Product line selection

With the objective of maximizing profit, a fast-moving consumer goods manufacturer has to choose the subset $W \subseteq \{1, ..., J\}$ of products to make and sell. In a given planning horizon (i.e., throughout a year), a selection generates $\mathrm{R}(W)$ gross profit, determined by the choices consumers make. On the other hand, the manufacturer needs to incur $\mathrm{C}(W)$ indirect costs to produce and sell the selected products, which depend on the production decisions made for each planning period (i.e., monthly). Expression (1) is the general objective function of this problem.

$$\max_{W} \quad \mathrm{R}(W) - \mathrm{C}(W) \tag{1}$$

Each product j has a contribution margin p_j and, during period t, $D_{jt}(W)$ units are sold, leading to $\mathrm{R}(W) = \sum_t \sum_j p_j D_{jt}(W)$. The demand for a product depends on W and, while it may not be profitable or even feasible for a manufacturer to satisfy the demand fully, the amounts sold $D_{jt}(W)$ will never exceed it. At each period t, the manufacturer will carry inventory I_{jt} with a unitary holding cost h_j. Additionally, a setup for j, denoted by binary variable Y_{jt} and resulting in a cost θ_j, is incurred to allow a quantity X_{jt} to be produced (with p_j including the variable production cost). The manufacturing plant has a capacity c_t available at each period t that is spent either producing or on changeovers. Setting up for a product j uses up τ_j available time, and each unit of the product takes r_j time to be produced. By computing the amounts to produce and stock that enable $D_{jt}(W)$ to be made available for sale, the manufacturer estimates the costs associated with selection $\mathrm{C}(W) = \sum_t \sum_j h_j I_{jt} + \sum_t \sum_j \theta_j Y_{jt}$.

Throughout the following subsections, we introduce the model for the PLS of FMCG. In the following section we provide a consumer response model able to handle both the substitution within the product categories that a manufacturer produces and the independence between them. The integration of the demand management model with the constraints in the subsection after that one, adds the multi-period production planning needed to account for the complexity costs of offering the line, including inventory which is crucial in capital-intensive manufacturing businesses. We also provide extensions for the formulation to include fixed administrative costs and shared setups.

Consumer response

Among all J products that the manufacturer can offer, some do not compete with each other for the same consumers. Let $\mathcal{N}^k$ be the subset of products that belong to an independent category k. A product $j \in \mathcal{N}^k$ can be only be substituted by others in $\mathcal{N}^k$ and options offered by competitors (denoted by $j = 0$). Considering d_t^k as the demand for category k in period t, and $S_j(W^k)$ the probability of a consumer selecting product j as function of the selection within its category $W^k = W \cap \mathcal{N}^k$, the demanded amount for the product will be computed as $d_t^k S_j(W^k)$. We assume that, within each category, consumer response follows an attraction model. As Figure 1 represents, the share of a

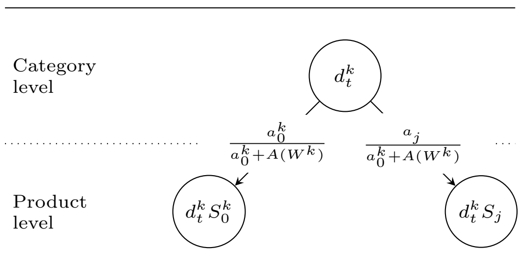

Figure 1: Demand structure for this problem. The category demand is determined exogenously. Inside subcategories, product share results from an attraction model.

product is computed as its attractiveness a_j divided by that of the assortment $A(W^k) = \sum_j a_j$, with $j \in W^k$, and the competition a_0^k in that subcategory. Hence, $S_j(W^k) = a_j/(a_0^k + A(W^k))$ for all k and $j \in \mathcal{N}^k$. Also, consumers cannot purchase products that are not selected, implying that $S_j = 0$ for any $j \notin W$. The following constraints capture these relations.

$$D_{jt} \leq d_t^k S_j \quad \forall t, k, j \in \mathcal{N}^k \tag{2}$$

$$S_j \leq W_j \quad \forall j \tag{3}$$

$$a_0^k S_j = a_j S_0^k - F_j \quad \forall k, j \in \mathcal{N}^k \tag{4}$$

$$F_j \leq a_j S_0^k \quad \forall k, j \in \mathcal{N}^k \tag{5}$$

$$S_0^k + \sum_{j \in \mathcal{N}^k} S_j = 1 \quad \forall k \tag{6}$$

$$W_j \in \{1, 0\} \quad \forall j \tag{7}$$

$$D_{jt}, S_j, S_0^k, F_j \in \mathbb{R}_0^+ \quad \forall t, k, j \in \mathcal{N}^k \tag{8}$$

Constraints (2)-(8) model consumer response. Constraints (2) set the demanded amounts as an upper limit on sales. Let $W_j = 1$ if $j \in W$ and $W_j = 0$ otherwise, constraints (3) only allow selected products to be demanded. Constraints (4) define that the ratios between the shares behave according to an attraction model, within each category. They ensure that the ratio between the share of a product and the share S_0^k of the competing options in its category is, at most, their attractiveness ratio a_j/a_0^k (i.e., $S_j/S_0^k \leq a_j/a_0^k$, where the equality is obtained by adding the slack variables F_j). Constraints (5) and (6) establish that, if the manufacturer does not offer any products in a subcategory, the competition will absorb all of the market share. In this situation, F_j in constraints (4) relax the relationship between the attractiveness and share ratios. Given that our objective encourages us to maximize the shares of selected products, and that the sum of all shares in a subcategory is limited by constraints (6), variables F_j will be null otherwise. Finally, constraints (7) and (8) set the domain of the variables. The problem of maximizing $\mathrm{R}(W)$ subject to constraints (3)-(8) is a simple one. The optimal selection can be determined by starting with an empty set, and adding the product with the highest margin to the choice set until no increase in revenue is possible.

Production planning

The following constraints capture the multi-period production planning, which greatly increases the complexity of the model.

$$X_{jt} \leq Y_{jt} K_{jt} \quad \forall t, j \tag{9}$$

$$\sum_j \tau_j Y_{jt} + r_j X_{jt} \leq c_t \quad \forall t \tag{10}$$

$$D_{jt} = I_{jt-1} + X_{jt} - I_{jt} \quad \forall t, j \tag{11}$$

$$Y_{jt} \in \{1, 0\} \quad \forall j, t \tag{12}$$

$$X_{jt}, I_{jt} \in \mathbb{R}_0^+ \quad \forall j, t \tag{13}$$

Constraints (9) - (13) model manufacturing. They impact the objective function both directly, with inventory and setup costs, and indirectly, by constraining the amounts that can be produced and, consequently, sold. Constraints (9) express that the plant must be setup for a product if any given quantity is to be manufactured. The upper bound for variables X_{jt} is the minimum between the maximum possible demand for the product j in remainder of the horizon, and the maximum possible output of j in period t. This implies that, in order for the formulation to be valid, constants K_{jt} need to satisfy $K_{jt} \geq \min\left\{ \sum_{t_2} d_{t_2}^k a_j/(a_j + a_0^k);\ (c_t - \tau_j - \tau^m)/r_j \right\}\ \forall t, j, t_2 \in \{t, ..., T\}$ and $k : j \in \mathcal{N}^k$. Constraints (10) model the plant's capacity, which is shared among all products and split between production and setups. Constraints (11) ensure inventory is balanced between periods by requiring sold products to be either produced in that period or stocked from a previous one. Together with the capacity limitations, these constraints impose a limit on the sales the manufacturer can make, so $D_{jt} = \min\{d_t^k S_j, I_{jt-1} + X_{jt}\}$, given that the objective will try to maximize D_{jt}. Lastly, constraints (12) and (13) define the domain of the variables.

With objective (1), subject to constraints (2)-(13), we introduce the mixed-integer linear formulation for the selection of fast-moving consumer goods. The formulation may be extended with implementation-based considerations. For cases where product development and administrative costs are not negligible, a fixed cost per product throughout the planning horizon should be considered. Let f_j be the fixed development and introduction cost associated with product j, the term $\sum_j f_j W_j$ should be added to C(W). It is also common for FMCG to share part of the changeover across a set of products. Similarly to the categories defined for the market structure, manufacturing can have families that share a portion of the setup procedure.

$$X_{jt} \leq Y_t^m K_{jt} \quad \forall m, t, j \in \mathcal{N}_m \tag{14}$$

$$\sum_m \tau^m Y_t^m + \sum_{j \in \mathcal{N}_m} \tau_j Y_{jt} + r_j X_{jt} \leq c_t \quad \forall t \tag{15}$$

With $\mathcal{N}_m$ being the set of products that belong to family m and $Y_t^m \in \{1, 0\}$ $\forall j, t$, constraints (14) ensure $Y_t^m = 1$ whenever a product of that family is produced in that period. Constraints (15) modify constraints (10) to include the setup times τ^m for the families. The constants in constraints (9) are also used in constraints (14) and the conditions for the formulation to be valid become $K_{jt} \geq \min\big\{ \sum_{t_2} d_{t_2}^k a_j/(a_j + a_0^k);\ (c_t - \tau_j - \tau^m)/r_j \big\}$ $\forall t, j$, with $t_2 \in \{t, ..., T\}$ and $(k, m) : j \in \mathcal{N}^k \cap \mathcal{N}_m$. To finally incorporate the effect of resource sharing, category setup costs θ^m are accounted for by adding the term $\sum_m \theta^m Y_t^m$ to C(W).

Instances

Instances are organized in three folders: real, solved and unsolved. In the solved folder, you find sixty instances for which the optimal objective value is known. The unsolved folder contains larger instances which were never tackled. Finally, the real folder contains data from a real production line. The data in each file is ordered as follows:

T	Planning horizon.
J	Number of potential products.
K	Number of categories.
M	Number of product families.
p_j	Margin of product j
h_j	Cost of holding a unit of j.
θ_j	Setup cost for product j.
τ_j	Setup time for product j.
a_j	Attractiveness of product j.
f_j	Fixed administrative cost for producing j.
r_j	Time that a unit of j takes to produce.
d_t	Demand in period t.
c_t	Capacity in period t.
s_t^k	Share of category k.
a_0^k	Attractiveness of the competition in category k.
τ^m	Setup time for family m.
θ^m	Setup cost for family m.
$\mathcal{N}_m$	Set of products belonging to family m.
$\mathcal{N}^k$	Set of products belonging to category k.