theta_k M_{k1} - \frac{\Theta_k [\widetilde{\Delta}_1]^2}{8 b_1^S}$, i.e., the fixed cost is sufficiently small that the platform will enter both sellers' product market. I assume a platform cannot make discriminating commitments. That is to say, a platform's decision pertains to the markets of both sellers. Thus, if a platform commits not to enter, then it

[14] Salop and Scheffman (1983) provides related conclusions.
[15] Each seller secures zero profit if he chooses not to sell on the platform.

cannot enter S1's market or S2's market. If a platform makes no commitment, then it can enter either seller's market or both markets.

Let the subscript "$\cdot$" denote P under MP.[16] Proposition 1 reports sellers' equilibrium profits and the monopolistic platform's equilibrium profit, and Lemma **??** records consumer surplus under MP when Condition FS holds.

Proposition 1. *Suppose Condition FS holds. In the monopolistic platform setting, both sellers sell on P and P enters both sellers' product markets in equilibrium. Sj's equilibrium profit is* $\frac{\Theta}{\beta_j^S}\left[\frac{(2+\Omega_{\cdot j})\,\Delta_{\cdot j}}{8+\Omega_{\cdot j}}\right]^2$*, and P's equilibrium profit is* $\Theta\, M_{\cdot 1} - F + \Theta\, M_{\cdot 2} - F$.

3.4 Main Case: Platform Competition (PC)

This section considers the case of primary interest in which two platforms compete to attract two independent sellers.

Platforms can differ in platform strength and selling strength in the product market. Pk's unit cost of imitating Sj's product (i.e., c_{kj}^P) affects both Pk's and Sj's ($k, j \in \{1,2\}$) selling strengths: as c_{kj}^P increases Pk's cost increases, and therefore, Pk's selling strength declines and Sj's selling strength increases (i.e., $\frac{\partial \overline{\Delta}_{kj}}{\partial c_{kj}^P} < 0$ and $\frac{\partial \Delta_{kj}}{\partial c_{kj}^P} > 0$).[17] Consequently, P1 is a stronger platform than P2 if $\Theta_1 > \Theta_2$. P1 is a stronger seller than P2 (i.e., $\overline{\Delta}_{1j} > \overline{\Delta}_{2j}$) if $c_{1j}^P < c_{2j}^P$.

If both platforms commit not to enter, Sj's ($j \in \{1,2\}$) selling strength is the same on the two platforms (i.e., $\frac{\partial \tilde{\Delta}_{kj}}{\partial c_{kj}^P} = 0$). If both platforms provide the same boost to Sj's sales (i.e., $\Theta_1 = \Theta_2$), then Sj is indifferent between selling on P1 and selling on P2. If P1 provides a stronger boost to Sj's sales than does P2 (i.e., $\Theta_1 > \Theta_2$), then Sj will sell on P1. Lemma 7 records this result formally.

Lemma 7. *Suppose Condition FS holds and* $\frac{\Theta_1}{\Theta_2} \geq 1$*. Further suppose both platforms commit not to enter. Then Sj (*$j \in \{1,2\}$*) is indifferent between selling on P1 and selling on P2 if* $\frac{\Theta_1}{\Theta_2} = 1$*, whereas Sj sells on P1 if* $\frac{\Theta_1}{\Theta_2} > 1$.

If neither platform commits not to enter, Condition FS ensures that Sj ($j \in \{1,2\}$) faces competition from the platform on which Sj sells. In the case where the stronger platform

[16] For example, $\overline{\Delta}_{\cdot j}$ denotes P's selling strength under MP. *[Remind that $\overline{\Delta}_{kj}$ denotes Pk's selling strength under Platform Competition]*

[17] I discuss another source of asymmetric platform competition ($c_{1j}^P = c_{2j}^P$ and $\eta_{1j} \neq \eta_{2j}$) in the Technical Appendix.

is the weaker seller (i.e., $\Theta_1 > \Theta_2$ and $c_{1j}^P > c_{2j}^P$), Sj sells on P1. This is the case because Sj's selling strength is higher when Sj sells on P1 than when Sj sells on P2 (i.e., $\Delta_{1j} > \Delta_{2j}$) because P1 has a higher cost of imitating Sj's product than does P2 (i.e., $c_{1j}^P > c_{2j}^P$).[18] In addition, P1 provides a stronger boost to Sj's sales than does P2 (i.e., $\Theta_1 > \Theta_2$). In the case where the stronger platform is the stronger seller (i.e., $\Theta_1 > \Theta_2$ and $c_{1j}^P < c_{2j}^P$), Sj faces a trade-off. On the one hand, P1 provides a stronger boost to Sj's sales than does P2 (i.e., $\Theta_1 > \Theta_2$). On the other hand, Sj faces more intense competition from P1 than from P2. This is the case because P1 faces a relatively low cost of imitating Sj's product (i.e., $c_{1j}^P < c_{2j}^P$), and therefore, P1 is a stronger seller than P2 (i.e., $\overline{\Delta}_{1j} > \overline{\Delta}_{2j}$). Consequently, S$j$ will: (i) sell on P1 if the advantage of selling on a strong platform outweighs the disadvantage of competing against a strong seller (i.e., $\frac{\Theta_1}{\Theta_2} > \left[\frac{\Delta_{2j}}{\Delta_{1j}}\right]^2$); and (ii) sell on P2 if the advantage of competing against a weak seller outweighs the disadvantage of selling on a weak platform (i.e., $\frac{\Theta_1}{\Theta_2} < \left[\frac{\Delta_{2j}}{\Delta_{1j}}\right]^2$). Lemma 8 records these results formally.

Lemma 8. *Suppose Condition FS holds and* $\frac{\Theta_1}{\Theta_2} \geq 1$*. Further suppose platforms both make no commitment. If* $\frac{c_{2j}^P}{c_{1j}^P} < 1$*, then Sj will sell on P1. If* $\frac{c_{2j}^P}{c_{1j}^P} > 1$*, then Sj will: (i) sell on P1 when* $\frac{\Theta_1}{\Theta_2} > \left[\frac{\Delta_{2j}}{\Delta_{1j}}\right]^2$*; and (ii) sell on P2 when* $\frac{\Theta_1}{\Theta_2} < \left[\frac{\Delta_{2j}}{\Delta_{1j}}\right]^2$*.*

If P1 commits not to enter and P2 makes no such commitment, Condition FS ensures that Sj ($j \in \{1, 2\}$) faces competition from P2 if Sj sells on P2. Therefore, Sj will sell on P1. This is the case because Sj's selling strength is higher as a monopolistic seller (i.e., selling on P1) than as a duopolistic seller (i.e., selling on P2). In addition, P1 provides a stronger boost to Sj's sales than does P2. Lemma 9 records this result formally.

Lemma 9. *Suppose Condition FS holds and* $\frac{\Theta_1}{\Theta_2} \geq 1$*. Further suppose P1 commits not to enter and P2 makes no commitment. Then Sj* ($j \in \{1, 2\}$) *will sell on P1.*

If P2 commits not to enter and P1 makes no commitment, Condition FS ensures that Sj ($j \in \{1, 2\}$) faces competition from P1 if Sj sells on P1. Sj faces a trade-off in this case. P1 provides a stronger boost to Sj's sales than does P2. However, Sj's selling strength is higher as a monopolistic seller (i.e., selling on P2) than as a duopolistic seller (i.e., selling on P1). Therefore, Sj will: (i) sell on P1 if the advantage of selling on a strong platform outweighs the disadvantage of competing against a platform (i.e., $\frac{\Theta_1}{\Theta_2} > \phi_j$); and (ii) sell on P2 if the advantage of no platform-seller competition outweighs the disadvantage of selling on a weak platform (i.e., $\frac{\Theta_1}{\Theta_2} < \phi_j$). Lemma 10 records these results formally.

[18] Recall that $\frac{\partial \Delta_{kj}}{\partial c_{kj}^P} > 0$.

Lemma 10. *Suppose Condition FS holds and* $\frac{\Theta_1}{\Theta_2} \geq 1$*. Further suppose P1 makes no commitment and P2 commits to no entry. Then Sj* ($j \in \{1, 2\}$) *will: (i) sell on P1 if* $\frac{\Theta_1}{\Theta_2} > \phi_j$*; and (ii) sell on P2 if* $\frac{\Theta_1}{\Theta_2} < \phi_j$*.*

Condition FS ensures that Sj competes against Pk if Sj sells on Pk and Pk makes no commitment ($j, k \in \{1, 2\}$). Consequently, Sj faces a trade-off when choosing a platform. On the one hand, the stronger platform provides the stronger boost to Sj's sales. On the other hand, the stronger the platform, the more intense platform-seller competition Sj faces because the stronger platform has less incentive to commit not to enter. Because platforms cannot make discriminating commitments, Pk also faces a trade-off. On the one hand, committing not to enter can enhance the likelihood of attracting sellers. On the other hand, if Sj always sells on Pk regardless of Pk's commitment, Pk secures more profit by making no commitment (and subsequently entering Sj's market) than by committing not to enter Sj's market.

Lemmas 7 and 9 imply that if the strong platform (P1) commits not to enter, then both sellers sell on the strong platform. This is the case because each seller benefits from a strong platform boost and no platform-seller competition when the seller sells on P1.

Lemmas 8 and 10 imply that if the strong platform (P1) makes no commitment, then each seller's platform choice depends on: (i) P2's commitment decision; (ii) P1's relative platform strength ($\frac{\Theta_1}{\Theta_2}$); and (iii) P1's relative selling strength ($\frac{\overline{\Delta}_{1j}}{\overline{\Delta}_{2j}}$). Lemmas 8 and 10 also provide three conclusions in the case where the strong platform (P1) makes no commitment. First, if P1's relative platform strength is sufficiently pronounced (i.e., $\frac{\Theta_1}{\Theta_2} > \phi_2$), then each seller sells on the strong platform (P1).[19] This is the case because the advantage of selling on a strong platform outweighs the disadvantage of the platform-seller competition. Second, if P1's relative platform strength is sufficiently limited (i.e., $\frac{\Theta_1}{\Theta_2} < \left[\frac{\Delta_{21}}{\Delta_{11}}\right]^2$) and P1's relative selling strength is sufficiently pronounced (i.e., $\frac{\overline{\Delta}_{1j}}{\overline{\Delta}_{2j}} > 1$), then each seller sells on the weak platform (P2).[20] This is the case because the disadvantage of competing against a strong seller outweighs the advantage of selling on a strong platform.[21] Third, if P1's relative platform strength is relatively pronounced (i.e., $\frac{\Theta_1}{\Theta_2} \in \left(\left[\frac{\Delta_{21}}{\Delta_{11}}\right]^2, \phi_2\right)$), then each seller's platform choice depends on P2's commitment decision and P1's relative selling strength.

[19] $c_1^S < c_2^S$, $\frac{\partial\left(\frac{\tilde{\Delta}_{2j}}{\Delta_{1j}}\right)}{\partial c_j^S} > 0$, and (17) imply that $\phi_2 > \phi_1$. Therefore, if $\frac{\Theta_1}{\Theta_2} > \phi_2$, then it must be the case that $\frac{\Theta_1}{\Theta_2} > \phi_1$.

[20] Recall that $\frac{\partial \overline{\Delta}_{kj}}{\partial c_{kj}^P} < 0$. Therefore, $\frac{\overline{\Delta}_{1j}}{\overline{\Delta}_{2j}} > 1 \Leftrightarrow \frac{c_{1j}^P}{c_{2j}^P} < 1$. $c_1^S < c_2^S$ and $\frac{\partial\left(\frac{\Delta_{2j}}{\Delta_{1j}}\right)}{\partial c_j^S} > 0$ imply that $\left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2 > \left[\frac{\Delta_{21}}{\Delta_{11}}\right]^2$. Therefore, if $\frac{\Theta_1}{\Theta_2} < \left[\frac{\Delta_{21}}{\Delta_{11}}\right]^2$, then it must be the case that $\frac{\Theta_1}{\Theta_2} < \left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2$.

[21] Because P1's relative selling strength is sufficiently pronounced (i.e., $\frac{\overline{\Delta}_{1j}}{\overline{\Delta}_{2j}} > 1$), P1 is a stronger seller than P2.

Because S2 is a weaker seller than S1 (i.e., $c_1^S < c_2^S$), two conclusions arise. First, the benefit from no platform-seller competition is greater for S2 than for S1 (i.e., $\phi_2 > \phi_1$) when P1 makes no commitment and P2 commits not to enter. Second, the benefit from reduced platform-seller competition is greater for S2 than for S1 (i.e., $\left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2 > \left[\frac{\Delta_{21}}{\Delta_{11}}\right]^2$) when neither platform makes a commitment and P2 is a weaker seller than P1.

I now introduce an assumption that ensures the value of P2's commitment is not trivial in the case where P1 is a stronger seller than P2.[22]

Assumption BC if $c_{1j}^P < c_{2j}^P$, then $\phi_1 > \left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2$ for $j \in \{1, 2\}$.

Assumption BC pertains to the setting where P1 is a stronger seller than P2 (i.e., $c_{1j}^P < c_{2j}^P$). The assumption states that in this setting, S1's benefit from no platform-seller competition when P1 makes no commitment and P2 commits not to enter exceeds S2's benefit from reduced platform-seller competition when neither platform makes a commitment (i.e., $\phi_1 > \left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2$).

If P1 is a weaker seller than P2 (i.e., $c_{1j}^P > c_{2j}^P$), then $\phi_2 > \phi_1 > 1 > \left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2 > \left[\frac{\Delta_{21}}{\Delta_{11}}\right]^2$. If P1 is a stronger seller than P2 (i.e., $c_{1j}^P < c_{2j}^P$), then Assumption BC ensures that $\phi_2 > \phi_1 > \left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2 > \left[\frac{\Delta_{21}}{\Delta_{11}}\right]^2 > 1$. Lemma 11 records these results formally.

Lemma 11. *Suppose Assumption BC hold. If $c_{1j}^P > c_{2j}^P$, then $\phi_2 > \phi_1 > 1 > \left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2 > \left[\frac{\Delta_{21}}{\Delta_{11}}\right]^2$. If $c_{1j}^P < c_{2j}^P$, then $\phi_2 > \phi_1 > \left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2 > \left[\frac{\Delta_{21}}{\Delta_{11}}\right]^2 > 1$.*

Proposition 2. *Suppose Condition FS and Assumption BC hold, and $\frac{\Theta_1}{\Theta_2} \geq 1$. Then in equilibrium: (i) if $\frac{\Theta_1}{\Theta_2} > \phi_2$, P1 makes no commitment, and both S1 and S2 sell on P1; (ii)*

[22] In the case where P1 is a stronger seller than P2 (i.e., $c_{1j}^P < c_{2j}^P$), equilibrium results do not vary with the sign of $\phi_1 - \left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2$ if P1's relative platform strength is sufficiently pronounced (i.e., $\frac{\Theta_1}{\Theta_2} > \phi_2$) or sufficiently limited (i.e., $\frac{\Theta_1}{\Theta_2} < \phi_1$). However, when P1's relative platform strength is relatively pronounced (i.e., $\frac{\Theta_1}{\Theta_2} \in (\phi_1, \phi_2)$), the sign of $\phi_1 - \left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2$ affects P2's commitment decision in equilibrium. If $\phi_1 > \left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2$, then S2's benefit from a strong platform boost exceeds the benefit from reduced platform-seller competition (i.e., $\frac{\Theta_1}{\Theta_2} > \left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2$), but is less than the benefit from no platform-seller competition (i.e., $\frac{\Theta_1}{\Theta_2} < \phi_2$). Therefore, P2 must commit not to enter to ensure that S2 will choose to sell on P2. If $\phi_1 < \left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2$, P2 also commits not to enter to ensure S2 sells on P2 when $\frac{\Theta_1}{\Theta_2} \in \left(\left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2, \phi_2\right)$. However, P2 can successfully attract S2 without committing not to enter when $\frac{\Theta_1}{\Theta_2} \in \left(\phi_1, \left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2\right)$ because S2's benefit from reduced platform-seller competition exceeds the benefit from a strong platform boost (i.e., $\frac{\Theta_1}{\Theta_2} < \left[\frac{\Delta_{22}}{\Delta_{12}}\right]^2$).

if $\frac{\Theta_1}{\Theta_2} \in (\phi_1, \phi_2)$*, P1 makes no commitment whereas P2 commits not to enter, and S1 sells on P1 whereas S2 sells on P2; (iii) if* $\frac{\Theta_1}{\Theta_2} \in (1, \phi_1)$*, P1 commits not to enter, and both S1 and S2 sell on P1; and (iv) if* $\frac{\Theta_1}{\Theta_2} = 1$*, both platforms commit not to enter, and each seller is indifferent between selling on P1 and selling on P2.*

Proposition 2 provides three primary conclusions. First, if P1's relative platform strength is sufficiently pronounced (i.e., if $\frac{\Theta_1}{\Theta_2} > \phi_2$), P1 makes no commitment, and both sellers sell on P1 and compete against P1. This is the case because each seller's benefit from a strong platform boost exceeds the benefit from no platform-seller competition (i.e., $\frac{\Theta_1}{\Theta_2} > \phi_2$ and $\frac{\Theta_1}{\Theta_2} > \phi_1$).[23] Second, as P1's relative platform strength declines (i.e., if $\frac{\Theta_1}{\Theta_2} \in (\phi_1, \phi_2)$), P2 commits not to enter to ensure that S2 will sell on P2. As a weak seller, S2 sells on P2 because S2's benefit from no platform-seller competition exceeds the benefit from a strong platform boost (i.e., $\frac{\Theta_1}{\Theta_2} < \phi_2$). As a strong seller, S1 sells on P1 because S1's benefit from a strong platform boost exceeds the benefit from no platform-seller competition (i.e., $\frac{\Theta_1}{\Theta_2} > \phi_1$). P1 makes no commitment because P1's benefit from imitating the strong seller's (i.e., S1's) popular product exceeds its benefit from attracting an additional seller (i.e., S2). Consequently, S2 faces no competition whereas S1 competes against P1. Third, if P1 and P2 are sufficiently similar (i.e., $\frac{\Theta_1}{\Theta_2} \in (1, \phi_1)$) or symmetric (i.e., $\frac{\Theta_1}{\Theta_2} = 1$) platforms, the relatively intense inter-platform competition compels both platforms to commit not to enter, in order to attract sellers. Consequently, both sellers face no competition.

Figure 1 illustrates these equilibrium outcomes. The figure indicates that as P1's relative platform strength ($\frac{\Theta_1}{\Theta_2}$) declines, platform-seller competition is reduced in each seller's market, and the reduction occurs more rapidly for the weak seller (S2) than for the strong seller (S1). This is the case because as P1's relative platform strength declines, platform competition becomes relatively intense, which compels platforms to commit not to enter in order to attract sellers. When $\frac{\Theta_1}{\Theta_2} > \phi_2$ (pronounced advantage range), each seller's benefit from a strong platform boost exceeds the benefit from no platform-seller competition (i.e., $\frac{\Theta_1}{\Theta_2} > \phi_j$ for $j \in \{1, 2\}$).[24] Therefore, P2's commitment does not affect sellers' platform choices. When $\frac{\Theta_1}{\Theta_2} \in (\phi_1, \phi_2)$ (moderate advantage range), S2's benefit from no platform-seller competition exceeds the benefit from a strong platform boost (i.e., $\frac{\Theta_1}{\Theta_2} < \phi_2$), whereas S1's benefit from a strong platform boost exceeds the benefit it derives from no platform-seller competition (i.e., $\frac{\Theta_1}{\Theta_2} > \phi_1$). Therefore, P2 can successfully attract S2 by committing not to enter. P1 makes no commitment because P1's benefit from imitating the strong seller's (i.e., S1) popular product exceeds P1's benefit from attracting an additional seller (i.e., S2). Consequently, platform-seller competition in S2's market is reduced (in the sense that S2 faces no rival seller in equilibrium) as $\frac{\Theta_1}{\Theta_2}$ declines from the pronounced advantage range to the moderate advantage range. When $\frac{\Theta_1}{\Theta_2} < \phi_1$ (limited advantage range), each seller's benefit from no platform-seller competition exceeds the benefit from a strong platform boost (i.e., $\frac{\Theta_1}{\Theta_2} < \phi_j$

[23] Recall that $\phi_1 < \phi_2$ from (**??**).

[24] Recall that the selling strength gain from no platform-seller competition is higher for the weak seller (S2) than for the strong seller (S1) (i.e., $\phi_2 > \phi_1$). Therefore, if $\frac{\Theta_1}{\Theta_2} > \phi_2$, then it must be the case that $\frac{\Theta_1}{\Theta_2} > \phi_1$.

for $j \in \{1,2\}$).[25] Therefore, P2 can successfully attract both sellers by committing not to enter if P1 makes no commitment. This fact compels P1 to commit not to enter in order to attract sellers. Consequently, platform-seller competition in S1's market is reduced (in the sense that S1 faces no rival seller in equilibrium) as $\frac{\Theta_1}{\Theta_2}$ declines from the moderate advantage range to the limited advantage range.

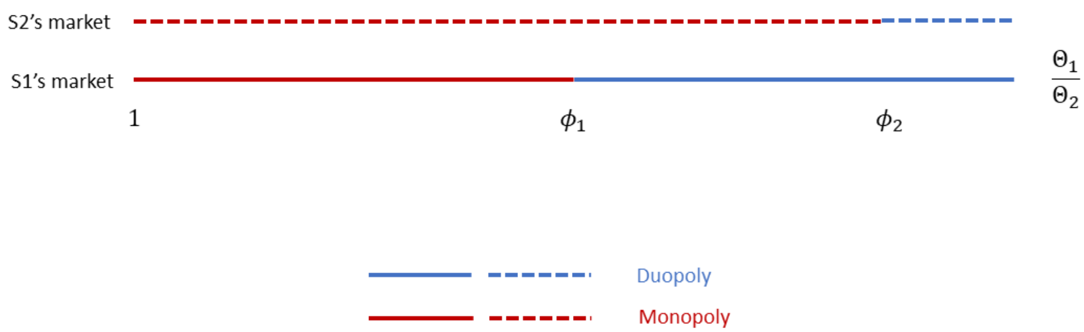

Figure 1: Equilibrium outcomes

3.5 Comparison of outcomes under MP and PC

In this section, I compare equilibrium outcomes in the benchmark setting with only one platform (MP) and in the setting of primary interest where two platforms compete (PC). In the ensuing discussion, the superscript "M" denotes the MP setting.[26]

When a seller chooses the platform on which it will sell, the seller considers how his sales are affected by platform strengths and the relative intensities of platform-seller competition. Condition FS ensures that Sj ($j \in \{1,2\}$) competes against P under MP. Increased platform competition might reduce the intensity of platform-seller competition in Sj's market because each platform's commitment decision under PC varies with the competitive strength (which encompasses platform strength and selling strength) of the rival platform.

A sufficiently weak rival platform (i.e., $\frac{\widetilde{\Theta}}{\Theta} < \frac{1}{\phi_2}$) will not affect sellers' platform choices under PC. Therefore, to ensure that the effect of platform competition is not trivial, the ensuing analysis focuses on settings in which the rival platform's ($\widetilde{\text{P}}$'s) relative platform

[25] Because $\phi_2 > \phi_1$, $\frac{\Theta_1}{\Theta_2} < \phi_1$ (i.e., S1's benefit from no platform-seller competition exceeds the benefit from a strong platform boost) implies that $\frac{\Theta_1}{\Theta_2} < \phi_2$ (i.e., S2's benefit from no platform-seller competition exceeds the benefit from a strong platform boost).

[26] For example, π_j^M and Π^M denote Sj's and P's equilibrium profits under MP.

strength is not too limited (i.e., $\frac{\widetilde{\Theta}}{\Theta} > \frac{1}{\phi_2}$).[27] The ensuing analysis focuses on settings in which the rival platform's ($\widetilde{\mathrm{P}}$'s) relative selling strength is weak (i.e., $\frac{\tilde{c}_j^P}{c_j^P} > 1$). The case where $\widetilde{\mathrm{P}}$'s relative selling strength is strong (i.e., $\frac{\tilde{c}_j^P}{c_j^P} < 1$) is analyzed in the Appendix and discussed in the concluding section. Figure 2 depicts $\widetilde{\mathrm{P}}$'s relative platform strength and relative selling strength.

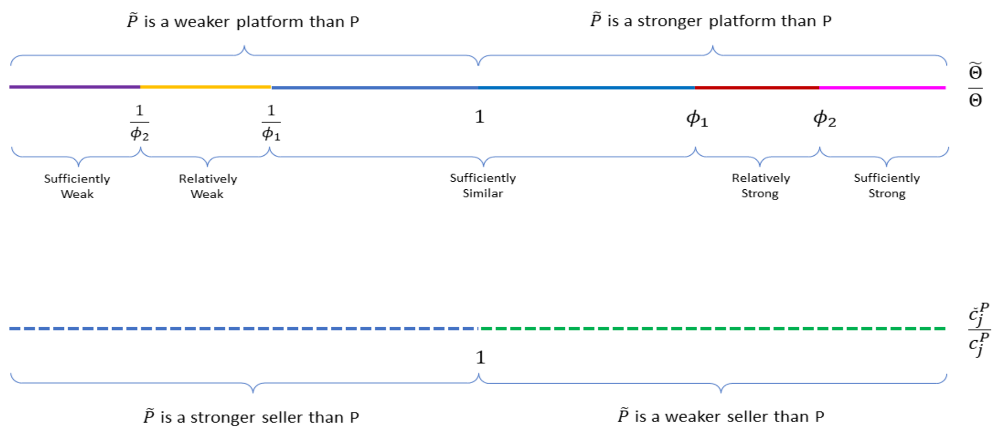

Figure 2: $\widetilde{\mathrm{P}}$'s relative platform strength and relative selling strength

Proposition 2 provides four conclusions. First, if P faces a relatively weak competing platform $\widetilde{\mathrm{P}}$ (i.e., if $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_2}, \frac{1}{\phi_1}\right)$), then $\widetilde{\mathrm{P}}$ commits not to enter to ensure it attracts the weak seller (S2).[28] Given this commitment, S2 sells on $\widetilde{\mathrm{P}}$ because S2's benefit from no platform-seller competition exceeds the benefit from a strong platform boost (i.e., $\phi_2 > \frac{\Theta}{\widetilde{\Theta}}$). P makes no commitment because P's benefit from imitating the strong seller's (i.e., S1) popular product exceeds P's benefit from attracting an additional seller (i.e., S2). S1 sells on P because S1's benefit from a strong platform boost exceeds the benefit from no platform-seller competition (i.e., $\frac{\Theta}{\widetilde{\Theta}} > \phi_1$). Therefore, increased platform competition reduces platform-seller competition in the sense that S2 faces no competition and S1 faces unchanged competition under PC, whereas each seller competes against P under MP.

Second, if P faces a sufficiently similar or symmetric competing platform $\widetilde{\mathrm{P}}$ (i.e., if $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_1}, \phi_1\right)$), then the relatively intense platform competition compels both platforms

[27] Proposition 2 reports that if $\widetilde{\mathrm{P}}$ is a sufficiently weaker platform than P (i.e., $\frac{\widetilde{\Theta}}{\Theta} < \frac{1}{\phi_2}$), then both sellers sell on P under MP and under PC. This is the case because each seller's disadvantage from selling on a sufficiently weak platform outweighs the advantage of no platform-seller competition.

[28] $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_2}, \frac{1}{\phi_1}\right)$ implies that $\frac{\Theta}{\widetilde{\Theta}} \in (\phi_1, \phi_2)$.

to commit not to enter in order to attract a seller.[29] Therefore, increased platform competition reduces platform-seller competition in the sense that each seller faces no competition under PC, whereas each seller competes against P under MP.

Third, if P faces a relatively strong competing platform $\widetilde{\mathrm{P}}$ (i.e., if $\frac{\widetilde{\Theta}}{\Theta} \in (\phi_1, \phi_2)$), then P commits not to enter to ensure it attracts the weak seller (S2). Given this commitment, S2 sells on P because S2's benefit from no platform-seller competition exceeds the benefit from a strong platform boost (i.e., $\phi_2 > \frac{\widetilde{\Theta}}{\Theta}$). $\widetilde{\mathrm{P}}$ makes no commitment because $\widetilde{\mathrm{P}}$'s benefit from imitating the strong seller's (i.e., S1) popular product exceeds $\widetilde{\mathrm{P}}$'s benefit from attracting an additional seller (i.e., S2). S1 sells on $\widetilde{\mathrm{P}}$ because S1's benefit from a strong platform boost exceeds the benefit from no platform-seller competition (i.e., $\frac{\widetilde{\Theta}}{\Theta} > \phi_1$). Consequently, because $\widetilde{\mathrm{P}}$ is a weaker seller than P (i.e., $\frac{\widetilde{c}_j^P}{c_j^P} > 1$), increased platform competition reduces platform-seller competition in the sense that S1 faces reduced competition and S2 faces no competition under PC, whereas each seller competes against P under MP.[30]

Fourth, if P faces a sufficiently strong competing platform $\widetilde{\mathrm{P}}$ (i.e., if $\frac{\widetilde{\Theta}}{\Theta} > \phi_2$), then $\widetilde{\mathrm{P}}$ makes no commitment and both sellers sell on $\widetilde{\mathrm{P}}$. This is the case because each seller's benefit from a strong platform boost exceeds the benefit from no platform-seller competition (i.e., $\frac{\widetilde{\Theta}}{\Theta} > \phi_j$ for $j \in \{1, 2\}$).[31] Therefore, because $\widetilde{\mathrm{P}}$ is a weaker seller than P (i.e., $\frac{\widetilde{c}_j^P}{c_j^P} > 1$), increased platform competition reduces platform-seller competition in the sense that each seller faces reduced competition under PC, whereas each seller competes against P under MP.[32]

Proposition 3 states these results formally.

Proposition 3. *Suppose Condition FS and Assumption BC hold. Further suppose P faces a competing platform $\widetilde{P}$ that is a weaker seller than P (i.e., $\frac{\widetilde{c}_j^P}{c_j^P} > 1$). Then increased platform competition reduces platform-seller competition in the sense that at least one seller faces no competition or reduced competition and no seller faces increased competition in the presence of platform competition.*

Figure 3 compares platform-seller competition under MP and PC. Solid lines represent S1's market and dashed lines represent S2's market. Recall that each seller competes against P under MP. Figure 3 indicates that if $\widetilde{\mathrm{P}}$ is a weaker seller than P, then increased platform competition reduces platform-seller competition in the sense that under PC: (i) one seller

[29] $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_1}, \phi_1\right)$ implies that $\frac{\Theta}{\widetilde{\Theta}} < \phi_1$ and $\frac{\widetilde{\Theta}}{\Theta} < \phi_1$.

[30] S1 faces reduced competition because S1 competes against a platform that is a weaker seller than P.

[31] Because $\phi_2 > \phi_1$, $\frac{\widetilde{\Theta}}{\Theta} > \phi_2$ ensures that $\frac{\widetilde{\Theta}}{\Theta} > \phi_1$.

[32] Each seller faces reduced competition because each seller competes against a platform that is a weaker seller than P.

faces unchanged competition and the other seller faces no competition (when $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_2}, \frac{1}{\phi_1}\right)$); or (ii) each seller faces no competition (when $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_1}, \phi_1\right)$); or (iii) one seller faces reduced competition and the other seller faces no competition (when $\frac{\widetilde{\Theta}}{\Theta} \in (\phi_1, \phi_2)$); or (iv) each seller faces reduced competition (when $\frac{\widetilde{\Theta}}{\Theta} > \phi_2$).[33]

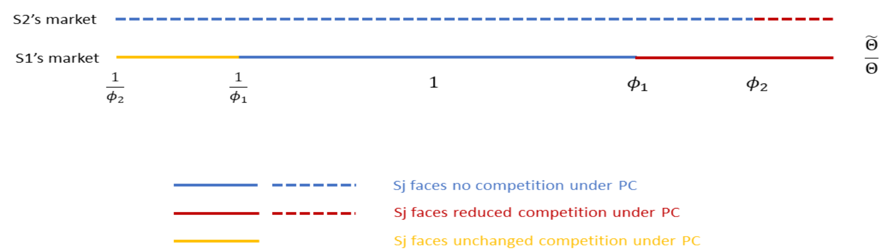

Figure 3: Platform-seller competition under MP and PC

Proposition 4 reports that increased platform competition either increases or does not change each seller's profit.[34] This is the case because each seller benefits from reduced platform-seller competition or no platform-seller competition under PC.[35]

Proposition 4. *Suppose Condition FS and Assumption BC hold. Further suppose P faces a competing platform $\widetilde{P}$ that is a weaker seller than P (i.e., $\frac{\widetilde{c}_j^P}{c_j^P} > 1$). Then $\pi_1 \geq \pi_1^M$ and $\pi_2 > \pi_2^M$, where the first inequality holds strictly unless $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_2}, \frac{1}{\phi_1}\right)$.*

Proposition 5 indicates that if P faces a competing platform that is a weaker seller than P (i.e., $\frac{\widetilde{c}_j^P}{c_j^P} > 1$), then increased platform competition reduces consumer surplus (i.e.,

[33] One seller faces unchanged competition when the seller competes against P both under MP and under PC. One seller faces reduced competition when the seller competes against P under MP whereas the seller competes against a platform that is a weaker seller than P under PC.

[34] Increased platform competition does not change S1's profit when $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_2}, \frac{1}{\phi_1}\right)$ because S1 competes against P both under MP and under PC.

[35] Recall that each seller competes against P under MP. Each seller benefits from reduced platform-seller competition under PC when the seller competes against a platform that is a weaker seller than P. Each seller benefits from no platform-seller competition under PC when the seller sells on a platform that commits not to enter.

$CS < CS^M$), unless $\widetilde{\mathrm{P}}$'s relative platform strength is sufficiently pronounced.[36] This is the case because increased platform competition reduces platform-seller competition and reduced platform-seller competition induces higher prices (i.e., $p_1^S \geq p_1^{SM}$ and $p_2^S > p_2^{SM}$). Proposition 6 records this result formally.

Proposition 5. *Suppose Condition FS and Assumption BC hold. Further suppose P faces a competing platform $\widetilde{P}$ that is a weaker seller than P (i.e., $\frac{\widetilde{c}_j^P}{c_j^P} > 1$). Then $CS < CS^M$ unless $\widetilde{P}$'s relative platform strength is sufficiently pronounced.*

Proposition 6. *Suppose Condition FS and Assumption BC hold. Further suppose P faces a competing platform $\widetilde{P}$ that is a weaker seller than P (i.e., $\frac{\widetilde{c}_j^P}{c_j^P} > 1$). Then $p_1^S \geq p_1^{SM}$ and $p_2^S > p_2^{SM}$, where the first inequality holds strictly unless $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_2}, \frac{1}{\phi_1}\right)$.*

If a seller faces competition from a platform both under MP and under PC, then increased platform competition reduces the commission the seller faces.[37] This is the case because under PC each seller competes against a platform ($\widetilde{\mathrm{P}}$) that is a relatively weak seller. A platform secures profit both from commission revenue and from retail revenue under platform-seller competition. If the platform is a relatively strong seller, then it focuses on securing retail revenue by raising its rival seller's cost and thereby shifting consumers' demand toward its product through a relatively high commission.[38] In contrast, if the platform is a relatively weak seller, then it focuses on securing commission revenue by encouraging the seller's sales through a relatively low commission.

If a seller faces competition from a platform under MP but faces no competition under PC, then increased platform competition increases the commission the seller faces.[39] This is the case because a seller's demand for the access to a platform is less sensitive to the commission as a monopolistic seller than as a duopolistic seller. Therefore, the more inelastic demand for the access to a platform increases the platform's profit-maximizing commission.

If a seller faces unchanged platform-seller competition under MP and under PC, then

[36] If $\widetilde{\mathrm{P}}$'s relative platform strength is sufficiently pronounced (i.e., $\frac{\widetilde{\Theta}}{\Theta} > \max\left\{\frac{\varsigma_{P1}}{\varsigma_{\widetilde{P}1}}, \frac{\varsigma_{P2}}{\varsigma_{\widetilde{P}2}}, \phi_{\widetilde{P}2}\right\}$), then increased platform competition increases consumer surplus (i.e., $CS > CS^M$). This is the case because the benefit consumers derive from the relatively strong platform boost exceeds the loss consumers suffer from higher prices due to reduced platform-seller competition.

[37] If $\frac{\widetilde{\Theta}}{\Theta} > \phi_j$, then S$j$ ($j \in \{1, 2\}$) competes against P under MP whereas Sj competes against $\widetilde{\mathrm{P}}$ under PC.

[38] Etro (2021) provides related discussions.

[39] If $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_1}, \phi_1\right)$, then S1 competes against P under MP whereas S1 faces no competition under PC. If $\frac{\widetilde{\Theta}}{\Theta} < \phi_2$, then S2 competes against P under MP whereas S2 faces no competition under PC.

increased platform competition does not change the prevailing commission.[40] Proposition 7 reports these results formally.

Proposition 7. *Suppose Condition FS and Assumption BC hold. Further suppose P faces a competing platform $\widetilde{P}$ that is a weaker seller than P (i.e., $\frac{\widetilde{c}_j^P}{c_j^P} > 1$). Then $w_j < w_j^M$ if $\frac{\widetilde{\Theta}}{\Theta} > \phi_j$ whereas $w_j \geq w_j^M$ if $\frac{\widetilde{\Theta}}{\Theta} < \phi_j$ ($j \in \{1, 2\}$), where the equality holds when $j = 1$ and $\frac{\widetilde{\Theta}}{\Theta} < \frac{1}{\phi_1}$.*

Proposition 8 indicates that if P faces a competing platform that is a weaker seller than P (i.e., $\frac{\widetilde{c}_j^P}{c_j^P} > 1$), then increased platform competition reduces total welfare (i.e., $SW < SW^M$) unless $\widetilde{\text{P}}$'s relative platform strength is sufficiently pronounced.[41] This is the case for three primary reasons. First, P suffers from increased platform competition. Second, increased platform competition reduces platform-seller competition and reduced platform-seller competition benefits sellers but hurts consumers due to higher prices. Third, the sum of the loss that platforms suffer from increased platform competition and the loss that consumers suffer from reduced platform-seller competition exceeds the benefit sellers derive from reduced platform-seller competition.

Proposition 8. *Suppose Condition FS and Assumption BC hold. Further suppose P faces a competing platform $\widetilde{P}$ that is a weaker seller than P (i.e., $\frac{\widetilde{c}_j^P}{c_j^P} > 1$). Then $SW < SW^M$ unless $\widetilde{P}$'s relative platform strength is sufficiently pronounced.*

4 Discussion and conclusion

It is not uncommon for a monopolistic platform to enter a third-party seller's product market and compete against the seller by introducing its private-label products. Platform

[40] If $\frac{\widetilde{\Theta}}{\Theta} < \frac{1}{\phi_1}$, then S1 competes against P both under MP and under PC. In this case, $w_1 = w_1^M$ because unchanged platform-seller competition induces no change in the prevailing commission.

[41] If $\widetilde{\text{P}}$'s relative platform strength is sufficiently pronounced, then increased platform competition increases total welfare. This is the case for three primary reasons. First, increased platform competition increases the aggregate profits of platforms if $\widetilde{\text{P}}$'s relative platform strength is sufficiently pronounced. This is the case because sellers sell on P under MP and sell on $\widetilde{\text{P}}$ under PC in this case. Therefore, increased platform competition benefits $\widetilde{\text{P}}$ but hurts P and the benefit that $\widetilde{\text{P}}$ derives exceeds the loss that P suffers. This is the case because $\widetilde{\text{P}}$ is a stronger platform and a weaker seller than P and the gain from the relatively pronounced platform strength exceeds the loss from the relatively weak selling strength. Second, Proposition 4 implies that increased platform competition increases the aggregate profits of sellers. Third, Proposition 5 implies that increased platform competition increases consumer surplus if $\widetilde{\text{P}}$'s relative platform strength is sufficiently pronounced.

entry reduces third-party sellers' profits because the platform's product imitation based on third-party sellers' sales data leads to unfair platform-seller competition.

If a platform faces a competing platform, each platform endogenously chooses its entry decision which affects the intensity of platform-seller competition, considering sellers' platform choices. The platform's relative strength and the intensity of platform-seller competition affect each seller's platform choice. If one platform's relative platform strength is sufficiently pronounced, both sellers compete against the strong platform. If one platform's relative platform strength is relatively pronounced, the strong seller competes against the strong platform and the weak seller secures the monopolistic profit on the weak platform. If two platforms are sufficiently similar or symmetric, the relatively intense inter-platform competition leads to no platform-seller competition.

My paper focuses on the case in which the competing platform is a relatively weak seller and finds that increased platform competition reduces platform-seller competition. Third-party sellers benefit from increased platform competition because of the reduced platform-seller competition. However, increased platform competition reduces consumer surplus and total welfare unless the competing platform is a sufficiently strong platform.

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6 Appendix

The Appendix sketches the proofs of the formal conclusions in the text. Detailed proofs are available in Online Appendix.

Proof of Lemma 1. Suppose Sj faces no competition from Pk when Sj sells on Pk ($j, k \in \{1, 2\}$). (11) implies that Sj's profit is given by ($j, k, i \in \{1, 2\}$, $k \neq i$):

$$\pi_j \;=\; \left[\, p_{kj}^S - w_{kj} - c_j^S \,\right] \Theta_k \, q_{kj}^S. \tag{20}$$

(5) and (20) imply Sj chooses p_j^S to Maximize $\left[\, p_{kj}^S - w_{kj} - c_j^S \,\right] \Theta_k \left[\, A_j - b_j^S \, p_{kj}^S \,\right]$:

$$\Rightarrow \quad \frac{\partial \pi_j}{\partial p_{kj}^S} \;=\; 0 \quad \Leftrightarrow \quad p_{kj}^S\left(w_{kj}\right) \;=\; \frac{A_j + b_j^S \, c_j^S + b_j^S \, w_{kj}}{2 \, b_j^S}. \tag{21}$$

(5) and (21) imply that consumers' initial demand for Sj's product is:

$$q_{kj}^S \;=\; A_j - b_j^S \, \frac{A_j + b_j^S \, c_j^S + b_j^S \, w_{kj}}{2 \, b_j^S} \;=\; \frac{\widetilde{\Delta}_{kj} - b_j^S \, w_{kj}}{2}. \tag{22}$$

(11) and (22) imply that Sj's sales are:

$$Q_{kj}^S \;=\; \Theta_k \, q_{kj}^S \;=\; \frac{\Theta_k \left[\, \widetilde{\Delta}_{kj} - b_j^S \, w_{kj} \,\right]}{2}. \tag{23}$$

(20), (21), and (23) imply that Sj's profit is:

$$\pi_j \;=\; \left[\, \frac{A_j + b_j^S \, c_j^S + b_j^S \, w_{kj}}{2 \, b_j^S} - w_{kj} - c_j^S \,\right] \frac{\Theta_k \left[\, \widetilde{\Delta}_{kj} - b_j^S \, w_{kj} \,\right]}{2} \;=\; \frac{\Theta_k}{b_j^S} \left[\, \frac{\widetilde{\Delta}_{kj} - b_j^S \, w_{kj}}{2} \,\right]^2. \;\blacksquare \tag{24}$$

Proof of Lemma 2. Suppose Sj faces no competition from Pk when Sj sells on Pk ($j, k \in$

$\{1,2\})$. (22) implies that Pk chooses w_{kj} to

$$\text{Maximize} \;\; \Pi_k \;=\; w_{kj}\,\Theta_k\,q^S_{kj} + \overline{\Pi}_{kl} \;=\; w_{kj}\,\Theta_k\,\frac{\widetilde{\Delta}_{kj} - b^S_j\,w_{kj}}{2} + \overline{\Pi}_{kl} \tag{25}$$

$$\Rightarrow\;\; \frac{\partial \Pi_k}{\partial w_{kj}} \;=\; 0 \;\;\Leftrightarrow\;\; \frac{\widetilde{\Delta}_{kj} - b^S_j\,w_{kj}}{2} - \frac{b^S_j\,w_{kj}}{2} \;=\; 0 \;\;\Leftrightarrow\;\; w_{kj} \;=\; \frac{\widetilde{\Delta}_{kj}}{2\,b^S_j}, \tag{26}$$

where $\overline{\Pi}_{kl}$ is the profit that Pk secures from Sl $(j,k,l \in \{1,2\},\ j \neq l)$, $\overline{\Pi}_{kl} > 0$ if Sl sells on Pk, and $\overline{\Pi}_{kl} = 0$ if Sl does not on Pk.[42] ■

<u>Proof of Lemma 3</u>. Suppose Sj faces no competition from Pk when Sj sells on Pk $(j,k \in \{1,2\})$. Lemmas 1 and 2 imply that consumers' initial demand for product j is

$$q^S_{kj} \;=\; \frac{\widetilde{\Delta}_{kj} - b^S_j\,w_{kj}}{2} \;=\; \frac{\widetilde{\Delta}_{kj} - b^S_j\,\frac{\widetilde{\Delta}_{kj}}{2\,b^S_j}}{2} \;=\; \frac{\widetilde{\Delta}_{kj} - \frac{\widetilde{\Delta}_{kj}}{2}}{2} \;=\; \frac{\widetilde{\Delta}_{kj}}{4}. \tag{27}$$

(11) and (27) imply that Sj's sales are:

$$Q^S_{kj} \;=\; \Theta_k\,q^S_{kj} \;=\; \frac{\Theta_k\,\widetilde{\Delta}_{kj}}{4}.$$

Lemma 1 and (27) imply that Sj's profit is

$$\pi_j \;=\; \frac{\Theta_k}{b^S_j}\left[\,q^S_{kj}\,\right]^2 \;=\; \frac{\Theta_k}{b^S_j}\left[\frac{\widetilde{\Delta}_{kj}}{4}\right]^2 \;=\; \frac{\Theta_k\left[\,\widetilde{\Delta}_{kj}\,\right]^2}{16\,b^S_j}. \tag{28}$$

Lemma 2 and (27) imply that Pk's profit from charging a commission from Sj is.

$$\Pi_k \;=\; w_{kj}\,\Theta_k\,q^S_{kj} \;=\; \Theta_k\,\frac{\widetilde{\Delta}_{kj}}{2\,b^S_j}\,\frac{\widetilde{\Delta}_{kj}}{4} \;=\; \frac{\Theta_k\left[\,\widetilde{\Delta}_{kj}\,\right]^2}{8\,b^S_j}.\;\; ■ \tag{29}$$

<u>Proof of Lemma 4</u>. Suppose Sj competes against Pk when Sj sells on Pk $(j,k \in \{1,2\})$. (1)

[42] $\overline{\Pi}_{kl}$ does not include w_{kj}.

and (2) imply that Pk's profit is

$$\Pi_k \;=\; \left[\, p^P_{kj} - c^P_{kj} \,\right] \Theta_k \, q^P_{kj} - F + w_{kj} \, \Theta_k \, q^S_{kj} + \overline{\Pi}_{kl} \tag{30}$$

$$= \; \left[\, p^P_{kj} - c^P_{kj} \,\right] \Theta_k \left[\, \theta_j \, \alpha_j - \beta^P_j \, p^P_{kj} + \eta_j \, p^S_{kj} \,\right] - F + w_{kj} \, \Theta_k \left[\, \alpha_j - \beta^S_j \, p^S_{kj} + \eta_j \, p^P_{kj} \,\right] + \overline{\Pi}_{kl}; \tag{31}$$

Sj's profit is

$$\pi_j \;=\; \left[\, p^S_{kj} - w_{kj} - c^S_j \,\right] \Theta_k \, q^S_{kj} \;=\; \left[\, p^S_{kj} - w_{kj} - c^S_j \,\right] \Theta_k \left[\, \alpha_j - \beta^S_j \, p^S_{kj} + \eta_j \, p^P_{kj} \,\right], \tag{32}$$

where $\overline{\Pi}_{kl}$ is the profit that Pk secures from Sl ($j, k, l \in \{1, 2\}$, $j \neq l$), $\overline{\Pi}_{kl} > 0$ if Sl sells on Pk, and $\overline{\Pi}_{kl} = 0$ if Sl does not on Pk.

(31) implies that Pk chooses its price p^P_{kj} to

$$\text{Maximize } \; \Pi_k \;=\; \left[\, p^P_{kj} - c^P_{kj} \,\right] \Theta_k \left[\, \theta_j \, \alpha_j - \beta^P_j \, p^P_{kj} + \eta_j \, p^S_{kj} \,\right] - F$$

$$+ \; w_{kj} \, \Theta_k \left[\, \alpha_j - \beta^S_j \, p^S_{kj} + \eta_j \, p^P_{kj} \,\right] + \overline{\Pi}_{kl} \tag{33}$$

$$\Rightarrow \; \frac{\partial \Pi_k}{\partial p^P_{kj}} \;=\; 0 \;\;\Leftrightarrow\;\; p^P_{kj} \;=\; \frac{\theta_j \, \alpha_j + \eta_j \, p^S_{kj} + \beta^P_j \, c^P_{kj} + w_{kj} \, \eta_j}{2 \, \beta^P_j}. \tag{34}$$

(32) implies that Sj chooses its price p^S_{kj} to

$$\text{Maximize } \; \pi_j \;=\; \left[\, p^S_{kj} - w_{kj} - c^S_j \,\right] \Theta_k \left[\, \alpha_j - \beta^S_j \, p^S_{kj} + \eta_j \, p^P_{kj} \,\right] \;\;\Rightarrow\;\; \frac{\partial \pi_j}{\partial p^S_{kj}} \;=\; 0$$

$$\Leftrightarrow \;\; \alpha_j - \beta^S_j \, p^S_{kj} + \eta_j \, p^P_{kj} - \beta^S_j \left[\, p^S_{kj} - w_{kj} - c^S_j \,\right] \;=\; 0 \;\;\Leftrightarrow\;\; p^S_{kj} \;=\; \frac{\alpha_j + \eta_j \, p^P_{kj} + \beta^S_j \left[\, w_{kj} + c^S_j \,\right]}{2 \, \beta^S_j}. \tag{35}$$

where $w_{kj} > 0$ is the commission that Sj faces.

(34) and (35) imply that:

$$p_{kj}^{P} \;=\; \frac{\theta_j\,\alpha_j + \beta_j^P\,c_{kj}^P + w_{kj}\,\eta_j}{2\,\beta_j^P} + \frac{\eta_j}{2\,\beta_j^P}\,p_{kj}^S$$

$$=\; \frac{\theta_j\,\alpha_j + \beta_j^P\,c_{kj}^P + w_{kj}\,\eta_j}{2\,\beta_j^P} + \frac{\eta_j}{2\,\beta_j^P}\,\frac{\alpha_j + \eta_j\,p_{kj}^P + \beta_j^S\left[\,w_{kj} + c_j^S\,\right]}{2\,\beta_j^S}$$

$$\Leftrightarrow\quad p_{kj}^{P} \;=\; \frac{\left[\,2\,\beta_j^S\,\theta_j + \eta_j\,\right]\alpha_j + 2\,\beta_j^S\,\beta_j^P\,c_{kj}^P + \eta_j\,\beta_j^S\,c_j^S\; + 3\,\beta_j^S\,\eta_j\,w_{kj}}{\Phi_{2kj}}. \tag{36}$$

(36) reflects (14). (35) and (36) imply that:

$$p_{kj}^{S} \;=\; \frac{\alpha_j + \beta_j^S\,c_j^S}{2\,\beta_j^S} + \frac{\beta_j^S\,w_{kj}}{2\,\beta_j^S} + \frac{\eta_j}{2\,\beta_j^S}\,p_{kj}^P$$

$$=\; \frac{\alpha_j + \beta_j^S\,c_j^S}{2\,\beta_j^S} + \frac{\beta_j^S\,w_{kj}}{2\,\beta_j^S} + \frac{\eta_j}{2\,\beta_j^S}\,\frac{\left[\,2\,\beta_j^S\,\theta_j + \eta_j\,\right]\alpha_j + 2\,\beta_j^S\,\beta_j^P\,c_{kj}^P + \eta_j\,\beta_j^S\,c_j^S\; + 3\,\beta_j^S\,\eta_j\,w_{kj}}{\Phi_{2kj}}$$

$$=\; \frac{\left[\,2\,\beta_j^P + \eta_j\,\theta_j\,\right]\alpha_j + 2\,\beta_j^P\,\beta_j^S\,c_j^S + \eta_j\,\beta_j^P\,c_{kj}^P}{\Phi_{2kj}} + \frac{w_{kj}}{2}\left[\,\frac{3\,\left(\,\eta_j\,\right)^2}{\Phi_{2kj}} + 1\,\right]. \tag{37}$$

(2), (36), (14), and (37) imply that consumers' initial demand for product j is:

$$q_{kj}^{S} \;=\; \alpha_j - \beta_j^S\left\{\,\frac{\left[\,2\,\beta_j^P + \eta_j\,\theta_j\,\right]\alpha_j + 2\,\beta_j^P\,\beta_j^S\,c_j^S + \eta_j\,\beta_j^P\,c_{kj}^P}{\Phi_{2kj}} + \frac{w_{kj}}{2}\left[\,\frac{3\,\left(\,\eta_j\,\right)^2}{\Phi_{2kj}} + 1\,\right]\right\}$$

$$+\;\eta_j\,\frac{\left[\,2\,\beta_j^S\,\theta_j + \eta_j\,\right]\alpha_j + 2\,\beta_j^S\,\beta_j^P\,c_{kj}^P + \eta_j\,\beta_j^S\,c_j^S\; + 3\,\beta_j^S\,\eta_j\,w_{kj}}{4\,\beta_j^P\,\beta_j^S - \left[\,\eta_j\,\right]^2}$$

$$=\; \frac{\beta_j^S}{\Phi_{2kj}}\left\{\,\left[\,2\,\beta_j^P + \eta_j\,\theta_j\,\right]\;\alpha_j + \eta_j\,\beta_j^P\,c_{kj}^P + c_j^S\left[\,\left(\,\eta_j\,\right)^2 - 2\,\beta_j^S\,\beta_j^P\,\right]\right\}$$

$$+\;\frac{\beta_j^S\,w_{kj}}{2}\left[\,\frac{3\,\left(\,\eta_j\,\right)^2}{\Phi_{2kj}} - 1\,\right]. \tag{38}$$

Observe that:

$$\left[\,2\,\beta_j^P + \eta_j\,\theta_j\,\right]\,\alpha_j + \eta_j\,\beta_j^P\,c_{kj}^P + c_j^S\left[\,\left(\eta_j\right)^2 - 2\,\beta_j^S\,\beta_j^P\,\right]$$

$$= \;\eta_j\left[\,\theta_j\,\alpha_j - \beta_j^P\,c_{kj}^P + c_j^S\,\eta_j\,\right] + 2\,\beta_j^P\left[\,\alpha_j + \eta_j\,c_{kj}^P - \beta_j^S\,c_j^S\,\right] \;=\; \eta_j\,\overline{\Delta}_{kj} + 2\,\beta_j^P\,\Delta_j. \quad (39)$$

(14), (38) and (39) imply that:

$$q_{kj}^S \;=\; \frac{\beta_j^S\left[\,\eta_j\,\overline{\Delta}_{kj} + 2\,\beta_j^P\,\Delta_{kj}\,\right]}{\Phi_{2kj}} + \frac{\beta_j^S\,w_{kj}}{2}\left[\frac{3\left(\eta_j\right)^2}{\Phi_{2kj}} - 1\right]$$

$$= \frac{\frac{\eta_j}{\beta_j^P}\,\overline{\Delta}_{kj} + 2\,\Delta_{kj}\; - 2\,\beta_j^S\left[\,1 - \Omega_j\,\right]\,w_{kj}}{4 - \Omega_j}. \quad (40)$$

(15) implies (40) can be written as:

$$q_{kj}^S \;=\; \frac{\frac{\beta_j^S\,\Omega_j}{\eta_j}\,\overline{\Delta}_{kj} + 2\,\Delta_{kj} - 2\,\beta_j^S\left[\,1 - \Omega_j\,\right]\,w_{kj}}{4 - \Omega_j}. \quad (41)$$

(11) and (41) imply that Sj's sales are:

$$Q_{kj}^S \;=\; \Theta_k\,q_{kj}^S \;=\; \frac{\Theta_k\left[\,\frac{\eta_j}{\beta_j^P}\,\overline{\Delta}_{kj} + 2\,\Delta_{kj} - 2\,\beta_j^S\left(\,1 - \Omega_j\,\right)\,w_{kj}\right]}{4 - \Omega_j}.$$

(14) and (37) imply that:

$$p_{kj}^S - w_{kj} - c_j^S \;=\; \frac{\left[\,2\,\beta_j^P + \eta_j\,\theta_j\,\right]\alpha_j + 2\,\beta_j^P\,\beta_j^S\,c_j^S + \eta_j\,\beta_j^P\,c_{kj}^P}{\Phi_{2kj}} + \frac{w_{kj}}{2}\left[\frac{3\,\left(\,\eta_j\,\right)^2}{\Phi_{2kj}} + 1\right] - w_{kj} - c_j^S$$

$$= \frac{\left[\,2\,\beta_j^P + \eta_j\,\theta_j\,\right]\alpha_j + \eta_j\,\beta_j^P\,c_{kj}^P + \left[\,\left(\eta_j\right)^2 - 2\,\beta_j^P\,\beta_j^S\,\right]\,c_j^S}{\Phi_{2kj}} + \frac{w_{kj}}{2}\left[\frac{3\,\left(\,\eta_j\,\right)^2}{\Phi_{2kj}} - 1\right]. \quad (42)$$

(14), (39) and (42) imply that:

$$p^S_{kj}-w_{kj}-c^S_j \;=\; \frac{\eta_j\,\overline{\Delta}_{kj}+2\,\beta^P_j\,\Delta_{kj}}{\Phi_{2kj}}+\frac{w_{kj}}{2}\left[\frac{3\,(\eta_j)^2}{\Phi_{2kj}}-1\right]$$

$$=\;\frac{\frac{\Omega_j}{\eta_j}\,\overline{\Delta}_{kj}+\frac{2}{\beta^S_j}\,\Delta_{kj}}{4-\Omega_j}+2\,w_{kj}\left[\frac{\Omega_j-1}{4-\Omega_j}\right]\;=\;\frac{\frac{\Omega_j}{\eta_j}\,\overline{\Delta}_{kj}+\frac{2}{\beta^S_j}\,\Delta_{kj}-2\,[\,1-\Omega_j\,]\;w_{kj}}{4-\Omega_j}. \tag{43}$$

(43) reflects (15). (41) and (43) imply that:

$$p^S_{kj}-w_{kj}-c^S_j \;=\; \frac{q^S_{kj}}{\beta^S_j}. \tag{44}$$

(32), (41), and (44) imply that:

$$\pi_j \;=\; [\,p^S_{kj}-w_{kj}-c^S_j\,]\,\Theta_k\,q^S_{kj} \;=\; \frac{q^S_{kj}}{\beta^S_j}\Theta_k\,q^S_{kj} \;=\; \frac{\Theta_k}{\beta^S_j}\,[\,q^S_{kj}\,]^2$$

$$=\;\frac{\Theta_k}{\beta^S_j}\left[\frac{\frac{\beta^S_j\,\Omega_j}{\eta_j}\,\overline{\Delta}_{kj}+2\,\Delta_{kj}-2\,\beta^S_j\,[\,1-\Omega_j\,]\;w_{kj}}{4-\Omega_j}\right]^2. \tag{45}$$

(1), (36), and (37) imply that

$$q^P_{kj} \;=\; \theta_j\,\alpha_j-\beta^P_j\,p^P_{kj}+\eta_j\,p^S_{kj}$$

$$=\;\theta_j\,\alpha_j-\beta^P_j\,\frac{[\,2\,\beta^S_j\,\theta_j+\eta_j\,]\,\alpha_j+2\,\beta^S_j\,\beta^P_j\,c^P_{kj}+\eta_j\,\beta^S_j\,c^S_j\;+3\,\beta^S_j\,\eta_j\,w_{kj}}{\Phi_{2j}}$$

$$+\;\eta_j\,\frac{[\,2\,\beta^P_j+\eta_j\,\theta_j\,]\,\alpha_j+2\,\beta^P_j\,\beta^S_j\,c^S_j+\eta_j\,\beta^P_j\,c^P_{kj}}{\Phi_{2j}}+\eta_j\,\frac{w_{kj}}{2}\left[\frac{3\,\left(\,\eta_j\,\right)^2}{\Phi_{2j}}+1\right]$$

$$=\;\frac{\beta^P_j}{\Phi_{2j}}\{\,\alpha_j\,[\,2\,\beta^S_j\,\theta_j+\eta_j\,]+c^P_{kj}\left[\,(\,\eta_j\,)^2-2\,\beta^P_j\,\beta^S_j\,\right]+\eta_j\,\beta^S_j\,c^S_j\,\}+\eta_j\,w_{kj}\left[\frac{(\,\eta_j\,)^2-\beta^P_j\,\beta^S_j}{\Phi_{2j}}\right]. \tag{46}$$

Observe that:

$$\alpha_j \left[2 \beta_j^S \theta_j + \eta_j \right] + c_{kj}^P \left[\left(\eta_j \right)^2 - 2 \beta_j^P \beta_j^S \right] + \eta_j \beta_j^S c_j^S$$

$$= 2 \beta_j^S \left[\theta_j \alpha_j - \beta_j^P c_{kj}^P + \eta_j c_j^S \right] + \eta_j \left[\alpha_j + c_{kj}^P \eta_j - \beta_j^S c_j^S \right] = 2 \beta_j^S \overline{\Delta}_{kj} + \eta_j \Delta_{kj}. \quad (47)$$

(14), (15), (46) and (47) imply that:

$$q_{kj}^P = \frac{\beta_j^P \left[2 \beta_j^S \overline{\Delta}_{kj} + \eta_j \Delta_{kj} \right]}{\Phi_{2j}} + \eta_j w_{kj} \left[\frac{\left(\eta_j \right)^2 - \beta_j^P \beta_j^S}{\Phi_{2j}} \right]$$

$$= \frac{2 \overline{\Delta}_{kj} + \frac{\eta_j}{\beta_j^S} \Delta_{kj}}{4 - \Omega_j} + \eta_j w_{kj} \left[\frac{\Omega_j - 1}{4 - \Omega_j} \right] = \frac{2 \overline{\Delta}_{kj} + \frac{\eta_j}{\beta_j^S} \Delta_{kj} - \eta_j w_{kj} \left[1 - \Omega_j \right]}{4 - \Omega_j}. \quad (48)$$

(11) and (48) imply that Pk's sales are:

$$Q_{kj}^P = \Theta_k q_{kj}^P = \frac{\Theta_k \left[2 \overline{\Delta}_{kj} + \frac{\eta_j}{\beta_j^S} \Delta_{kj} - \eta_j \left(1 - \Omega_j \right) w_{kj} \right]}{4 - \Omega_j}. \quad \blacksquare \quad (49)$$

<u>Proof of Lemma 5</u>. Suppose Sj competes against Pk when Sj sells on Pk ($j, k \in \{1, 2\}$). Pk's profit is

$$\Pi_k = \left[p_{kj}^P - c_{kj}^P \right] \Theta_k q_{kj}^P - F + w_{kj} \Theta_k q_{kj}^S + \overline{\Pi}_{kl}, \quad (50)$$

where $\overline{\Pi}_{kl}$ is the profit that Pk secures from Sl ($j, k, l \in \{1, 2\}$, $j \neq l$), $\overline{\Pi}_{kl} > 0$ if Sl sells on Pk, and $\overline{\Pi}_{kl} = 0$ if Sl does not on Pk.[43]

(36) implies that

$$p_{kj}^P - c_{kj}^P = \frac{\left[2 \beta_j^S \theta_j + \eta_j \right] \alpha_j + 2 \beta_j^S \beta_j^P c_{kj}^P + \eta_j \beta_j^S c_j^S + 3 \beta_j^S \eta_j w_{kj}}{\Phi_{2j}} - c_{kj}^P$$

[43] $\overline{\Pi}_{kl}$ does not include w_{kj}.

$$= \frac{\left[\, 2\,\beta_j^S\,\theta_j + \eta_j\,\right]\alpha_j + c_{kj}^P\left[\,\left(\,\eta_j\,\right)^2 - 2\,\beta_j^S\,\beta_j^P\,\right] + \eta_j\,\beta_j^S\,c_j^S \,+ 3\,\beta_j^S\,\eta_j\,w_{kj}}{\Phi_{2j}}. \tag{51}$$

(14), (15), (47) and (51) imply that:

$$p_{kj}^P - c_{kj}^P \;=\; \frac{2\,\beta_j^S\,\overline{\Delta}_{kj} + \eta_j\,\Delta_{kj} + 3\,\beta_j^S\,\eta_j\,w_{kj}}{\Phi_{2j}} \;=\; \frac{2\,\beta_j^S\,\overline{\Delta}_{kj} + \eta_j\,\Delta_{kj} + 3\,\beta_j^S\,\eta_j\,w_{kj}}{4\,\beta_j^P\,\beta_j^S - \left[\eta_j\right]^2}$$

$$= \frac{\frac{2}{\beta_j^P}\,\overline{\Delta}_{kj} + \frac{\eta_j}{\beta_j^P\,\beta_j^S}\,\Delta_{kj} + \frac{3\,\eta_j}{\beta_j^P}\,w_{kj}}{4 - \frac{\left[\eta_j\right]^2}{\beta_j^P\,\beta_j^S}} \;=\; \frac{\frac{2}{\beta_j^P}\,\overline{\Delta}_{kj} + \frac{\eta_j}{\beta_j^P\,\beta_j^S}\,\Delta_{kj} + \frac{3\,\eta_j}{\beta_j^P}\,w_{kj}}{4 - \Omega_j}. \tag{52}$$

(48) and (52) imply that:

$$\beta_{kj}^P\left[\,p_{kj}^P - c_{kj}^P\,\right] \;=\; \frac{2\,\overline{\Delta}_{kj} + \frac{\eta_j}{\beta_j^S}\,\Delta_{kj}}{4 - \Omega_j} + \frac{3\,\eta_j\,w_{kj}}{4 - \Omega_j} \;=\; q_{kj}^P + \frac{\eta_j\,w_{kj}\left[\,1 - \Omega_j\,\right]}{4 - \Omega_j} + \frac{3\,\eta_j\,w_{kj}}{4 - \Omega_j}$$

$$= \; q_{kj}^P + \frac{\eta_j\,w_{kj}\left[\,1 - \Omega_j + 3\,\right]}{4 - \Omega_j} \;=\; q_{kj}^P + \eta_j\,w_{kj}. \tag{53}$$

(53) implies that:

$$p_{kj}^P - c_{kj}^P \;=\; \frac{q_{kj}^P + \eta_j\,w_{kj}}{\beta_j^P}. \tag{54}$$

(50), (54) imply that:

$$\Pi_k \;=\; \left[\frac{q_{kj}^P + \eta_j\,w_{kj}}{\beta_j^P}\right]\Theta_k\,q_{kj}^P - F + w_{kj}\,\Theta_k\,q_{kj}^S + \overline{\Pi}_{kl}$$

$$= \; \frac{\Theta_k}{\beta_j^P}\left[\,q_{kj}^P\,\right]^2 + w_{kj}\,\Theta_k\left[\frac{\eta_j}{\beta_j^P}\,q_{kj}^P + q_{kj}^S\right] - F + \overline{\Pi}_{kl}. \tag{55}$$

(55) implies that Pk chooses w_{kj} to

$$\text{Maximize } \;\Pi_k \;=\; \frac{\Theta_k}{\beta_j^P}\left[\,q_{kj}^P\,\right]^2 + w_{kj}\,\Theta_k\left[\frac{\eta_j}{\beta_j^P}\,q_{kj}^P + q_{kj}^S\right] - F + \overline{\Pi}_{kl} \;\;\Rightarrow\;\; \frac{\partial\Pi_k}{\partial w_{kj}} \;=\; 0, \tag{56}$$

where $\overline{\Pi}_{kl}$ is the profit that Pk secures from Sl ($j,k,l \in \{1,2\}$, $j \neq l$), $\overline{\Pi}_{kl} > 0$ if Sl sells on

Pk, $\overline{\Pi}_{kl} = 0$ if Sl does not on Pk, and $\overline{\Pi}_{kl}$ does not include w_{kj}.

(56) implies that:

$$\frac{2}{\beta_j^P}\, q_{kj}^P \frac{\partial q_{kj}^P}{\partial w_{kj}} + \frac{\eta_j}{\beta_j^P}\, q_{kj}^P + q_{kj}^S + w_{kj} \left[\frac{\eta_j}{\beta_j^P} \frac{\partial q_{kj}^P}{\partial w_{kj}} + \frac{\partial q_{kj}^S}{\partial w_{kj}} \right] \;=\; 0. \tag{57}$$

(40) and (48) imply that:

$$\frac{\partial q_{kj}^P}{\partial w_{kj}} \;=\; -\,\frac{\eta_j \,[\, 1 - \Omega_j \,]}{4 - \Omega_j} \text{ and } \frac{\partial q_{kj}^S}{\partial w_{kj}} \;=\; -\,\frac{2\,\beta_j^S \,[\, 1 - \Omega_j \,]}{4 - \Omega_j}. \tag{58}$$

(57) and (58) imply that:

$$-\frac{\eta_j \,[\, 1 - \Omega_j \,]}{4 - \Omega_j} \frac{2}{\beta_j^P}\, q_{kj}^P + \frac{\eta_j}{\beta_j^P}\, q_{kj}^P + q_{kj}^S + w_{kj} \left[-\frac{\eta_j \,(\, 1 - \Omega_j \,)}{4 - \Omega_j} \frac{\eta_j}{\beta_j^P} - \frac{2\,\beta_j^S \,(\, 1 - \Omega_j \,)}{4 - \Omega_j} \right] \;=\; 0$$

$$\Leftrightarrow \;\; \frac{\eta_j}{\beta_j^P} \left[1 - \frac{2\,(\, 1 - \Omega_j \,)}{4 - \Omega_j} \right] q_{kj}^P + q_{kj}^S - w_{kj} \frac{[\, 1 - \Omega_j \,]}{4 - \Omega_j} \left[\frac{\left(\eta_j \right)^2}{\beta_j^P} + 2\,\beta_j^S \right] \;=\; 0$$

$$\Leftrightarrow \;\; w_{kj} \;=\; \frac{1}{2\,[\, 1 - \Omega_j \,]} \left\{ \frac{\Omega_j \,\overline{\Delta}_{kj}}{\eta_j} + \frac{\left[\, 8 + (\Omega_j)^2 \,\right] \,\Delta_{kj}}{\beta_j^S \,[\, 8 + \Omega_j \,]} \right\}. \;\; \blacksquare \tag{59}$$

<u>Proof of Lemma 6</u>. Suppose Sj competes against Pk when Sj sells on Pk ($j, k \in \{1, 2\}$). Lemmas 4 and 5 imply that Sj's sales are:

$$Q_{kj}^S \;=\; \frac{\Theta_k \left[\frac{\eta_j}{\beta_j^P} \,\overline{\Delta}_{kj} + 2\,\Delta_{kj} \right]}{4 - \Omega_j} - \frac{\Theta_k \, 2\,\beta_j^S \,[\, 1 - \Omega_j \,]}{4 - \Omega_j}\, w_{kj}$$

$$= \; \frac{\Theta_k}{4 - \Omega_j}\, \Delta_{kj} \left[\frac{8 + 2\,\Omega_j - (\Omega_j)^2}{8 + \Omega_j} \right] \;=\; \frac{\Theta_k}{4 - \Omega_j}\, \Delta_{kj} \frac{[\, 4 - \Omega_j \,]\, [\, 2 + \Omega_j \,]}{8 + \Omega_j} \;=\; \frac{\Theta_k \,[\, 2 + \Omega_j \,]\; \Delta_{kj}}{8 + \Omega_j}. \tag{60}$$

The first equality in (**??**) reflects $\frac{\eta_j}{\beta_j^P} = \frac{\beta_j^S \,\Omega_j}{\eta_j}$ because (15) implies that $\frac{\beta_j^S \,\Omega_j}{\eta_j} = \frac{\beta_j^S \left[\eta_j \right]^2}{\eta_j \,\beta_j^P \,\beta_j^S} =$

$\frac{\eta_j}{\beta_j^P}$. (60) implies that

$$q_{kj}^S \;=\; \frac{[\,2+\Omega_j\,]\;\Delta_{kj}}{8+\Omega_j}. \tag{61}$$

Lemma 4 and (61) imply that Sj's profit is

$$\pi_j^S \;=\; \frac{\Theta_k}{\beta_j^S}\left[\,q_{kj}^S\,\right]^2 \;=\; \frac{\Theta_k}{\beta_j^S}\left[\frac{(\,2+\Omega_j\,)\;\Delta_{kj}}{8+\Omega_j}\right]^2. \tag{62}$$

Lemmas 4 and 5 imply that Pk's sales are:

$$\begin{aligned}
Q_{kj}^P \;&=\; \frac{\Theta_k\left[\,2\,\overline{\Delta}_{kj}+\frac{\eta_j}{\beta_j^S}\,\Delta_{kj}\right]}{4-\Omega_j} - \frac{\Theta_k\,\eta_j\,[\,1-\Omega_j\,]}{4-\Omega_j}\,w_{kj} \\
&=\; \frac{\Theta_k\left[\,2\,\overline{\Delta}_{kj}+\frac{\eta_j}{\beta_j^S}\,\Delta_{kj}\right]}{4-\Omega_j} - \frac{\Theta_k\,\eta_j\,[\,1-\Omega_j\,]}{2\,[\,1-\Omega_j\,]\,[\,4-\Omega_j\,]}\left\{\frac{\Omega_j\,\overline{\Delta}_{kj}}{\eta_j}+\frac{\left[\,8+(\Omega_j)^2\,\right]\;\Delta_{kj}}{\beta_j^S\,(\,8+\Omega_j\,)}\right\} \\
&=\; \frac{\Theta_k\,\overline{\Delta}_{kj}}{2}+\frac{\eta_j\,\Theta_k\,\Delta_{kj}}{\beta_j^S\,[\,4-\Omega_j\,]}\,\frac{[\,4-\Omega_j\,]\,[\,2+\Omega_j\,]}{2\,[\,8+\Omega_j\,]} \;=\; \frac{\Theta_k\,\overline{\Delta}_{kj}}{2}+\frac{\eta_j\,\Theta_k\,[\,2+\Omega_j\,]\;\Delta_{kj}}{2\beta_j^S\,[\,8+\Omega_j\,]}.
\end{aligned} \tag{63}$$

(63) implies that:

$$q_{kj}^P \;=\; \frac{\overline{\Delta}_{kj}}{2}+\frac{\eta_j\,[\,2+\Omega_j\,]\;\Delta_{kj}}{2\,\beta_j^S\,[\,8+\Omega_j\,]}. \tag{64}$$

(61) and (63) imply that:

$$Q_{kj}^P \;=\; \frac{\Theta_k\,\overline{\Delta}_{kj}}{2}+\frac{\eta_j}{2\,\beta_j^S}\,Q_{kj}^S. \tag{65}$$

(60) and (65) imply that

$$Q_{kj}^P \;>\; Q_{kj}^S \;\;\Leftrightarrow\;\; \frac{\Theta_k\,\overline{\Delta}_{kj}}{2}+\frac{\eta_j}{2\,\beta_j^S}\,Q_{kj}^S \;>\; Q_{kj}^S \;\;\Leftrightarrow\;\; \frac{\overline{\Delta}_{kj}}{\Delta_{kj}} \;>\; \frac{2+\Omega_j}{8+\Omega_j}\left[\,2-\frac{\eta_j}{\beta_j^S}\,\right]. \tag{66}$$

Observe that:

$$\frac{\partial\left(\frac{2+\Omega_j}{8+\Omega_j}\right)}{\partial\Omega_j} \;=\; \frac{8+\Omega_j-[\,2+\Omega_j\,]}{[\,8+\Omega_j\,]^2} \;=\; \frac{6}{[\,8+\Omega_j\,]^2} \;>\; 0. \tag{67}$$

(67) implies that for $\Omega_j \in (0,1)$,

$$\frac{2+\Omega_j}{8+\Omega_j} \;<\; \max \frac{2+\Omega_j}{8+\Omega_j} \;=\; \frac{2+1}{8+1} \;=\; \frac{1}{3}. \tag{68}$$

Because $2 - \frac{\eta_j}{\beta_j^S} < 2$, (68) implies that for $\Omega_j \in (0,1)$,

$$\frac{2+\Omega_j}{8+\Omega_j} \left[2 - \frac{\eta_j}{\beta_j^S} \right] \;<\; \frac{2}{3}. \tag{69}$$

(66) and (69) imply that for $\Omega_j \in (0,1)$,

$$Q_{kj}^P \;>\; Q_{kj}^S \;\text{ if }\; \frac{\overline{\Delta}_{kj}}{\Delta_{kj}} \;>\; 1. \tag{70}$$

(55), (61), (64), and Lemma 5 imply that Pk's profit from the commission it collects from Sj and from entering Sj's product market is:

$$\Pi_k \;=\; \frac{\Theta_k}{\beta_j^P} \left[q_{kj}^P \right]^2 + w_{kj}\, \Theta_k \left[\frac{\eta_j}{\beta_j^P}\, q_{kj}^P + q_{kj}^S \right] - F$$

$$= \frac{\Theta_k}{\beta_j^P} \left[\frac{\overline{\Delta}_{kj}}{2} + \frac{\eta_j \left[2+\Omega_j \right] \Delta_{kj}}{2\, \beta_j^S \left[8+\Omega_j \right]} \right]^2$$

$$+ \frac{\Theta_k}{2\left[1-\Omega_j \right]} \left\{ \frac{\Omega_j\, \overline{\Delta}_{kj}}{\eta_j} + \frac{\left[8 + (\Omega_j)^2 \right] \Delta_{kj}}{\beta_j^S \left(8+\Omega_j \right)} \right\}$$

$$\cdot \left\{ \frac{\eta_j}{\beta_j^P} \left[\frac{\overline{\Delta}_{kj}}{2} + \frac{\eta_j \left(2+\Omega_j \right) \Delta_{kj}}{2\, \beta_j^S \left(8+\Omega_j \right)} \right] + \frac{\left[2+\Omega_j \right] \Delta_{kj}}{8+\Omega_j} \right\} - F$$

$$= \frac{\Theta_k}{2\left(1-\Omega_j \right)} \left\{ \frac{\left(\overline{\Delta}_{kj} \right)^2}{2\, \beta_j^P} + \frac{\left(\Delta_{kj} \right)^2 \left(2+\Omega_j \right) \Omega_j}{2\, \beta_j^S \left(8+\Omega_j \right)^2} \left[26 - \Omega_j + 2\,(\Omega_j)^2 \right] \right.$$

$$\left. + \frac{\Omega_j\, \overline{\Delta}_{kj}\, \Delta_{kj}}{\eta_j \left(8+\Omega_j \right)} \left[6 + 2\,\Omega_j + (\Omega_j)^2 \right] \right\} - F. \;\blacksquare$$

Proof of Proposition 1. Suppose Condition FS holds. Lemma 3 implies that P's profit is

$\frac{\Theta\left[\widetilde{\Delta}_{\cdot j}\right]^2}{8\,b_j^S}$, if S$j$ ($j \in \{1,2\}$) sells product j on P and P does not enter Sj's product market. Lemma 6 implies that P's profit is $\Theta\, M_{\cdot j} - F$, if Sj ($j \in \{1,2\}$) sells product j on P and P enters Sj's product market. Because Condition FS holds, $\Theta\, M_{\cdot j} - F > \frac{\Theta\left[\widetilde{\Delta}_{\cdot j}\right]^2}{8\,b_j^S}$, i.e., P secures a higher profit by entering Sj's market than "no entry". Therefore, if Sj sells on P, P will enter Sj's market, Lemma 6 implies that P's profit is $\Theta\, M_{\cdot j} - F$ and Sj's profit is $\frac{\Theta}{\beta_j^S}\left[\frac{(2+\Omega_{\cdot j})\,\Delta_{\cdot j}}{8+\Omega_{\cdot j}}\right]^2$.

Therefore, knowing P's entry decisions, Sj ($j \in \{1,2\}$) will choose to sell on P because he secures a positive profit if he sells on P (i.e., $\frac{\Theta}{\beta_j^S}\left[\frac{(2+\Omega_{\cdot j})\,\Delta_{\cdot j}}{8+\Omega_{\cdot j}}\right]^2 > 0$) while he secures zero profit if he does not sell on P, regardless of the other seller's choice.

Therefore, in equilibrium, both S1 and S2 sell on P, and P enters each seller's market. Lemma 6 implies Sj's profit is $\frac{\Theta}{\beta_j^S}\left[\frac{(2+\Omega_{\cdot j})\,\Delta_{\cdot j}}{8+\Omega_{\cdot j}}\right]^2$, and P's profit is $\Theta\, M_{\cdot 1} - F + \Theta\, M_{\cdot 2} - F$. ■

Proof of Lemma 7. Lemma 3 implies that Sj's profit is $\frac{\Theta_k\left[\widetilde{\Delta}_{kj}\right]^2}{16\,b_j^S}$ if Sj sells on Pk ($j, k \in \{1,2\}$). (12) implies that

$$\widetilde{\Delta}_{kj} \;=\; \Delta_{kj} + \frac{\eta_j}{\beta_j^P}\,\overline{\Delta}_{kj} \;=\; \alpha_j - \beta_j^S\, c_j^S + \eta_j\, c_{kj}^P + \frac{\eta_j}{\beta_j^P}\left[\,\theta_j\,\alpha_j - \beta_j^P\, c_{kj}^P + \eta_j\, c_j^S\,\right] \qquad (71)$$

(71) implies that

$$\frac{\partial \widetilde{\Delta}_{kj}}{\partial c_{kj}^P} \;=\; \eta_j - \frac{\eta_j}{\beta_j^P}\,\beta_j^P \;=\; 0. \qquad (72)$$

(72) implies that $\widetilde{\Delta}_{1j} = \widetilde{\Delta}_{2j}$. Consequently, if $\Theta_1 = \Theta_2$, then Sj is indifferent between selling on P1 and selling on P2; if $\Theta_1 > \Theta_2$, then Sj sells on P1. ■

Proof of Lemma 8. Lemma 6 implies that Sj's profit is $\frac{\Theta_k}{\beta_j^S}\left[\frac{(2+\Omega_j)\,\Delta_{kj}}{8+\Omega_j}\right]^2$ if Sj sells on Pk ($j, k \in \{1,2\}$). Therefore,

$$\frac{\Theta_1}{\beta_j^S}\left[\frac{(\,2+\Omega_j\,)\;\Delta_{1j}}{8+\Omega_j}\right]^2 \;\gtreqless\; \frac{\Theta_2}{\beta_j^S}\left[\frac{(\,2+\Omega_j\,)\;\Delta_{2j}}{8+\Omega_j}\right]^2 \;\;\Leftrightarrow\;\; \frac{\Theta_1}{\Theta_2} \;\gtreqless\; \left[\frac{\Delta_{2j}}{\Delta_{1j}}\right]^2.$$

Observe that

$$\frac{\Delta_{2j}}{\Delta_{1j}} \gtreqless 1 \quad \Leftrightarrow \quad \frac{c_{2j}^{P}}{c_{1j}^{P}} \gtreqless 1.$$

First suppose $\frac{c_{2j}^{P}}{c_{1j}^{P}} < 1$. Then S$j$ will sell on P1 because $\frac{\Theta_1}{\Theta_2} \geq 1 > \left[\frac{\Delta_{2j}}{\Delta_{1j}}\right]^2$.

Next suppose $\frac{c_{2j}^{P}}{c_{1j}^{P}} > 1$. Then S$j$ will: (i) sell on P1 when $\frac{\Theta_1}{\Theta_2} > \left[\frac{\Delta_{2j}}{\Delta_{1j}}\right]^2$; and (ii) sell on P2 when $\frac{\Theta_1}{\Theta_2} < \left[\frac{\Delta_{2j}}{\Delta_{1j}}\right]^2$. ■

<u>Proof of Lemma 9</u>. Condition FS ensures that P2 will enter Sj's market if Sj sells on P2 ($j \in \{1, 2\}$). Lemma 3 implies that Sj's profit is $\frac{\Theta_1 [\widetilde{\Delta}_{1j}]^2}{16\, b_j^S}$ if Sj sells on P1. Lemma 6 implies that Sj's profit is $\frac{\Theta_2}{\beta_j^S}\left[\frac{(2+\Omega_j)\,\Delta_{2j}}{8+\Omega_j}\right]^2$ if Sj sells on P2. (10) implies that:

$$\frac{\Theta_1 \left[\widetilde{\Delta}_{1j}\right]^2}{16\, b_j^S} > \frac{\Theta_2}{\beta_j^S}\left[\frac{(2+\Omega_j)\,\Delta_{2j}}{8+\Omega_j}\right]^2 \quad \Leftrightarrow \quad \frac{\Theta_1}{\Theta_2} > \left[\frac{4\sqrt{1-\Omega_j}\,(2+\Omega_j)}{8+\Omega_j}\frac{\Delta_{2j}}{\widetilde{\Delta}_{1j}}\right]^2. \tag{73}$$

(73) holds because

$$\frac{\Theta_1}{\Theta_2} \geq 1 > \left[\frac{4\sqrt{1-\Omega_j}\,(2+\Omega_j)}{8+\Omega_j}\frac{\Delta_{2j}}{\widetilde{\Delta}_{1j}}\right]^2. \tag{74}$$

The last inequality in (74) holds because

$$\frac{4\sqrt{1-\Omega_j}\,[2+\Omega_j]}{8+\Omega_j} < 1 \quad \text{and} \quad \frac{\Delta_{2j}}{\widetilde{\Delta}_{1j}} < 1. \tag{75}$$

The first inequality in (75) holds because

$$\frac{\partial\left(\sqrt{1-\Omega_j}\frac{4[2+\Omega_j]}{8+\Omega_j}\right)}{\partial\Omega_j} = \sqrt{1-\Omega_j}\,\frac{\partial\left(\frac{4[2+\Omega_j]}{8+\Omega_j}\right)}{\partial\Omega_j} + \frac{\partial\left(\sqrt{1-\Omega_j}\right)}{\partial\Omega_j}\,\frac{4[2+\Omega_j]}{8+\Omega_j}$$

$$= \frac{24\sqrt{1-\Omega_j}}{[8+\Omega_j]^2} - \frac{2[2+\Omega_j]}{\sqrt{1-\Omega_j}\,[8+\Omega_j]} = \frac{2}{8+\Omega_j}\left[\frac{12\sqrt{1-\Omega_j}}{8+\Omega_j} - \frac{[2+\Omega_j]}{\sqrt{1-\Omega_j}}\right] < 0, \tag{76}$$

and therefore, for $\Omega_j \in (0,1)$,

$$\frac{4\sqrt{1-\Omega_j}\,[\,2+\Omega_j\,]}{8+\Omega_j} \;<\; \max \frac{4\sqrt{1-\Omega_j}\,[\,2+\Omega_j\,]}{8+\Omega_j} \;=\; \frac{4\sqrt{1-0}\,[\,2+0\,]}{8+0} \;=\; 1. \tag{77}$$

The last inequality in (76) holds because

$$\frac{12\sqrt{1-\Omega_j}}{8+\Omega_j} \;<\; \frac{[\,2+\Omega_j\,]}{\sqrt{1-\Omega_j}} \;\Leftrightarrow\; 12\,[\,1-\Omega_j\,] \;<\; [\,2+\Omega_j\,]\,[\,8+\Omega_j\,]$$

$$\Leftrightarrow\; 12 - 12\,\Omega_j \;<\; 16 + [\Omega_j]^2 + 10\,\Omega_j \;\Leftrightarrow\; 4 + [\Omega_j]^2 + 22\,\Omega_j \;>\; 0.$$

The last inequality in (75) holds because

$$\widetilde{\Delta}_{1j} \;=\; \widetilde{\Delta}_{2j} \;>\; \Delta_{2j}. \tag{78}$$

The equality in (78) reflects (72) and the inequality in (78) reflects (12). ■

Proof of Lemma 10. Condition FS ensures that P1 will enter Sj's market if Sj sells on P1 ($j \in \{1,2\}$). Lemma 3 implies that Sj's profit is $\frac{\Theta_2 [\,\widetilde{\Delta}_{2j}\,]^2}{16\,b^S_{2j}}$ if Sj sells on P2. Lemma 6 implies that Sj's profit is $\frac{\Theta_1}{\beta^S_j}\left[\frac{(2+\Omega_j\,)\,\Delta_{1j}}{8+\Omega_j}\right]^2$ if Sj sells on P1. (10) and (17) imply that:

$$\frac{\Theta_1}{\beta^S_j}\left[\frac{(\,2+\Omega_j\,)\;\Delta_{1j}}{8+\Omega_j}\right]^2 \;\gtreqless\; \frac{\Theta_2\left[\,\widetilde{\Delta}_{2j}\,\right]^2}{16\,b^S_{2j}} \;\Leftrightarrow\; \frac{\Theta_1}{\Theta_2} \;\gtreqless\; \phi_j. \tag{79}$$

(77) implies that $\frac{8+\Omega_j}{4\sqrt{1-\Omega_j}[2+\Omega_j\,]} > 1$. (12) and (72) imply that $\widetilde{\Delta}_{2j} = \widetilde{\Delta}_{1j} > \Delta_{1j}$. Therefore,

$$\phi_j \;>\; 1. \tag{80}$$

(79) and (80) imply that Sj will: (i) sell on P1 if $\frac{\Theta_1}{\Theta_2} > \phi_j$; and (ii) sell on P2 if $\frac{\Theta_1}{\Theta_2} < \phi_j$. ■

Proof of Proposition 2. Condition FS ensures that each platform enters each seller's market if the platform makes no commitment. Since S1 and S2 sell independent products, S1's choice of platform is independent of S2's choice of platform.

Case I. $c^P_{1j} < c^P_{2j}$.

First suppose $\frac{\Theta_1}{\Theta_2} > \phi_2$. Lemmas 7 - 10 and 11 imply that both S1 and S2 sells on P1, regardless of the platforms' commitments. Condition FS ensures P1 enters each seller's market. Therefore, Lemmas 3 and 6 imply that: (i) P1's profit is $\Theta_1 M_{11} - F + \Theta_1 M_{12} - F$ if P1 makes no commitment; and (ii) P1's profit is $\frac{\Theta_1 [\widetilde{\Delta}_{11}]^2}{8 b^S_1} + \frac{\Theta_1 [\widetilde{\Delta}_{12}]^2}{8 b^S_2}$ if P1 commits not to enter. Condition FS ensures that $\Theta_1 M_{1j} - F > \frac{\Theta_1 [\widetilde{\Delta}_{1j}]^2}{8 b^S_j}$, i.e., P1 secures more profit by making no commitment than by committing not to enter. Therefore, in equilibrium, P1 makes no commitment, and both S1 and S2 sell on P1 if $\frac{\Theta_1}{\Theta_2} > \phi_2$.

Next suppose $\frac{\Theta_1}{\Theta_2} \in (\phi_1, \phi_2)$. Lemmas 7 - 10 and 11 imply that S1 sells on P1, regardless of the platforms' commitments.

If P2 makes no commitment, Lemmas 8, 9 and 11 imply that S2 sells on P1. Condition FS and Lemmas 3 and 6 imply that P1 secures more profit by making no commitment than by committing not to enter in this case.

If P2 commits not to enter, Lemmas 7 and 10 imply that S2: (i) sells on P1 if P1 commits not to enter; and (ii) sells on P2 if P1 makes no commitment. Lemmas 3 and 6 imply that P1's profit is: (i) $\frac{\Theta_1 [\widetilde{\Delta}_{11}]^2}{8 b^S_1} + \frac{\Theta_1 [\widetilde{\Delta}_{12}]^2}{8 b^S_2}$ if P1 commits not to enter; and (ii) $\Theta_1 M_{11} - F$ if P1 makes no commitment. Therefore, P1 secures more profit by making no commitment than by committing not to enter in this case because

$$\Theta_1 M_{11} - F \ > \ \frac{\Theta_1 \left[\widetilde{\Delta}_{11}\right]^2}{8 b^S_1} + \frac{\Theta_1 \left[\widetilde{\Delta}_{12}\right]^2}{8 b^S_2} \ \Leftrightarrow \ F \ < \ \Theta_1 M_{11} - \frac{\Theta_1 \left[\widetilde{\Delta}_{11}\right]^2}{8 b^S_1} - \frac{\Theta_1 \left[\widetilde{\Delta}_{12}\right]^2}{8 b^S_2}.$$

If P1 commits not to enter, Lemmas 7 and 9 imply that S2 sells on P1, regardless of P2's commitment.

If P1 makes no commitment, Lemmas 8 and 10 imply that: (i) S2 sells on P1 if P2 makes no commitment; and (ii) S2 sells on P2 if P2 commits not to enter. Therefore, P2 secures more profit by committing not to enter than by making no commitment.

Consequently, in equilibrium, P1 makes no commitment whereas P2 commits not to enter, and S1 sells on P1 whereas S2 sells on P2, if $\frac{\Theta_1}{\Theta_2} \in (\, \phi_1, \phi_2\,)$.

Next suppose $\frac{\Theta_1}{\Theta_2} \in \left(\left[\frac{\Delta_{22}}{\Delta_{12}} \right]^2, \phi_1 \right)$. If P2 makes no commitment, Lemmas 8, 9 and 11 imply that S1 and S2 both sell on P1, regardless of P1's commitment. Lemmas 3 and 6 imply that P1's profit is: (i) $\frac{\Theta_1 [\, \widetilde{\Delta}_{11}\,]^2}{8\, b_1^S} + \frac{\Theta_1 [\, \widetilde{\Delta}_{12}\,]^2}{8\, b_2^S}$ if P1 commits not to enter; and (ii) $\Theta_1 \, M_{11} - F + \Theta_1 \, M_{12} - F$ if P1 makes no commitment. Condition FS ensures that P1 secures more profit by making no commitment than by committing not to enter in this case.

If P2 commits not to enter, Lemmas 7, 10 and 11 imply that Sj ($j \in \{1, 2\}$): (i) sells on P1 if P1 commits not to enter; and (ii) sells on P2 if P1 makes no commitment. Therefore, P1 secures more profit by committing not to enter than by making no commitment in this case.

If P1 commits not to enter, Lemmas 7, 9 and 11 imply that both S1 and S2 sell on P1, regardless of P2's commitment.

If P1 makes no commitment, Lemmas 8, 10, and 11 imply that Sj ($j \in \{1, 2\}$): (i) sells on P1 if P2 makes no commitment; and (ii) sells on P2 if P2 commits not to enter. Therefore, P2 secures more profit by committing not to enter than by making no commitment in this case.

Consequently, in equilibrium, both P1 and P2 commit not to enter, and both S1 and S2 sell on P1, if $\frac{\Theta_1}{\Theta_2} \in \left(\left[\frac{\Delta_{22}}{\Delta_{12}} \right]^2, \phi_1 \right)$.

Next suppose $\frac{\Theta_1}{\Theta_2} \in \left(\left[\frac{\Delta_{21}}{\Delta_{11}} \right]^2, \left[\frac{\Delta_{22}}{\Delta_{12}} \right]^2 \right)$. If P2 makes no commitment, Lemmas 8, 9, and 11 imply that S1 sells on P1 and S2: (i) sells on P1 if P1 commits not to enter; and (ii) sells on P2 if P1 makes no commitment. Lemmas 3 and 6 imply that P1's profit is: (i) $\frac{\Theta_1 \left[\widetilde{\Delta}_{11} \right]^2}{8\, b_1^S} + \frac{\Theta_1 \left[\widetilde{\Delta}_{12} \right]^2}{8\, b_2^S}$ if P1 commits not to enter; and (ii) $\Theta_1 \, M_{11} - F$ if P1 makes no commitment. Therefore, P1 secures more profit by making no commitment than by committing not to enter in this case because

$$\Theta_1 \, M_{11} - F \;>\; \frac{\Theta_1 \left[\widetilde{\Delta}_{11} \right]^2}{8\, b_1^S} + \frac{\Theta_1 \left[\widetilde{\Delta}_{12} \right]^2}{8\, b_2^S} \;\Leftrightarrow\; F \;<\; \Theta_1 \, M_{11} - \frac{\Theta_1 \left[\widetilde{\Delta}_{11} \right]^2}{8\, b_1^S} - \frac{\Theta_1 \left[\widetilde{\Delta}_{12} \right]^2}{8\, b_2^S} .$$

If P2 commits not to enter, Lemmas 7, 10, and 11 imply that Sj ($j \in \{1, 2\}$): (i) sells on P1 if P1 commits not to enter; and (ii) sells on P2 if P1 makes no commitment. Therefore, P1 secures more profit by committing not to enter than by making no commitment in this case.

If P1 commits not to enter, Lemmas 7, 9, and 11 imply that both S1 and S2 sell on P1, regardless of P2's commitment.

If P1 makes no commitment, Lemmas 8, 10, and 11 imply that S2 sells on P2 and S1: (i) sells on P1 if P2 makes no commitment; and (ii) sells on P2 if P2 commits not to enter. Lemmas 3 and 6 imply that P2's profit is: (i) $\frac{\Theta_2 \left[\widetilde{\Delta}_{21} \right]^2}{8\, b_1^S} + \frac{\Theta_2 \left[\widetilde{\Delta}_{22} \right]^2}{8\, b_2^S}$ if P2 commits not to enter; and (ii) $\Theta_2 \, M_{22} - F$ if P2 makes no commitment. Therefore, P2 secures more profit by committing not to enter than by making no commitment in this case because

$$\Theta_2 \, M_{22} - F \;<\; \frac{\Theta_2 \left[\widetilde{\Delta}_{21} \right]^2}{8\, b_1^S} + \frac{\Theta_2 \left[\widetilde{\Delta}_{22} \right]^2}{8\, b_2^S} \;\Leftrightarrow\; F \;>\; \Theta_2 \, M_{22} - \frac{\Theta_2 \left[\widetilde{\Delta}_{21} \right]^2}{8\, b_1^S} - \frac{\Theta_2 \left[\widetilde{\Delta}_{22} \right]^2}{8\, b_2^S} .$$

Consequently, in equilibrium, both P1 and P2 commit not to enter, and both S1 and S2 sell on P1, if $\frac{\Theta_1}{\Theta_2} \in \left(\left[\frac{\Delta_{21}}{\Delta_{11}} \right]^2, \left[\frac{\Delta_{22}}{\Delta_{12}} \right]^2 \right)$.

Next suppose $\frac{\Theta_1}{\Theta_2} < \left[\frac{\Delta_{21}}{\Delta_{11}} \right]^2$. If P2 makes no commitment, Lemmas 8, 9, and 11 imply that Sj ($j \in \{1, 2\}$): (i) sells on P1 if P1 commits not to enter; and (ii) sells on P2 if P1 makes no commitment. Therefore, P1 secures more profit by committing not to enter than by making no commitment in this case.

If P2 commits not to enter, Lemmas 7, 10, and 11 imply that Sj ($j \in \{1,2\}$): (i) sells on P1 if P1 commits not to enter; and (ii) sells on P2 if P1 makes no commitment. Therefore, P1 secures more profit by committing not to enter than by making no commitment in this case.

If P1 commits not to enter, Lemmas 7, 9, and 11 imply that both S1 and S2 sell on P1, regardless of P2's commitment.

If P1 makes no commitment, Lemmas 8, 10, and 11 imply that both S1 and S2 sell on P2, regardless of P2's commitment. Lemmas 3 and 6 imply that P2's profit is: (i) $\frac{\Theta_2 \left[\widetilde{\Delta}_{21}\right]^2}{8\, b_1^S} + \frac{\Theta_2 \left[\widetilde{\Delta}_{22}\right]^2}{8\, b_2^S}$ if P2 commits not to enter; and (ii) $\Theta_2 \, M_{21} - F + \Theta_2 \, M_{22} - F$ if P2 makes no commitment. Condition FS ensures that P2 secures more profit by making no commitment than by committing not to enter in this case.

Consequently, in equilibrium, P1 commits not to enter, and both S1 and S2 sell on P1, if $\frac{\Theta_1}{\Theta_2} < \left[\frac{\Delta_{21}}{\Delta_{11}}\right]^2$.

Finally, suppose $\frac{\Theta_1}{\Theta_2} = 1$. Lemma 7 implies that if both platforms commit not to enter, then Sj ($j \in \{1,2\}$) is indifferent between selling on P1 and selling on P2. Lemma 8 implies that if platforms both make no commitment, then Sj sells on P2. Lemma 9 implies that if P1 commits not to enter and P2 makes no commitment, then Sj sells on P1. Lemma 10 implies that if P1 makes no commitment and P2 commits to no entry, then Sj sells on P2.

If P2 makes no commitment, Sj: (i) sells on P1 if P1 commits not to enter; and (ii) sells on P2 if P1 makes no commitment. Therefore, P1 secures more profit by committing not to enter than by making no commitment in this case.

If P2 commits not to enter, Sj: (i) is indifferent between selling on P1 and selling on P2 if P1 commits not to enter; and (ii) sells on P2 if P1 makes no commitment. Therefore, P1 secures more profit by committing not to enter than by making no commitment in this case.

If P1 commits not to enter, Sj: (i) is indifferent between selling on P1 and selling on P2 if P2 commits not to enter; and (ii) sells on P1 if P2 makes no commitment. Therefore, P2 secures more profit by committing not to enter than by making no commitment in this case.

Consequently, in equilibrium, both platforms commit not to enter, and each seller is indifferent between selling on P1 and selling on P2, if $\frac{\Theta_1}{\Theta_2} = 1$.

<u>Case II</u>. $c_{1j}^P > c_{2j}^P$.

First suppose $\frac{\Theta_1}{\Theta_2} > \phi_2$. Lemmas 7 - 10 and 11 imply that both S1 and S2 sell on P1, regardless of the platforms' commitments. Condition FS ensures P1 enters each seller's market. Therefore, Lemmas 3 and 6 imply that: (i) P1's profit is $\Theta_1 M_{11} - F + \Theta_1 M_{12} - F$ if P1 makes no commitment; and (ii) P1's profit is $\frac{\Theta_1 [\widetilde{\Delta}_{11}]^2}{8 b_1^S} + \frac{\Theta_1 [\widetilde{\Delta}_{12}]^2}{8 b_2^S}$ if P1 commits not to enter. Condition FS ensures that $\Theta_1 M_{1j} - F > \frac{\Theta_1 [\widetilde{\Delta}_{1j}]^2}{8 b_j^S}$, i.e., P1 secures more profit by making no commitment than by committing to no entry. Therefore, in equilibrium, P1 makes no commitment, and both S1 and S2 sell on P1 if $\frac{\Theta_1}{\Theta_2} > \phi_2$.

Next suppose $\frac{\Theta_1}{\Theta_2} \in (\phi_1, \phi_2)$. Lemmas 7 - 10 and 11 imply that S1 sells on P1, regardless of the platforms' commitments.

If P2 makes no commitment, Lemmas 8, 9 imply that S2 sells on P1. Condition FS and Lemmas 3 and 6 imply that P1 secures more profit by making no commitment than by committing not to enter in this case.

If P2 commits not to enter, Lemmas 7 and 10 imply that S2: (i) sells on P1 if P1 commits not to enter; and (ii) sells on P2 if P1 makes no commitment. Lemmas 3 and 6 imply that P1's profit is: (i) $\frac{\Theta_1 [\widetilde{\Delta}_{11}]^2}{8 b_1^S} + \frac{\Theta_1 [\widetilde{\Delta}_{12}]^2}{8 b_2^S}$ if P1 commits not to enter; and (ii) $\Theta_1 M_{11} - F$ if

P1 makes no commitment. Therefore, Therefore, P1 secures more profit by making no commitment than by committing not to enter in this case because

$$\Theta_1 \, M_{11} - F \;>\; \frac{\Theta_1 \left[\widetilde{\Delta}_{11} \right]^2}{8 \, b_1^S} + \frac{\Theta_1 \left[\widetilde{\Delta}_{12} \right]^2}{8 \, b_2^S} \;\Leftrightarrow\; F \;<\; \Theta_1 \, M_{11} - \frac{\Theta_1 \left[\widetilde{\Delta}_{11} \right]^2}{8 \, b_1^S} - \frac{\Theta_1 \left[\widetilde{\Delta}_{12} \right]^2}{8 \, b_2^S}.$$

If P1 commits not to enter, Lemmas 7 and 9 imply that S2 sells on P1, regardless of P2's commitment.

If P1 makes no commitment, Lemmas 8 and 10 imply that: (i) S2 sells on P1 if P2 makes no commitment; and (ii) S2 sells P2 if P2 commits not to enter. Therefore, P2 secures more profit by committing not to enter than by making no commitment.

Consequently, in equilibrium, P1 makes no commitment while P2 commits not to enter, and S1 sells on P1 while S2 sells on P2, if $\frac{\Theta_1}{\Theta_2} \in (\, \phi_1, \phi_2 \,)$.

Next suppose $\frac{\Theta_1}{\Theta_2} < \phi_1$. If P2 makes no commitment, Lemmas 8 and 9 imply that both S1 and S2 sell on P1, regardless of P1's commitment. Condition FS and Lemmas 3 and 6 imply that P1 secures more profit by making no commitment than by committing not to enter in this case.

If P2 commits not to enter, Lemmas 7 and 10 imply that both S1 and S2 sell: (i) on P1 if P1 commits not to enter; and (ii) on P2 if P1 makes no commitment. Therefore, P1 secures more profit by committing not to enter than by making no commitment in this case.

If P1 makes no commitment, Lemmas 8 and 10 imply that both S1 and S2 sell: (i) on P1 if P2 makes no commitment; and (ii) on P2 if P2 commits not to enter. Therefore, P2 secures more profit by committing not to enter than by making no commitment.

If P1 commits not to enter, Lemmas 7 and 9 imply that both S1 and S2 sell on P1, regardless of P2's commitment.

Therefore, in equilibrium, both P1 and P2 commit not to enter and both S1 and S2 sell on P1 if $\frac{\Theta_1}{\Theta_2} < \phi_1$.

Finally, suppose $\frac{\Theta_1}{\Theta_2} = 1$. Lemma 7 implies that if both platforms commit not to enter, then Sj ($j \in \{1, 2\}$) is indifferent between selling on P1 and selling on P2. Lemma 8 implies that if platforms both make no commitment, then Sj sells on P1. Lemma 9 implies that if P1 commits not to enter and P2 makes no commitment, then Sj sells on P1. Lemma 10 implies that if P1 makes no commitment and P2 commits to no entry, then Sj sells on P2.

If P2 makes no commitment, Sj sells on P1, regardless of P1's commitment.

If P2 commits not to enter, Sj: (i) is indifferent between selling on P1 and selling on P2 if P1 commits not to enter; and (ii) sells on P2 if P1 makes no commitment. Therefore, P1 secures more profit by committing not to enter than by making no commitment in this case.

If P1 commits not to enter, Sj: (i) is indifferent between selling on P1 and selling on P2 if P2 commits not to enter; and (ii) sells on P1 if P2 makes no commitment. Therefore, P2 secures more profit by committing not to enter than by making no commitment in this case.

If P1 makes no commitment, Sj: sells on P1 if P2 makes no commitment; and (ii) sells on P2 if P2 commits to no entry. Therefore, P2 secures more profit by committing not to enter than by making no commitment in this case.

Consequently, in equilibrium, both platforms commit not to enter, and each seller is indifferent between selling on P1 and selling on P2, if $\frac{\Theta_1}{\Theta_2} = 1$. ■

<u>Proof of Proposition 3</u>. Proposition 1 implies that Sj ($j \in \{1, 2\}$) competes against P under MP.

First suppose P faces a competing platform $\widetilde{\text{P}}$ that is a stronger platform (i.e., $\frac{\widetilde{\Theta}}{\Theta} > 1$) under PC, where $\widetilde{\Theta}$ denotes $\widetilde{\text{P}}$'s platform strength.

Case I. $\frac{\widetilde{\Theta}}{\Theta} > \phi_2$.

Proposition 2 implies that each seller competes against $\widetilde{\mathrm{P}}$ under PC. Because $\widetilde{\mathrm{P}}$ is a weaker seller than P (i.e., $\frac{\widetilde{c}_j^P}{c_j^P} > 1$), increased platform competition reduces platform-seller competition in the sense that each seller faces reduced competition under PC, whereas each seller competes against P under MP.[44]

Case II. $\frac{\widetilde{\Theta}}{\Theta} \in (\,\phi_1, \phi_2\,)$.

Proposition 2 implies that S1 competes against $\widetilde{\mathrm{P}}$ whereas S2 faces no competition under PC. Because $\widetilde{\mathrm{P}}$ is a weaker seller than P (i.e., $\frac{\widetilde{c}_j^P}{c_j^P} > 1$), increased platform competition reduces platform-seller competition in the sense that S1 faces reduced competition and S2 faces no competition under PC, whereas each seller competes against P under MP.

Case III. $\frac{\widetilde{\Theta}}{\Theta} \in (\,1, \phi_1\,)$.

Proposition 2 implies that each seller faces no competition under PC. Therefore, increased platform competition reduces platform-seller competition in the sense that each seller faces no competition under PC, whereas each seller competes against P under MP.

Next suppose P faces a symmetric competing platform $\widetilde{\mathrm{P}}$ (i.e., $\frac{\widetilde{\Theta}}{\Theta} = 1$) under PC. Proposition 2 implies that each seller faces no competition under PC. Therefore, increased platform competition reduces platform-seller competition in the sense that each seller faces no competition under PC, whereas each seller competes against P under MP.

Finally, suppose P faces a competing platform $\widetilde{\mathrm{P}}$ that is a weaker platform (i.e., $\frac{\widetilde{\Theta}}{\Theta} < 1$) under PC.

Case I. $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_1}, 1\right)$.

In this case, $\frac{\Theta}{\widetilde{\Theta}} \in (\,1, \phi_1\,)$. Proposition 2 implies that each seller faces no competition under PC. Therefore, increased platform competition reduces platform-seller competition

[44] $\widetilde{c}_j^P$ denotes $\widetilde{\mathrm{P}}$'s cost of imitating Sj's product, and c_j^P denotes P's cost of imitating Sj's product.

in the sense that each seller faces no competition under PC, whereas each seller competes against P under MP.

Case II. $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_2}, \frac{1}{\phi_1} \right)$.

In this case, $\frac{\Theta}{\widetilde{\Theta}} \in (\, \phi_1, \phi_2 \,)$. Proposition 2 implies that S1 competes against P whereas S2 faces no competition under PC. Therefore, increased platform competition reduces platform-seller competition in the sense that S2 faces no competition and S1 faces unchanged competition under PC, whereas each seller competes against P under MP. ■

Proof of Proposition 4. Proposition 1 indicates that Sj's ($j \in \{1, 2\}$) equilibrium profit under MP is

$$\pi_j^M \;=\; \frac{\Theta}{\beta_j^S} \left[\frac{(\, 2 + \Omega_j \,)\; \Delta_{Pj}}{8 + \Omega_j} \right]^2 . \tag{81}$$

First suppose P faces a competing platform $\widetilde{\text{P}}$ that is a stronger platform (i.e., $\frac{\widetilde{\Theta}}{\Theta} > 1$) under PC.

Case I. $\frac{\widetilde{\Theta}}{\Theta} > \phi_2$.

Proposition 2 implies that each seller competes against $\widetilde{\text{P}}$. Lemma 6 implies that Sj's equilibrium profit is

$$\pi_j \;=\; \frac{\widetilde{\Theta}}{\beta_j^S} \left[\frac{(\, 2 + \Omega_j \,)\; \Delta_{\widetilde{P}j}}{8 + \Omega_j} \right]^2 . \tag{82}$$

(81) and (82) imply that:

$$\pi_j \;>\; \pi_j^M \quad \Leftrightarrow \quad \widetilde{\Theta} \left[\Delta_{\widetilde{P}j} \right]^2 \;>\; \Theta \left[\Delta_{Pj} \right]^2 . \tag{83}$$

The last inequality in (83) holds because $\frac{\widetilde{\Theta}}{\Theta} > \phi_2 > 1$ and $\Delta_{\widetilde{P}j} > \Delta_{Pj}$ (due to $\frac{\widetilde{c}_j^P}{c_j^P} > 1$).

Case II. $\frac{\widetilde{\Theta}}{\Theta} \in (\, \phi_1, \phi_2 \,)$.

Proposition 2 implies that S1 competes against $\widetilde{\text{P}}$ whereas S2 faces no competition on P. Lemmas 3 and 6 imply that S1's equilibrium profit is $\pi_1 = \frac{\widetilde{\Theta}}{\beta_1^S} \left[\frac{(\, 2+\Omega_1 \,)\, \Delta_{\widetilde{P}1}}{8+\Omega_1} \right]^2$ and S2's equilibrium profit is $\pi_2 = \frac{\Theta[\widetilde{\Delta}_{P2}]^2}{16\, b_2^S}$. Therefore, (83) implies $\pi_1 > \pi_1^M$. (10) and (81) imply that:

$$\pi_2 \;>\; \pi_2^M \quad \Leftrightarrow \quad \frac{\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{16\,\beta_2^S\,[\,1-\Omega_2\,]} \;>\; \frac{\Theta}{\beta_2^S}\left[\frac{(\,2+\Omega_2\,)\;\Delta_{P2}}{8+\Omega_2}\right]^2$$

$$\Leftrightarrow \quad \left[\frac{\widetilde{\Delta}_{P2}}{\Delta_{P2}}\right]^2 \;>\; \left[\frac{4\sqrt{1-\Omega_2}\,(\,2+\Omega_2\,)}{8+\Omega_2}\right]^2. \tag{84}$$

(84) holds because $\frac{\widetilde{\Delta}_{P2}}{\Delta_{P2}} > 1$ and $\left[\frac{4\sqrt{1-\Omega_2}(\,2+\Omega_2\,)}{8+\Omega_2}\right]^2 < 1$ (from (77)).

Case III. $\frac{\widetilde{\Theta}}{\Theta} \in (\,1, \phi_1\,)$.

Proposition 2 implies that each seller sells on $\widetilde{\mathrm{P}}$ and faces no competition. Lemma 3 implies that Sj's equilibrium profit is $\pi_j = \frac{\widetilde{\Theta}[\widetilde{\Delta}_{\widetilde{P}j}]^2}{16\, b_j^S}$. (81) implies that:

$$\pi_j \;>\; \pi_j^M \quad \Leftrightarrow \quad \frac{\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}j}\right]^2}{16\, b_j^S} \;>\; \frac{\Theta}{\beta_j^S}\left[\frac{(\,2+\Omega_j\,)\;\Delta_{Pj}}{8+\Omega_j}\right]^2. \tag{85}$$

(85) holds because

$$\frac{\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}j}\right]^2}{16\, b_j^S} \;>\; \frac{\Theta\left[\widetilde{\Delta}_{Pj}\right]^2}{16\, b_j^S} \;>\; \frac{\Theta}{\beta_j^S}\left[\frac{(\,2+\Omega_j\,)\;\Delta_{Pj}}{8+\Omega_j}\right]^2. \tag{86}$$

The first inequality in (86) reflects $\widetilde{\Theta} > \Theta$ and $\widetilde{\Delta}_{\widetilde{P}j} = \widetilde{\Delta}_{Pj}$ (due to $\frac{\partial \widetilde{\Delta}_{kj}}{\partial c_{kj}^P} = 0$). The second inequality in (86) reflects (84).

Next suppose P faces a symmetric competing platform $\widetilde{\mathrm{P}}$ (i.e., $\frac{\widetilde{\Theta}}{\Theta} = 1$). Proposition 2 implies that each seller faces no competition and is indifferent between selling on P and selling on $\widetilde{\mathrm{P}}$ under PC. Lemma 3 implies that Sj's equilibrium profit is

$$\pi_j \;=\; \frac{1}{2}\frac{\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}j}\right]^2}{16\, b_j^S} + \frac{1}{2}\frac{\Theta\left[\widetilde{\Delta}_{Pj}\right]^2}{16\, b_j^S} \;>\; \frac{1}{2}\frac{\Theta\left[\widetilde{\Delta}_{Pj}\right]^2 + \Theta\left[\widetilde{\Delta}_{Pj}\right]^2}{16\, b_j^S} \;=\; \frac{\Theta\left[\widetilde{\Delta}_{Pj}\right]^2}{16\, b_j^S}. \tag{87}$$

The inequality in (87) reflects $\widetilde{\Theta} > \Theta$ and $\widetilde{\Delta}_{\widetilde{P}j} = \widetilde{\Delta}_{Pj}$ (due to $\frac{\partial \widetilde{\Delta}_{kj}}{\partial c^P_{kj}} = 0$). (81), (86), and (87) imply that $\pi_j > \pi^M_j$.

Finally, suppose P faces a competing platform $\widetilde{\text{P}}$ that is a weaker platform (i.e., $\frac{\widetilde{\Theta}}{\Theta} < 1$) under PC.

<u>Case I</u>. $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_1}, 1 \right)$.

In this case, $\frac{\Theta}{\widetilde{\Theta}} \in (\, 1, \phi_1\,)$. Proposition 2 implies that each seller sells on P and faces no competition under PC. Lemma 3 implies that Sj's equilibrium profit is $\pi_j = \frac{\Theta \left[\, \widetilde{\Delta}_{Pj} \,\right]^2}{16\, b^S_j}$. (81) and (86) imply that $\pi_j > \pi^M_j$.

<u>Case II</u>. $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_2}, \frac{1}{\phi_1} \right)$.

In this case, $\frac{\Theta}{\widetilde{\Theta}} \in (\, \phi_1, \phi_2\,)$. Proposition 2 implies that S1 competes against P whereas S2 sells on $\widetilde{\text{P}}$ and faces no competition under PC. Lemmas 3 and 6 imply that S1's equilibrium profit is $\pi_1 = \frac{\Theta}{\beta^S_1} \left[\frac{(\, 2+\Omega_1\,)\, \Delta_{P1}}{8+\Omega_1} \right]^2$ and S2's equilibrium profit is $\pi_2 = \frac{\widetilde{\Theta} [\widetilde{\Delta}_{\widetilde{P}2}]^2}{16\, b^S_2}$. (81) implies that $\pi_1 = \pi^M_1$. (85) implies that $\pi_2 > \pi^M_2$. ■

<u>Proof of Proposition 5</u>. (1) implies that if Sj sells on Pk and competes against Pk ($j, k \in \{1, 2\}$), then:

$$p^S_{kj} \;=\; \frac{q^P_{kj}}{\eta_j} - \frac{\theta_j\, \alpha_j}{\eta_j} + \frac{\beta^P_j\, p^P_{kj}}{\eta_j}. \tag{88}$$

(2) and (88) imply that:

$$q^S_{kj} \;=\; \alpha_j - \frac{\beta^S_j\, q^P_{kj}}{\eta_j} + \frac{\beta^S_j\, \theta_j\, \alpha_j}{\eta_j} - \frac{\beta^S_j\, \beta^P_j\, p^P_{kj}}{\eta_j} + \eta_j\, p^P_{kj}$$

$$\Leftrightarrow \;\; p^P_{kj} \;=\; \frac{\eta_j\, q^S_{kj} + \beta^S_j\, q^P_{kj} - \alpha_j \left[\, \eta_j + \beta^S_j\, \theta_j \,\right]}{\left[\eta_j\right]^2 - \beta^S_j\, \beta^P_j}. \tag{89}$$

(88) and (89) imply that:

$$p^S_{kj} \;=\; \frac{q^P_{kj}}{\eta_j} - \frac{\theta_j\, \alpha_j}{\eta_j} + \frac{\beta^P_j}{\eta_j}\, \frac{\eta_j\, q^S_{kj} + \beta^S_j\, q^P_{kj} - \alpha_j \left[\, \eta_j + \beta^S_j\, \theta_j \,\right]}{\left[\eta_j\right]^2 - \beta^S_j\, \beta^P_j}$$

$$= \frac{\left[\eta_j\right]^2 q_{kj}^P - \eta_j \, \alpha_j \left[\theta_j \, \eta_j + \beta_j^P\right] + \beta_j^P \, \eta_j \, q_{kj}^S}{\eta_j \left[\left(\eta_j\right)^2 - \beta_j^S \, \beta_j^P\right]} = \frac{\eta_j \, q_{kj}^P + \beta_j^P \, q_{kj}^S - \alpha_j \left[\theta_j \, \eta_j + \beta_j^P\right]}{\left[\eta_j\right]^2 - \beta_j^S \, \beta_j^P}. \tag{90}$$

(89) and (90) imply that if S1 competes against Pk and S2 competes against Pi ($k, i \in \{1, 2\}$), then consumer surplus is:

$$CS = U\left(Q_{k1}^{*P}, Q_{i2}^{*P}, Q_{k1}^{*S}, Q_{i2}^{*S}\right) - p_{k1}^{*P} \, Q_{k1}^{*P} - p_{i2}^{*P} \, Q_{i2}^{*P} - p_{k1}^{*S} \, Q_{k1}^{*S} - p_{i2}^{*S} \, Q_{i2}^{*S} \tag{91}$$

$$\begin{aligned}
= \; & \frac{\frac{1}{2} \, \eta_1 \, \frac{Q_{k1}^{*S} \, Q_{k1}^{*P}}{\Theta_k} + \frac{1}{2} \, \beta_1^S \frac{\left[Q_{k1}^{*P}\right]^2}{\Theta_k} - \alpha_1 \left[\eta_1 + \beta_1^S \, \theta_1\right] Q_{k1}^{*P}}{\left[\eta_1\right]^2 - \beta_1^S \, \beta_1^P} \\
& + \frac{\frac{1}{2} \, \eta_2 \, \frac{Q_{i2}^{*S} \, Q_{i2}^{*P}}{\Theta_i} + \frac{1}{2} \, \beta_2^S \frac{\left[Q_{i2}^{*P}\right]^2}{\Theta_i} - \alpha_2 \left[\eta_2 + \beta_2^S \, \theta_2\right] Q_{i2}^{*P}}{\left[\eta_2\right]^2 - \beta_2^S \, \beta_2^P} \\
& + \frac{\frac{1}{2} \, \eta_1 \, \frac{Q_{k1}^{*S} \, Q_{k1}^{*P}}{\Theta_k} + \frac{1}{2} \, \beta_1^P \frac{\left[Q_{k1}^{*S}\right]^2}{\Theta_k} - \alpha_1 \left[\theta_1 \, \eta_1 + \beta_1^P\right] Q_{k1}^{*S}}{\left[\eta_1\right]^2 - \beta_1^S \, \beta_1^P} \\
& + \frac{\frac{1}{2} \, \eta_2 \, \frac{Q_{i2}^{*P} \, Q_{i2}^{*S}}{\Theta_i} + \frac{1}{2} \, \beta_2^P \frac{\left[Q_{i2}^{*S}\right]^2}{\Theta_i} - \alpha_2 \left[\theta_2 \, \eta_2 + \beta_2^P\right] Q_{i2}^{*S}}{\left[\eta_2\right]^2 - \beta_2^S \, \beta_2^P} \\
& - p_{k1}^{*P} \, Q_{k1}^{*P} - p_{i2}^{*P} \, Q_{i2}^{*P} - p_{k1}^{*S} \, Q_{k1}^{*S} - p_{i2}^{*S} \, Q_{i2}^{*S}
\end{aligned} \tag{92}$$

$$= \Theta_k \, \frac{\eta_1 \, q_{k1}^{*P} \, q_{k1}^{*S} + \frac{1}{2} \, \beta_1^S \left[q_{k1}^{*P}\right]^2 + \frac{1}{2} \, \beta_1^P \left[q_{k1}^{*S}\right]^2}{\beta_1^S \, \beta_1^P - \left[\eta_1\right]^2} + \Theta_i \, \frac{\eta_2 \, q_{i2}^{*P} \, q_{i2}^{*S} + \frac{1}{2} \, \beta_2^S \left[q_{i2}^{*P}\right]^2 + \frac{1}{2} \, \beta_2^P \left[q_{i2}^{*S}\right]^2}{\beta_2^S \, \beta_2^P - \left[\eta_2\right]^2}. \tag{93}$$

Lemma 6 implies that if Sj competes against Pk, then:

$$q_{kj}^{*S} = \frac{Q_{kj}^{*S}}{\Theta_k} = \frac{\left[2 + \Omega_j\right] \Delta_{kj}}{8 + \Omega_j} \text{ and } q_{kj}^{*P} = \frac{Q_{kj}^{*P}}{\Theta_k} = \frac{\overline{\Delta}_{kj}}{2} + \frac{\eta_j \left[2 + \Omega_j\right] \Delta_{kj}}{2\beta_j^S \left[8 + \Omega_j\right]}. \tag{94}$$

(93) and (94) imply that if S1 competes against Pk and S2 competes against Pi ($k, i \in \{1, 2\}$), then consumer surplus is:

$$CS = \Theta_k \, \varsigma_{k1} + \Theta_i \, \varsigma_{i2}, \tag{95}$$

where for $j \in \{1,2\}$,

$$\varsigma_{kj} \;\equiv\; \frac{1}{\beta_j^S\,\beta_j^P\,[\,1-\Omega_j\,]}\left\{\frac{\eta_j\,[\,2+\Omega_j\,]\;\Delta_{kj}}{8+\Omega_j}\left[\frac{\overline{\Delta}_{kj}}{2}+\frac{\eta_j\,(\,2+\Omega_j\,)\;\Delta_{kj}}{2\beta_j^S\,(\,8+\Omega_j\,)}\right]\right.$$

$$\left.+\;\frac{\beta_j^S}{2}\left[\frac{\overline{\Delta}_{kj}}{2}+\frac{\eta_j\,(\,2+\Omega_j\,)\;\Delta_{kj}}{2\beta_j^S\,(\,8+\Omega_j\,)}\right]^2+\frac{\beta_j^P}{2}\left[\frac{(\,2+\Omega_j\,)\;\Delta_{kj}}{8+\Omega_j}\right]^2\right\}. \tag{96}$$

Observe that:

$$\frac{\partial\left(\frac{\overline{\Delta}_{kj}}{2}+\frac{\eta_j(\,2+\Omega_j\,)\,\Delta_{kj}}{2\beta_j^S(\,8+\Omega_j\,)}\right)}{\partial c_{kj}^P} \;=\; \frac{1}{2}\frac{\partial\overline{\Delta}_{kj}}{\partial c_{kj}^P}+\frac{\eta_j\,[\,2+\Omega_j\,]}{2\beta_j^S\,[\,8+\Omega_j\,]}\frac{\partial\Delta_{kj}}{\partial c_{kj}^P} \;=\; -\,\frac{\beta_j^P}{2}+\frac{\eta_j\,[\,2+\Omega_j\,]}{2\beta_j^S\,[\,8+\Omega_j\,]}\eta_j$$

$$=\;\frac{\beta_j^P}{2}\left[\frac{2\,\Omega_j+(\Omega_j)^2\,-8-\Omega_j}{8+\Omega_{kj}}\right] \;=\; \frac{\beta_j^P}{2}\left[\frac{\Omega_j+(\Omega_j)^2\,-8}{8+\Omega_j}\right] \;<\; 0; \tag{97}$$

$$\frac{\partial\left(\frac{(\,2+\Omega_j\,)\,\Delta_{kj}}{8+\Omega_j}\right)}{\partial c_{kj}^P} \;=\; \frac{2+\Omega_j}{8+\Omega_j}\frac{\partial\Delta_{kj}}{\partial c_{kj}^P} \;=\; \frac{\eta_j\,[\,2+\Omega_j\,]}{8+\Omega_j} \;>\; 0; \tag{98}$$

$$\frac{\partial\widetilde{\Delta}_{kj}}{\partial c_{kj}^P} \;=\; \frac{\partial\left(\Delta_{kj}+\frac{\eta_j}{\beta_j^P}\,\overline{\Delta}_{kj}\right)}{\partial c_{kj}^P} \;=\; \frac{\partial\Delta_{kj}}{\partial c_{kj}^P}+\frac{\eta_j}{\beta_j^P}\frac{\partial\overline{\Delta}_{kj}}{\partial c_{kj}^P} \;=\; \eta_j-\frac{\eta_j}{\beta_j^P}\,\beta_j^P \;=\; 0. \tag{99}$$

The inequality in (97) holds because $\Omega_j \in (0,1)$. (96) and (97) - (99) imply that:

$$\frac{\partial\varsigma_{kj}}{\partial c_{kj}^P} \;\stackrel{s}{=}\; \frac{\eta_j\,[\,2+\Omega_j\,]}{8+\Omega_j}\left\{\frac{\partial\Delta_{kj}}{\partial c_{kj}^P}\left[\frac{\overline{\Delta}_{kj}}{2}+\frac{\eta_j\,(\,2+\Omega_j\,)\;\Delta_{kj}}{2\beta_j^S\,(\,8+\Omega_j\,)}\right]+\Delta_{kj}\frac{\partial\left(\frac{\overline{\Delta}_{kj}}{2}+\frac{\eta_j(\,2+\Omega_j\,)\,\Delta_{kj}}{2\beta_j^S(\,8+\Omega_j\,)}\right)}{\partial c_{kj}^P}\right\} \tag{100}$$

$$+\;\frac{\beta_j^S}{2}2\left[\frac{\overline{\Delta}_{kj}}{2}+\frac{\eta_j\,(\,2+\Omega_j\,)\;\Delta_{kj}}{2\beta_j^S\,(\,8+\Omega_j\,)}\right]\frac{\partial\left(\frac{\overline{\Delta}_{kj}}{2}+\frac{\eta_j(\,2+\Omega_j\,)\,\Delta_{kj}}{2\beta_j^S(\,8+\Omega_j\,)}\right)}{\partial c_{kj}^P}$$

$$+\;\frac{\beta_j^P}{2}2\left[\frac{(\,2+\Omega_j\,)\;\Delta_{kj}}{8+\Omega_j}\right]\frac{\partial\left(\frac{(\,2+\Omega_j\,)\,\Delta_{kj}}{8+\Omega_j}\right)}{\partial c_{kj}^P}$$

$$=\;\beta_j^S\,\beta_j^P\left[\frac{\overline{\Delta}_{kj}}{2}+\frac{\eta_j\,(\,2+\Omega_j\,)\;\Delta_{kj}}{2\beta_j^S\,(\,8+\Omega_j\,)}\right]\left[\frac{5\,\Omega_j+3\,(\Omega_j)^2-8}{2\,(\,8+\Omega_j\,)}\right]$$

$$+\ \frac{\beta_j^P\,\eta_j\,\Delta_{kj}\,[\,2+\Omega_j\,]}{2\,[\,8+\Omega_j\,]}\left[\frac{3\,\Omega_j+(\Omega_j)^2-4}{8+\Omega_j}\right]\ <\ 0. \tag{101}$$

(100) holds because $\Omega_j \in (0,1)$. (101) holds because

$$5\,\Omega_{kj}+3\,(\Omega_{kj})^2-8\ <\ 0\ \text{ and }\ 3\,\Omega_{kj}+(\Omega_{kj})^2-4\ <\ 0. \tag{102}$$

(102) holds because both $5\,\Omega_{kj}+3\,(\Omega_{kj})^2$ and $3\,\Omega_{kj}+(\Omega_{kj})^2$ increase in $\Omega_{kj} \in (0,1)$, and thus, $5\,\Omega_{kj}+3\,(\Omega_{kj})^2 < 5\times 1+3\,(1)^2 = 8$ and $3\,\Omega_{kj}+(\Omega_{kj})^2 < 3\times 1+1^2 = 4$.

(5) implies that if Sj sells on Pk ($j,k \in \{1,2\}$) and faces no competition, then:

$$q_{kj}^S\ =\ A_j - b_j^S\,p_{kj}^S\ \ \Leftrightarrow\ \ p_{kj}^S = \frac{A_j}{b_j^S} - \frac{q_{kj}^S}{b_j^S}. \tag{103}$$

(103) implies that if S1 sells on Pk, S2 sells on Pi ($k,i \in \{1,2\}$), and each seller faces no competition, then consumer surplus is:

$$CS\ =\ U\left(Q_{k1}^{*S},Q_{i2}^{*S}\right) - p_{k1}^{*S}\,Q_{k1}^{*S} - p_{i2}^{*S}\,Q_{i2}^{*S} \tag{104}$$

$$=\ \frac{\Theta_k}{b_1^S}\left[q_{k1}^{*S}\right]^2 + \frac{\Theta_i}{b_2^S}\left[q_{i2}^{*S}\right]^2 - \frac{\Theta_k}{2\,b_1^S}\left[q_{k1}^{*S}\right]^2 - \frac{\Theta_i}{2\,b_2^S}\left[q_{i2}^{*S}\right]^2\ =\ \frac{\Theta_k}{2\,b_1^S}\left[q_{k1}^{*S}\right]^2 + \frac{\Theta_i}{2\,b_2^S}\left[q_{i2}^{*S}\right]^2. \tag{105}$$

Lemma 3 implies that if Sj sells on Pk ($k,i \in \{1,2\}$) and faces no competition, then:

$$q_{kj}^{*S}\ =\ \frac{Q_j^{*S}}{\Theta_k}\ =\ \frac{\widetilde{\Delta}_{kj}}{4}. \tag{106}$$

(105) and (106) imply that if S1 sells on Pk, S2 sells on Pi ($k,i \in \{1,2\}$), and each seller faces no competition, then consumer surplus is:

$$CS\ =\ \frac{\Theta_k}{2\,b_1^S}\left[\frac{\widetilde{\Delta}_{k1}}{4}\right]^2 + \frac{\Theta_i}{2\,b_2^S}\left[\frac{\widetilde{\Delta}_{i2}}{4}\right]^2\ =\ \frac{\Theta_k}{2\,\beta_1^S\,[\,1-\Omega_1\,]}\left[\frac{\widetilde{\Delta}_{k1}}{4}\right]^2 + \frac{\Theta_i}{2\,\beta_2^S\,[\,1-\Omega_2\,]}\left[\frac{\widetilde{\Delta}_{i2}}{4}\right]^2. \tag{107}$$

The second equality in (107) reflects (10).

(89), (90), and (103) imply that if S1 competes against Pk and S2 sells on Pi and faces no competition ($k, i \in \{1, 2\}$), then consumer surplus is:

$$CS = U\left(Q_{k1}^{*P}, Q_{k1}^{*S}, Q_{i2}^{*S}\right) - p_{k1}^{*P} Q_{k1}^{*P} - p_{k1}^{*S} Q_{k1}^{*S} - p_{i2}^{*S} Q_{i2}^{*S} \tag{108}$$

$$= \frac{\frac{1}{2}\eta_1 \frac{Q_{k1}^{*S} Q_{k1}^{*P}}{\Theta_k} + \frac{1}{2}\beta_1^S \frac{\left[Q_{k1}^{*P}\right]^2}{\Theta_k} - \alpha_1\left[\eta_1 + \beta_1^S \theta_1\right] Q_{k1}^{*P}}{[\eta_1]^2 - \beta_1^S \beta_1^P}$$

$$+ \frac{\frac{1}{2}\eta_1 \frac{Q_{k1}^{*S} Q_{k1}^{*P}}{\Theta_k} + \frac{1}{2}\beta_1^P \frac{\left[Q_{k1}^{*S}\right]^2}{\Theta_k} - \alpha_1\left[\theta_1 \eta_1 + \beta_1^P\right] Q_{k1}^{*S}}{[\eta_1]^2 - \beta_1^S \beta_1^P}$$

$$+ \frac{A_2}{b_2^S} Q_{i2}^{*S} - \frac{\left[Q_{i2}^{*S}\right]^2}{2\, b_2^S \,\Theta_i} - p_{k1}^{*P} Q_{k1}^{*P} - p_{k1}^{*S} Q_{k1}^{*S} - p_{i2}^{*S} Q_{i2}^{*S} \tag{109}$$

$$= \Theta_k \frac{\eta_1 q_{k1}^{*P} q_{k1}^{*S} + \frac{1}{2}\beta_1^S \left[q_{k1}^{*P}\right]^2 + \frac{1}{2}\beta_1^P \left[q_{k1}^{*S}\right]^2}{\beta_1^S \beta_1^P - [\eta_1]^2} + \frac{\Theta_i \left[q_{i2}^{*S}\right]^2}{2\, b_2^S}. \tag{110}$$

(94), (96), (106), and (110) imply that if S1 competes against Pk and S2 sells on Pi and faces no competition ($k, i \in \{1, 2\}$), then consumer surplus is:

$$CS = \Theta_k \varsigma_{k1} + \frac{\Theta_i}{2\, b_2^S}\left[\frac{\widetilde{\Delta}_{i2}}{4}\right]^2 = \Theta_k \varsigma_{k1} + \frac{\Theta_i}{2\,\beta_2^S\,[1 - \Omega_2]}\left[\frac{\widetilde{\Delta}_{i2}}{4}\right]^2. \tag{111}$$

The second equality in (111) reflects (10).

(12) and (96) imply that:

$$\varsigma_{kj} > \frac{1}{2\,\beta_j^S\,[1 - \Omega_j]}\left[\frac{\widetilde{\Delta}_{kj}}{4}\right]^2 \tag{112}$$

$$\Leftrightarrow \quad \frac{4\,[5\,\Omega_j + 4]\,[2 + \Omega_j]^2 - [8 + \Omega_j]^2}{4\,[8 + \Omega_j]^2}\beta_j^P\,[\Delta_{kj}]^2 + \frac{[16 + 11\,\Omega_j]\,\eta_j\,\Delta_{kj}\,\overline{\Delta}_{kj}}{2\,[8 + \Omega_j]}$$

$$+ \frac{\beta_j^P \, \beta_j^S \, [\, 4 - \Omega_j \,] \, \left[\overline{\Delta}_{kj} \right]^2}{4 \, \beta_j^P} \;>\; 0. \tag{113}$$

(113) holds because $\Omega_j \in (0,1)$ and

$$4 \, [\, 5 \, \Omega_j + 4 \,] \, [\, 2 + \Omega_j \,]^2 - [\, 8 + \Omega_j \,]^2 \;>\; 0 \quad \Leftrightarrow \quad 4 \, [\, 5 \, \Omega_j + 4 \,] \, [\, 2 + \Omega_j \,]^2 \;>\; [\, 8 + \Omega_j \,]^2$$

$$\Leftrightarrow \quad 128 \, \Omega_j + 20 \, [\Omega_j]^3 + 95 \, [\Omega_j]^2 \;>\; 0. \tag{114}$$

(114) holds because $\Omega_j \in (0,1)$.

Proposition 1 implies that each seller competes against P under MP. Therefore, (95) implies that:

$$CS^M \;=\; \Theta \, \varsigma_{P1} + \Theta \, \varsigma_{P2}, \tag{115}$$

where ς_{Pj} is given by (96).

First suppose P faces a competing platform $\widetilde{\mathrm{P}}$ that is a stronger platform (i.e., $\frac{\widetilde{\Theta}}{\Theta} > 1$) under PC, where $\widetilde{\Theta}$ denotes $\widetilde{\mathrm{P}}$'s platform strength. (101) and $\frac{\widetilde{c}_j^P}{c_j^P} > 1$ imply that:

$$\frac{\varsigma_{Pj}}{\varsigma_{\widetilde{P}j}} \;>\; 1. \tag{116}$$

<u>Case I</u>. $\frac{\widetilde{\Theta}}{\Theta} > \max \left\{ \frac{\varsigma_{P1}}{\varsigma_{\widetilde{P}1}}, \frac{\varsigma_{P2}}{\varsigma_{\widetilde{P}2}}, \phi_{\widetilde{P}2} \right\}$.

Because $\frac{\widetilde{\Theta}}{\Theta} > \phi_{\widetilde{P}2}$, Proposition 2 implies that each seller competes against $\widetilde{\mathrm{P}}$ under PC. Therefore, (95) implies that:

$$CS \;=\; \widetilde{\Theta} \, \varsigma_{\widetilde{P}1} + \widetilde{\Theta} \, \varsigma_{\widetilde{P}2}, \tag{117}$$

where $\varsigma_{\widetilde{P}j}$ is given by (96).

Because $\frac{\widetilde{\Theta}}{\Theta} > \frac{\varsigma_{Pj}}{\varsigma_{\widetilde{P}j}}$, $\widetilde{\Theta} \, \varsigma_{\widetilde{P}j} > \Theta \, \varsigma_{Pj}$. Therefore, (115) and (117) imply that $CS > CS^M$ in this case.

Case II. $\frac{\widetilde{\Theta}}{\Theta} \in \left(1, \min\left\{\frac{\varsigma_{P1}}{\varsigma_{\widetilde{P}1}}, \frac{\varsigma_{P2}}{\varsigma_{\widetilde{P}2}}, \phi_{\widetilde{P}2}\right\}\right)$.

First suppose $\phi_{\widetilde{P}1} < \min\left\{\frac{\varsigma_{P1}}{\varsigma_{\widetilde{P}1}}, \frac{\varsigma_{P2}}{\varsigma_{\widetilde{P}2}}\right\}$.

If $\frac{\widetilde{\Theta}}{\Theta} \in \left(\phi_{\widetilde{P}1}, \min\left\{\frac{\varsigma_{P1}}{\varsigma_{\widetilde{P}1}}, \frac{\varsigma_{P2}}{\varsigma_{\widetilde{P}2}}, \phi_{\widetilde{P}2}\right\}\right)$, then $\frac{\widetilde{\Theta}}{\Theta} \in \left(\phi_{\widetilde{P}1}, \phi_{\widetilde{P}2}\right)$. Proposition 2 implies that S1 competes against $\widetilde{\mathrm{P}}$ whereas S2 sells on P and faces no competition under PC. Therefore, (111) implies that:

$$CS \;=\; \widetilde{\Theta}\,\varsigma_{\widetilde{P}1} + \frac{\Theta}{2\,\beta_2^S\,[\,1-\Omega_2\,]}\left[\frac{\widetilde{\Delta}_{P2}}{4}\right]^2. \tag{118}$$

Because $\frac{\widetilde{\Theta}}{\Theta} < \frac{\varsigma_{P1}}{\varsigma_{\widetilde{P}1}}$, $\widetilde{\Theta}\,\varsigma_{\widetilde{P}1} < \Theta\,\varsigma_{P1}$. (112) implies that $\frac{1}{2\,\beta_2^S[1-\Omega_2]}\left[\frac{\widetilde{\Delta}_{P2}}{4}\right]^2 < \varsigma_{P2}$. Therefore, (115) and (118) imply that $CS < CS^M$ in this case.

If $\frac{\widetilde{\Theta}}{\Theta} \in \left(1, \phi_{\widetilde{P}1}\right)$, Proposition 2 implies that each seller sells on $\widetilde{\mathrm{P}}$ and faces no competition under PC. Therefore, (107) implies that:

$$CS \;=\; \frac{\widetilde{\Theta}}{2\,\beta_1^S\,[\,1-\Omega_1\,]}\left[\frac{\widetilde{\Delta}_{\widetilde{P}1}}{4}\right]^2 + \frac{\widetilde{\Theta}}{2\,\beta_2^S\,[\,1-\Omega_2\,]}\left[\frac{\widetilde{\Delta}_{\widetilde{P}2}}{4}\right]^2. \tag{119}$$

(112) implies that $\frac{\widetilde{\Theta}}{2\,\beta_j^S[1-\Omega_j]}\left[\frac{\widetilde{\Delta}_{\widetilde{P}j}}{4}\right]^2 < \widetilde{\Theta}\,\varsigma_{\widetilde{P}j}$. Because $\frac{\widetilde{\Theta}}{\Theta} < \frac{\varsigma_{Pj}}{\varsigma_{\widetilde{P}j}}$, $\widetilde{\Theta}\,\varsigma_{\widetilde{P}j} < \Theta\,\varsigma_{Pj}$. Therefore, $\frac{\widetilde{\Theta}}{2\,\beta_j^S[1-\Omega_j]}\left[\frac{\widetilde{\Delta}_{\widetilde{P}j}}{4}\right]^2 < \Theta\,\varsigma_{Pj}$. Therefore, (115) and (119) imply that $CS < CS^M$ in this case.

Next suppose $\phi_{\widetilde{P}1} \geq \min\left\{\frac{\varsigma_{P1}}{\varsigma_{\widetilde{P}1}}, \frac{\varsigma_{P2}}{\varsigma_{\widetilde{P}2}}\right\}$. (**??**) implies that $\phi_{\widetilde{P}2} > \min\left\{\frac{\varsigma_{P1}}{\varsigma_{\widetilde{P}1}}, \frac{\varsigma_{P2}}{\varsigma_{\widetilde{P}2}}\right\}$. Therefore, $\min\left\{\frac{\varsigma_{P1}}{\varsigma_{\widetilde{P}1}}, \frac{\varsigma_{P2}}{\varsigma_{\widetilde{P}2}}, \phi_{\widetilde{P}2}\right\} = \min\left\{\frac{\varsigma_{P1}}{\varsigma_{\widetilde{P}1}}, \frac{\varsigma_{P2}}{\varsigma_{\widetilde{P}2}}\right\}$. Because $\frac{\widetilde{\Theta}}{\Theta} \in \left(1, \min\left\{\frac{\varsigma_{P1}}{\varsigma_{\widetilde{P}1}}, \frac{\varsigma_{P2}}{\varsigma_{\widetilde{P}2}}\right\}\right)$, $\frac{\widetilde{\Theta}}{\Theta} \in \left(1, \phi_{\widetilde{P}1}\right)$. Proposition 2 implies that each seller sells on $\widetilde{\mathrm{P}}$ and faces no competition under PC. Therefore, consumer surplus is given by (119). Therefore, (115) and (119) imply that $CS < CS^M$ in this case.

Next suppose P faces a symmetric competing platform $\widetilde{\mathrm{P}}$ (i.e., $\frac{\widetilde{\Theta}}{\Theta} = 1$) under PC. Proposition 2 implies that each seller is indifferent between selling on P and selling on $\widetilde{\mathrm{P}}$ and each seller faces no competition under PC. Therefore, (107) implies that:

$$CS \;=\; \frac{1}{2}\,\frac{\widetilde{\Theta}}{2\,\beta_1^S\,[\,1-\Omega_1\,]}\left[\frac{\widetilde{\Delta}_{\widetilde{P}1}}{4}\right]^2 + \frac{1}{2}\,\frac{\Theta}{2\,\beta_1^S\,[\,1-\Omega_1\,]}\left[\frac{\widetilde{\Delta}_{P1}}{4}\right]^2$$

$$+\;\frac{1}{2}\,\frac{\widetilde{\Theta}}{2\,\beta_2^S\,[\,1-\Omega_2\,]}\left[\frac{\widetilde{\Delta}_{\widetilde{P}2}}{4}\right]^2 + \frac{1}{2}\,\frac{\Theta}{2\,\beta_2^S\,[\,1-\Omega_2\,]}\left[\frac{\widetilde{\Delta}_{P2}}{4}\right]^2. \tag{120}$$

(112) implies that $\frac{\Theta}{2\,\beta_j^S[1-\Omega_j\,]}\left[\frac{\widetilde{\Delta}_{Pj}}{4}\right]^2 < \Theta\,\varsigma_{Pj}$ and $\frac{\widetilde{\Theta}}{2\,\beta_j^S[1-\Omega_j\,]}\left[\frac{\widetilde{\Delta}_{\widetilde{P}j}}{4}\right]^2 < \widetilde{\Theta}\,\varsigma_{\widetilde{P}j}$. Because $\frac{\widetilde{\Theta}}{\Theta} = 1$, (116) implies that $\frac{\widetilde{\Theta}}{\Theta} < \frac{\varsigma_{Pj}}{\varsigma_{\widetilde{P}j}}$, and thus, $\widetilde{\Theta}\,\varsigma_{\widetilde{P}j} < \Theta\,\varsigma_{Pj}$. Therefore, $\frac{\widetilde{\Theta}}{2\,\beta_j^S[1-\Omega_j\,]}\left[\frac{\widetilde{\Delta}_{\widetilde{P}j}}{4}\right]^2 < \Theta\,\varsigma_{Pj}$. Therefore, for $j \in \{1,2\}$

$$\frac{1}{2}\,\frac{\widetilde{\Theta}}{2\,\beta_j^S\,[\,1-\Omega_j\,]}\left[\frac{\widetilde{\Delta}_{\widetilde{P}j}}{4}\right]^2 + \frac{1}{2}\,\frac{\Theta}{2\,\beta_j^S\,[\,1-\Omega_j\,]}\left[\frac{\widetilde{\Delta}_{Pj}}{4}\right]^2 \;<\; \frac{\Theta}{2}\,\varsigma_{Pj} + \frac{\Theta}{2}\,\varsigma_{Pj} \;=\; \Theta\,\varsigma_{Pj}. \tag{121}$$

Therefore, (115), (120), and (121) imply that $CS < CS^M$ in this case.

Finally, suppose P faces a competing platform $\widetilde{\mathrm{P}}$ that is a weaker platform (i.e., $\frac{\widetilde{\Theta}}{\Theta} < 1$) under PC.

<u>Case I</u>. $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_{P1}}, 1\right)$.

In this case, $\frac{\Theta}{\widetilde{\Theta}} \in (\,1, \phi_1\,)$. Proposition 2 implies that each seller sells on P and faces no competition under PC. Therefore, (107) implies that:

$$CS \;=\; \frac{\Theta}{2\,\beta_1^S\,[\,1-\Omega_1\,]}\left[\frac{\widetilde{\Delta}_{P1}}{4}\right]^2 + \frac{\Theta}{2\,\beta_2^S\,[\,1-\Omega_2\,]}\left[\frac{\widetilde{\Delta}_{P2}}{4}\right]^2. \tag{122}$$

(112) implies that $\frac{1}{2\,\beta_j^S[1-\Omega_j\,]}\left[\frac{\widetilde{\Delta}_{Pj}}{4}\right]^2 < \varsigma_{Pj}$ for $j \in \{1,2\}$. Therefore, (115) and (122) imply that $CS < CS^M$ in this case.

<u>Case II</u>. $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_{P2}}, \frac{1}{\phi_{P1}}\right)$.

In this case, $\frac{\Theta}{\widetilde{\Theta}} \in (\,\phi_1, \phi_2\,)$. Proposition 2 implies that S1 competes against P whereas S2 sells on $\widetilde{\text{P}}$ and faces no competition under PC. Therefore, (111) implies that:

$$CS \;=\; \Theta\,\varsigma_{P1} + \frac{\widetilde{\Theta}}{2\,\beta_2^S\,[\,1-\Omega_2\,]} \left[\frac{\widetilde{\Delta}_{\widetilde{P}2}}{4}\right]^2 . \tag{123}$$

(112) implies that $\frac{\widetilde{\Theta}}{2\,\beta_2^S[1-\Omega_2]} \left[\frac{\widetilde{\Delta}_{\widetilde{P}2}}{4}\right]^2 < \widetilde{\Theta}\,\varsigma_{\widetilde{P}2}$. (116) and $\frac{\widetilde{\Theta}}{\Theta} < 1$ imply that $\frac{\widetilde{\Theta}}{\Theta} < \frac{\varsigma_{Pj}}{\varsigma_{\widetilde{P}j}}$, and thus, $\widetilde{\Theta}\,\varsigma_{\widetilde{P}j} < \Theta\,\varsigma_{Pj}$. Therefore, $\frac{\widetilde{\Theta}}{2\,\beta_2^S[1-\Omega_2]} \left[\frac{\widetilde{\Delta}_{\widetilde{P}2}}{4}\right]^2 < \Theta\,\varsigma_{P2}$. Therefore, (115) and (123) imply that $CS < CS^M$ in this case. ■

<u>Proof of Proposition 6</u>. (90) and (94) imply that if Sj sells on Pk and competes against Pk ($j, k \in \{1, 2\}$), then in equilibrium:

$$p_{kj}^{*S} \;=\; \frac{1}{[\eta_j]^2 - \beta_j^S\,\beta_j^P} \left\{ \eta_j \left[\frac{\overline{\Delta}_{kj}}{2} + \frac{\eta_j\,(\,2+\Omega_j\,)\,\Delta_{kj}}{2\,\beta_j^S\,(\,8+\Omega_j\,)} \right] + \frac{\beta_j^P\,[\,2+\Omega_j\,]\,\Delta_{kj}}{8+\Omega_j} - \alpha_j\,[\,\theta_j\,\eta_j + \beta_j^P\,] \right\}$$

$$= \frac{1}{\beta_j^S\,\beta_j^P\,[\,1-\Omega_j\,]} \left[\alpha_j\,\left(\,\theta_j\,\eta_j + \beta_j^P\,\right) - \frac{\eta_j\,\overline{\Delta}_{kj}}{2} - \frac{\beta_j^P\,(\,2+\Omega_j\,)^2\,\Delta_{kj}}{2\,(\,8+\Omega_j\,)} \right]. \tag{124}$$

(103) and (106) imply that if Sj sells on Pk ($j, k \in \{1, 2\}$) and faces no competition, then in equilibrium:

$$p_{kj}^{*S} \;=\; \frac{A_j}{b_j^S} - \frac{\widetilde{\Delta}_{kj}}{4\,b_j^S} \;=\; \frac{4\,A_j - \widetilde{\Delta}_{kj}}{4\,b_j^S} \;=\; \frac{4\,A_j - \widetilde{\Delta}_{kj}}{4\,\beta_j^S\,[\,1-\Omega_j\,]}. \tag{125}$$

The last equality in (125) reflects (10).

Let p_j^{SM} denote Sj's equilibrium price under MP and p_j^S denote Sj's equilibrium price under PC. Proposition 1 implies that Sj ($j \in \{1, 2\}$) competes against P under MP. Therefore, (124) implies that:

$$p_j^{SM} \;=\; \frac{1}{\beta_j^S\,\beta_j^P\,[\,1-\Omega_j\,]} \left[\alpha_j\,\left(\,\theta_j\,\eta_j + \beta_j^P\,\right) - \frac{\eta_j\,\overline{\Delta}_{Pj}}{2} - \frac{\beta_j^P\,(\,2+\Omega_j\,)^2\,\Delta_{Pj}}{2\,(\,8+\Omega_j\,)} \right]. \tag{126}$$

First suppose P faces a competing platform $\widetilde{\text{P}}$ that is a stronger platform (i.e., $\frac{\widetilde{\Theta}}{\Theta} > 1$)

under PC.

Case I. $\frac{\widetilde{\Theta}}{\Theta} > \phi_2$.

Proposition 2 implies that Sj ($j \in \{1, 2\}$) competes against $\widetilde{\mathrm{P}}$ under PC. Therefore, (124) implies that:

$$p_j^S \;=\; \frac{1}{\beta_j^S\,\beta_j^P\,[\,1-\Omega_j\,]}\left[\alpha_j\left(\theta_j\,\eta_j+\beta_j^P\right)-\frac{\eta_j\,\overline{\Delta}_{\widetilde{P}j}}{2}-\frac{\beta_j^P\,(\,2+\Omega_j\,)^2\,\Delta_{\widetilde{P}j}}{2\,(\,8+\Omega_j\,)}\right]. \tag{127}$$

Observe that:

$$\frac{\partial\left(\frac{\eta_j\,\overline{\Delta}_{kj}}{2}+\frac{\beta_j^P\,(\,2+\Omega_j\,)^2\,\Delta_{kj}}{2(\,8+\Omega_j\,)}\right)}{\partial c_{kj}^P} \;=\; \frac{\eta_j}{2}\,\frac{\partial\overline{\Delta}_{kj}}{\partial c_{kj}^P}+\frac{\beta_j^P\,[\,2+\Omega_j\,]^2}{2\,[\,8+\Omega_j\,]}\,\frac{\partial\Delta_{kj}}{\partial c_{kj}^P} \;=\; -\frac{\eta_j\,\beta_j^P}{2}+\frac{\eta_j\,\beta_j^P\,[\,2+\Omega_j\,]^2}{2\,[\,8+\Omega_j\,]}$$

$$\stackrel{s}{=}\; -1+\frac{[\,2+\Omega_j\,]^2}{8+\Omega_j} \;=\; \frac{[\,2+\Omega_j\,]^2-[\,8+\Omega_j\,]}{8+\Omega_j} \;=\; \frac{4+4\,\Omega_j+[\Omega_j]^2-8-\Omega_j}{8+\Omega_j} \;=\; \frac{3\,\Omega_j+[\Omega_j]^2-4}{8+\Omega_j} \;<\; 0. \tag{128}$$

The inequality in (128) holds because $3\,\Omega_j+[\Omega_j]^2-4$ increases in $\Omega_j \in (0,1)$, and therefore, $3\,\Omega_j+[\Omega_j]^2-4 < \max 3\,\Omega_j+[\Omega_j]^2-4 = 3*1+[1]^2-4 = 0$.

(126) - (128) imply that $p_j^S > p_j^{SM}$ in this case because $\frac{\widetilde{c}_j^P}{c_j^P} > 1$.

Case II. $\frac{\widetilde{\Theta}}{\Theta} \in (\,\phi_1, \phi_2\,)$.

Proposition 2 implies that S1 competes against $\widetilde{\mathrm{P}}$ whereas S2 sells on P and faces no competition under PC. Therefore, (124) implies that:

$$p_1^S \;=\; \frac{1}{\beta_1^S\,\beta_1^P\,[\,1-\Omega_1\,]}\left[\alpha_1\left(\theta_1\,\eta_1+\beta_1^P\right)-\frac{\eta_1\,\overline{\Delta}_{\widetilde{P}1}}{2}-\frac{\beta_1^P\,(\,2+\Omega_1\,)^2\,\Delta_{\widetilde{P}1}}{2\,(\,8+\Omega_1\,)}\right]. \tag{129}$$

(126), (128), and (129) imply that $p_1^S > p_1^{SM}$ in this case because $\frac{\widetilde{c}_j^P}{c_j^P} > 1$. (125) implies that:

$$p_2^S \;=\; \frac{4\,A_2-\widetilde{\Delta}_{P2}}{4\,\beta_2^S\,[\,1-\Omega_2\,]}. \tag{130}$$

(126) and (130) imply that:

$$p_2^S \; > \; p_2^{SM} \quad \Leftrightarrow \quad \frac{4\,A_2 - \widetilde{\Delta}_{P2}}{4\,\beta_2^S\,[\,1-\Omega_2\,]} \; > \; \frac{1}{\beta_2^S\,\beta_2^P\,[\,1-\Omega_2\,]}\left[\alpha_2\left(\theta_2\,\eta_2+\beta_2^P\,\right) - \frac{\eta_2\,\overline{\Delta}_{P2}}{2} - \frac{\beta_2^P\,(\,2+\Omega_2\,)^2\,\Delta_{P2}}{2\,(\,8+\Omega_2\,)}\right] \tag{131}$$

$$\Leftrightarrow \quad \frac{7\,\Omega_2 + 2\,[\Omega_2]^2}{[\,8+\Omega_2\,]}\beta_2^P\,\Delta_{P2} + \eta_2\,\overline{\Delta}_{P2} \; > \; 0. \tag{132}$$

<u>Case III</u>. $\frac{\widetilde{\Theta}}{\Theta} \in (\,1, \phi_1\,)$.

Proposition 2 implies that Sj ($j \in \{1,2\}$) sells on $\widetilde{\text{P}}$ and faces no competition under PC. Therefore, (125) implies that:

$$p_j^S \; = \; \frac{4\,A_j - \widetilde{\Delta}_{\widetilde{P}j}}{4\,\beta_j^S\,[\,1-\Omega_j\,]}. \tag{133}$$

(99) and (131) imply that:

$$\frac{4\,A_j - \widetilde{\Delta}_{\widetilde{P}j}}{4\,\beta_j^S\,[\,1-\Omega_j\,]} \; = \; \frac{4\,A_j - \widetilde{\Delta}_{Pj}}{4\,\beta_j^S\,[\,1-\Omega_j\,]} \; > \; \frac{1}{\beta_j^S\,\beta_j^P\,[\,1-\Omega_j\,]}\left[\alpha_j\left(\theta_j\,\eta_j+\beta_j^P\,\right) - \frac{\eta_j\,\overline{\Delta}_{Pj}}{2} - \frac{\beta_j^P\,(\,2+\Omega_j\,)^2\,\Delta_{Pj}}{2\,(\,8+\Omega_j\,)}\right]. \tag{134}$$

(126), (133), and (134) imply that $p_j^S > p_j^{SM}$ in this case.

Next suppose P faces a symmetric competing platform $\widetilde{\text{P}}$ (i.e., $\frac{\widetilde{\Theta}}{\Theta} = 1$) under PC. Proposition 2 implies that Sj ($j \in \{1,2\}$) is indifferent between selling on P and selling on $\widetilde{\text{P}}$ and Sj faces no competition under PC. (125) implies that if Sj sells on P, then:

$$p_j^S \; = \; \frac{4\,A_j - \widetilde{\Delta}_{Pj}}{4\,\beta_j^S\,[\,1-\Omega_j\,]}; \tag{135}$$

if Sj sells on $\widetilde{\text{P}}$, then:

$$p_j^S \; = \; \frac{4\,A_j - \widetilde{\Delta}_{\widetilde{P}j}}{4\,\beta_j^S\,[\,1-\Omega_j\,]}. \tag{136}$$

(126), (134), (135), and (136) imply that $p_j^S > p_j^{SM}$ in this case.

Finally, suppose P faces a competing platform $\widetilde{\text{P}}$ that is a weaker platform (i.e., $\frac{\widetilde{\Theta}}{\Theta} < 1$) under PC.

Case I. $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_1}, 1 \right)$.

In this case, $\frac{\Theta}{\widetilde{\Theta}} \in (\, 1, \phi_1 \,)$. Proposition 2 implies that Sj ($j \in \{1, 2\}$) sells on P and faces no competition under PC. Therefore, (125) implies that p_j^S is given by (135). (126), (134), and (135) imply that $p_j^S > p_j^{SM}$ in this case.

Case II. $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_2}, \frac{1}{\phi_1} \right)$.

In this case, $\frac{\Theta}{\widetilde{\Theta}} \in (\, \phi_1, \phi_2 \,)$. Proposition 2 implies that S1 competes against P whereas S2 sells on $\widetilde{\text{P}}$ and faces no competition under PC. Therefore, (124) implies that:

$$p_1^S \;=\; \frac{1}{\beta_1^S \, \beta_1^P \, [\, 1 - \Omega_1 \,]} \left[\alpha_1 \left(\theta_1 \, \eta_1 + \beta_1^P \right) - \frac{\eta_1 \, \overline{\Delta}_{P1}}{2} - \frac{\beta_1^P \, (\, 2 + \Omega_1 \,)^2 \, \Delta_{P1}}{2 \, (\, 8 + \Omega_1 \,)} \right]. \tag{137}$$

(126) and (137) imply that $p_1^S = p_1^{SM}$ in this case. (125) implies that:

$$p_2^S \;=\; \frac{4 \, A_2 - \widetilde{\Delta}_{\widetilde{P}2}}{4 \, \beta_2^S \, [\, 1 - \Omega_2 \,]}. \tag{138}$$

(126), (134), and (138) imply that $p_2^S > p_2^{SM}$ in this case. ■

Proof of Proposition 7. Let w_j denote the commission Sj faces under PC and w_j^M denote the commission Sj faces under MP ($j \in \{1, 2\}$). Proposition 1 implies that Sj ($j \in \{1, 2\}$) competes against P under MP. Lemma 5 implies that:

$$w_j^M \;=\; \frac{1}{2 \, [\, 1 - \Omega_j \,]} \left\{ \frac{\Omega_j \, \overline{\Delta}_{Pj}}{\eta_j} + \frac{\left[\, 8 + (\Omega_j)^2 \,\right] \, \Delta_{Pj}}{\beta_j^S \, [\, 8 + \Omega_j \,]} \right\}. \tag{139}$$

First suppose P faces a competing platform $\widetilde{\text{P}}$ that is a stronger platform (i.e., $\frac{\widetilde{\Theta}}{\Theta} > 1$) under PC.

Case I. $\frac{\widetilde{\Theta}}{\Theta} > \phi_2$.

Proposition 2 implies that Sj ($j \in \{1, 2\}$) competes against $\widetilde{\text{P}}$ under PC. Lemma 5 implies that:

$$w_j = \frac{1}{2\,[\,1-\Omega_j\,]} \left\{ \frac{\Omega_j\,\overline{\Delta}_{\widetilde{P}j}}{\eta_j} + \frac{\left[\,8+(\Omega_j)^2\,\right]\,\Delta_{\widetilde{P}j}}{\beta_j^S\,[\,8+\Omega_j\,]} \right\}. \tag{140}$$

Observe that:

$$\frac{\partial\left(\frac{\Omega_j\,\overline{\Delta}_{kj}}{\eta_j}+\frac{\left[8+(\Omega_j)^2\right]\Delta_{kj}}{\beta_j^S[8+\Omega_j]}\right)}{\partial c_{kj}^P} = \frac{\Omega_j}{\eta_j}\frac{\partial\overline{\Delta}_{kj}}{\partial c_{kj}^P} + \frac{\left[\,8+(\Omega_j)^2\,\right]}{\beta_j^S\,[\,8+\Omega_j\,]}\frac{\partial\Delta_{kj}}{\partial c_{kj}^P} = -\frac{\Omega_j\,\beta_j^P}{\eta_j} + \frac{\eta_j\left[\,8+(\Omega_j)^2\,\right]}{\beta_j^S\,[\,8+\Omega_j\,]}$$

$$\stackrel{s}{=} \frac{8+[\Omega_j]^2-8-\Omega_j}{\eta_j\,\beta_j^S\,[\,8+\Omega_j\,]} = -\frac{\Omega_j\,[\,1-\Omega_j\,]}{\eta_j\,\beta_j^S\,[\,8+\Omega_j\,]} < 0. \tag{141}$$

The inequality in (141) holds because $\Omega_j \in (0,1)$. (139) - (141) imply that $w_j < w_j^M$ if $\frac{\widetilde{\Theta}}{\Theta} > \phi_2$ because $\frac{\widetilde{c}_j^P}{c_j^P} > 1$.

<u>Case II</u>. $\frac{\widetilde{\Theta}}{\Theta} \in (\,\phi_1, \phi_2\,)$.

Proposition 2 implies that S1 competes against $\widetilde{\mathrm{P}}$ whereas S2 sells on P and faces no competition under PC. Lemmas 2 and 5 imply that:

$$w_1 = \frac{1}{2\,[\,1-\Omega_1\,]} \left\{ \frac{\Omega_1\,\overline{\Delta}_{\widetilde{P}1}}{\eta_1} + \frac{\left[\,8+(\Omega_1)^2\,\right]\,\Delta_{\widetilde{P}1}}{\beta_1^S\,[\,8+\Omega_1\,]} \right\} \text{ and } w_2 = \frac{\widetilde{\Delta}_{P2}}{2\,b_2^S} = \frac{\widetilde{\Delta}_{P2}}{2\,\beta_2^S\,[\,1-\Omega_2\,]}. \tag{142}$$

The last equality in (142) reflects (10). (139), (141), and (142) imply that $w_1 < w_1^M$ if $\frac{\widetilde{\Theta}}{\Theta} \in (\,\phi_1, \phi_2\,)$ because $\frac{\widetilde{c}_1^P}{c_1^P} > 1$. (139) and (142) imply that:

$$w_2 > w_2^M \Leftrightarrow \frac{\widetilde{\Delta}_{P2}}{2\,\beta_2^S\,[\,1-\Omega_2\,]} > \frac{1}{2\,[\,1-\Omega_2\,]} \left\{ \frac{\Omega_2\,\overline{\Delta}_{P2}}{\eta_2} + \frac{\left[\,8+(\Omega_2)^2\,\right]\,\Delta_{P2}}{\beta_2^S\,[\,8+\Omega_2\,]} \right\} \tag{143}$$

$$\Leftrightarrow \frac{\Omega_2-[\Omega_2]^2}{8+\Omega_2}\,\Delta_{P2} + \frac{[\eta_2]^2-[\eta_2]^2}{\beta_2^P\,\eta_2}\,\overline{\Delta}_{P2} > 0 \Leftrightarrow \frac{\Omega_2\,[\,1-\Omega_2\,]}{8+\Omega_2}\,\Delta_{P2} > 0. \tag{144}$$

<u>Case III</u>. $\frac{\widetilde{\Theta}}{\Theta} \in (\,1, \phi_1\,)$.

Proposition 2 implies that Sj ($j \in \{1,2\}$) sells on $\widetilde{\mathrm{P}}$ and faces no competition under PC.

Lemma 2 and (10) imply that:

$$w_j \;=\; \frac{\widetilde{\Delta}_{\widetilde{P}j}}{2\, b_j^S} \;=\; \frac{\widetilde{\Delta}_{\widetilde{P}j}}{2\, \beta_j^S \, [\, 1 - \Omega_j \,]} . \tag{145}$$

(99) and (143) imply that

$$\frac{\widetilde{\Delta}_{\widetilde{P}j}}{2\, \beta_j^S \, [\, 1 - \Omega_j \,]} \;=\; \frac{\widetilde{\Delta}_{Pj}}{2\, \beta_j^S \, [\, 1 - \Omega_j \,]} \;>\; \frac{1}{2\, [\, 1 - \Omega_j \,]} \left\{ \frac{\Omega_j \, \overline{\Delta}_{Pj}}{\eta_j} + \frac{\left[\, 8 + (\Omega_j)^2 \,\right] \, \Delta_{Pj}}{\beta_j^S \, [\, 8 + \Omega_j \,]} \right\} . \tag{146}$$

Therefore, (139), (145), and (146) imply that $w_j > w_j^M$ if $\frac{\widetilde{\Theta}}{\Theta} \in (\, 1, \phi_1 \,)$.

Next suppose P faces a symmetric competing platform $\widetilde{\mathrm{P}}$ (i.e., $\frac{\widetilde{\Theta}}{\Theta} = 1$) under PC. Proposition 2 implies that Sj ($j \in \{1, 2\}$) is indifferent between selling on P and selling on $\widetilde{\mathrm{P}}$ and Sj faces no competition under PC. If Sj sells on P, Lemma 2 and (10) imply that:

$$w_j \;=\; \frac{\widetilde{\Delta}_{Pj}}{2\, b_j^S} \;=\; \frac{\widetilde{\Delta}_{Pj}}{2\, \beta_j^S \, [\, 1 - \Omega_j \,]} . \tag{147}$$

(139), (143), and (147) imply that $w_j > w_j^M$ in this case. If Sj sells on $\widetilde{\mathrm{P}}$, Lemma 2 and (10) imply that w_j is given by (145). (139), (145), and (146) imply that $w_j > w_j^M$ in this case.

Finally, suppose P faces a competing platform $\widetilde{\mathrm{P}}$ that is a weaker platform (i.e., $\frac{\widetilde{\Theta}}{\Theta} < 1$) under PC.

<u>Case I</u>. $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_1}, 1 \right)$.

In this case, $\frac{\Theta}{\widetilde{\Theta}} \in (\, 1, \phi_1 \,)$. Proposition 2 implies that Sj ($j \in \{1, 2\}$) sells on P and faces no competition under PC. Lemma 2 and (10) imply that w_j is given by (147). (139), (143), and (147) imply that $w_j > w_j^M$ if $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_1}, 1 \right)$.

<u>Case II</u>. $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_2}, \frac{1}{\phi_1} \right)$.

In this case, $\frac{\Theta}{\overline{\Theta}} \in (\,\phi_1, \phi_2\,)$. Proposition 2 implies that S1 competes against P whereas S2 sells on $\widetilde{\mathrm{P}}$ and faces no competition under PC. Lemmas 2 and 5 imply that:

$$w_1 \;=\; \frac{1}{2\,[\,1-\Omega_1\,]} \left\{ \frac{\Omega_1\,\overline{\Delta}_{P1}}{\eta_1} + \frac{\left[\,8+(\Omega_1)^2\,\right]\,\Delta_{P1}}{\beta_1^S\,[\,8+\Omega_1\,]} \right\} \;\text{ and }\; w_2 \;=\; \frac{\widetilde{\Delta}_{\widetilde{P}2}}{2\,b_2^S} \;=\; \frac{\widetilde{\Delta}_{\widetilde{P}2}}{2\,\beta_2^S\,[\,1-\Omega_2\,]}. \tag{148}$$

The last equality in (148) reflects (10). (139) and (148) imply that $w_1 = w_1^M$. (145). (139), (146), and (148) imply that $w_2 > w_2^M$. ■

<u>Proof of Proposition 8</u>. (30), (32), and (91) imply that if S1 competes against Pk and S2 competes against Pi ($k, i \in \{1,2\}$), then social welfare is:

$$SW \;=\; \Pi_k + \Pi_i + \pi_1 + \pi_2 + CS$$

$$=\; -\,c_{k1}^P\,\Theta_k\,q_{k1}^{*P} - c_{i2}^P\,\Theta_i\,q_{i2}^{*P} - c_1^S\,\Theta_k\,q_{k1}^{*S} - c_2^S\,\Theta_i\,q_{i2}^{*S} + U\left(Q_{k1}^{*P}, Q_{i2}^{*P}, Q_{k1}^{*S}, Q_{i2}^{*S}\right) - 2\,F, \tag{149}$$

where q_{k1}^{*P}, q_{i2}^{*P}, q_{k1}^{*S}, and q_{i2}^{*S} are given by (94). (149) holds because $Q_{kj}^{*P} = \Theta_k\,q_{kj}^{*P}$ and $Q_{kj}^{*S} = \Theta_k\,q_{kj}^{*S}$. (92) imply that:

$$U\left(Q_{k1}^{*P}, Q_{i2}^{*P}, Q_{k1}^{*S}, Q_{i2}^{*S}\right) \;=\; \frac{\frac{1}{2}\,\eta_1\,\frac{Q_{k1}^{*S}\,Q_{k1}^{*P}}{\Theta_k} + \frac{1}{2}\,\beta_1^S\,\frac{\left[Q_{k1}^{*P}\right]^2}{\Theta_k} - \alpha_1\left[\,\eta_1 + \beta_1^S\,\theta_1\,\right]\,Q_{k1}^{*P}}{[\eta_1]^2 - \beta_1^S\,\beta_1^P}$$

$$+\;\frac{\frac{1}{2}\,\eta_2\,\frac{Q_{i2}^{*S}\,Q_{i2}^{*P}}{\Theta_i} + \frac{1}{2}\,\beta_2^S\,\frac{\left[Q_{i2}^{*P}\right]^2}{\Theta_i} - \alpha_2\left[\,\eta_2 + \beta_2^S\,\theta_2\,\right]\,Q_{i2}^{*P}}{[\eta_2]^2 - \beta_2^S\,\beta_2^P}$$

$$+\;\frac{\frac{1}{2}\,\eta_1\,\frac{Q_{k1}^{*S}\,Q_{k1}^{*P}}{\Theta_k} + \frac{1}{2}\,\beta_1^P\,\frac{\left[Q_{k1}^{*S}\right]^2}{\Theta_k} - \alpha_1\left[\,\theta_1\,\eta_1 + \beta_1^P\,\right]\,Q_{k1}^{*S}}{[\eta_1]^2 - \beta_1^S\,\beta_1^P}$$

$$+\;\frac{\frac{1}{2}\,\eta_2\,\frac{Q_{i2}^{*P}\,Q_{i2}^{*S}}{\Theta_i} + \frac{1}{2}\,\beta_2^P\,\frac{\left[Q_{i2}^{*S}\right]^2}{\Theta_i} - \alpha_2\left[\,\theta_2\,\eta_2 + \beta_2^P\,\right]\,Q_{i2}^{*S}}{[\eta_2]^2 - \beta_2^S\,\beta_2^P}$$

$$=\;\Theta_k\,\frac{\alpha_1\left[\,\eta_1 + \beta_1^S\,\theta_1\,\right]\,q_{k1}^{*P} + \alpha_1\left[\,\theta_1\,\eta_1 + \beta_1^P\,\right]\,q_{k1}^{*S} - \eta_1\,q_{k1}^{*P}\,q_{k1}^{*S} - \frac{1}{2}\,\beta_1^S\left[q_{k1}^{*P}\right]^2 - \frac{1}{2}\,\beta_1^P\left[q_{k1}^{*S}\right]^2}{\beta_1^S\,\beta_1^P\,[\,1-\Omega_1\,]}$$

$$+\ \Theta_i\,\frac{\alpha_2\left[\,\eta_2+\beta_2^S\,\theta_2\,\right]\,q_{i2}^{*P}+\alpha_2\left[\,\theta_2\,\eta_2+\beta_2^P\,\right]\,q_{i2}^{*S}-\eta_2\,q_{i2}^{*P}\,q_{i2}^{*S}-\frac{1}{2}\,\beta_2^S\left[q_{i2}^{*P}\right]^2-\frac{1}{2}\,\beta_2^P\left[q_{i2}^{*S}\right]^2}{\beta_2^S\,\beta_2^P\left[\,1-\Omega_2\,\right]}. \tag{150}$$

(149) and (150) imply that if S1 competes against Pk and S2 competes against Pi ($k,i\in\{1,2\}$), then social welfare is:

$$\begin{aligned}
SW\ =\ & -c_{k1}^P\,\Theta_k\,q_{k1}^{*P}-c_{i2}^P\,\Theta_i\,q_{i2}^{*P}-c_1^S\,\Theta_k\,q_{k1}^{*S}-c_2^S\,\Theta_i\,q_{i2}^{*S}-2\,F \\
& +\ \Theta_k\,\frac{\alpha_1\left[\,\eta_1+\beta_1^S\,\theta_1\,\right]\,q_{k1}^{*P}+\alpha_1\left[\,\theta_1\,\eta_1+\beta_1^P\,\right]\,q_{k1}^{*S}-\eta_1\,q_{k1}^{*P}\,q_{k1}^{*S}-\frac{1}{2}\,\beta_1^S\left[q_{k1}^{*P}\right]^2-\frac{1}{2}\,\beta_1^P\left[q_{k1}^{*S}\right]^2}{\beta_1^S\,\beta_1^P\left[\,1-\Omega_1\,\right]} \\
& +\ \Theta_i\,\frac{\alpha_2\left[\,\eta_2+\beta_2^S\,\theta_2\,\right]\,q_{i2}^{*P}+\alpha_2\left[\,\theta_2\,\eta_2+\beta_2^P\,\right]\,q_{i2}^{*S}-\eta_2\,q_{i2}^{*P}\,q_{i2}^{*S}-\frac{1}{2}\,\beta_2^S\left[q_{i2}^{*P}\right]^2-\frac{1}{2}\,\beta_2^P\left[q_{i2}^{*S}\right]^2}{\beta_2^S\,\beta_2^P\left[\,1-\Omega_2\,\right]} \\
=\ & \Theta_k\,q_{k1}^{*P}\left[\frac{\alpha_1\left(\,\eta_1+\beta_1^S\,\theta_1\,\right)}{\beta_1^S\,\beta_1^P\,(\,1-\Omega_1\,)}-c_{k1}^P\right]+\Theta_i\,q_{i2}^{*P}\left[\frac{\alpha_2\left(\,\eta_2+\beta_2^S\,\theta_2\,\right)}{\beta_2^S\,\beta_2^P\,(\,1-\Omega_2\,)}-c_{i2}^P\right] \\
& +\ \Theta_k\,q_{k1}^{*S}\left[\frac{\alpha_1\left(\,\theta_1\,\eta_1+\beta_1^P\,\right)}{\beta_1^S\,\beta_1^P\,(\,1-\Omega_1\,)}-c_1^S\right]+\Theta_i\,q_{i2}^{*S}\left[\frac{\alpha_2\left(\,\theta_2\,\eta_2+\beta_2^P\,\right)}{\beta_2^S\,\beta_2^P\,(\,1-\Omega_2\,)}-c_2^S\right]-2\,F \\
& -\ \Theta_k\,\frac{\eta_1\,q_{k1}^{*P}\,q_{k1}^{*S}+\frac{1}{2}\,\beta_1^S\left[q_{k1}^{*P}\right]^2+\frac{1}{2}\,\beta_1^P\left[q_{k1}^{*S}\right]^2}{\beta_1^S\,\beta_1^P\left[\,1-\Omega_1\,\right]}-\Theta_i\,\frac{\eta_2\,q_{i2}^{*P}\,q_{i2}^{*S}+\frac{1}{2}\,\beta_2^S\left[q_{i2}^{*P}\right]^2+\frac{1}{2}\,\beta_2^P\left[q_{i2}^{*S}\right]^2}{\beta_2^S\,\beta_2^P\left[\,1-\Omega_2\,\right]}.
\end{aligned} \tag{151}$$

Observe that:

$$\begin{aligned}
\frac{\alpha_j\left[\,\eta_j+\beta_j^S\,\theta_j\,\right]}{\beta_j^S\,\beta_j^P\left[\,1-\Omega_j\,\right]}-c_{kj}^P\ &=\ \frac{\alpha_j\left[\,\eta_j+\beta_j^S\,\theta_j\,\right]-\beta_j^S\,\beta_j^P\left[\,1-\Omega_j\,\right]c_{kj}^P}{\beta_j^S\,\beta_j^P\left[\,1-\Omega_j\,\right]} \\
&=\ \frac{\eta_j\left[\,\alpha_j+\eta_j\,c_{kj}^P-\beta_j^S\,c_j^S\,\right]+\beta_j^S\left[\,\theta_j\,\alpha_j-\beta_j^P\,c_{kj}^P+\eta_j\,c_j^S\,\right]}{\beta_j^S\,\beta_j^P\left[\,1-\Omega_j\,\right]}\ =\ \frac{\eta_j\,\Delta_{kj}+\beta_j^S\,\overline{\Delta}_{kj}}{\beta_j^S\,\beta_j^P\left[\,1-\Omega_j\,\right]};
\end{aligned} \tag{152}$$

$$\frac{\alpha_j\left[\,\theta_j\,\eta_j+\beta_j^P\,\right]}{\beta_j^S\,\beta_j^P\left[\,1-\Omega_j\,\right]}-c_j^S\ =\ \frac{\alpha_j\left[\,\theta_j\,\eta_j+\beta_j^P\,\right]-\beta_j^S\,\beta_j^P\left[\,1-\Omega_j\,\right]c_j^S}{\beta_j^S\,\beta_j^P\left[\,1-\Omega_j\,\right]}\ =\ \frac{\beta_j^P\,A_j-\beta_j^P\,b_j^S\,c_j^S}{\beta_j^S\,\beta_j^P\left[\,1-\Omega_j\,\right]} \tag{153}$$

$$=\ \frac{\widetilde{\Delta}_{kj}}{\beta_j^S\left[\,1-\Omega_j\,\right]}\ =\ \frac{\Delta_{kj}+\frac{\eta_j}{\beta_j^P}\,\overline{\Delta}_{kj}}{\beta_j^S\left[\,1-\Omega_j\,\right]}\ =\ \frac{\beta_j^P\,\Delta_{kj}+\eta_j\,\overline{\Delta}_{kj}}{\beta_j^S\,\beta_j^P\left[\,1-\Omega_j\,\right]}. \tag{154}$$

The last equality in (153) reflects (7) and (10). The first equality in (154) holds because

$\widetilde{\Delta}_{kj} \equiv A_j - b_j^S c_j^S$. The second equality in (154) reflects (12).

(151), (152), and (153) imply that if S1 competes against Pk and S2 competes against Pi ($k, i \in \{1, 2\}$), then social welfare is:

$$\begin{aligned} SW \;=\; & \Theta_k\, q_{k1}^{*P}\, \frac{\eta_1\, \Delta_{k1} + \beta_1^S\, \overline{\Delta}_{k1}}{\beta_1^S\, \beta_1^P\, [\, 1 - \Omega_1\,]} + \Theta_i\, q_{i2}^{*P}\, \frac{\eta_2\, \Delta_{i2} + \beta_2^S\, \overline{\Delta}_{i2}}{\beta_2^S\, \beta_2^P\, [\, 1 - \Omega_2\,]} - 2\, F \\ & + \Theta_k\, q_{k1}^{*S}\, \frac{\beta_1^P\, \Delta_{k1} + \eta_1\, \overline{\Delta}_{k1}}{\beta_1^S\, \beta_1^P\, [\, 1 - \Omega_1\,]} + \Theta_i\, q_{i2}^{*S}\, \frac{\beta_2^P\, \Delta_{i2} + \eta_2\, \overline{\Delta}_{i2}}{\beta_2^S\, \beta_2^P\, [\, 1 - \Omega_2\,]} \\ & - \Theta_k\, \frac{\eta_1\, q_{k1}^{*P}\, q_{k1}^{*S} + \frac{1}{2}\, \beta_1^S\, \left[q_{k1}^{*P} \right]^2 + \frac{1}{2}\, \beta_1^P\, \left[q_{k1}^{*S} \right]^2}{\beta_1^S\, \beta_1^P\, [\, 1 - \Omega_1\,]} - \Theta_i\, \frac{\eta_2\, q_{i2}^{*P}\, q_{i2}^{*S} + \frac{1}{2}\, \beta_2^S\, \left[q_{i2}^{*P} \right]^2 + \frac{1}{2}\, \beta_2^P\, \left[q_{i2}^{*S} \right]^2}{\beta_2^S\, \beta_2^P\, [\, 1 - \Omega_2\,]} \end{aligned} \tag{155}$$

where q_{k1}^{*P}, q_{i2}^{*P}, q_{k1}^{*S}, and q_{i2}^{*S} are given by (94).

(94), (96), and (155) imply that if S1 competes against Pk and S2 competes against Pi ($k, i \in \{1, 2\}$), then social welfare is:

$$SW \;=\; \Theta_k\, \kappa_{k1} + \Theta_i\, \kappa_{i2} - 2\, F, \tag{156}$$

where

$$\kappa_{kj} \;\equiv\; \left[\frac{\overline{\Delta}_{kj}}{2} + \frac{\eta_j\, (\, 2 + \Omega_j\,)\, \Delta_{kj}}{2\beta_j^S\, (\, 8 + \Omega_j\,)} \right] \frac{\eta_j\, \Delta_{kj} + \beta_j^S\, \overline{\Delta}_{kj}}{\beta_j^S\, \beta_j^P\, [\, 1 - \Omega_j\,]} + \frac{[\, 2 + \Omega_j\,]\, \Delta_{kj}}{8 + \Omega_j} \frac{\beta_j^P\, \Delta_{kj} + \eta_j\, \overline{\Delta}_{kj}}{\beta_j^S\, \beta_j^P\, [\, 1 - \Omega_j\,]} - \varsigma_{kj}, \tag{157}$$

and ς_{kj} is given by (96).

(96) and (157) imply that:

$$\begin{aligned} \kappa_{kj} \;=\; & \left[\frac{\overline{\Delta}_{kj}}{2} + \frac{\eta_j\, (\, 2 + \Omega_j\,)\, \Delta_{kj}}{2\beta_j^S\, (\, 8 + \Omega_j\,)} \right] \frac{\eta_j\, \Delta_{kj} + \beta_j^S\, \overline{\Delta}_{kj}}{\beta_j^S\, \beta_j^P\, [\, 1 - \Omega_j\,]} + \frac{[\, 2 + \Omega_j\,]\, \Delta_{kj}}{8 + \Omega_j} \frac{\beta_j^P\, \Delta_{kj} + \eta_j\, \overline{\Delta}_{kj}}{\beta_j^S\, \beta_j^P\, [\, 1 - \Omega_j\,]} \\ & - \frac{1}{\beta_j^S\, \beta_j^P\, [\, 1 - \Omega_j\,]} \left\{ \frac{\eta_j\, [\, 2 + \Omega_j\,]\, \Delta_{kj}}{8 + \Omega_j} \left[\frac{\overline{\Delta}_{kj}}{2} + \frac{\eta_j\, (\, 2 + \Omega_j\,)\, \Delta_{kj}}{2\beta_j^S\, (\, 8 + \Omega_j\,)} \right] \right. \end{aligned}$$

$$+\ \frac{\beta_j^S}{2}\left[\frac{\overline{\Delta}_{kj}}{2}+\frac{\eta_j\,(\,2+\Omega_j\,)\ \Delta_{kj}}{2\beta_j^S\,(\,8+\Omega_j\,)}\right]^2+\frac{\beta_j^P}{2}\left[\frac{(\,2+\Omega_j\,)\ \Delta_{kj}}{8+\Omega_j}\right]^2\Bigg\}$$

$$= \frac{1}{\beta_j^S\,\beta_j^P\,[\,1-\Omega_j\,]}\left[\frac{\overline{\Delta}_{kj}}{2}+\frac{\eta_j\,(\,2+\Omega_j\,)\ \Delta_{kj}}{2\beta_j^S\,(\,8+\Omega_j\,)}\right]\left[\eta_j\,\Delta_{kj}+\beta_j^S\,\overline{\Delta}_{kj}-\frac{\eta_j\,(\,2+\Omega_j\,)\ \Delta_{kj}}{8+\Omega_j}\right]$$

$$+\frac{[\,2+\Omega_j\,]\ \Delta_{kj}}{8+\Omega_j}\,\frac{\beta_j^P\,\Delta_{kj}+\eta_j\,\overline{\Delta}_{kj}}{\beta_j^S\,\beta_j^P\,[\,1-\Omega_j\,]}$$

$$-\ \frac{1}{2\,\beta_j^P\,[\,1-\Omega_j\,]}\left[\frac{\overline{\Delta}_{kj}}{2}+\frac{\eta_j\,(\,2+\Omega_j\,)\ \Delta_{kj}}{2\beta_j^S\,(\,8+\Omega_j\,)}\right]^2-\frac{1}{2\,\beta_j^S\,[\,1-\Omega_j\,]}\left[\frac{(\,2+\Omega_j\,)\ \Delta_{kj}}{8+\Omega_j}\right]^2 \tag{158}$$

Observe that:

$$\frac{1}{\beta_j^S\,\beta_j^P\,[\,1-\Omega_j\,]}\left[\frac{\overline{\Delta}_{kj}}{2}+\frac{\eta_j\,(\,2+\Omega_j\,)\ \Delta_{kj}}{2\,\beta_j^S\,(\,8+\Omega_j\,)}\right]\left[\eta_j\,\Delta_{kj}+\beta_j^S\,\overline{\Delta}_{kj}-\frac{\eta_j\,(\,2+\Omega_j\,)\ \Delta_{kj}}{8+\Omega_j}\right]$$

$$= \frac{1}{\beta_j^S\,\beta_j^P\,[\,1-\Omega_j\,]}\left[\frac{\overline{\Delta}_{kj}}{2}+\frac{\eta_j\,(\,2+\Omega_j\,)\ \Delta_{kj}}{2\,\beta_j^S\,(\,8+\Omega_j\,)}\right]\left[\eta_j\,\Delta_{kj}\left(1-\frac{2+\Omega_j}{8+\Omega_j}\right)+\beta_j^S\,\overline{\Delta}_{kj}\right]$$

$$= \frac{\eta_j\,\Delta_{kj}\,\overline{\Delta}_{kj}}{2\,\beta_j^S\,\beta_j^P\,[\,1-\Omega_j\,]}+\frac{\left[\overline{\Delta}_{kj}\right]^2}{2\,\beta_j^P\,[\,1-\Omega_j\,]}+\frac{3\,\Omega_j\,[\,2+\Omega_j\,]\ [\Delta_{kj}]^2}{\beta_j^S\,[\,1-\Omega_j\,]\,[\,8+\Omega_j\,]^2}; \tag{159}$$

$$\frac{[\,2+\Omega_j\,]\ \Delta_{kj}}{8+\Omega_j}\,\frac{\beta_j^P\,\Delta_{kj}+\eta_j\,\overline{\Delta}_{kj}}{\beta_j^S\,\beta_j^P\,[\,1-\Omega_j\,]} = \frac{[\,2+\Omega_j\,]\ \Delta_{kj}}{8+\Omega_j}\,\frac{\beta_j^P\,\Delta_{kj}}{\beta_j^S\,\beta_j^P\,[\,1-\Omega_j\,]}+\frac{[\,2+\Omega_j\,]\ \Delta_{kj}}{8+\Omega_j}\,\frac{\eta_j\,\overline{\Delta}_{kj}}{\beta_j^S\,\beta_j^P\,[\,1-\Omega_j\,]}$$

$$= -\ \frac{\left[\overline{\Delta}_{kj}\right]^2}{8\,\beta_j^P\,[\,1-\Omega_j\,]}-\frac{[\,2+\Omega_j\,]^2\,[\,4+\Omega_j\,]\ [\Delta_{kj}]^2}{8\,\beta_j^S\,[\,1-\Omega_j\,]\,[\,8+\Omega_j\,]^2}-\frac{\eta_j\,[\,2+\Omega_j\,]\ \Delta_{kj}\,\overline{\Delta}_{kj}}{4\,\beta_j^P\,[\,1-\Omega_j\,]\,\beta_j^S\,[\,8+\Omega_j\,]}. \tag{160}$$

(158) - (160) imply that:

$$\kappa_{kj} = \frac{\eta_j\,\Delta_{kj}\,\overline{\Delta}_{kj}}{2\,\beta_j^S\,\beta_j^P\,[\,1-\Omega_j\,]}+\frac{\left[\overline{\Delta}_{kj}\right]^2}{2\,\beta_j^P\,[\,1-\Omega_j\,]}+\frac{3\,\Omega_j\,[\,2+\Omega_j\,]\ [\Delta_{kj}]^2}{\beta_j^S\,[\,1-\Omega_j\,]\,[\,8+\Omega_j\,]^2}$$

$$+\ \frac{[\,2+\Omega_j\,]\ [\Delta_{kj}]^2}{\beta_j^S\,[\,1-\Omega_j\,]\,[\,8+\Omega_j\,]}+\frac{\eta_j\,[\,2+\Omega_j\,]\ \Delta_{kj}\,\overline{\Delta}_{kj}}{\beta_j^S\,\beta_j^P\,[\,1-\Omega_j\,]\,[\,8+\Omega_j\,]}$$

$$-\ \frac{\left[\overline{\Delta}_{kj}\right]^2}{8\,\beta_j^P\,[\,1-\Omega_j\,]}-\frac{[\,2+\Omega_j\,]^2\,[\,4+\Omega_j\,]\ [\Delta_{kj}]^2}{8\,\beta_j^S\,[\,1-\Omega_j\,]\,[\,8+\Omega_j\,]^2}-\frac{\eta_j\,[\,2+\Omega_j\,]\ \Delta_{kj}\,\overline{\Delta}_{kj}}{4\,\beta_j^P\,[\,1-\Omega_j\,]\,\beta_j^S\,[\,8+\Omega_j\,]}$$

$$= \frac{3\left[\overline{\Delta}_{kj}\right]^2}{8\,\beta_j^P\,[\,1-\Omega_j\,]} + \frac{[\,2+\Omega_j\,]\,[\Delta_{kj}]^2}{\beta_j^S\,[\,1-\Omega_j\,]\,[\,8+\Omega_j\,]}\,\frac{[\,2+\Omega_j\,]\,[\,28-\Omega_j\,]}{8\,[\,8+\Omega_j\,]} + \frac{\eta_j\,[\,22+5\,\Omega_j\,]\,\Delta_{kj}\,\overline{\Delta}_{kj}}{4\,\beta_j^S\,\beta_j^P\,[\,8+\Omega_j\,]\,[\,1-\Omega_j\,]}$$

$$= \frac{3\left[\overline{\Delta}_{kj}\right]^2}{8\,\beta_j^P\,[\,1-\Omega_j\,]} + \frac{[\,2+\Omega_j\,]^2\,[\,28-\Omega_j\,]\,[\Delta_{kj}]^2}{8\,\beta_j^S\,[\,1-\Omega_j\,]\,[\,8+\Omega_j\,]^2} + \frac{\eta_j\,[\,22+5\,\Omega_j\,]\,\Delta_{kj}\,\overline{\Delta}_{kj}}{4\,\beta_j^S\,\beta_j^P\,[\,8+\Omega_j\,]\,[\,1-\Omega_j\,]}. \tag{161}$$

(161) implies that:

$$\frac{\partial\kappa_{kj}}{\partial c_{kj}^P} = \frac{1}{4\,[\,1-\Omega_j\,]}\left[\frac{6\,\overline{\Delta}_{kj}}{2\,\beta_j^P}\,\frac{\partial\overline{\Delta}_{kj}}{\partial c_{kj}^P} + \frac{(\,2+\Omega_j\,)^2\,(\,28-\Omega_j\,)\;2\,\Delta_{kj}}{2\,\beta_j^S\,(\,8+\Omega_j\,)^2}\,\frac{\partial\Delta_{kj}}{\partial c_{kj}^P} + \frac{\eta_j\,(\,22+5\,\Omega_j\,)}{\beta_j^S\,\beta_j^P\,(\,8+\Omega_j\,)}\,\frac{\partial\left(\Delta_{kj}\,\overline{\Delta}_{kj}\right)}{\partial c_{kj}^P}\right]$$

$$= \overline{\Delta}_{kj}\left[\frac{\Omega_j\,(\,22+5\,\Omega_j\,)}{8+\Omega_j} - 3\right] + \frac{\eta_j\,\Delta_{kj}}{\beta_j^S\,[\,8+\Omega_j\,]}\left[\frac{(\,2+\Omega_j\,)^2\,(\,28-\Omega_j\,)}{8+\Omega_j} - (\,22+5\,\Omega_j\,)\right] \;<\; 0. \tag{162}$$

The inequality in (162) holds because

$$\frac{\Omega_j\,[\,22+5\,\Omega_j\,]}{8+\Omega_j} - 3 \;<\; 0 \;\;\Leftrightarrow\;\; 22\,\Omega_j + 5\,[\Omega_j]^2 - 24 - 3\,\Omega_j \;<\; 0 \;\;\Leftrightarrow\;\; 19\,\Omega_j + 5\,[\Omega_j]^2 - 24 \;<\; 0$$

$$\Leftrightarrow\;\; [\,5\,\Omega_j + 24\,]\,[\,\Omega_j - 1\,] \;<\; 0, \text{ and} \tag{163}$$

$$\frac{[\,2+\Omega_j\,]^2\,[\,28-\Omega_j\,]}{8+\Omega_j} - [\,22+5\,\Omega_j\,] \;<\; 0 \;\;\Leftrightarrow\;\; [\,2+\Omega_j\,]^2\,[\,28-\Omega_j\,] - [\,8+\Omega_j\,]\,[\,22+5\,\Omega_j\,] \;<\; 0. \tag{164}$$

(163) holds because $\Omega_j \in (0,1)$. (164) holds because $[\,2+\Omega_j\,]^2\,[\,28-\Omega_j\,] - [\,8+\Omega_j\,]\,[\,22+5\,\Omega_j\,]$ increases in $\Omega_j \in (0,1)$, and thus, $[\,2+\Omega_j\,]^2\,[\,28-\Omega_j\,] - [\,8+\Omega_j\,]\,[\,22+5\,\Omega_j\,] < \max\,[\,2+\Omega_j\,]^2\,[\,28-\Omega_j\,] - [\,8+\Omega_j\,]\,[\,22+5\,\Omega_j\,] = [\,2+1\,]^2\,[\,28-1\,] - [\,8+1\,]\,[\,22+5*1\,] = 0$.

Because $\frac{\widetilde{c}_j^P}{c_j^P} > 1$, (162) implies that:

$$\frac{\kappa_{Pj}}{\kappa_{\widetilde{P}j}} \;>\; 1. \tag{165}$$

(12) and (17) imply that:

$$
\begin{aligned}
\kappa_{kj} &> \frac{7\left[\widetilde{\Delta}_{kj}\right]^2}{32\,\beta_j^S\,[\,1-\Omega_j\,]} \\
\Leftrightarrow \;& \frac{3\left[\overline{\Delta}_{kj}\right]^2}{8\,\beta_j^P\,[\,1-\Omega_j\,]} + \frac{[\,2+\Omega_j\,]^2\,[\,28-\Omega_j\,]\,[\Delta_{kj}]^2}{8\,\beta_j^S\,[\,1-\Omega_j\,]\,[\,8+\Omega_j\,]^2} + \frac{\eta_j\,[\,22+5\,\Omega_j\,]\,\Delta_{kj}\,\overline{\Delta}_{kj}}{4\,\beta_j^S\,\beta_j^P\,[\,8+\Omega_j\,]\,[\,1-\Omega_j\,]} \\
&> \frac{7\left[\,\Delta_{kj}+\frac{\eta_j}{\beta_j^P}\,\overline{\Delta}_{kj}\,\right]^2}{32\,\beta_j^S\,[\,1-\Omega_j\,]} \\
\Leftrightarrow \;& \frac{[\,12-7\Omega_j\,]\left[\overline{\Delta}_{kj}\right]^2}{8\,\beta_j^P} + \frac{4\,[\,2+\Omega_j\,]^2\,[\,28-\Omega_j\,]-7\,[\,8+\Omega_j\,]^2}{8\,\beta_j^S\,[\,8+\Omega_j\,]^2}\,[\Delta_{kj}]^2 \\
&+ \frac{4\,\eta_j\,[\,22+5\,\Omega_j\,]-7\,\eta_j\,[\,8+\Omega_j\,]}{4\,\beta_j^S\,\beta_j^P\,[\,8+\Omega_j\,]}\,\Delta_{kj}\,\overline{\Delta}_{kj} \;>\; 0. \qquad (166)
\end{aligned}
$$

(166) holds because for $\Omega_j \in (0,1)$

$$
\frac{4\,[\,2+\Omega_j\,]^2\,[\,28-\Omega_j\,]-7\,[\,8+\Omega_j\,]^2}{8\,\beta_j^S\,[\,8+\Omega_j\,]^2} > 0 \;\Leftrightarrow\; 4\,[\,2+\Omega_j\,]^2\,[\,28-\Omega_j\,] > 7\,[\,8+\Omega_j\,]^2
$$

$$
\Leftrightarrow\; 89\,[\Omega_j]^2-4\,[\Omega_j]^3+320\,\Omega_j > 0 \;\Leftrightarrow\; 89\,\Omega_j-4\,[\Omega_j]^2+320 > 0, \text{ and}
$$

$$
\frac{4\,\eta_j\,[\,22+5\,\Omega_j\,]-7\,\eta_j\,[\,8+\Omega_j\,]}{4\,\beta_j^S\,\beta_j^P\,[\,8+\Omega_j\,]} > 0 \;\Leftrightarrow\; 4\,\eta_j\,[\,22+5\,\Omega_j\,] > 7\,\eta_j\,[\,8+\Omega_j\,]
$$

$$
\Leftrightarrow\; 4\,[\,22+5\,\Omega_j\,] > 7\,[\,8+\Omega_j\,] \Leftrightarrow\; 88+20\,\Omega_j > 56+7\,\Omega_j \;\Leftrightarrow\; 32+13\,\Omega_j > 0\,.
$$

(20), (25), and (104) imply that if S1 sells on Pk, S2 sells on Pi ($k,i\in\{1,2\}$), and each seller faces no competition, then social welfare is:

$$
\begin{aligned}
SW &= \Pi_k+\Pi_i+\pi_1+\pi_2+CS \\
&= w_{k1}^*\,\Theta_k\,q_{k1}^{*S}+w_{i2}^*\,\Theta_i\,q_{i2}^{*S}+\left[\,p_{k1}^{*S}-w_{k1}^*-c_1^S\,\right]\Theta_k\,q_{k1}^{*S}+\left[\,p_{i2}^{*S}-w_{i2}^*-c_2^S\,\right]\Theta_i\,q_{i2}^{*S} \\
&\quad + U\left(Q_{k1}^{*S},Q_{i2}^{*S}\right)-p_{k1}^{*S}\,Q_{k1}^{*S}-p_{i2}^{*S}\,Q_{i2}^{*S}
\end{aligned}
$$

$$= \; -\, c_1^S\, \Theta_k\, q_{k1}^{*S} - c_2^S\, \Theta_i\, q_{i2}^{*S} + U\left(Q_{k1}^{*S}, Q_{i2}^{*S}\right), \tag{167}$$

where q_{k1}^{*S} and q_{i2}^{*S} are given by (106). (104) and (**??**) imply that:

$$\begin{aligned} U\left(Q_{k1}^{*S}, Q_{i2}^{*S}\right) &= \frac{A_1}{b_1^S}\, Q_{k1}^{*S} + \frac{A_2}{b_2^S}\, Q_{i2}^{*S} - \frac{1}{2}\left[\frac{\left(Q_{k1}^{*S}\right)^2}{b_1^S\, \Theta_k} + \frac{\left(Q_{i2}^{*S}\right)^2}{b_2^S\, \Theta_i}\right] \\ &= \frac{A_1}{b_1^S}\, \Theta_k\, q_{k1}^{*S} + \frac{A_2}{b_2^S}\, \Theta_i\, q_{i2}^{*S} - \frac{1}{2}\, \frac{\Theta_k \left(q_{k1}^{*S}\right)^2}{b_1^S} - \frac{1}{2}\, \frac{\Theta_i \left(q_{i2}^{*S}\right)^2}{b_2^S}, \end{aligned} \tag{168}$$

where q_{k1}^{*S} and q_{i2}^{*S} are given by (106). (167) and (168) imply that if S1 sells on Pk, S2 sells on Pi ($k, i \in \{1, 2\}$), and each seller faces no competition, then social welfare is:

$$\begin{aligned} SW &= \; -\, c_1^S\, \Theta_k\, q_{k1}^{*S} - c_2^S\, \Theta_i\, q_{i2}^{*S} + \frac{A_1}{b_1^S}\, \Theta_k\, q_{k1}^{*S} + \frac{A_2}{b_2^S}\, \Theta_i\, q_{i2}^{*S} - \frac{1}{2}\, \frac{\Theta_k \left[q_{k1}^{*S}\right]^2}{b_1^S} - \frac{1}{2}\, \frac{\Theta_i \left[q_{i2}^{*S}\right]^2}{b_2^S} \\ &= \Theta_k\, q_{k1}^{*S}\left[\frac{A_1}{b_1^S} - c_1^S\right] + \Theta_i\, q_{i2}^{*S}\left[\frac{A_2}{b_2^S} - c_2^S\right] - \frac{\Theta_k \left[q_{k1}^{*S}\right]^2}{2\, b_1^S} - \frac{\Theta_i \left[q_{i2}^{*S}\right]^2}{2\, b_2^S} \\ &= \frac{\Theta_k\, \widetilde{\Delta}_{k1}\, 7\, \widetilde{\Delta}_{k1}}{32\, \beta_1^S\, [\, 1 - \Omega_1\,]} + \frac{\Theta_i\, \widetilde{\Delta}_{i2}\, 7\, \widetilde{\Delta}_{i2}}{32\, \beta_2^S\, [\, 1 - \Omega_2\,]} \; = \; \frac{7\, \Theta_k \left[\widetilde{\Delta}_{k1}\right]^2}{32\, \beta_1^S\, [\, 1 - \Omega_1\,]} + \frac{7\, \Theta_i \left[\widetilde{\Delta}_{i2}\right]^2}{32\, \beta_2^S\, [\, 1 - \Omega_2\,]}. \end{aligned} \tag{169}$$

(20), (25), (30), (32), and (108) imply that if S1 competes against Pk and S2 sells on Pi and faces no competition ($k, i \in \{1, 2\}$), then social welfare is:

$$\begin{aligned} SW &= \Pi_k + \Pi_i + \pi_1 + \pi_2 + CS \\ &= \left[\, p_{k1}^{*P} - c_{k1}^P\, \right] \Theta_k\, q_{k1}^{*P} - F + w_{k1}^*\, \Theta_k\, q_{k1}^{*S} + w_{i2}^*\, \Theta_i\, q_{i2}^{*S} + \left[\, p_{k1}^{*S} - w_{k1}^* - c_1^S\, \right] \Theta_k\, q_{k1}^{*S} \\ &\quad + \left[\, p_{i2}^{*S} - w_{i2}^* - c_2^S\, \right] \Theta_i\, q_{i2}^{*S} + U\left(Q_{k1}^{*P}, Q_{k1}^{*S}, Q_{i2}^{*S}\right) - p_{k1}^{*P}\, Q_{k1}^{*P} - p_{k1}^{*S}\, Q_{k1}^{*S} - p_{i2}^{*S}\, Q_{i2}^{*S} \\ &= \; -\, c_{k1}^P\, \Theta_k\, q_{k1}^{*P} - c_1^S\, \Theta_k\, q_{k1}^{*S} - \Theta_i\, c_2^S\, q_{i2}^{*S} + U\left(Q_{k1}^{*P}, Q_{k1}^{*S}, Q_{i2}^{*S}\right) - F, \end{aligned} \tag{170}$$

where q_{k1}^{*P} and q_{k1}^{*S} are given by (94) and q_{i2}^{*S} is given by (106). (108) and (109) imply that:

$$U\left(Q_{k1}^{*P}, Q_{k1}^{*S}, Q_{i2}^{*S}\right) = \frac{\frac{1}{2}\,\eta_1\,\frac{Q_{k1}^{*S}\,Q_{k1}^{*P}}{\Theta_k} + \frac{1}{2}\,\beta_1^S\,\frac{\left[Q_{k1}^{*P}\right]^2}{\Theta_k} - \alpha_1\left[\,\eta_1 + \beta_1^S\,\theta_1\,\right]\,Q_{k1}^{*P}}{\left[\eta_1\right]^2 - \beta_1^S\,\beta_1^P}$$

$$+ \frac{\frac{1}{2}\,\eta_1\,\frac{Q_{k1}^{*S}\,Q_{k1}^{*P}}{\Theta_k} + \frac{1}{2}\,\beta_1^P\,\frac{\left[Q_{k1}^{*S}\right]^2}{\Theta_k} - \alpha_1\left[\,\theta_1\,\eta_1 + \beta_1^P\,\right]\,Q_{k1}^{*S}}{\left[\eta_1\right]^2 - \beta_1^S\,\beta_1^P} + \frac{A_2}{b_2^S}\,Q_{i2}^{*S} - \frac{\left[Q_{i2}^{*S}\right]^2}{2\,b_2^S\,\Theta_i}$$

$$= \frac{-\eta_1\,\Theta_k\,q_{k1}^{*P}\,q_{k1}^{*S} - \frac{\beta_1^S\,\Theta_k}{2}\left[q_{k1}^{*P}\right]^2 - \frac{\beta_1^P\,\Theta_k}{2}\left[q_{k1}^{*S}\right]^2 + \alpha_1\left[\,\eta_1 + \beta_1^S\,\theta_1\,\right]\Theta_k\,q_{k1}^{*P} + \alpha_1\left[\,\theta_1\,\eta_1 + \beta_1^P\,\right]\Theta_k\,q_{k1}^{*S}}{\beta_1^S\,\beta_1^P\left[\,1 - \Omega_1\,\right]}$$

$$+ \frac{A_2}{b_2^S}\,\Theta_i\,q_{i2}^{*S} - \frac{\Theta_i\left[q_{i2}^{*S}\right]^2}{2\,b_2^S}, \tag{171}$$

where q_{k1}^{*P} and q_{k1}^{*S} are given by (94) and q_{i2}^{*S} is given by (106). (171) implies that (170) can be written as:

$$SW = -c_{k1}^P\,\Theta_k\,q_{k1}^{*P} - c_1^S\,\Theta_k\,q_{k1}^{*S} - \Theta_i\,c_2^S\,q_{i2}^{*S} - F + \frac{A_2}{b_2^S}\,\Theta_i\,q_{i2}^{*S} - \frac{\Theta_i\left[q_{i2}^{*S}\right]^2}{2\,b_2^S}$$

$$+ \frac{-\eta_1\,\Theta_k\,q_{k1}^{*P}\,q_{k1}^{*S} - \frac{\beta_1^S\,\Theta_k}{2}\left[q_{k1}^{*P}\right]^2 - \frac{\beta_1^P\,\Theta_k}{2}\left[q_{k1}^{*S}\right]^2 + \alpha_1\left[\,\eta_1 + \beta_1^S\,\theta_1\,\right]\Theta_k\,q_{k1}^{*P} + \alpha_1\left[\,\theta_1\,\eta_1 + \beta_1^P\,\right]\Theta_k\,q_{k1}^{*S}}{\beta_1^S\,\beta_1^P\left[\,1 - \Omega_1\,\right]}$$

$$= \Theta_k\,q_{k1}^{*P}\left[\frac{\alpha_1\left(\,\eta_1 + \beta_1^S\,\theta_1\,\right)}{\beta_1^S\,\beta_1^P\left(\,1 - \Omega_1\,\right)} - c_{k1}^P\right] + \Theta_k\,q_{k1}^{*S}\left[\frac{\alpha_1\left(\,\theta_1\,\eta_1 + \beta_1^P\,\right)}{\beta_1^S\,\beta_1^P\left(\,1 - \Omega_1\,\right)} - c_1^S\right] + \Theta_i\,q_{i2}^{*S}\left[\frac{A_2}{b_2^S} - c_2^S\right]$$

$$- \Theta_k\,\frac{\eta_1\,q_{k1}^{*P}\,q_{k1}^{*S} + \frac{1}{2}\,\beta_1^S\left[q_{k1}^{*P}\right]^2 + \frac{1}{2}\,\beta_1^P\left[q_{k1}^{*S}\right]^2}{\beta_1^S\,\beta_1^P\left[\,1 - \Omega_1\,\right]} - F - \frac{\Theta_i\left[q_{i2}^{*S}\right]^2}{2\,b_2^S}. \tag{172}$$

(152), (153) imply that (172) can be written as:

$$SW = \Theta_k\,q_{k1}^{*P}\,\frac{\eta_1\,\Delta_{k1} + \beta_1^S\,\overline{\Delta}_{k1}}{\beta_1^S\,\beta_1^P\left[\,1 - \Omega_1\,\right]} + \Theta_k\,q_{k1}^{*S}\,\frac{\beta_1^P\,\Delta_{k1} + \eta_1\,\overline{\Delta}_{k1}}{\beta_1^S\,\beta_1^P\left[\,1 - \Omega_1\,\right]} + \Theta_i\,q_{i2}^{*S}\left[\frac{A_2 - b_2^S\,c_2^S}{b_2^S}\right]$$

$$- \Theta_k\,\frac{\eta_1\,q_{k1}^{*P}\,q_{k1}^{*S} + \frac{1}{2}\,\beta_1^S\left[q_{k1}^{*P}\right]^2 + \frac{1}{2}\,\beta_1^P\left[q_{k1}^{*S}\right]^2}{\beta_1^S\,\beta_1^P\left[\,1 - \Omega_1\,\right]} - F - \frac{\Theta_i\left[q_{i2}^{*S}\right]^2}{2\,b_2^S}. \tag{173}$$

(94), (106), and (96), (157), and (173) imply that if S1 competes against Pk and S2 sells on Pi and faces no competition ($k, i \in \{1,2\}$), then social welfare is:

$$SW = \Theta_k \, \kappa_{k1} + \Theta_i \, q_{i2}^{*S} \left[\frac{A_2 - b_2^S \, c_2^S}{b_2^S} \right] - \frac{\Theta_i \left[q_{i2}^{*S} \right]^2}{2 \, b_2^S} - F$$

$$= \Theta_k \, \kappa_{k1} + \frac{\Theta_i \left[\widetilde{\Delta}_{i2} \right]^2}{4 \, \beta_2^S \, [\, 1 - \Omega_2 \,]} - \frac{\Theta_i \left[\widetilde{\Delta}_{i2} \right]^2}{32 \, \beta_2^S \, [\, 1 - \Omega_2 \,]} - F \;=\; \Theta_k \, \kappa_{k1} + \frac{7 \, \Theta_i \left[\widetilde{\Delta}_{i2} \right]^2}{32 \, \beta_2^S \, [\, 1 - \Omega_2 \,]} - F. \tag{174}$$

(16) and (161) imply that:

$$\kappa_{kj} - M_{kj} = \frac{3 \left[\overline{\Delta}_{kj} \right]^2}{8 \, \beta_j^P \, [\, 1 - \Omega_j \,]} + \frac{[\, 2 + \Omega_j \,]^2 \, [\, 28 - \Omega_j \,] \, [\Delta_{kj}]^2}{8 \, \beta_j^S \, [\, 1 - \Omega_j \,] \, [\, 8 + \Omega_j \,]^2} + \frac{\eta_j \, [\, 22 + 5 \, \Omega_j \,] \; \Delta_{kj} \, \overline{\Delta}_{kj}}{4 \, \beta_j^S \, \beta_j^P \, [\, 8 + \Omega_j \,] \, [\, 1 - \Omega_j \,]}$$

$$- \frac{1}{2 \, [\, 1 - \Omega_j \,]} \left\{ \frac{\left(\overline{\Delta}_{kj} \right)^2}{2 \, \beta_j^P} + \frac{(\Delta_{kj})^2 \, (\, 2 + \Omega_j \,) \, \Omega_j}{2 \, \beta_j^S \, (\, 8 + \Omega_j \,)^2} \left[26 - \Omega_j + 2 \, (\Omega_j)^2 \right] \right.$$

$$\left. + \frac{\Omega_j \, \overline{\Delta}_{kj} \, \Delta_{kj}}{\eta_j \, (\, 8 + \Omega_j \,)} \left[6 + 2 \, \Omega_j + (\Omega_j)^2 \right] \right\}. \tag{175}$$

(175) implies that:

$$2 \, [\, 1 - \Omega_j \,] \, [\, \kappa_{kj} - M_{kj} \,]$$

$$= \frac{3 \left[\overline{\Delta}_{kj} \right]^2}{4 \, \beta_j^P} + \frac{[\, 2 + \Omega_j \,]^2 \, [\, 28 - \Omega_j \,] \, [\Delta_{kj}]^2}{4 \, \beta_j^S \, [\, 8 + \Omega_j \,]^2} + \frac{\eta_j \, [\, 22 + 5 \, \Omega_j \,] \; \Delta_{kj} \, \overline{\Delta}_{kj}}{2 \, \beta_j^S \, \beta_j^P \, [\, 8 + \Omega_j \,]}$$

$$- \left\{ \frac{\left(\overline{\Delta}_{kj} \right)^2}{2 \, \beta_j^P} + \frac{(\Delta_{kj})^2 \, (\, 2 + \Omega_j \,) \, \Omega_j}{2 \, \beta_j^S \, (\, 8 + \Omega_j \,)^2} \left[26 - \Omega_j + 2 \, (\Omega_j)^2 \right] + \frac{\Omega_j \, \overline{\Delta}_{kj} \, \Delta_{kj}}{\eta_j \, (\, 8 + \Omega_j \,)} \left[6 + 2 \, \Omega_j + (\Omega_j)^2 \right] \right\}$$

$$= \frac{\left[\overline{\Delta}_{kj} \right]^2}{4 \, \beta_j^P} + \frac{[\, 2 + \Omega_j \,] \, [\, 5 - 2 \, \Omega_j \,]}{2 \, \eta_j \, [\, 8 + \Omega_j \,]} \Omega_j \, \overline{\Delta}_{kj} \, \Delta_{kj} + \frac{56 - 26 \, \Omega_j + [\Omega_j]^2 - 4 \, [\Omega_j]^3}{4 \, \beta_j^S \, [\, 8 + \Omega_j \,]^2} \, [\, 2 + \Omega_j \,] \, [\Delta_{kj}]^2 . \tag{176}$$

Observe that:

$$M_{kj} - \frac{\left[\widetilde{\Delta}_{kj} \right]^2}{8 \, \beta_j^S \, [\, 1 - \Omega_j \,]} \;<\; \kappa_{kj} - \frac{7 \left[\widetilde{\Delta}_{kj} \right]^2}{32 \, \beta_j^S \, [\, 1 - \Omega_j \,]} \;\Leftrightarrow\; \frac{7 \left[\widetilde{\Delta}_{kj} \right]^2}{32 \, \beta_j^S \, [\, 1 - \Omega_j \,]} - \frac{\left[\widetilde{\Delta}_{kj} \right]^2}{8 \, \beta_j^S \, [\, 1 - \Omega_j \,]} \;<\; \kappa_{kj} - M_{kj} \tag{177}$$

$$\Leftrightarrow \quad \frac{3\left[\widetilde{\Delta}_{kj}\right]^2}{32\,\beta_j^S\,[\,1-\Omega_j\,]} \;<\; \kappa_{kj} - M_{kj} \quad \Leftrightarrow \quad \frac{3\left[\widetilde{\Delta}_{kj}\right]^2}{16\,\beta_j^S} \;<\; 2\,[\,1-\Omega_j\,]\,[\,\kappa_{kj} - M_{kj}\,]\,. \tag{178}$$

The last inequality in (178) holds because (176) implies that:

$$\frac{3\left[\widetilde{\Delta}_{kj}\right]^2}{16\,\beta_j^S} \;<\; 2\,[\,1-\Omega_j\,]\,[\,\kappa_{kj} - M_{kj}\,]$$

$$\Leftrightarrow \quad \frac{\left[\overline{\Delta}_{kj}\right]^2}{16\,\beta_j^P} + \frac{4\,[\,2+\Omega_j\,]\,[\,5-2\,\Omega_j\,] - 3\,[\,8+\Omega_j\,]}{8\,\eta_j\,[\,8+\Omega_j\,]}\Omega_j\,\overline{\Delta}_{kj}\,\Delta_{kj}$$

$$+ \;\frac{4\left[\,56 - 26\,\Omega_j + (\Omega_j)^2 - 4\,(\Omega_j)^3\,\right][\,2+\Omega_j\,] - 3\,[\,8+\Omega_j\,]^2}{16\,\beta_j^S\,[\,8+\Omega_j\,]^2}\,[\Delta_{kj}]^2 \;>\; 0. \tag{179}$$

Observe that:

$$4\,[\,2+\Omega_j\,]\,[\,5-2\,\Omega_j\,] - 3\,[\,8+\Omega_j\,] \;=\; [\,8+4\,\Omega_j\,]\,[\,5-2\,\Omega_j\,] - 24 - 3\,\Omega_j$$

$$= \; 256 - 32\,\Omega_j - 99\,[\Omega_j]^2 - 28\,[\Omega_j]^3 - 16\,[\Omega_j]^4\,. \tag{180}$$

(179) - (180) imply that:

$$\frac{3\left[\widetilde{\Delta}_{kj}\right]^2}{16\,\beta_j^S} \;<\; 2\,[\,1-\Omega_j\,]\,[\,\kappa_{kj} - M_{kj}\,]$$

$$\Leftrightarrow \quad \frac{\left[\overline{\Delta}_{kj}\right]^2}{16\,\beta_j^P} + \frac{16 - 8\,[\Omega_j]^2 + \Omega_j}{8\,\eta_j\,[\,8+\Omega_j\,]}\Omega_j\,\overline{\Delta}_{kj}\,\Delta_{kj}$$

$$+ \;\frac{256 - 32\,\Omega_j - 99\,[\Omega_j]^2 - 28\,[\Omega_j]^3 - 16\,[\Omega_j]^4}{16\,\beta_j^S\,[\,8+\Omega_j\,]^2}\,[\Delta_{kj}]^2 \;>\; 0. \tag{181}$$

(181) holds because for $\Omega_j \in (0,1)$

$$16 - 8\,[\Omega_j]^2 + \Omega_j \;>\; 0 \;\;\text{and} \tag{182}$$

$$256 - 32\,\Omega_j - 99\,[\Omega_j]^2 - 28\,[\Omega_j]^3 - 16\,[\Omega_j]^4 \;>\; 0. \tag{183}$$

(182) holds because $16 - 8\,[\Omega_j]^2 + \Omega_j$ decreases in $\Omega_j \in (0,1)$, and thus, $16 - 8\,[\Omega_j]^2 + \Omega_j > \min 16 - 8\,[\Omega_j]^2 + \Omega_j = 16 - 8\,[1]^2 + 1 = 9 > 0$. (183) holds because $256 - 32\,\Omega_j - 99\,[\Omega_j]^2 - 28\,[\Omega_j]^3 - 16\,[\Omega_j]^4$ decreases in $\Omega_j \in (0,1)$, and thus, $256 - 32\,\Omega_j - 99\,[\Omega_j]^2 - 28\,[\Omega_j]^3 - 16\,[\Omega_j]^4 > \min 256 - 32\,\Omega_j - 99\,[\Omega_j]^2 - 28\,[\Omega_j]^3 - 16\,[\Omega_j]^4 = 256 - 32 * 1 - 99\,[1]^2 - 28\,[1]^3 - 16\,[1]^4 = 81 > 0$.

(177) implies that:

$$M_{P1} - \frac{\left[\widetilde{\Delta}_{P1}\right]^2}{8\,\beta_1^S\,[\,1 - \Omega_1\,]} \;<\; \kappa_{P1} - \frac{7\left[\widetilde{\Delta}_{P1}\right]^2}{64\,\beta_1^S\,[\,1 - \Omega_1\,]}; \tag{184}$$

$$\Theta\,M_{P2} - \frac{\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{8\,\beta_2^S\,[\,1 - \Omega_2\,]} \;<\; \Theta\,\kappa_{P2} - \frac{7\,\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{32\,\beta_2^S\,[\,1 - \Omega_2\,]}. \tag{185}$$

Condition FS ensures that $F < \Theta\,M_{P1} - \frac{\Theta\left[\widetilde{\Delta}_{P1}\right]^2}{8\,b_1^S} = \Theta\,M_{P1} - \frac{\Theta\left[\widetilde{\Delta}_{P1}\right]^2}{8\,\beta_1^S[1-\Omega_1]}$ and $F < \Theta\,M_{P2} - \frac{\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{8\,b_2^S} = \Theta\,M_{P2} - \frac{\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{8\,\beta_2^S[1-\Omega_2]}$. Therefore, (184) and (185) imply that:

$$F \;<\; \Theta\,\kappa_{P1} - \frac{7\,\Theta\left[\widetilde{\Delta}_{P1}\right]^2}{64\,\beta_1^S\,[\,1 - \Omega_1\,]}; \tag{186}$$

$$F \;<\; \Theta\,\kappa_{P2} - \frac{7\,\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{32\,\beta_2^S\,[\,1 - \Omega_2\,]}. \tag{187}$$

Proposition 1 implies that each seller competes against P under MP. Therefore, (156) implies that:

$$SW^M \;=\; \Theta\,\kappa_{P1} + \Theta\,\kappa_{P2} - 2\,F, \tag{188}$$

where κ_{kj} is given by (161).

First suppose P faces a competing platform $\widetilde{\mathrm{P}}$ that is a stronger platform (i.e., $\frac{\widetilde{\Theta}}{\Theta} > 1$) under PC.

Case I. $\frac{\widetilde{\Theta}}{\Theta} > \max\left\{\frac{\kappa_{P1}}{\kappa_{\widetilde{P}1}}, \frac{\kappa_{P2}}{\kappa_{\widetilde{P}2}}, \phi_{\widetilde{P}2}\right\}$.

Because $\frac{\widetilde{\Theta}}{\Theta} > \phi_{\widetilde{P}2}$, Proposition 2 implies that each seller competes against $\widetilde{\mathrm{P}}$ under PC. Therefore, (156) implies that:

$$SW \;=\; \widetilde{\Theta}\,\kappa_{\widetilde{P}1} + \widetilde{\Theta}\,\kappa_{\widetilde{P}2} - 2\,F, \tag{189}$$

where κ_{kj} is given by (161).

Because $\frac{\widetilde{\Theta}}{\Theta} > \max\left\{\frac{\kappa_{P1}}{\kappa_{\widetilde{P}1}}, \frac{\kappa_{P2}}{\kappa_{\widetilde{P}2}}\right\}$, $\frac{\widetilde{\Theta}}{\Theta} > \frac{\kappa_{Pj}}{\kappa_{\widetilde{P}j}}$ ($j \in \{1, 2\}$), and thus, $\widetilde{\Theta}\,\kappa_{\widetilde{P}j} > \Theta\,\kappa_{Pj}$. Therefore, (188) and (189) imply that $SW > SW^M$ in this case.

Case II. $\frac{\widetilde{\Theta}}{\Theta} \in \left(1, \min\left\{\frac{\kappa_{P1}}{\kappa_{\widetilde{P}1}}, \frac{\kappa_{P2}}{\kappa_{\widetilde{P}2}}, \phi_{\widetilde{P}2}\right\}\right)$.

First suppose $\phi_{\widetilde{P}1} < \min\left\{\frac{\kappa_{P1}}{\kappa_{\widetilde{P}1}}, \frac{\kappa_{P2}}{\kappa_{\widetilde{P}2}}\right\}$.

If $\frac{\widetilde{\Theta}}{\Theta} \in \left(\phi_{\widetilde{P}1}, \min\left\{\frac{\kappa_{P1}}{\kappa_{\widetilde{P}1}}, \frac{\kappa_{P2}}{\kappa_{\widetilde{P}2}}, \phi_{\widetilde{P}2}\right\}\right)$, then $\frac{\widetilde{\Theta}}{\Theta} \in \left(\phi_{\widetilde{P}1}, \phi_{\widetilde{P}2}\right)$. Proposition 2 implies that S1 competes against $\widetilde{\mathrm{P}}$ whereas S2 sells on P and faces no competition under PC. Therefore, (174) implies that:

$$SW \;=\; \widetilde{\Theta}\,\kappa_{\widetilde{P}1} + \frac{7\,\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{32\,\beta_2^S\,[\,1 - \Omega_2\,]} - F. \tag{190}$$

Because $\frac{\widetilde{\Theta}}{\Theta} < \min\left\{\frac{\kappa_{P1}}{\kappa_{\widetilde{P}1}}, \frac{\kappa_{P2}}{\kappa_{\widetilde{P}2}}\right\}$, $\frac{\widetilde{\Theta}}{\Theta} < \frac{\kappa_{P1}}{\kappa_{\widetilde{P}1}}$, and thus,

$$\widetilde{\Theta}\,\kappa_{\widetilde{P}1} \;<\; \Theta\,\kappa_{P1}. \tag{191}$$

(166) implies that $\frac{7\left[\widetilde{\Delta}_{P2}\right]^2}{32\,\beta_2^S\,[1-\Omega_2\,]} < \kappa_{P2}$. Therefore, (115) and (118) imply that in this case:

$$SW \;<\; SW^M \quad\Leftrightarrow\quad \widetilde{\Theta}\,\kappa_{\widetilde{P}1} + \frac{7\,\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{32\,\beta_2^S\,[\,1 - \Omega_2\,]} - F \;<\; \Theta\,\kappa_{P1} + \Theta\,\kappa_{P2} - 2\,F$$

$$\Leftrightarrow\; F \;<\; \Theta\,\kappa_{P1} - \widetilde{\Theta}\,\kappa_{\widetilde{P}1} + \Theta\,\kappa_{P2} - \frac{7\,\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{32\,\beta_2^S\,[\,1 - \Omega_2\,]}. \tag{192}$$

(192) holds because (187) and (191) imply that $\Theta\,\kappa_{P1} - \widetilde{\Theta}\,\kappa_{\widetilde{P}1} > 0$, and thus,

$$F \;<\; \Theta\,\kappa_{P2} - \frac{7\,\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]} \;<\; \Theta\,\kappa_{P1} - \widetilde{\Theta}\,\kappa_{\widetilde{P}1} + \Theta\,\kappa_{P2} - \frac{7\,\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]}.$$

If $\frac{\widetilde{\Theta}}{\Theta} \in \left(1, \phi_{\widetilde{P}1}\right)$, Proposition 2 implies that each seller sells on $\widetilde{\mathrm{P}}$ and faces no competition under PC. Therefore, (169) implies that:

$$SW \;=\; \frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}1}\right]^2}{32\,\beta_1^S\,[\,1-\Omega_1\,]} + \frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]}. \tag{193}$$

(166) implies that $\frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}j}\right]^2}{32\,\beta_j^S[1-\Omega_j]} < \widetilde{\Theta}\,\kappa_{\widetilde{P}j}$. Because $\frac{\widetilde{\Theta}}{\Theta} < \phi_{\widetilde{P}1} < \min\left\{\frac{\kappa_{P1}}{\kappa_{\widetilde{P}1}}, \frac{\kappa_{P2}}{\kappa_{\widetilde{P}2}}\right\}$, $\frac{\widetilde{\Theta}}{\Theta} < \frac{\kappa_{Pj}}{\kappa_{\widetilde{P}j}}$ ($j \in \{1,2\}$), and thus,

$$\widetilde{\Theta}\,\kappa_{\widetilde{P}j} \;<\; \Theta\,\kappa_{Pj}. \tag{194}$$

Therefore, $\frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}j}\right]^2}{32\,\beta_j^S[1-\Omega_j]} < \Theta\,\kappa_{Pj}$. Condition FS ensures that $F < \widetilde{\Theta}\,M_{\widetilde{P}2} - \frac{\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}2}\right]^2}{8\,b_2^S} = \widetilde{\Theta}\,M_{\widetilde{P}2} - \frac{\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}2}\right]^2}{8\,\beta_2^S[1-\Omega_2]}$ and $F < \widetilde{\Theta}\,M_{\widetilde{P}1} - \frac{\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}1}\right]^2}{8\,b_1^S} = \widetilde{\Theta}\,M_{\widetilde{P}1} - \frac{\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}1}\right]^2}{8\,\beta_1^S[1-\Omega_1]}$. Therefore, (177) implies that

$$F \;<\; \widetilde{\Theta}\,\kappa_{\widetilde{P}1} - \frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}1}\right]^2}{32\,\beta_1^S\,[\,1-\Omega_1\,]} \;\text{ and }\; F \;<\; \widetilde{\Theta}\,\kappa_{\widetilde{P}2} - \frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]}. \tag{195}$$

(194) and (195) imply that:

$$F \;<\; \Theta\,\kappa_{P1} - \frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}1}\right]^2}{32\,\beta_1^S\,[\,1-\Omega_1\,]} \;\text{ and }\; F \;<\; \Theta\,\kappa_{P2} - \frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]}. \tag{196}$$

Therefore, (188) and (193) imply that in this case:

$$SW \;<\; SW^M \quad\Leftrightarrow\quad \frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}1}\right]^2}{32\,\beta_1^S\,[\,1-\Omega_1\,]} + \frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]} \;<\; \Theta\,\kappa_{P1} + \Theta\,\kappa_{P2} - 2\,F$$

$$\Leftrightarrow \quad 2\,F \;<\; \Theta\,\kappa_{P1} - \frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}1}\right]^2}{32\,\beta_1^S\,[\,1-\Omega_1\,]} + \Theta\,\kappa_{P2} - \frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]}. \tag{197}$$

(196) implies that (197) holds.

Next suppose $\phi_{\widetilde{P}1} \geq \min\left\{\frac{\kappa_{P1}}{\kappa_{\widetilde{P}1}}, \frac{\kappa_{P2}}{\kappa_{\widetilde{P}2}}\right\}$. (**??**) implies that $\phi_{\widetilde{P}2} > \min\left\{\frac{\kappa_{P1}}{\kappa_{\widetilde{P}1}}, \frac{\kappa_{P2}}{\kappa_{\widetilde{P}2}}\right\}$. Therefore, $\min\left\{\frac{\kappa_{P1}}{\kappa_{\widetilde{P}1}}, \frac{\kappa_{P2}}{\kappa_{\widetilde{P}2}}, \phi_{\widetilde{P}2}\right\} = \min\left\{\frac{\kappa_{P1}}{\kappa_{\widetilde{P}1}}, \frac{\kappa_{P2}}{\kappa_{\widetilde{P}2}}\right\}$. Therefore, $\frac{\widetilde{\Theta}}{\Theta} < \min\left\{\frac{\kappa_{P1}}{\kappa_{\widetilde{P}1}}, \frac{\kappa_{P2}}{\kappa_{\widetilde{P}2}}, \phi_{\widetilde{P}2}\right\} = \min\left\{\frac{\kappa_{P1}}{\kappa_{\widetilde{P}1}}, \frac{\kappa_{P2}}{\kappa_{\widetilde{P}2}}\right\} \leq \phi_{\widetilde{P}1}$. Because $\frac{\widetilde{\Theta}}{\Theta} > 1$ in this case, then $\frac{\widetilde{\Theta}}{\Theta} \in \left(1, \phi_{\widetilde{P}1}\right)$. Proposition 2 implies that each seller sells on $\widetilde{\mathrm{P}}$ and faces no competition under PC. Therefore, consumer surplus is given by (193). (188), (193), and (197) imply that $SW < SW^M$ in this case.

Next suppose P faces a symmetric competing platform $\widetilde{\mathrm{P}}$ (i.e., $\frac{\widetilde{\Theta}}{\Theta} = 1$) under PC. Proposition 2 implies that each seller is indifferent between selling on P and selling on $\widetilde{\mathrm{P}}$ and each seller faces no competition under PC. Therefore, (169) implies that:

$$SW \;=\; \frac{1}{2}\,\frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}1}\right]^2}{32\,\beta_1^S\,[\,1-\Omega_1\,]} + \frac{1}{2}\,\frac{7\,\Theta\left[\widetilde{\Delta}_{P1}\right]^2}{32\,\beta_1^S\,[\,1-\Omega_1\,]} + \frac{1}{2}\,\frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]} + \frac{1}{2}\,\frac{7\,\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]}$$

$$= \frac{1}{2}\,\frac{7\,\Theta\left[\widetilde{\Delta}_{P1}\right]^2}{32\,\beta_1^S\,[\,1-\Omega_1\,]} + \frac{1}{2}\,\frac{7\,\Theta\left[\widetilde{\Delta}_{P1}\right]^2}{32\,\beta_1^S\,[\,1-\Omega_1\,]} + \frac{1}{2}\,\frac{7\,\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]} + \frac{1}{2}\,\frac{7\,\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]} \tag{198}$$

$$= \frac{7\,\Theta\left[\widetilde{\Delta}_{P1}\right]^2}{32\,\beta_1^S\,[\,1-\Omega_1\,]} + \frac{7\,\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]}. \tag{199}$$

(198) holds because $\frac{\widetilde{\Theta}}{\Theta} = 1$ and $\widetilde{\Delta}_{\widetilde{P}j} = \widetilde{\Delta}_{Pj}$. Therefore, (188) and (199) imply that in this case:

$$SW \;<\; SW^M \quad\Leftrightarrow\quad \frac{7\,\Theta\left[\widetilde{\Delta}_{P1}\right]^2}{32\,\beta_1^S\,[\,1-\Omega_1\,]} + \frac{7\,\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]} \;<\; \Theta\,\kappa_{P1} + \Theta\,\kappa_{P2} - 2\,F$$

$$\Leftrightarrow \quad 2\,F \;<\; \Theta\,\kappa_{P1} - \frac{7\,\Theta\left[\widetilde{\Delta}_{P1}\right]^2}{32\,\beta_1^S\,[\,1-\Omega_1\,]} + \Theta\,\kappa_{P2} - \frac{7\,\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]}. \tag{200}$$

(186) and (187) imply that (200) holds.

Finally, suppose P faces a competing platform $\widetilde{\mathrm{P}}$ that is a weaker platform (i.e., $\frac{\widetilde{\Theta}}{\Theta} < 1$) under PC.

Case I. $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_{P1}}, 1 \right)$.

In this case, $\frac{\Theta}{\widetilde{\Theta}} \in (\, 1, \phi_{P1} \,)$. Proposition 2 implies that each seller sells on P and faces no competition under PC. Therefore, (169) implies that:

$$SW \;=\; \frac{7\,\Theta\left[\widetilde{\Delta}_{P1}\right]^2}{32\,\beta_1^S\,[\,1-\Omega_1\,]} + \frac{7\,\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]}. \tag{201}$$

(166) implies that $\frac{7\left[\widetilde{\Delta}_{Pj}\right]^2}{32\,\beta_j^S[\,1-\Omega_j\,]} < \kappa_{Pj}$ for $j \in \{1, 2\}$. Therefore, (188) and (201) imply that:

$$SW \;<\; SW^M \;\;\Leftrightarrow\;\; \frac{7\,\Theta\left[\widetilde{\Delta}_{P1}\right]^2}{32\,\beta_1^S\,[\,1-\Omega_1\,]} + \frac{7\,\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]} \;<\; \Theta\,\kappa_{P1} + \Theta\,\kappa_{P2} - 2\,F$$

$$\Leftrightarrow\;\; 2\,F \;<\; \Theta\,\kappa_{P1} - \frac{7\,\Theta\left[\widetilde{\Delta}_{P1}\right]^2}{32\,\beta_1^S\,[\,1-\Omega_1\,]} + \Theta\,\kappa_{P2} - \frac{7\,\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]}. \tag{202}$$

(186) and (187) imply that (202) holds.

Case II. $\frac{\widetilde{\Theta}}{\Theta} \in \left(\frac{1}{\phi_{P2}}, \frac{1}{\phi_{P1}} \right)$.

In this case, $\frac{\Theta}{\widetilde{\Theta}} \in (\, \phi_{P1}, \phi_{P2} \,)$. Proposition 2 implies that S1 competes against P whereas S2 sells on $\widetilde{\mathrm{P}}$ and faces no competition under PC. Therefore, (174) implies that:

$$SW \;=\; \Theta\,\kappa_{P1} + \frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]} - F. \tag{203}$$

(166) implies that $\frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}2}\right]^2}{32\,\beta_2^S[\,1-\Omega_2\,]} < \widetilde{\Theta}\,\kappa_{\widetilde{P}2}$. (165) and $\frac{\widetilde{\Theta}}{\Theta} < 1$ imply that $\frac{\widetilde{\Theta}}{\Theta} < \frac{\kappa_{Pj}}{\kappa_{\widetilde{P}j}}$, and thus, $\widetilde{\Theta}\,\kappa_{\widetilde{P}j} < \Theta\,\kappa_{Pj}$. Therefore, $\frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}2}\right]^2}{32\,\beta_2^S[\,1-\Omega_2\,]} < \Theta\,\kappa_{Pj}$. Therefore, (188) and (203) imply that in

this case:

$$SW \;<\; SW^M \quad \Leftrightarrow \quad \Theta\,\kappa_{P1} + \frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]} - F \;<\; \Theta\,\kappa_{P1} + \Theta\,\kappa_{P2} - 2\,F$$

$$\Leftrightarrow \quad F \;<\; \Theta\,\kappa_{P2} - \frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}2}\right]^2}{32\,\beta_2^S\,[\,1-\Omega_2\,]}. \tag{204}$$

Because $\frac{\widetilde{\Theta}}{\Theta} < 1$ in this case and $\widetilde{\Delta}_{\widetilde{P}2} = \widetilde{\Delta}_{P2}$ (since $\frac{\partial \widetilde{\Delta}_{kj}}{\partial c_{kj}^P} = 0$), then $\Theta\,\kappa_{P2} - \frac{7\,\widetilde{\Theta}\left[\widetilde{\Delta}_{\widetilde{P}2}\right]^2}{32\,\beta_2^S[\,1-\Omega_2\,]} > \Theta\,\kappa_{P2} - \frac{7\,\Theta\left[\widetilde{\Delta}_{P2}\right]^2}{32\,\beta_2^S[\,1-\Omega_2\,]}$. Therefore, (187) implies that (204) must hold. ■