Time on the market and list prices in "hot" real estate markets

Eric Schmidbauer* and Dmitry Lubensky†

September 27, 2022

Abstract

We study the features of "hot" real estate markets and provide a theoretical foundation for empirically puzzling observations. For example, in a market in which many buyers are competing for a relatively scarce set of houses, it is not clear a priori that this will resolve with extremely fast sales at high prices instead of slower sales and even higher prices. We present a simple auction model of the housing market in which list prices partially commit the seller, requiring acceptance of the highest offer at or above the list price but allowing rejection of offers below it. In equilibrium more buyers lead to outcomes generally recognized as consistent with a hot market: list and sales prices increase, the ratio of sales to list price increases, and time-on-the-market decreases.

Keywords: hot markets, list prices, time on the market, sales to list price ratio, auctions

JEL Classification: R00, R21 D44

*Corresponding author. University of Central Florida, College of Business Administration. 4336 Scorpius St., Orlando, FL 32816. P: 407.823.1204. eschmidb@ucf.edu.

†Amazon. P: 847.858.0328. dmitry.lubensky@gmail.com

1 Introduction

It is commonly observed that in "hot" real estate markets average sales prices increase but also the average time a house is on the market decreases. This may seem puzzling since sellers might prefer to wait and receive even higher prices. Conversely, in cold markets average sales prices are depressed and time-on-the-market increases. Why then don't sellers reduce their asking price in order to consumate a sale more quickly?

Popular notions of a hot housing market include a rising average list price, a high and rising sales-to-list-price ratio, declining time on the market, and higher transaction volumes (Carrillo 2013; Krainer 2001; Dale-Johnson and Hamilton 1998; Chernobai and Hossain 2012). Within theoretical models these observable consequences of market hotness are tied to market fundamentals such as the ratio of buyers to sellers (Novy-Marx 2009; Li and Yavas 2015). Adopting this same notion of market hotness, in this paper we present a novel theoretical channel by which a hot market leads to a change in the commonly cited empirical metrics listed above in the expected direction.

We model list prices as partially committing the seller in that offers at or above list price must be accepted while those below list can be rejected (justification for this assumption is provided below). Compared to a standard auction, this induces more aggressive bidding both because those that would have bid slightly below the list price now have incentive to at least match the list price, and because bidders that bid below the list price try to induce the seller to accept even though he is not required to. When raising the list price the seller faces a tradeoff between pushing down the bids of some bids just below the list price and increasing the bids of those above it. We show it is always optimal to set a list price that is interior (i.e., positive but below the highest possible buyer valuation). Finally, we show that an increase in the number of bidders increases both list prices and sales prices, but that sales prices increase faster and so the ratio of sales to list price also increases.[1] The probability of a sale also increases in the number of buyers, which we interpret as shorter time on the market and a higher volume of transactions.

Due to the important role the list price has on the chance of a sale and the resulting sales price in our model we further discuss its impact here. In one strand of the literature the list price is assumed to function as a binding price ceiling which affects a consumer's choice to engage in costly search to learn his valuation for the item for sale (Chen and Rosenthal 1996a; Chen and Rosenthal 1996b; Arnold 1999; Lester, Visschers, and Wolthoff 2017; Stacey 2019). However, Han and Strange (2016) present ample evidence that sales prices often do exceed the list price for homes. Neither can the list price reasonably be said to function as a floor, as when a seller sets a reserve price in an auction. In addition to being inconsistent with empirical observation, there is also the counter-intuitive theoretical prediction that the seller's optimal reserve price in an auction is *independent* of the number of bidders (Krishna, 2002). What then is the role of a list price? In this paper we assume it entails partial commitment: the seller must accept offers at or above list price but is free to reject offers below it. This assumption has been made elsewhere in the literature (Han and Strange 2016; Albrecht, Gautier, and Vroman 2016) and is justified by the fact that although there is usually not a legal requirement for offers above list to be accepted, "the listing contract with a real estate agent creates a partial commitment of a similar nature by requiring the seller to pay the agent's commission if the seller rejects an unrestricted offer equal to or greater than the asking price. Furthermore, there may be behavioral reasons why a seller may feel committed to the asking price," such as failing to conform to generally accepted notions of good faith bargaining (Han &

[1] In fact, any exogenous change that stochastically increases the distribution of the maximum valuation of the bidders will have a qualitatively similar effect as an increase in bidders. Thus, for example, if a neighborhood becomes more trendy and this increases each bidder's distribution of values in the sense of first order stochastic dominance, this too can be regarded as a hotter market within our model since it has the same mathematical effect as does more bidders.

Strange, 2016). Thus in our model the high bidder whose bid exceeds the list price is guaranteed to win the auction whereas if the high bid is below the list price it might not. A seller will choose to reject bids below the list price if he thinks his discounted payoff from continuing to stay in the market and solicit new bids is higher.

Related literature

Han and Strange (2016) explore the role of the list price on home sales in a search model with a partial commitment assumption on the list price similar to that which we use. In their model a lower list price results in higher expected surplus to a bidder from searching and so draws in more bidders. However, it also risks selling the house at a lower price in the event that there is limited interest in the house among the bidders that show up. Han and Strange go on to provide empirical evidence that matches their prediction that a lower list price increases the number bids a seller receives. Our papers differ in that a large focus in Han and Strange is on how list price affects the search process, which we treat as exogenous, with less focus on how it affects the ultimate sales price, which is our main focus. After search has occurred in Han and Strange it is assumed that buyers' valuations are common knowledge and in most cases Nash bargaining ensues with the list price playing no role. However, in the event that the list price is between the highest- and second-highest buyer's valuation no bargaining occurs and instead the sales price equals the list price. In contrast, in our model buyers always retain private information about their valuations and the list price always affects a buyer's optimal bid.

Khezr and Menezes (2018) examine a model similar to ours in which the seller posts a list price and might keep the object that is for sale if all bids are below the list price. However, we assume that the list price commits the seller to accepting the highest offer at or above the list price whereas in Khezr and Menezes is the seller is able to remove the house from the market in this case. In their model the list price instead serves as a threshold that determines when a "run off" auction will be held among bidders that met the list price in the first round. Thus they allow for bidding wars but this is the only way in which the sales price can end up being higher than the list price. Our analysis also differs in that we examine how metrics of market hotness are affected by the number of buyers in the market.

Novy-Marx (2009) examines the same questions we do through a two-sided search model. Sellers' responses to exogenous shocks are shown to amplify the effect of the initial shock and thus make metrics such as sales prices and time on the market very sensitive to market fundamentals. For example, if more buyers enter the market this increases sellers' bargaining strength allowing them to sell more quickly, and since each sold house is not guaranteed to be replaced by another new listing the stock of sellers in the market decreases. That in turn increases the ratio of buyers to sellers, thereby amplifying the initial shock. Krainer (2001) likewise addresses the question as to why it should be that hot markets lead to rising prices, shorter selling times and a higher volume of transactions. He develops a search model in which exogenous shocks cause a homeowner to become a seller and then simultaneously search for a new home. As both a buyer and seller, the agent optimizes by trading off the cost of additional search with the discounted expected benefit from continuing to search. Krain shows that price does not fully adjust to changes in market conditions, rather average selling times also serves as an adjustment mechanism. While both the models of Novy-Marx (2009) and Krainer (2001) explore how market hotness affects observed metrics such as sales price, time on the market, etc., our mechanisms are different. In their models the list price plays no role whereas in our model it crucially affects bidding behavior.

The list price has also been modeled as a potential signal of the seller's private information, as in De Wit and Van der Klaauw (2013) and Albrecht et al. (2016). This information may include hidden characteristics of the house or neighborhood, as well as seller traits such as willingness to negotiate, risk preference, financial constraints and time discounting rate.

In a somewhat different context Burguet and Sakovics (1996) consider an auction in which there is not full commitment to the reserve price. They assume that if no bid exceeds the reserve price then the item does not go unsold but rather a second auction is held without a reserve price. They show that when a bidder must incur a cost to participate in the auction (such as learning their valuation), that upon observing the first auction failed to sell the good a potential bidder can infer the first set of bidders had relatively low values so that entering the second auction might be worth the cost of doing so. This increased entry can then increase seller revenue and welfare.

2 Model

There is a single seller and $n \geq 2$ bidders.[2] The bidders have iid valuations $v \sim U[0,1]$ and have utility $u = v - p$ from winning the auction and utility 0 from losing, where p is the winning bid. The seller holds a first price sealed bid auction that includes naming a list price r. The list price is similar to a reserve price in that if the highest bid is at least as high as r then the high bid must be accepted. However, if the high bid is below r then it is accepted with probability $\alpha(p)$, which is determined as follows. After receiving bids but before accepting or rejecting offers, the seller gets an independent signal $x \sim U[0,1]$ that equals the value of his outside option (which can be interpretted as the expected discounted payoff next period if he does not accept any offers now). The seller accepts a bid below r only if it exceeds his outside option. Letting $p = \max_i\{p_i\}$, from the buyers' perspective the probability the house sells is $\alpha(p)$ is 1 if $p \geq r$ and is p if $p < r$.

[2] While we assume that the number of bidders is common knowledge, one can straightforwardly adapt arguments in Harstad, Kagel, and Levin (1990) to allow for bidders to have uncertainty about the number of other bidders they face in the auction.

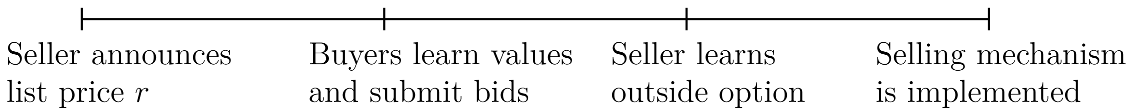


Figure 1: Timing of the game

3 Equilibrium

We first consider the subgame where r is already determined. Note that for the special case $r = 0$ we have the familiar first price sealed bid auction with no reserve price and thus $b(v) = \frac{n-1}{n}$, resulting in expected revenue $\frac{n-1}{n+1}$. Now suppose $r > 0$. Conjecture a symmetric equilibrium with increasing bid function $b(v)$ and inverse bid function b^{-1}. We initially restrict attention to $v \in [0, b^{-1}(r))$; that is, to bidders with valuations low enough that they will bid below the list price r. Then a bidder's expected payoff from submitting bid p is

$$\pi(p; v) \;=\; \underbrace{\left[F\left(b^{-1}\left(p\right)\right)\right]^{n-1}}_{\substack{\text{Probability of being} \\ \text{the high bidder}}} \;\times\; \underbrace{F(p)}_{\substack{\text{Probability of} \\ \text{exceeding the} \\ \text{seller's outside} \\ \text{option}}} \;\times\; \underbrace{(v-p)}_{\substack{\text{Payoff from} \\ \text{winning}}} . \tag{1}$$

Line (1) closesly resembles the payoff of a bidder in a standard first price auction, with the difference being that here the probability of winning the auction is reduced by the requirement that the bid p exceed the seller's outside option. Substituting $F(v) = v$ (since all values are iid uniform on $[0, 1]$), differentiating and setting to 0 gives

$$\frac{d\pi}{dp} \;=\; \frac{(n-1)\; b^{-1}(p)^{n-2}\; p(v-p)}{b'\left(b^{-1}(p)\right)} + b^{-1}(p)^{n-1}(v-2p) \;=\; 0. \tag{2}$$

Let p^* be a solution to line (2). Thus for $b(v)$ to be a best response we must have $b(v) = p^*$ for all v in our restricted domain $[0, b^{-1}(r))$. Substituting this into line (2) for p and solving for $b'(v)$ gives the differential equation

$$b'(v) = \frac{(n-1)\ b(v)\ (v - b(v))}{2v\ b(v) - v^2}. \tag{3}$$

Imposing the initial condition $b(0) = 0$ (since it is dominated for type 0 to submit a positive bid), line (3) admits solution $b\,(v) = \left(\frac{n}{n+1}\right) v$.

It is interesting to observe that here bidding is linear in the value, as in the case of a standard first price auction. In fact, with n risk-neutral bidders with iid $U[0,1]$ valuations it is well-known that bidders use shading factor $\frac{n-1}{n}$ in the standard auction. Since $\frac{n}{n+1} > \frac{n-1}{n}$ we conclude bidders bid more aggressively in our context. This is because a marginally higher bid not only increases the chance that a bidder submits a higher bid than the others, as is true in a standard auction, but it also increases the probability that the bid exceeds the seller's outside option so that a sale will occur.

We have thus far shown there is an interval of types $[0, b^{-1}(r))$ that use bid function $b\,(v) = \left(\frac{n}{n+1}\right) v$. We now argue there exists an interval of bids in a left neighborhood of r that are never submitted by any bidder. This is because bid r has a discretely higher probability of winning the auction than bids slightly below r, owing to our assumption about how the list price functions. In equilibrium there must exist a bidder valuation x such that this bidder is indifferent between submitting bid $b\,(x) = \left(\frac{n}{n+1}\right) x$ and r. We can determine this value of x as follows. First, by bidding r type x wins with probability $F(x)^{n-1}$ (i.e., when all $n-1$ other bidders have a lower value), whereas the bid $\left(\frac{n}{n+1}\right) x$ wins when all other bidders have a lower value *and* the seller's outside option is below the bid of $\left(\frac{n}{n+1}\right) x$, which has probability $F(x)^{n-1}\ F\left(\left(\frac{n}{n+1}\right) x\right)$. Thus the expected payoff to type x from bidding $\left(\frac{n}{n+1}\right) x$ is $F(x)^{n-1}\ F\left(\left(\frac{n}{n+1}\right) x\right) \left(x - \left(\frac{n}{n+1}\right) x\right)$ whereas from bid r it is $F(x)^{n-1}(x-r)$. Substituting $F(v) = v$, reducing, and setting these payoffs equal yields the indifference condition

$$\frac{n}{(n+1)^2}x^{n+1} = x^n - rx^{n-1}. \tag{4}$$

Dividing through by x^{n-1} leaves a quadratic with solutions $x = \frac{(n+1)^2 \pm (n+1)\sqrt{(n+1)^2-4rn}}{2n}$.[3] Since the positive root is always greater than 1 for $r \in [0,1]$ we select the negative root, and denote it by $\tilde{x} \equiv \frac{(n+1)^2-(n+1)\sqrt{(n+1)^2-4rn}}{2n}$.

Thus we have found a discontinuity in the bid function, with $b(v) = \left(\frac{n}{n+1}\right)x$ on $[0,\tilde{x})$ and $b(\tilde{x}) = r$. All that remains is to determine the bid function on $(\tilde{x},1]$. By construction types in this interval bid greater than the list price r and so the high bidder will always win as in a standard first price auction. Type v's expected payoff from submitting bid p is $\pi = \left[F\left(b^{-1}\left(p\right)\right)\right]^{n-1}(v-p)$. Differentiating and setting to 0 gives the differential equation

$$b'(v) = \frac{(n-1)(v-b(v))}{v},$$

which has solution $b\left(v\right) = \frac{n-1}{n}v + \frac{k}{v^{n-1}}$, where k is a constant that is determined by an initial condition. Imposing $b\left(\tilde{x}\right) = r$, we can find k as follows:

$$b(\tilde{x}) = r = \frac{n-1}{n}\tilde{x} + \frac{k}{\tilde{x}^{n-1}} \rightarrow k = r\tilde{x}^{n-1} - \frac{n-1}{n}\tilde{x}^n.$$

Substituting for k in $b\left(v\right) = \frac{n-1}{n}v + \frac{k}{v^{n-1}}$ gives $b(v) = \left(\frac{n-1}{n}\right)v + \frac{r\tilde{x}^{n-1}}{v^{n-1}} - \left(\frac{n-1}{n}\right)\frac{\tilde{x}^n}{v^{n-1}}$ on $(\tilde{x},1]$. We have now solved for the equilibrium bid function. Figure 2 depicts this function for the $n=2$ case while the proposition below summarizes our findings.

Finally, note that the inequality $\left(\frac{n}{n+1}\right)\tilde{x} < r = b\left(\tilde{x}\right)$ holds when $\tilde{x} \leq 1 \Leftrightarrow r \leq$

[3] Although $x = 0$ also solves line (4) this cannot represent an equilibrium except in the case where $r = 0$ too. If instead $r > 0$ but $x = 0$ then types in a left neighborhood of $b^{-1}(r)$ (who therefore bid less than r) could profitably deviate to the marginally higher bid r since this would lead to a discretely higher probability of winning the auction.

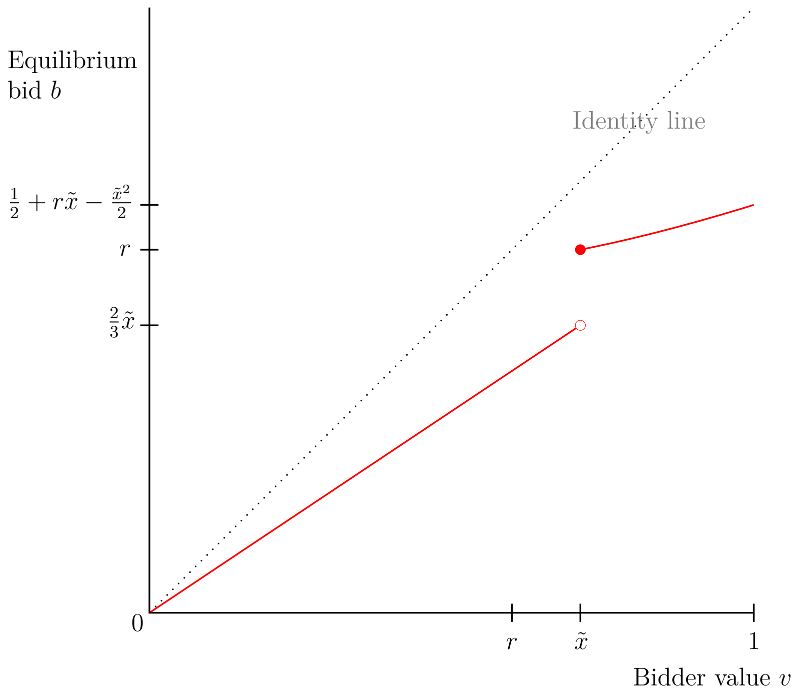


Figure 2: Equilibrium bid function with 2 bidders and iid valuations $v_i \sim U[0,1]$, where $\tilde{x} = \frac{9-3\sqrt{9-8r}}{4}$.

$\frac{n^2+n+1}{(n+1)^2}$, so that there is a discontinuity in the bid function at valuation $\tilde{x}$. Whenever $r > \frac{n^2+n+1}{(n+1)^2}$ we have $\tilde{x}$ greater than the upper bound of the type distribution and so there is no discontinuity in the bid function: all types bid $\left(\frac{n}{n+1}\right) v$.

Proposition 1 *Let n and r be given and define $\tilde{x} = \frac{(n+1)^2-(n+1)\sqrt{(n+1)^2-4rn}}{2n} > r$. There is a symmetric equilibrium in which each bidder uses bid function*

$$b(v) = \begin{cases} \left(\frac{n}{n+1}\right) v & \text{if } v < \tilde{x} \\ \left(\frac{n-1}{n}\right) v + \frac{r\tilde{x}^{n-1}}{v^{n-1}} - \left(\frac{n-1}{n}\right) \frac{\tilde{x}^n}{v^{n-1}} & \text{if } v \geq \tilde{x} \end{cases}.$$

There are a few important features of the equilibrium bidding function to point out. First, a calculation shows $b(\tilde{x}) = r > \left(\frac{n}{n+1}\right)\tilde{x}$, and thus there is a discontinuity at $\tilde{x}$. This arises due to the fact that bids just below the list price r have a discretely lower probability of being accepted than bids at or above r. Thus if bidding below r

the bid must be sufficiently below r to "be worth it", i.e. the higher payoff resulting from the acceptance of a much lower bid needs to offset the discretely lower probability of winning. Second, the bidding function is steeper for $v \in [0, \tilde{x})$ than $v \geq \tilde{x}$. Referring back to equation (1), when v is below the threshold $\tilde{x}$ a marginally higher bid both increases the probability of beating other bidders and exceeding the seller's outside option, whereas when v exceeds $\tilde{x}$ the outside option is not operative and so this second benefit from a higher bid is absent. Hence, bids increase faster in bidder valuation for $v < \tilde{x}$. Finally, we establish how the bidding function changes with parameters n and r in the remark below, and relegate the proof of its non-obvious components to the appendix.

Remark 1

(i). The location of the bidding discontinuity, $\tilde{x}$, is decreasing in n and approaches r in the limit. Further, $\tilde{x}$ is increasing in r, with $\tilde{x} = 0$ when $r = 0$ and $\tilde{x} = 1$ when $r = \frac{n^2+n+1}{(n+1)^2}$.

(ii). The bidding function is strictly increasing in n and weakly increasing in r. More specifically, $\frac{b(v)}{dn} > 0$ for all v while $\frac{b(v)}{dr} > 0$ when $v \geq \tilde{x}$.

The facts regarding $b(v)$ will be useful in the next section where we determine the seller's optimal list price r.

Optimal selection of r

Before presenting the next proposition we discuss the intuition behind the seller's optimal choice of list price r. We first point out that inducing a standard first price auction by setting $r = 0$ will not be optimal for the seller. This follows easily since we have previously observed that when $r > 0$ all types below $\tilde{x}$ bid higher than they would in a standard first price auction, and so by picking $r = \frac{n^2+n+1}{(n+1)^2} < 1$ and inducing

$\tilde{x} = 1$ the seller will cause all types $v \in [0, 1]$ to bidder higher. Thus selecting $r = 0$ cannot be revenue maximizing for the seller.

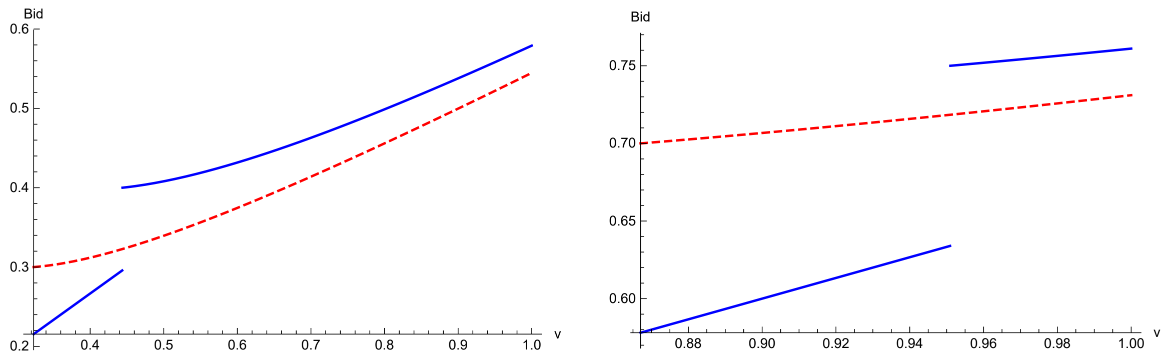
While $r = 0$ is not optimal for the seller neither is the maximal list price $r = 1$. This is because increasing the list price r involves a tradeoff for the seller, wherein it pushes up some bids and pushes down others. To see this tradeoff more clearly, let $\hat{F}$ denote the distribution of the maximum value of the n bidders, recall the value of the seller's outside option $x \sim U[0, 1]$, and since $b(v)$ is a piecewise function define $b_1(v) = \left(\frac{n}{n+1}\right) v$ for $v \leq \tilde{x}$ and $b_2(v) = b(v)$ for $v \geq \tilde{x}$. We can write total revenue as coming from two parts, the first when the highest bidder's value is below the threshold $\tilde{x}$ and second where it is above that threshold:

$$R \;=\; \int_0^1 \left(\int_0^{\tilde{x}} \max\{b_1(v), x\} \; d\hat{F}(v) \right) dx \;+\; \int_{\tilde{x}}^1 b_2(v) \; d\hat{F}(v). \tag{5}$$

Note that when the highest valuation is below $\tilde{x}$ it is possible the seller takes his outside option, and for this reason we must integrate over the possible values of x for v below $\tilde{x}$. Differentiating with respect to r and collecting terms,

$$\frac{dR}{dr} \;=\; \underbrace{\int_0^{\tilde{x}} \frac{\partial b_1}{\partial r} \; d\hat{F}(v)}_{\substack{=0, \text{ unchanged} \\ \text{bidding for } v < \tilde{x}}} + \underbrace{\int_0^1 \left(\frac{\partial \tilde{x}}{\partial r} \Big(\max\{b_1(\tilde{x}), x\} - b_2(\tilde{x}) \Big) \; d\hat{F}(\tilde{x}) \right) dx}_{\substack{\lesseqgtr 0, \text{ lower bidding by type } \tilde{x} \text{ but} \\ \text{greater availability of outside option}}} + \underbrace{\int_{\tilde{x}}^1 \frac{\partial b_2(v)}{\partial r} \; d\hat{F}(v)}_{\substack{>0, \text{ higher bidding} \\ \text{for } v > \tilde{x}}}.$$

The derivative consists of the three bracketed terms. The first term evaluates to 0 since bidding is not a function of list price for types with valuables below $\tilde{x}$. However, types above this threshold bid higher when r increases and thus the third bracketed term above is positive. The second term is more complicated because a higher list price causes the threshold type's bid to jump down (from r to $\frac{2}{3}\tilde{x}$, as seen in Figure 2), which lowers revenue, but also expands the region over which the seller can exercise the outside option, since $\frac{\partial \tilde{x}}{\partial r} > 0$. The net effect is ambiguous in general and will depend on the current value of r.



(a) In dashed red, $(r = 0.3, \tilde{x} \approx 0.32)$. In blue, $(r' = 0.4, \tilde{x}' \approx 0.44)$.

(b) In dashed red, $(r = 0.7, \tilde{x} \approx 0.87)$. In blue, $(r' = 0.75, \tilde{x}' \approx 0.95)$.

Figure 3: Bidding functions on $[\tilde{x}, 1]$ for different values of r when $n = 2$

Considering small discrete changes is also instructive. In Figure 3a we plot the equilibrium bid function for $r = 0.3$ in dashed red and a marginally higher $r' = 0.4$ in blue for the case of two bidders. Each bid function has its own jump threshold, which evaluate to approximately $\tilde{x} = 0.32$ for $r = 0.3$ and $\tilde{x}' = 0.44$ for $r' = 0.4$. Since both bid functions equal $\frac{2}{3}v$ below $\tilde{x}$ we truncate the graph below that point to focus on where the bid functions differ.

When increasing r from 0.3 to 0.4 bidders with values in $[\tilde{x}, \tilde{x}']$ now have a discretely lower probability of winning since their bids will be below r, thus causing their bids to be discretely lower than when $r = 0.3$. This effect lowers the seller's revenue and corresponds to the blue line segment below the red curve in the figure. However, there are two offsetting effects that increase revenue. First, with marginally higher r the seller is now able to exercise his outside option when the high bid lands in $[\tilde{x}, \tilde{x}']$, in the event that it would be advantageous to do so. Second, as previously mentioned, the bid function is increasing in r for bidders with valuations above the threshold $\tilde{x}$. This corresponds to the segment of the blue curve that exceeds the red curve in the figure.

We have seen that the seller will select $r > 0$ and that higher r induces a tradeoff between pushing down some bids and increasing others, among other things. We now

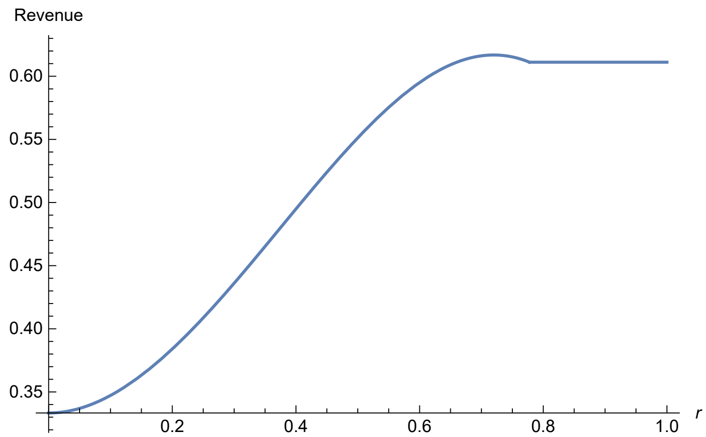


Figure 4: Expected payoff of the seller as a function of reserve price $r \in [0, 1]$ when there are 2 bidders and iid valuations $v_i \sim U[0, 1]$. Numerical calculations show a maximizer occurs at $r = 0.72$ resulting in expected revenue of 0.616833.

give some intuition for why r^* will be interior. First, higher r shifts the interval $[\tilde{x}, \tilde{x}']$ to the right, and so it is both less likely the seller's outside option will be exercised and on average less beneficial when it is. Second and relatedly, for high enough $r < 1$ we have $\tilde{x} < 1$ but $\tilde{x}' = 1$, and so the region on which bids are pushed up gets smaller and approaches 0. For example, when $n = 2$ a list price of $\frac{7}{9}$ induces a threshold of 1 and thus revenue is constant in r when $r > \frac{7}{9}$. See Figures 3b and 4.

Proposition 2 *The revenue maximizing list price is increasing in the number of bidders n. In particular, $r^* = \frac{9n^2+12n+12}{9n^2+24n+16}$, which results in bidding threshold $\tilde{x} = \frac{3n+3}{3n+4}$ that is also increasing in n.*

Proof See the appendix. ■

An explicit expression for the maximized value of revenue as a function of n can be found by substituting r^* into line (9) in the appendix and reducing. However, this expression is unwieldy so we omit it here. Instead, in Figure 5 we plot the seller's expected revenue when selecting r^* for bidders ranging from 2 up to 50.

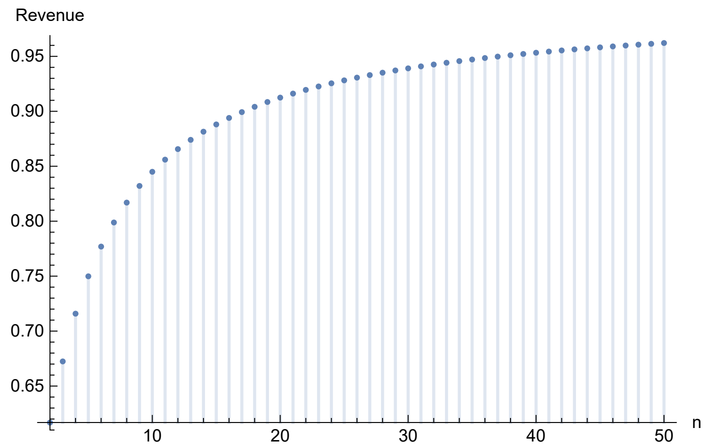


Figure 5: The seller's expected revenue when selecting the optimal list price, for $n \in \{2, ..., 50\}$.

4 Metrics of market hotness

In this section we consider comparative statics with respect to the number of bidders, n, and relate them to commonly used notions of market hotness. It should be noted that within the model it is the distribution of the maximum value of the n bidders that determines the equilibrium bid function, and this distribution is stochastically increasing in n in the sense of first order stochastic dominance. Thus for the purposes of determining the sellers payoff and optimal list price we can interpret any other unmodeled variable that would stochastically increase the bidders' valuations as equivalent to an increase in n. Such unmodeled variables might include an increase in bidders' income or an increase in the trendiness of the neighborhood in which the seller's house is located, for example.

As discussed in the Introduction, one widely used metric of market hotness is the ratio of sales prices to list prices. We have also argued here that the list price functions somewhat as a reserve price. It is interesting to note then, that in a standard first price auction (in which the reserve price fully binds) the seller's optimal reserve price depends on the bidders' distribution of values and the seller's own value for the house, *but not on the number of bidders.*[4] Thus if an increase in the number of bidders pushes up the expected sales price this would automatically increase the ratio of sales to list prices. However, if list prices also increase with the number of bidders, as has been suggested in this paper, then it is not immediately clear in what direction the sales to list price ratio will move. Below we will show that in our model the optimal list price increases slower in the number of bidders than the expected sales price increases, and thus the sales to list price ratio is increasing in n.

Average list prices and sales prices

We must first calculate the sales price, or more precisely the expected sales price conditional on a sale. A sale fails to occur whenever the highest bid is both below the list price r and the seller's outside option. Thus the probability of a sale is $1 - \int_0^{\tilde{x}} (1 - F_s(b(v)))\, nv^{n-1} dv$. Substituting $b(v) = \frac{n}{n+1}v$ and using the expression for revenue from bidders given in line (7), the expected revenue raised from bidders conditional on a sale occurring is

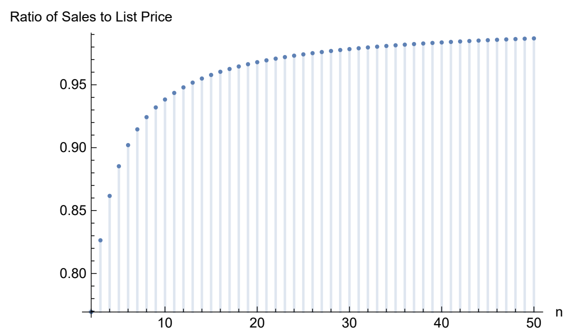
$$\frac{\left[\int_0^{\tilde{x}} F_s\left(\frac{n}{n+1}v\right)\left(\frac{n}{n+1}v\right) nv^{n-1} dv\right] + \left[\int_{\tilde{x}}^1 \left(\left(\frac{n-1}{n}\right) v + \frac{r\tilde{x}^{n-1}}{v^{n-1}} - \left(\frac{n-1}{n}\right)\frac{\tilde{x}^n}{v^{n-1}}\right) nv^{n-1}\, dv\right]}{1 - \int_0^{\tilde{x}} \left(1 - F_s\left(\frac{n}{n+1}v\right)\right) nv^{n-1} dv}. \tag{6}$$

One can then divide this by the seller's list price as calculated in Proposition 2 to obtain the sales to list price ratio.

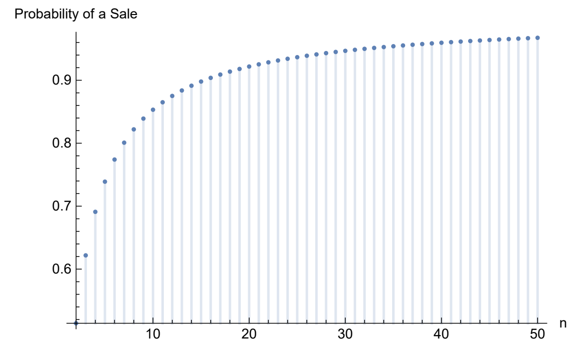
Proposition 3 *The list price, the expected sales price, and the ratio of sales price to list price are all increasing in the number of bidders n.*

Proof Proposition 2 established that the seller's optimal list price r^* is increasing

[4] This is a well-known and yet at first surprising result from auction theory. If bidders are symmetric and the hazard rate function associated with their distribution of values is denoted $\lambda(x)$, and the seller's valuation of the object is x_0, then the optimal reserve price r^* solves $r^* - \frac{1}{\lambda(r^*)} = x_0$, and thus does not depend on n (Krishna, 2002).



(a) The ratio of the sales price to list price



(b) The probability of a sale

Figure 6: Metrics of market hotness as a function of the number of bidders, $n \in \{2, ..., 50\}$

in n. Substituting r^* and its corresponding $\tilde{x}$ into line (6) and reducing shows that the expected sales price increases in n. Another direct calculation shows the ratio of these two is also increasing in n. ■

See Figure 6a for a depiction of the sales-to-list price ratio for bidders ranging from 2 up to 50.

Time on the market and transaction volume

Although we use a static model one can interpret a higher probability of a sale as lower time on the market and higher transaction volume.

Proposition 4 *The probability of a sale increases in n.*

Proof The probability of a sale evaluates to $1 - \frac{3^n}{(n+1)^{n-1}(3n+4)^{n+1}(7n+4)}$, which upon inspection is seen to increase in n. ■

See Figure 6b for the probability of a sale for bidders ranging from 2 up to 50.

Testable implications

Propositions 3 and 4 present theoretical predictions in line with empirical observation and other theoretical models in the literature (albeit through a novel theoretical mechanism). One distinguishable feature of our model is the gap in the equilibrium bidding function (see Figure 2) below the list price r. We can quite easily calculate the size of this gap as a function of the number of bidders, assuming the seller selects the optimal list price as given in Proposition 2:

$$\begin{aligned} r - \frac{n}{n+1}\tilde{x} &= \frac{9n^2+12n+12}{9n^2+24n+16} - \frac{n}{n+1}\,\frac{3n+3}{3n+4} \\ &= \frac{12}{(3n+4)^2}, \end{aligned}$$

which is decreasing in n.

Proposition 5 *The interval in which no bids are placed,* $(\frac{n}{n+1}\tilde{x}, r)$*, decreases in size as the number of bidders increases.*

5 Conclusion

In this paper we have shown that in hot markets (those in which the number of buyers has increased or their distribution of valuations has stochastically increased in the sense of FOSD) average sales prices increase but list prices increase faster. This arises in a model in which the list price has partial commitment power- offers above the list price must be accepted but those below are only accepted with an exogenous probability that is increasing in the bid.

6 Appendix

Proof of Remark 1 Part(i). Straightforward, though tedious, calculations establish the results on $\tilde{x}$.

Part (ii). When $v < \tilde{x}$ we have $\frac{b(v)}{dn} = \frac{n}{n+1} > 0$. When $v \geq \tilde{x}$, [COME BACK]. Thus each piece of the piecewise function $b(v)$ increases in n. Finally, since the point of discontinuity $\tilde{x}$ decreases in n and $b_1(\tilde{x}) < b_2(\tilde{x})$ this implies $\frac{b(v)}{dn} > 0$ for all v.

To see that $\frac{b(v)}{dr} > 0$ when $v \geq \tilde{x}$, on the relevant domain we calculate

$$\frac{\partial b()}{\partial r} = \frac{1}{v^{n-1}} \left(\tilde{x}^{n-1} + (n-1)\tilde{x}^{n-2}\frac{d\tilde{x}}{dr}r - \frac{n-1}{n} n\tilde{x}^{n-1}\frac{d\tilde{x}}{dr} \right).$$

Substituting for $\tilde{x}$ and $\frac{d\tilde{x}}{dr} = \frac{n+1}{\sqrt{(n+1)^2 - 4rn}} > 0$ and reducing gives the result. ■

Proof of Proposition 2 The proof takes the following steps. First, we write down the expression for the seller's expected revenue as a function of $\tilde{x}$ (which itself is determined by r). Next, by substituting from line (4) and rejecting the solution $r = 0$ we simplify the expression for revenue down to a *quadratic* function of $\tilde{x}$, which easily gives us two candidate solutions. We then argue that one of these solutions is outside the feasible range and confirm the other is a maximizer, not a minimizer.

We take the expression for revenue from line (5) and substitute $d\hat{F}(v) = nx^{n-1}\,dv$, evaluate the max operator, and rearrange terms to show revenue received from outside option and from bidders:

$$\begin{aligned} R \;=\; & \underbrace{\left[\int_0^{\tilde{x}} \left(\int_{\frac{n}{n+1}v}^{1} v_s dv_s \right) nv^{n-1}\, dv \right]}_{\text{Revenue from the outside option}} \quad + \qquad (7) \\ & \underbrace{\left[\int_0^{\tilde{x}} \left(\frac{n}{n+1} v \right)^2 nv^{n-1}\, dv + \int_{\tilde{x}}^{1} \left(\left(\frac{n-1}{n} \right) v + \frac{r\tilde{x}^{n-1}}{v^{n-1}} - \left(\frac{n-1}{n} \right) \frac{\tilde{x}^n}{v^{n-1}} \right) nv^{n-1}\, dv \right]}_{\text{Revenue from bidders}} \\ =\; & \left[\frac{\tilde{x}^n}{2} - \frac{1}{2} \left(\frac{n}{n+1} \right)^2 n \frac{\tilde{x}^{n+2}}{n+2} \right] \\ & + \left[\left(\left(\frac{n}{n+1} \right)^2 n \frac{\tilde{x}^{n+2}}{n+2} \right) + \left(\frac{n-1}{n+1}(1 - \tilde{x}^{n+1}) + nr\tilde{x}^{n-1} - (nr + n - 1)\tilde{x}^n - (n-1)\tilde{x}^{n+1} \right) \right] \\ =\; & \left(\frac{n^3}{2(n+1)^2(n+2)} \right) \tilde{x}^{n+2} + \left(\frac{n^2 - n}{n+1} \right) \tilde{x}^{n+1} + \left(\frac{3}{2} - nr - n \right) \tilde{x}^n + nr\tilde{x}^{n-1} + \frac{n-1}{n+1}. \end{aligned}$$

To simplify matters we drop the tilde on x and employ the notation $c_1 = \left(\frac{n^3}{2(n+1)^2(n+2)}\right)$, $c_2 = \left(\frac{n^2-n}{n+1}\right)$, $c_3 = \left(\frac{3}{2} - n\right)$, and $c_4 = \frac{n-1}{n+1}$. Thus

$$R = c_1 x^{n+2} + c_2 x^{n+1} + c_3 x^n - nrx^n + nrx^{n-1} + c_4. \tag{8}$$

Next, line (4) implies

$$\begin{aligned} -nrx^n &= \frac{n^2}{(n+1)^2} x^{n+2} - nx^{n+1}, \text{ and} \\ nrx^{n-1} &= \frac{-n^2}{(n+1)^2} x^{n+1} + nx^n, \end{aligned}$$

which upon substitution into line (8) gives:

$$\begin{aligned} R &= c_1 x^{n+2} + c_2 x^{n+1} + c_3 x^n + \frac{n^2}{(n+1)^2} x^{n+2} - nx^{n+1} + \frac{-n^2}{(n+1)^2} x^{n+1} + nx^n + c_4 \\ &= c_1' x^{n+2} + c_2' x^{n+1} + c_3' x^n + c_4, \end{aligned} \tag{9}$$

where $c_1' = \left(c_1 + \frac{n^2}{(n+1)^2}\right)$, $c_2' = \left(c_2 - n - \frac{n^2}{(n+1)^2}\right)$, and $c_3' = (c_3 + n)$. Next,

$$\begin{aligned} \frac{dR}{dr} &= c_1' (n+2) x^{n+1} \frac{dx}{dr} + c_2' (n+1) x^n \frac{dx}{dr} + c_3' n x^{n-1} \frac{dx}{dr} \\ &= c_1'' x^{n+1} \frac{dx}{dr} + c_2'' x^n \frac{dx}{dr} + c_3'' x^{n-1} \frac{dx}{dr}, \end{aligned} \tag{10}$$

where $c_1'' = c_1' (n+2)$, $c_2'' = c_2' (n+1)$, and $c_3'' = c_3' n$. Thus

$$\frac{dR}{dr} = 0 \quad \Longleftarrow \quad c_1'' x^{n+1} + c_2'' x^n + c_3'' x^{n-1} = 0. \tag{11}$$

By the discussion preceding Proposition 2, $x = 0$ is not an optimizer and so the problem reduces to the quadratic

$$c_1'' x^2 + c_2'' x + c_3'' = 0, \tag{12}$$

which has solutions

$$\frac{1}{2c_1''}\left(-c_2''+\sqrt{(c_2'')^2-4c_1''c_3''}\right) \quad \text{and} \quad \frac{1}{2c_1''}\left(-c_2''-\sqrt{(c_2'')^2-4c_1''c_3''}\right). \tag{13}$$

For clarity, after substituting for c_i and c_i' $(i = 1, 2, 3)$ and reducing the coefficients become $c_1'' = \frac{3n^3+4n^2}{2(n+1)^2} > 0$, $c_2'' = -2n - \frac{n^2}{n+1} < 0$, and $c_3'' = \frac{3}{2}n > 0$. Substituting in for these values and reducing gives the two solutions $-\frac{(n+1)^2}{3n^3+4n^2}\left(-2n - \frac{n^2}{(n+1)} \pm \sqrt{\frac{4n^2}{(n+1)^2}}\right)$, which upon further simplification reduces to $\frac{n+1}{n} > 1$ and $\frac{3n+3}{3n+4} \in [0, 1]$. We reject the former since it is outside the permissible range. Substituting $\frac{3n+3}{3n+4}$ into the expression for $\tilde{x}$ from Proposition 1 and solving for r gives $r = \frac{9n^2+12n+12}{9n^2+24n+16}$.

We now argue the second derivative of R is negative at $x = \frac{3n+3}{3n+4}$ and thus this critical point is a maximizer, not a minimizer. Differentiating line (10) gives

$$\begin{aligned}
\frac{d^2R}{dr^2} &= c_1''(n+1)\,x^n\left(\frac{dx}{dr}\right)^2 + c_1''\frac{d^2x}{dr^2}x^{n+1} \\
&\quad + c_2''nx^{n-1}\left(\frac{dx}{dr}\right)^2 + c_2''\frac{d^2x}{dr^2}x^n \\
&\quad + c_3''(n-1)\,x^{n-2}\left(\frac{dx}{dr}\right)^2 + c_3''\frac{d^2x}{dr^2}x^{n-1} \\
&= x^{n+1}\left(c_1''\frac{d^2x}{dr^2}\right) + x^n\left(c_2''\frac{d^2x}{dr^2} + c_1''(n+1)\left(\frac{dx}{dr}\right)^2\right) \\
&\quad + x^{n-1}\left(c_3''\frac{d^2x}{dr^2} + c_2''n\left(\frac{dx}{dr}\right)^2\right) + x^{n-2}\left(c_3''(n-1)\left(\frac{dx}{dr}\right)^2\right).
\end{aligned}$$

We now rearrange terms so that we can substitute from line (11):

$$\begin{aligned}
\frac{d^2R}{dr^2} &= \frac{d^2x}{dr^2}\left(c_1''x^{n+1} + c_2''x^n + c_3''x^{n-1}\right) \\
&\quad + (n-1)\left(\frac{dx}{dr}\right)^2\left(c_1''x^n + c_2''x^{n-1} + c_3''x^{n-2}\right) \\
&\quad + 2c_1''\left(\frac{dx}{dr}\right)^2 x^n + c_2''\left(\frac{dx}{dr}\right)^2 x^{n-1}
\end{aligned}$$

Thus when the first order condition in line (11) is satisfied the first two lines above evaluate to 0 and thus the second derivative reduces to

$$\frac{d^2R}{dr^2} = 2c_1''\left(\frac{dx}{dr}\right)^2 x^n + c_2''\left(\frac{dx}{dr}\right)^2 x^{n-1},$$

which we claim is negative. Since $\left(\frac{dx}{dr}\right)^2 > 0$ it suffices to establish $2c_1''x^n + c_2''x^{n-1} < 0$, which upon substitution for c_1'', c_2'', and $x = \frac{3n+3}{3n+4}$ and much simplification is shown to be equivalent to $3n + 2 > 0$, which is true.

Finally, a direct calculation of $\frac{dr^*}{dn}$ and $\frac{d\tilde{x}}{dn}$ reveals that both are positive. ■

References

Albrecht, J., Gautier, P. A., & Vroman, S. (2016). Directed search in the housing market. *Review of Economic Dynamics*, *19*, 218–231.

Arnold, M. A. (1999). Search, bargaining and optimal asking prices. *Real estate economics*, *27*(3), 453–481.

Burguet, R., & Sakovics, J. (1996). Reserve prices without commitment. *Games and Economic Behavior*, *15*(2), 149–164.

Carrillo, P. E. (2013). To sell or not to sell: Measuring the heat of the housing market. *Real estate economics*, *41*(2), 310–346.

Chen, Y., & Rosenthal, R. W. (1996a). Asking prices as commitment devices. *International Economic Review*, 129–155.

Chen, Y., & Rosenthal, R. W. (1996b). On the use of ceiling-price commitments by monopolists. *The RAND Journal of Economics*, 207–220.

Chernobai, E., & Hossain, T. (2012). House search duration in hot and cold residential markets. *Real Estate Economics*, *40*(4), 809–841.

Dale-Johnson, D., & Hamilton, S. W. (1998). Housing market conditions, listing choice and mls market share. *Real Estate Economics*, *26*(2), 275–307.

De Wit, E. R., & Van der Klaauw, B. (2013). Asymmetric information and list-price reductions in the housing market. *Regional Science and Urban Economics*, *43*(3), 507–520.

Han, L., & Strange, W. C. (2016). What is the role of the asking price for a house? *Journal of Urban Economics*, *93*, 115–130.

Harstad, R. M., Kagel, J. H., & Levin, D. (1990). Equilibrium bid functions for auctions with an uncertain number of bidders. *Economics Letters*, *33*(1), 35–40.

Khezr, P., & Menezes, F. (2018). Auctions with an asking price. *International Journal of Game Theory*, *47*(4), 1329–1350.

Krainer, J. (2001). A theory of liquidity in residential real estate markets. *Journal of urban Economics*, *49*(1), 32–53.

Krishna, V. (2002). *Auction theory*. Academic Press.

Lester, B., Visschers, L., & Wolthoff, R. (2017). Competing with asking prices. *Theoretical Economics*, *12*(2), 731–770.

Li, Y., & Yavas, A. (2015). Residential brokerage in hot and cold markets. *The Journal of Real Estate Finance and Economics*, *51*(1), 1–21.

Novy-Marx, R. (2009). Hot and cold markets. *Real Estate Economics*, *37*(1), 1–22.

Stacey, D. (2019). Posted prices, search and bargaining. *Review of Economic Dy-*

namics, *33*, 85–104.