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Product line selection of fast-moving consumer goods[☆]

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ABSTRACT

The fast-moving consumer goods sector relies on economies of scale. However, its assortments have been overextended as a means of market share appropriation and top-line growth. This paper studies the selection of the optimal set of products for fast-moving consumer goods producers to offer, as there is no previous model for product line selection that satisfies the requirements of the sector. Our mixed-integer programming model combines a multi-category attraction model with a capacitated lot-sizing problem, shared setups and safety stock. The multi-category attraction model predicts how the demand for each product responds to changes within the assortment. The capacitated lot-sizing problem allows us to account for the indirect production costs associated with different assortments. As seasonality is prevalent in consumer goods sales, the production plan optimally weights the trade-off between stocking finished goods from a long run with performing shorter runs with additional setups. Finally, the safety stock extension addresses the effect of the demand uncertainty associated with each assortment. With the computational experiments, we assess the value of our approach using data based on a real case. Our findings suggest that the benefits of a tailored approach are at their highest in scenarios typical fast-moving consumer goods industry: when capacity is tight, demand exhibits seasonal patterns and high service levels are required. This also occurs when the firm has a strong competitive position and consumer price-sensitivity is low. By testing the approach in two real-world instances, we show that this decision should not be made based on the current myopic industry practices. Lastly, our approach obtains profits of up to 9.4% higher than the current state-of-the-art models for product line selection.

1. Introduction

Fast-moving consumer goods (FMCG) producers are offering suboptimal assortments. The sector is composed of capital-intensive plants that manufacture low-margin high-volume products, such as beverages and toiletries. Investment sizes associated with these kinds of plants make capacity a hard-constraint. Additionally, the demand for goods such as beverages exhibits a clear seasonal pattern, further complicating capacity allocation. Nevertheless, product introductions have been widely used for market share appropriation and entry deterrence [1], and broad product lines cut economies of scale short, nibbling at the already thin margins practised within the sector. It is also common for manufacturers to find themselves producing multiple categories (e.g. beer, water and juices), which further complicates operations.

Firms matching the introductions of their rivals are led into a competitive trap where operational costs are high, the size of the market does not catch up with the size of the line, and delisting the wrong products potentially results in ruin. Consultants currently recommend such firms to focus on their core product portfolio while taking advantage of high-margin consumer trends, such as organic packaged goods [2]. The competitive trap creates an opening for a manufacturer to become price and quality leader by selecting the optimal product line and concentrating its efforts on the products that the consumers really seek. Practitioners are aware of this fact and a recent trend for stock-keeping-unit (SKU) rationalization. Still, in the absence of models to assist their product line decisions, some FMCG firms myopically make product removals only taking share or margin into account, and not its impact on operations and the sales of other products. With the increasing cooperation between retailers and manufacturers [3,4] along with several FMCG brands opting for direct-to-consumer (D2C) channels, such as brand-owned online stores [5] or even brick-and-mortar stores exclusively for their products, the urgency for these models increases.

Optimal lot sizing and scheduling can only do so much, and product line selection (PLS) can only do so much without it. There is ample research on planning the production of FMCG [6]. However, with the dissemination of these approaches, higher-level decisions need to be optimized for substantial competitive advantages over other firms to arise. Still, current product line selection

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models disregard some subtleties required to assess the performance of a FMCG line accurately. Even the approaches with the most sophisticated production planning models [7,8] are single-period. They consider a fixed production frequency, neglecting how inventory levels can be adjusted to compensate for variations in demand (i.e., dealing with seasonality) which is critical for FMCG [9]. Furthermore, these approaches tend to oversimplify consumer response to a single-choice process. Although it fits the high-involvement purchases of hard goods (e.g., cars, computers), it is unreasonable to assume that a consumer segment will wholly converge to a single FMCG option. Attraction models define the share of a product as its attractiveness divided by that of the whole assortment (including competition), which corresponds better to consumer behaviour on such low-involvement purchases. Consumer choice was first tackled this way in PLS by Schön [10], with no work contemplating multiple product categories. Still, the gap for PLS models with attraction consumer response and multi-period production planning remains unfilled.

In this paper, we propose a mixed-integer programming (MIP) model for the PLS of FMCG and quantify its benefits when compared with the state-of-the-art models and heuristics that practitioners use. For consumer response, we develop a linear formulation for the multi-category attraction model, which can simulate low-involvement purchases even if the manufacturer produces different categories within a plant. While the demanded amounts depend on the selection and are defined by the multi-category attraction model, production costs and quantities are estimated based on the well-known capacitated lot-sizing problem. As required for high-margin low-volume products, the model accounts for the right number of setups and the amounts of stock that are carried over, thus resulting in a more accurate cost estimation than previous PLS models. Moreover, the model also takes into account the costs of an inventory build-up when anticipating an increase in demand, and the potential benefits of carrying inventory instead of having to set up for a product when demand is at a trough. While it is not our objective to determine the production plan, it serves to assess the indirect costs associated with a product line and hence, obtain a feasible and cost-efficient product line.

The MIP model is the central contribution of this work. We complement it by proposing a safety stock modelling approach, addressing the demand uncertainty associated with a product line and its impact on holding costs. By using our model to perform computational experiments, we assess its value and derive managerial insights. Using generated instances, we show the conditions under which an integrated and multi-period model outperforms the state-of-the-art approach and the typical heuristics used by practitioners. The results make evident that our approach has the most advantage when capacity is tight, consumers seek quality, service level requirements are high, and the firm is a market leader seasonal categories. With real instances, we assess the magnitude of the benefits that our approach can bring to a real production line, relative to previous methods. They suggest that the decision should not be left to myopic sales-driven heuristics, as the profit resulting from an optimal selection surpasses that of the heuristics by 125.3% in a real case. They further show that our multi-period approach, which accounts for the setup-inventory trade-offs, beats the state-of-the-art by up to 9.4%.

The remainder of this paper is structured as follows. Section 2 provides a focused review of the PLS literature, emphasizing the different models and reflecting the gap for our work. In Section 3, we present the model for the PLS of FMCG, an extension to include the costs of safety stock, and an assessment of the computational performance of the model. Section 4 describes and exposes the results of the computational experiment designed to evaluate each contribution. Section 5 exhibits the results of the application of the model to real-world instances. We consolidate the insights derived from the results of the application of the model to both test and real-world instances in Section 6. Finally, Section 7 summarizes the conclusions and contributions of this research and makes future issues evident to explore.

2. Literature review

Interest in PLS arose from marketing research, with the realization that there are complex reactions from the market to the offered assortment. The literature on the problem traces back to Monroe et al. [11], who introduce a model for the addition and removal of products over a planning horizon, of which the resulting revenue depends on the pairwise interactions between the on-shelf products. Locander and Scannell [12] introduce a model to rank products for elimination using the covariances between the returns of each product for accounting synergies. A mathematical model for product selection, assuming a single-choice process and considering setup costs, is provided by Shugan et al. [13]. Green and Krieger [14] consolidate the problem by presenting models for the selection of a product line of a specified size, in the perspectives of both buyer and seller. With the buyer as the decision-maker, the objective is to maximize the utility of the assortment, given a single-choice model. In the seller's problem, the value of each segment is weighted in the decision, and a product is sold only provided its utility for the consumer exceeds that of the *status quo* (having a reservation value). McBride and Zufryden [15] formally apply mixed-integer programming to solve the seller's problem and extend it by incorporating fixed costs. Likewise, Dobson and Kalish [16,17] consider fixed costs associated with each product and add the pricing decision to the perspective of the seller. Additionally, the authors compare heuristics for the buyer's problem and propose decomposing the seller's problem into selection and pricing for tractability. From this point on, research on this topic either attempts to incorporate detailed manufacturing implications into models with single-choice or aims to model consumer behaviour more efficiently and meaningfully with attraction consumer response.

The importance of detailed manufacturing modelling derives from the weight of the underlying upstream costs associated with the variety in the assortment. Using single-choice consumer response, some authors developed approaches that make more sensible selections by increasing the detail in which operational costs are estimated. Dobson and Yano [18] substitute the simple fixed and variable costs in their version of the seller's model for ones associated with resource and process choice. Raman and Chhajed [19] propose a model integrating the design decision (choosing the features and attribute levels that compose a product) and price, and the assignment of production processes to attribute levels, given associated fixed and variable costs. Without pricing and product design, Morgan et al. [7] take on resource sharing by including family setups and decide on the production frequency, which enables a rough estimate of the trade-off between setup and production inventory costs. As extensions to their model, the authors provide capacity constraints and a framework to include multiple planning horizons with no inventory carry-over. The PLS problem with pricing and the make-to-order or make-to-stock decision is introduced by Dobson and Yano [8]. In this formulation capacity is explicitly modelled, with demand being parametric and affected by delivery time at a linear rate. However, the demanded amounts do not depend on the offered assortment. Day and Venkataramanan [20] extend the profit-maximizing problem from Dobson and Kalish [17] with family setups, segment sizes and a reservation price for consumers to make a purchase. The authors also compare the performance of a discrete-pricing reduction approach with genetic algorithms to solve this problem.

Single-choice can be limiting in both tractability and modelling power. As it states that a homogeneous segment will holistically go for the same alternative, it requires a large number of segments to represent a market realistically, especially if purchases require little engagement. Some PLS research tries to mitigate these implications with the use of attraction models. The multinomial logit (MNL) can be seen as a particular case of attraction models. It was introduced to PLS by Aydin and Ryan [21], who decide on pricing and divide the purchase process into arrival and choice. Chen and Hausman [22], on a technical note, undertake the product line selection problem with attraction consumer response, when the number of products to choose from is bounded. The authors show that, without fix costs, the optimal line is the solution to the linear relaxation of the problem. Steiner and Hruschka [23] present a nonlinear program for product and product line design and selection, using MNL choice probability. They consider fixed and variable costs, dependent on the attribute levels of each product, and solve the problem using a genetic algorithm. Kraus and Yano [24] present a formulation with continuous pricing that computes attractiveness as the utility-price surplus.

More recently, some authors have made developments in manufacturing modelling with attraction consumer response. Hopp and Xu [25] study a product line design, selection and pricing problem without economies of scale. The authors study the influence of reducing product introduction costs through modularity in the overall assortment strategy of a company. Schön [10] extends the PLS and pricing problem with attraction consumer response is extended by with resource sharing and fixed costs associated with resources. The model supports multiple segments, allowing price discrimination, and is the first work in PLS using linear constraints to implement the attraction model. In a technical note, Schön [26] extends her work by considering price as a bounded continuous variable and associating fix costs with capacitated attribute levels. In both works, the author takes advantage of the problem structure to make the solution tractable.

In a different research stream, Bertsimas and Mišić [27] tackle the uncertainty caused by the mismatch between choice modelling and consumer response by developing a robust approach for PLS. The authors consider that both distinct parameter estimations and structurally distinct models can constitute the uncertainty set. Hence, the approach can protect retailers from losses due to both incorrectly modelling and parameterizing consumer behaviour. In [28], the authors present a new formulation for the PLS problem with single-choice and a decomposition approach for solving it. The authors prove that the formulation provides a stronger linear relaxation than previous ones, and provide computational experiments to demonstrate the magnitude of the benefits of their approach. By not covering manufacturing issues, this research moves away from the needs of the FMCG sector. Therefore, two papers emerge as the state-of-the-art of PLS research. Morgan et al. [7], with single-choice consumer response, propose the most thorough model to account for manufacturing costs. With attraction consumer response, Schön [26] has the most advanced production modelling.

Despite these advances, there are still gaps in both marketing and manufacturing fronts holding back the use of the aforementioned research for decision making, notably, by FMCG producers. Given the extent of the lines, one FMCG plant can manufacture products that do not fall under the same market category, and therefore do not compete with each other. These categories have negligible demand interactions between them, so basic attraction models, as Schön [26] uses, do not describe well such a line. This work extends the marketing front by introducing a linear formulation for a multi-category attraction model for consumer response. The model makes it possible to represent the behaviour of several independent categories sharing the capacity of a single plant.

Moreover, the current manufacturing cost estimates are simplistic for capital-intensive low-margin industry cases, where capacity cannot be relaxed, and inventory management is critical . Morgan et al. [7], use a production frequency variable to assess the significance of the setup and inventory costs on the selection of the optimal product line. Nevertheless, extending a single-period model with production frequency is not enough to account for demand variations in a capacity-constrained scenario [29]. introduce a (single-choice) multi-period assortment problem with order and inventory costs. Our model extends the manufacturing modelling front of PLS literature by considering this multi-period setting with capacity constraints, enabling it to correctly account for setup and holding costs under demand variations. This is crucial, as the sales of many FMCG categories frequently exhibit seasonal patterns.

Lastly, and as explored by Chong et al. [30] for delisting a single product, demand predictability depends on the products that are selected. Therefore, we introduce a safety stock modelling approach that can be used to estimate the inventory needed to handle the demand uncertainty associated with a product line. In the assortment literature, Bernales et al. [31] apply a single-period assortment planning model with safety stock to rationalize the stock-keeping-units of an information technology retailer. However, their approach to safety stock considers a fixed coefficient of variation, which does not depend on the share that the product gets.

3. Mathematical model

With the objective of maximizing profit, a fast-moving consumer goods manufacturer has to choose the subset $W \subseteq \{1, \dots, J\}$ of products to make and sell. In a given planning horizon (i.e., throughout a year), a selection generates $\mathrm{R}(W)$ gross profit, determined by the choices consumers make. On the other hand, the manufacturer needs to incur $\mathrm{C}(W)$ indirect costs to produce and sell the selected products, which depend on the production decisions made for each planning period (i.e., monthly). Expression (3.1) is the general objective function of this problem.

$$\max_{W} \mathrm{R}(W) - \mathrm{C}(W) \tag{3.1}$$

Each product j has a contribution margin p_j and, during period t, $D_{jt}(W)$ units are sold, leading to $\mathrm{R}(W) = \sum_t \sum_j p_j D_{jt}(W)$. The demand for a product depends on W and, while it may not be profitable or even feasible for a manufacturer to satisfy the demand fully, the amounts sold $D_{jt}(W)$ will never exceed it. At each period t, the manufacturer will carry inventory I_{jt} with a unitary holding cost h_j. Additionally, a setup for j, denoted by binary variable Y_{jt} and resulting in a cost θ_j, is incurred to allow a quantity X_{jt} to be produced (with p_j including the variable production cost). The manufacturing plant has a capacity c_t available at each period t that is spent either producing or on changeovers. Setting up for a product j uses up τ_j available time, and each unit of the product takes r_j time to be produced. By computing the amounts to produce and stock that enable $D_{jt}(W)$ to be made available for sale, the manufacturer estimates the costs associated with selection $\mathrm{C}(W) = \sum_t \sum_j h_j I_{jt} + \sum_t \sum_j \theta_j Y_{jt}$.

Throughout the following subsections, we introduce the model for the PLS of FMCG. In Subsection 3.1 we provide a consumer response model able to handle both the substitution within the product categories that a manufacturer produces and the independence between them. The integration of the demand management model with the constraints in Subsection 3.2, adds the multi-period production planning needed to account for the costs of hedging demand variations with inventory, which is crucial in such capital-intensive businesses. We also provide extensions for our formulation to include fixed administrative costs and shared setup. Subsection 3.3 provides an approach to safety stock modelling, and to include the respective costs in the selection problem. Then, in

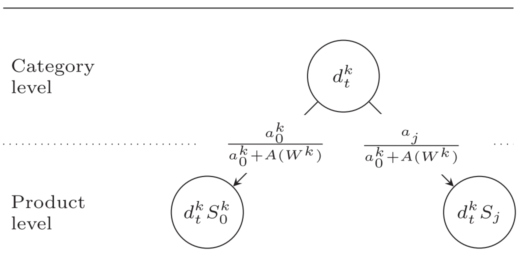

Fig. 1. Demand structure for this problem. The category demand is determined exogenously. Inside subcategories, product share results from an attraction model.

Subsection 3.4, we display the bounds of each parameter for the generation of the virtual instances and make a brief assessment of the performance of the formulation. We have centralized the notation used throughout this section in Appendix B.

3.1. Consumer response

Among all J products that the manufacturer can offer, some do not compete with each other for the same consumers. Let $\mathcal{N}^k$ be the subset of products that belong to an independent category k. A product $j \in \mathcal{N}^k$ can be only be substituted by others in $\mathcal{N}^k$ and options offered by competitors (denoted by $j = 0$). Considering d_t^k as the demand for category k in period t, and $S_j(W^k)$ the probability of a consumer selecting product j as function of the selection within its category $W^k = W \cap \mathcal{N}^k$, the demanded amount for the product will be computed as $d_t^k S_j(W^k)$. We assume that, within each category, consumer response follows an attraction model, which is a generalization of the MNL and the multiplicative competitive interaction model (MCI) [32]. As Fig. 1 represents, the share of a product is computed as its attractiveness a_j divided by that of the assortment $A(W^k) = \sum_j a_j$, with $j \in W^k$, and the competition a_0^k in that subcategory. Hence, $S_j(W^k) = a_j/(a_0^k + A(W^k))$ for all k and $j \in \mathcal{N}^k$. Also, consumers cannot purchase products that are not selected, implying that $S_j = 0$ for any $j \notin W$. The following constraints capture these relations, with $D_{jt}(W)$ is noted as D_{jt} since the formulation makes the dependence on W explicit.

$$D_{jt} \le d_t^k S_j \quad \forall t, k, j \in \mathcal{N}^k \tag{3.2}$$

$$S_j \le W_j \quad \forall j \tag{3.3}$$

$$a_0^k S_j = a_j S_0^k - F_j \quad \forall k, j \in \mathcal{N}^k \tag{3.4}$$

$$F_j \le a_j S_0^k \quad \forall k, j \in \mathcal{N}^k \tag{3.5}$$

$$S_0^k + \sum_{j \in \mathcal{N}^k} S_j = 1 \quad \forall k \tag{3.6}$$

$$W_j \in \{1, 0\} \quad \forall j \tag{3.7}$$

$$D_{jt}, S_j, S_0^k, F_j \in \mathbb{R}_0^+ \quad \forall t, k, j \in \mathcal{N}^k \tag{3.8}$$

Constraints (3.2)-(3.8) model consumer response. Constraints (3.2) set the demanded amounts as an upper limit on sales. Let $W_j = 1$ if $j \in W$ and $W_j = 0$ otherwise, constraints (3.3) only allow selected products to be demanded. Constraints (3.4) define that the ratios between the shares behave according to an attraction model, within each category. Similarly to the scale constraints in the sales-based linear program of Gallego et al. [33], they ensure that the ratio between the share of a product and the share S_0^k of the competing options in its category is, at most, their attractiveness ratio a_j/a_0^k (i.e., $S_j/S_0^k \le a_j/a_0^k$, where the equality is obtained by adding the slack variables F_j). Constraints (3.5) and (3.6) establish that, if the manufacturer does not offer any products in a subcategory, the competition will absorb all of the market share. In this situation, F_j in constraints (3.4) relax the relationship between the attractiveness and share ratios. Given that our objective encourages us to maximize the shares of selected products, and that the sum of all shares in a subcategory is limited by constraints (3.6), variables F_j will be null otherwise. Finally, constraints (3.7) and (3.8) set the domain of the variables. Let U_j denote the utility of product j, with its observable V_j and unobservable ε_j components. For the MCI consumer behavior $a_j = V_j$, since $U_j = V_j \varepsilon_j$ and $\ln \varepsilon_j$ follows the Type I extreme value distribution. For the MNL $a_j = \exp V_j$, since $U_j = V_j + \varepsilon_j$ and ε_j follows the same Type I extreme value distribution. The problem of maximizing $\mathrm{R}(W)$ subject to constraints (3.3)-(3.8) is a simple one. The optimal selection can be determined by starting with an empty set, and adding the product with the highest margin to the choice set until no increase in revenue is possible. This was first shown by Talluri and Van Ryzin [34].

3.2. Production planning

The following constraints capture the multi-period production planning, which greatly increases the complexity of the model.

$$X_{jt} \le Y_{jt} K_{jt} \quad \forall t, j \tag{3.9}$$

$$\sum_j \tau_j Y_{jt} + r_j X_{jt} \le c_t \quad \forall t \tag{3.10}$$

$$D_{jt} = I_{jt-1} + X_{jt} - I_{jt} \quad \forall t, j \tag{3.11}$$

$$Y_{jt} \in \{1, 0\} \quad \forall j, t \tag{3.12}$$

$$X_{jt}, I_{jt} \in \mathbb{R}_0^+ \quad \forall j, t \tag{3.13}$$

Constraints (3.9) - (3.13) model manufacturing as a capacitated lot sizing problem [35]. They impact the objective function both directly, with inventory and setup costs, and indirectly, by constraining the amounts that can be produced and, consequently, sold. Constraints (3.9) express that the plant must be setup for a product if any given quantity is to be manufactured. The upper bound for variables X_{jt} is the minimum between the maximum possible demand for the product j in remainder of the horizon, and the maximum possible output of j in period t. This implies that, in order for the formulation to be valid, constants K_{jt} need to satisfy $K_{jt} \ge \min\left\{ \sum_{t_2} d_{t_2}^k a_j/(a_j + a_0^k);\ (c_t - \tau_j - \tau^m)/r_j \right\}$ $\forall t, j, t_2 \in \{t, \ldots, T\}$ and $k : j \in \mathcal{N}^k$. Constraints (3.10) model the plant's capacity, which is shared among all products and split between production and setups. Constraints (3.11) ensure inventory is balanced between periods by requiring sold products to be either produced in that period or stocked from a previous one. Together with the capacity limitations, these constraints impose a limit on the sales the manufacturer can make, so $D_{jt} = \min\{d_t^k S_j, I_{jt-1} + X_{jt}\}$, given that the objective will try to maximize D_{jt}. Lastly, constraints (3.12) and (3.13) define the domain of the variables.

With objective (3.1), subject to constraints (3.2)-(3.13), we introduce the mixed-integer linear formulation for the selection of fast-moving consumer goods. The formulation may be extended with implementation-based considerations. For cases where product development and administrative costs are not negligible, a fixed cost per product throughout the planning horizon should be considered. Let f_j be the fixed development and introduction cost associated with product j, the term $\sum_j f_j W_j$ should be added to $\mathrm{C}(W)$.

It is also common for FMCG to share part of the changeover across a set of products. Similarly to the categories defined for the market structure, manufacturing can have families that share a portion of the setup procedure.

$$X_{jt} \leq Y_t^m K_{jt} \quad \forall m, t, j \in \mathcal{N}_m \tag{3.14}$$

$$\sum_m \tau^m Y_t^m + \sum_{j \in \mathcal{N}_m} \tau_j Y_{jt} + r_j X_{jt} \leq c_t \quad \forall t \tag{3.15}$$

With $\mathcal{N}_m$ being the set of products that belong to family m and $Y_t^m \in \{1,0\}\ \forall j,t,$ constraints (3.14) ensure $Y_t^m = 1$ whenever a product of that family is produced in that period. Constraints (3.15) modify constraints (3.10) to include the setup times τ^m for the families. The constants in constraints (3.9) are also used in constraints (3.14) and the conditions for the formulation to be valid become $K_{jt} \geq \min\left\{\sum_{t_2} d_{t_2}^k a_j/(a_j + a_0^k);\ (c_t - \tau_j - \tau^m)/r_j\right\}\ \forall t, j,$ with $t_2 \in \{t, \ldots, T\}$ and $(k, m) : j \in \mathcal{N}^k \cap \mathcal{N}_m$. To finally incorporate the effect of resource sharing, category setup costs θ^m are accounted for by adding the term $\sum_m \theta^m Y_t^m$ to C(W).

3.3. Safety stock

Since our model is deterministic, it disregards the inventory costs that are incurred to deal with demand unpredictability and how the selection affects them. To counteract this, we simulate the most widespread safety stock policies by requiring inventory levels to be a number l of standard deviations above the average demand for a product (α service level).

We assume that the number of consumers N_t^k that purchase a unit of category k in time period t is a normally distributed random variable with average d_t^k and standard deviation σ_t^k. We also consider that the selection of a given product $j \in \mathcal{N}^k$ by a consumer is the result of a binomial trial with $S_j(W^k)$ probability of success, as proposed by Chong et al. [30]. For a given selection within category k, W^k, the variance of the Bernoulli variable B_j that denotes that a consumer selects product $j \in \mathcal{N}^k$ is computed as $S_j(1 - S_j)$, with its expected value being S_j. Then, the amount of product $j \in \mathcal{N}^k$ sold in time period t is the product of the two random variables $N_t^k B_j$, with $\mathrm{E}[N_t^k B_j] = d_t^k S_j$ and $\mathrm{Var}(N_t^k B_j) = {\sigma_t^k}^2 S_j(1 - S_j) + {\sigma_t^k}^2 {S_j}^2 + S_j(1 - S_j){d_t^k}^2$. With the plant having a production lead time of Λ periods, non-linear constraints (3.16) enforce the safety stock policy.

$$I_{jt} \geq l\sqrt{\left({\sigma_t^k}^2 S_j(1 - S_j) + {\sigma_t^k}^2 {S_j}^2 + S_j(1 - S_j){d_t^k}^2\right)\Lambda} \quad \forall t, k, j \in \mathcal{N}^k \tag{3.16}$$

In order to keep the advantages of a linear formulation, we break each constraint into N linear segments, using $N+1$ borders. Implementing this approximation requires the addition of constraints (3.18)-(3.21) to the original model.

$$I_{jt} \geq y_{0t}^{kn} + \delta y_t^{kn} S_j - M_t^{kn}\left(1 - Z_j^n\right) \quad \forall t, n, k, j \in \mathcal{N}^k \tag{3.17}$$

$$S_j \leq 1 + \left(x_0^{n+1} - 1\right) Z_j^n \quad \forall j, n \tag{3.18}$$

$$S_j \geq x_0^n Z_j^n \quad \forall j, n \tag{3.19}$$

$$\sum_n Z_j^n = 1 \quad \forall j \tag{3.20}$$

$$Z_j^n \in \{1, 0\} \quad \forall j, n \tag{3.21}$$

Let y_{0t}^{kn} denote the intersection of the nth linear segment with the vertical axis and δy_t^{kn} its slope for subcategory k and period t. Constraints (3.17) impose the linear approximation of the safety stock policy. Working with constants M_t^{kn}, binary variables Z_j^n ensure that, for product j, the only potentially active constraint is the one relative to the segment where S_j is placed. The formulation requires that $M_t^{kn} \geq y_{0t}^{kn} + \delta y_t^{kn}$ for segments where the slope is positive and $M_t^{kn} \geq y_{0j}^n$ otherwise. Constraints (3.18) and (3.19) set $Z_j^n = 1$ if $S_j \in \left[x_0^n, x_0^{n+1}\right]$ and $Z_j^n = 0$ otherwise. Constraints (3.20) ensure that S_j is placed at one segment. Finally, constraints (3.21) define Z_j^n as binary variables.

3.4. Computational performance

To assess the tractability and scalability of the approach, we have generated problem instances with different numbers of products and planning periods. We compare the quality of the solutions reached by the MIP model with those reached by its linear relaxation, along with the time elapsed to obtain each solution. We evaluate the impact of the number of products by using five sets of ten instances with the number of periods $T = 12$ being constant across the sets and the number of products $J \in \{8, 10, 12, 14, 16\}$. Likewise, to measure impact of the number of products we use sets of instances with $J = 12$ and $T \in \{4, 6, 8, 10, 12\}$. In this experiment, capacity is constrained to the maximum time that it can take to produce the whole line without setup times, or $\max_t\{\sum_k d_t^k \sum_{j \in \mathcal{N}^k} r_j a_j/(a_0^k + A(W^k))\}$ with $W = \{1, \ldots, J\}$.

For all these instances, we sample the values of the model parameters using uniform distributions. The bounds of the distributions are based on real-world data and are displayed in Table 1. Product families (with respect to setups) correspond to the demand categories, thus sets $\mathcal{N}^k = \mathcal{N}_m$ for every $k = m$. To generate these sets, we assign to each product j a value of a random variable uniformly distributed between zero and one, U(0,1). Then, we iterate through the set of products and make partitions when the value is less than $\frac{1}{3}$. The resulting partitions correspond to the subcategories. This process generates a non-deterministic number of categories while averaging three products per subcategory. Additionally, we generate category demand $d_t^k = d_t s_t^k$ where, for each period, s_t^k is obtained by dividing K random U(0,1) variables by their sum.

Regarding safety stock, we place borders at $(1 - \cos n\pi/N)/2$, with $n \in \{1, \ldots, N\}$. Since the standard deviation of a Bernoulli variable, as a function of the probability of a successful trial, has the shape of a semicircle, this division would cut it into equally sized slices. The linear constraints are defined by the connections between adjacent intersections of the function with a border, resulting in an inner approximation. This is shown by the dashed lines in Fig. 2, where we illustrate this procedure. Throughout this work production lead time $\Lambda = \frac{1}{8}$ and the number of segments in the piece-wise linearization $N = 4$, which guarantees an approxi-

Table 1
Parameters for instance generation, and bounds for the uniform distribution.

Parameter	Description	Lower bound	Upper bound
d_t	Market size	400.0	500.0
σ_t^k	Standard deviation	$\sqrt{s_t^k} \times 60.0$	$\sqrt{s_t^k} \times 150.0$
p_j	Margin	6.0	12.0
h_j	Holding cost	1.6	3.6
f_j	Fix cost	22.5	110
θ_j	Setup cost	6.0	32.4
θ^m	Family setup cost	12.0	72.0
τ_j	Setup time	1.0	1.0
τ^m	Family setup time	3.0	3.0
a_j	Attraction	12.0	48.0
r_j	Resource utilization	0.4	1.0
a_0^k	Competition	$\lvert\mathcal{N}^k\rvert \times 24.0$	$\lvert\mathcal{N}^k\rvert \times 96.0$

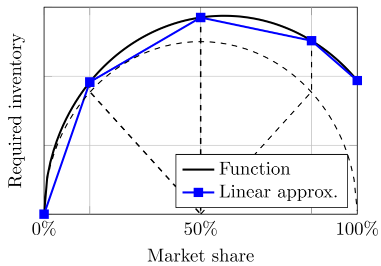

Fig. 2. Illustrative example of piecewise linearization. An inner linear approximation is made for the required inventory, as a function of the share of a product within its subcategory.

Table 2
Results for the tractability tests without safety stock.

T	J	Fractional solution time (s)	Integer solution time (s)	Integrality gap
4	12	0.00	168.53	54.0%
6	12	0.01	197.76	43.4%
8	12	0.01	194.05	41.3%
10	12	0.01	229.47	38.2%
12	8	0.00	81.67	28.9%
12	10	0.01	155.74	29.8%
12	12	0.01	295.70	38.0%
12	14	0.01	913.73	42.5%
12	16	0.01	4664.29	53.2%

mation (i.e., area below the non-linear function covered by the linear one) of at least 90.6% for the instance generation parameters. The resulting test instances are available online [36].

All experiments were conducted on four cores of an Intel Xeon E5-2450 processor and 16 GB of memory allocated from a computer cluster running Version 7 of the Scientific Linux distribution. The optimization problems were solved using IBM ILOG CPLEX 12.8, and the experiments were implemented in the C++ programming language using the Concert Technology library as an interface with the optimizer.

Table 2 exhibits the results of the experiment for the fully extended model without safety stock. The first and second columns regard the number of products and periods. The continuous and integer solution times, in seconds, follow. In the last column, we display the integrality gap as the ratio between the difference in objective values of the fractional and the integer solutions, and the value obtained by the integer solution ($\pi_f/\pi_i - 1$, with π_f and π_i being the profit values for the fractional and integer solutions). As it stands, the model contains $2J + 3TJ + K$ positive continuous variables and $2J + TJ + M$ binaries. Their behaviour is delimited by $3J + 3TJ + T + K$ constraints, besides those that define the domain of the variables. The results show that integer solution times scale exponentially with both the number of products and periods, with fractional solution times being negligible. The integrality gap increases with the number of products and decreases with the number of periods. In the linear relaxation of the problem, fractional product selections and setups reduce the fixed costs associated with the inclusion of each product and the setup costs needed to produce them. Moreover, a part of the capacity that, in integer solutions, would be used for setups becomes free to be used for production. As the fixed costs are one-shot and setups are usually concentrated in early periods, both these effects get diluted over longer planning horizons.

The previous model needs to be supplemented with JN binary variables and $J + 2JN + TJN$ constraints (excluding those defining the domain of the new variables) to include safety stock. Table 3 displays the computational performance results for the fully extended model with safety stock. Overall, the inclusion of safety

Table 3
Results for the tractability tests with safety stock.

T	J	Fractional solution time (s)	Integer solution time (s)	Integrality gap
4	12	0.01	352.58	133.1%
6	12	0.02	1098.75	107.8%
8	12	0.02	1187.78	98.3%
10	12	0.02	1573.56	94.8%
12	8	0.01	94.98	79.3%
12	10	0.02	249.95	81.3%
12	12	0.02	2994.95	97.6%
12	14	0.05	3110.34	97.7%
12	16	0.06	19087.91	105.4%

stock makes the problem harder to solve with the table showing higher integer solution times. The fractional values of the segment selection variables for the linearization lower the value of the right-hand side of constraints (3.17), potentially deactivating them. This leads to integrality gaps higher across the board than when safety stock is not accounted for.

4. Model assessment

In this section, we expose and analyse the results of the experiment aimed to measure the performance of our approach. We test the value of the integration of the decisions, of considering multiple production periods, and of safety-stock modelling by comparing our results with those of approaches that disregard these features. The comparison enables us to study the impact of production and demand conditions on the benefits of each model or solution method. To estimate the value of our contribution, and gauge the influence of the external parameters on our conclusions, we conduct computational experiments on 480 instances resulting from 20 templates generated using the bounds from Table 1. For all templates, $J = 12$ and $T = 6$, which correspond to a relatively small line and planning horizon when compared to a real-world case. The templates can be found online [36]. Throughout the tests, we change how limiting the plant's capacity is, how intense the competition is, the demand pattern that categories exhibit, and the α service level that the company is required to guarantee. Furthermore, customers can exhibit different behavioural patterns according to their priorities. For luxury items, a higher price tag can translate into more demand for a product, as consumers are sensitive to quality. The opposite is expected for common consumption goods. Varying the distribution of the attractiveness throughout the product line can simulate this effect. With this analysis, we extend our conclusions, as well as test how robust each method is to deviations against the predicted behaviour.

Table 4 outlines the experiment. The first column regards the contribution for which we are testing. For each test, we compare the resulting profit and product selections made by each method. The second column displays the external and internal parameters

Table 4
Computational experiment design including the number of product selection decisions.

Test	Condition	Number of levels	Number of instances	Number of decisions
Integration	Capacity	6	120	1440
	Market priority	3	60	720
	Dynamicity	3	60	720
Multi-period	Capacity	6	120	1440
	Seasonality	3	60	720
Safety-stock	Competitiveness	6	120	1440
	Service level	3	60	720

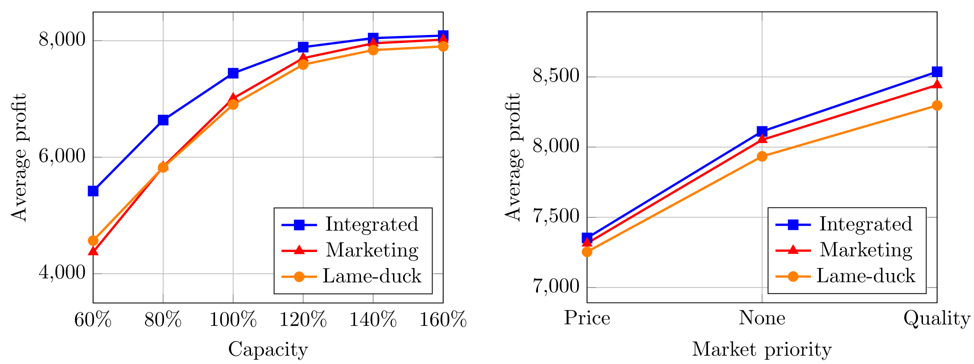

Fig. 3. Results for the integrated approach versus gross-profit optimisation and iterative removal (trimming the lame duck). On the left, the average profit is presented as a function of capacity. On the right, as a function of the priority of the market.

for which sensitivity analysis was conducted, with the considered number of levels exhibited in the third column. The fourth column is the number of instances, computed as the number of templates multiplied by the number of levels and the last column, relative to the number of decisions, allows the comparison of the selections of each approach to be clearer in the following subsections. In the table, we denote period-to-period variations in demand as dynamicity.

4.1. The value of the integrated approach

Historically, sales managers dictate which products a company sells. To assess the value of an integrated decision, we test the performance of two heuristics used in practice against that of the integrated optimisation model. In these product line strategies, marketing dictates the products to sell and the target market shares. Production seeks to cost-efficiently (optimally) comply with marketing's manufacturing needs, given the sales target for each product. We consider marketing only has access to administrative and development cost data. To make the performance of the lines comparable, we inject the selection made by the heuristics into the integrated model, so that it computes the profit that the line generates.

In the marketing-driven optimization heuristic, marketing determines the product line that maximizes gross profit after fixed costs by solving the problem with the objective $\sum_t \sum_j p_j D_{jt} - \sum_j f_j W_j$ subject to constraints (3.2)-(3.8). The solution is then injected into the integrated approach to evaluate profit. Algorithm 1

Algorithm 1 Pseudo-code for marketing-driven optimization.

procedure MARKETINGOPTIMIZATION()
  $W \leftarrow$ marketingOptimization();
  profit $\leftarrow$ integratedOptimization(W);

(Marketing, in Fig. 3), describes this procedure where marketing commands production. Trimming the lame-duck [30] is a heuristic that, starting from a full line, removes the worst performing product according to a myopic criterion (e.g., margin, share or revenue). It is a common product removal strategy that firms opt for when attempting to deal with cannibalisation and manufacturing issues. This strategy is conservative, as it can choose a product line containing all of the manufacturer's products if none are worth to remove. In this implementation, we consider the choice criterion to be the gross profit of a product j after its fixed administrative and development costs, $\sum_t p_j D_{jt} - f_j$.

After choosing which product to trim out, marketing consults production to understand the operational impact of the removal. If an increase in profit is expected, the removal is accepted, and marketing will search for another product to remove. Otherwise, the removal is cancelled, and the local optimum is reached. The pseudo-code in Algorithm 2 (Lame-duck, in Fig. 3) shows the pro-

Algorithm 2 Pseudo-code for iteratively removing the worst performers.

procedure REMOVEWORSTPERFORMERS(*criterion*)
  $W \leftarrow \vec{1}$;
  currentProfit $\leftarrow$ integratedOptimization(W);
  repeat
    incumbentProfit = *currentProfit*;
    $W \leftarrow$ removeWorst(W, *criterion*);
    currentProfit $\leftarrow$ integratedOptimization(W);
  until *currentProfit* $<$ *incumbentProfit*;
  profit $\leftarrow$ *incumbentProfit*;

cedure of trimming the lame duck. By fixing product selection W to a vector of ones, the algorithm starts by evaluating the performance of the full line. Then, we remove the product with the worst performance from the line and recalculate profit using the integrated model. If profit increases, this procedure is repeated. If not, the line is reverted to that of the previous iteration, and the procedure ends. The heuristic depicts some degree of cooperation between the departments as, although marketing still commands, at each step of the PLS process, the feedback from production has the final say on the decision.

Fig. 3 exposes the results of each approach as a function of capacity and the evaluation that consumers make on price and quality. On the left, the performance of each solution method as a function of capacity is plotted. The integrated approach corresponds to the model exposed in Section 3 with development and administrative costs and resource sharing. Our base unit for capacity (100%) is computed as described in Subsection 3.4. To isolate this effect, a random attractiveness-price distribution was considered. Similarly, to isolate the effects of the priorities of the market, as represented on the right side of the figure, production is not constrained by capacity. We simulate quality-prioritising markets by ensuring that a product with a higher margin has a greater attractiveness value, and reverse this association to mimic price-sensitive markets. In the case with no market priority, the association between attractiveness and price is random.

The plot on the left shows that an integrated approach is most beneficial when capacity is the most constrained. We note that when capacity is not constraining, and the market does not have a specific priority, keeping the full line results in 31.5% less profit than using the product lines recommended by our model. For this set of instances, when capacity is at 160% of the production time of the full line, profits obtained from the optimisation for gross

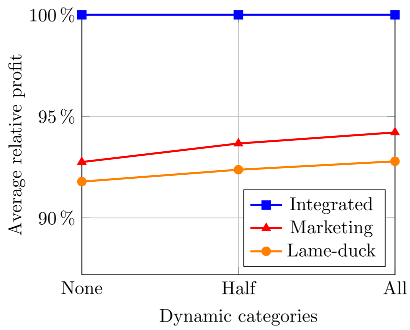

Fig. 4. Results for the integrated approach versus gross-profit optimization and iterative removal (trimming the lame duck). Average relative profit presented as a function of the number of dynamic categories, with capacity at 100%.

profit are, on average, 0.9% lower than those obtained by the integrated approach. For the iterative removal of the worst products, this difference is of 2.3%. These values rise to 19.3% and 15.7% when capacity is at 60% of the full line, noting that the iterative worst removal outperforms gross-profit optimisation when capacity is below 80%. The difference between both non-integrated heuristics diminishes with how constraining capacity is. The iterative removal manages to beat gross-profit optimisation in fifteen instances and tie in one, for the scenario where capacity is most constrained. When capacity is most loose, the iterative removal heuristic is worse in eight cases and ties in nine.

On the right, we observe that the profit difference between the approaches increases with how much quality is prioritised over price. In price-prioritising markets, while product lines resulting from gross-profit optimisation manage to profit 0.5% below the optimal profit value, lines chosen by the iterative marketing-led removal result in a 1.3% difference. In quality-prioritising markets, the same metrics present the values of 1.1% and 2.8%. The marketing-driven heuristic is not dominant across all instances, however the number of instances in which it outperforms the iterative removal increases with quality prioritisation.

Fig. 4 shows the evolution of the benefits of the integrated approach with the number of categories for which demand varies throughout the planning horizon, with capacity being constrained to the 100% level. The relative profit of the approaches is displayed for static demand (none of the categories is dynamic), for uniform demand (all categories are dynamic, which corresponds to the unaltered templates), and for the half level (where demand in half of the categories, rounded up, is static). The results show that, unintuitively, the heuristics perform relatively better if demand is dynamic. In this case, trimming the lame-duck leads to profits of 1.0 *p.p.* higher than if demand were static. This difference, when optimising for gross profit is of 1.5 *p.p.* To justify these results, we recall that, to assess the performance of each heuristic, we inject its product selection into the integrated approach with the profit it generates being calculated assuming an optimal production plan. With dynamic demand, the approach manages to make better trade-offs between piling up inventory and making changeovers, as opportunities will arise to setup at a demand peak, produce, and hold additional product to cover a demand trough without additional setups. Moreover, gross profit slightly increases with demand variations, since these trade-offs open up some capacity that otherwise would be lost setting up, with this effect being constant throughout the approaches. Therefore, the benefits of the integrated approach decrease with how dynamic demand is because, by finding better fits in the production plan, indirect production costs become a smaller slice of gross profit.

Both heuristics increasingly overestimate product line breadth with capacity, with this effect being more prominent in the solutions provided by the marketing-driven heuristic. This is expected as the heuristic does not account for capacity (does not affect the solution), and the iterative removal only accounts for it indirectly. Moreover, for all cases, the optimal product line was contained in the solution provided by gross-profit optimisation. Concerning both heuristics, the more quality is prioritised by the consumers, the bigger the divergence from the optimal product line. For the lame-duck, while 15.8% of the 720 product choices differ when the market prioritises price, 24.6% of the choices differ when the market seeks quality. Although with lesser differences from the optimal line, this effect still manifests when optimising for gross profit, with the respective values being of 5.0% and 10.8%. The differences between the product lines proposed by the heuristics are most significant when the market has no specific priority, and demand is static. While the size of the lines selected by the gross-profit optimisation heuristic does not vary with the number of dynamic categories, the lines selected by the lame-duck heuristic and the integrated approach become slightly larger. This is expected since setups can be better utilised in the dynamic scenario, and the two approaches have some insight into operations. At 42.1% of different selections, the lines of lame-duck and the integrated approach for the half level are the ones that differ the most. When demand is static, 41.3% of the choices the lame-duck heuristic makes are sub-optimal. When demand is dynamic, the proportion lowers to 39.6%. Regarding the marketing-driven heuristic, the percentages become 32.1% and 29.6% for the static and dynamic scenarios, respectively.

4.2. The value of multi-period planning

The value of a multi-period approach is connected to the trade-off between setups and inventory, given non-stationary demand. If capacity is considered, additional profit can be generated from more detailed production planning by fitting the right products into the line. In this subsection, we measure the impact that dividing the horizon of the production plan into multiple periods can have on the selection of the product line. We compare the performance of and choices made by a single-period approach to those of the multi-period approach corresponding to the model exposed in Section 3 with development and administrative costs, and resource sharing.

For the single-period approach, one period encompasses the whole planning horizon and might include multiple production cycles. We evaluate the optimal solution W for each possible production frequency by inputting it into the multi-period model, with the maximum number of cycles being the initial number of periods. Finally, we pick out the best and worst-performing production frequencies ω to contend with the selection from the multi-period approach. To convert the multi-period instance into single-period, demand and capacity are reevaluated as $d = \sum_t d_t$ and $c = \sum_t c_t$. Furthermore, $s^k = \sum_t s^k_t / T$. In each production on cycle, the necessary setups for each product in W must be executed, and the average inventory is half of the cycle's production (linear depletion rate). We implement this by considering setup costs and times as $\omega T \theta_j$ and $\omega T \tau_j$ for each product and $\omega T \theta^m$ and $\omega T \tau^m$ for each manufacturing category. Additionally, holding costs are reformulated as $2 h_j T / \omega$. After the conversion, the model exposed in Section 3 can solve the single-period instance. Nevertheless, the single-period model, where production planning is similar to that of Morgan et al. [7] and our multi-category attraction model estimates demand response, is provided in Appendix A.

Capacity and the demand pattern are the most relevant factors when considering the trade-off between stocking and setting up in a situation where demand is not stationary. Fig. 5 shows the results of this analysis as a function of these factors. We exhibit the profit resulting from the multi-period optimal product line, against that

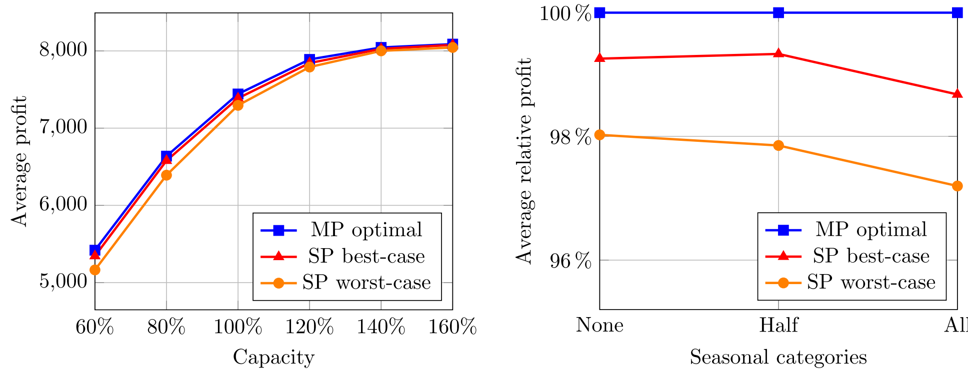

Fig. 5. Results for the multi-period approach versus the single period approaches with the best and worst production frequencies. On the left, average profit is presented as a function of capacity. On the right, average relative profit is shown as a function of the number of seasonal categories, with capacity at 100%.

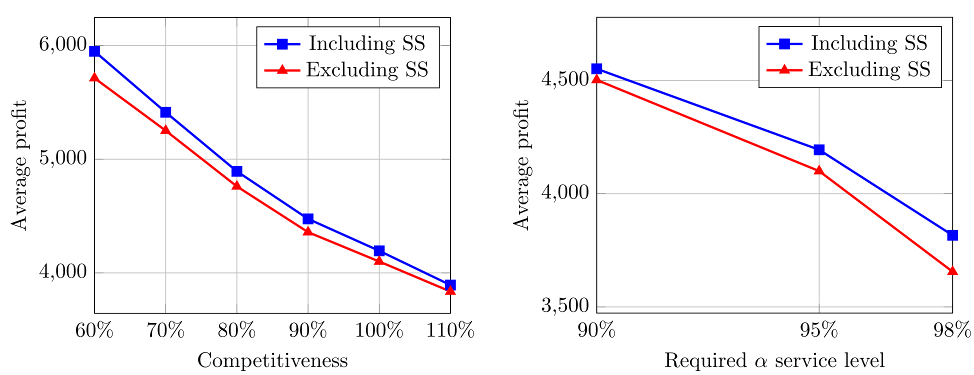

Fig. 6. Results for the integrated approach with and without accounting for safety-stock (SS). On the left, the average profit is presented as a function of market competitiveness. On the right, as a function of the required α service level.

of the single-period approach. We include profit for the best and worst production frequencies.

The left side of the figure deals shows profit as a function of capacity. As in the previous subsection, the base capacity level is the maximum time that the full line could take to produce, without setups. Having a time-between-runs $1/\omega = 1$ consistently results in the worst profits. As expected, for cases where capacity is tightly constrained, higher times between runs lead to better results. When the capacity is at 60% and 80% of its baseline, a $1/\omega = 5$ and $1/\omega = 3$ are the best performers, whereas $1/\omega = 2$ leads to the best profits in less restricted cases. Comparing the best and worst cases for production frequency, the multi-period model leads to improvements in profit inferior to 0.5% in the situation with the loosest capacity. In a constrained scenario, we estimate the multi-period approach to lead to product lines that are 1.4% to 4.7% more profitable. This denotes the increasing value of multi-period PLS with a decreasing capacity, which stems from the augmented importance of managing the setup-inventory trade-off. This value is strictly increasing for a given time-between-runs.

The right side of the figure exhibits profit for three proportions of seasonal categories, with a capacity level of 100%. When none of the categories is seasonal, instances correspond to the unaltered templates. In the half level, half of the categories (rounded up) have a seasonal pattern that is implemented by moving half of the demand from the first two periods into the fourth and fifth ones. In the last level, all of the categories exhibit this seasonal pattern. For all demand patterns, $1/\omega = 1$ is to the worst single-period production frequency and $1/\omega = 2$ the best. For all approaches, holding costs increase steeply with seasonality. The results show that the benefits of a multi-period approach increase with the number of seasonal categories. For the case where every category is seasonal, the profit advantage of the model is of 1.3% for the best frequency and of 2.8% for the worst. The advantage becomes 0.7% and 2.0% for the respective best and worst production frequencies. When half of the categories exhibit the seasonal pattern, the best production frequency obtains similar results to when demand is uniform, and the worst frequency leads to results 0.2 *p.p.* lower.

In the worst-case, 11.8% of the choices the single-period method makes differ from the optimal multi-period approach across the tests involving capacity. This amounts to 134 product exclusions and 34 product inclusions that are suboptimal. By considering setting up every period, this scenario is undershooting the number of products that the line withstands. The opposite is done by the best-case single-period approach, which slightly overshoots the number of products, in all scenarios but the most constrained. This translates into 74 inclusions and 54 exclusions that differ from the multi-period approach, totaling of 5.1% of different choices. The best-case includes all products worst-case approach does for the two situations where capacity is most lax. More so, it only does not include 30 products that the worst-case approach does the total of 1440 product choices. Inversely, it includes 152 products that the worst-case single-period method does not. While the single-period approaches maintain the size of the line with an increase in the number of seasonal categories, the multi-period approach selects smaller lines. With 15.0% of the choices being made differently, the worst single-period and the multi-period approaches lead to the most distinct solutions. Then, with 13.8% of different choices, stand the single-period approaches. With 9.9% of the choices made differently, the multi-period and the best single period approaches select the most identical lines. The half level increases the differences between the lines chosen by the worst single-period by and the multi-period by 1.5 *p.p.* It also decreases the differences between single-period approaches by 1.3 *p.p.* and by 1.6 *p.p.* between the best single-period and the multi-period approaches. Again, the single-period with the worst frequency undershoots the line and the one with the best production frequency slightly overshoots it.

4.3. The value of safety stock modelling

Safety stock must be taken into consideration as service levels must be guaranteed. In this subsection, we compare how considering a safety-stock policy influences product selection. Concerning the piece-wise linearization used to model safety stock, we established $N = 4$, trading the added complexity of the binary variables over the fit of the linearised function, and define the lead time parameter $\Lambda = \frac{1}{8}$. To appraise the value of including safety stock in the model, we compare the profits obtained from the product lines chosen when the model regards and does not regard safety stock, in a situation where it must be assured. Here, advantages arise from taking the variability of the sales and the costs associated with the additional inventory into account.

The most influential parameters to consider in this assessment are service level and market competitiveness. While the service level directly influences the amount of inventory needed to fulfil the policy's requirements, market competitiveness influences the share of the company, which can affect the variability of the demand for its products. On the left of Fig. 6, the results of both approaches are compared as a function of competitiveness, considering a base service level of 95%. In the right side of the figure, profit is plotted as a function of the α service level requirements, with competitiveness at its base level (corresponding to a market share of 33% while offering the full product line and meaning that $\sum_k a_0^k = 2\sum_j a_j$).

The importance of accounting for safety stock in the decision decreases with the strength of the competition and the consequent reduction in the market share of the product line. Revenue is asymptotically decreasing with the strength of the competition, and this is reflected in profit. Regarding the base case for both factors, excluding safety-stock leads to product lines 2.2% less profitable. Furthermore, the expected profit difference between both approaches for the situation when competitiveness is at 60% is 4.0%. This value falls to 1.5% when the competition is 110% of the basis. Contrarily, the importance of including safety-stock policies in the model increases with the service level requirements. When a 90% α service level is required, including safety-stock constraints in the model leads to 1.1% better results. This value rises to 4.2% with a 98% service level requirement.

Expectedly, the number of products included in the line is overestimated when not considering the safety-stock policy. In a total of 1920 decisions, 165 inclusions and 7 exclusions separated the selection made by each approach. Congruently with the variation in profit, the percentage of different product choices decreases with competitiveness, from 15.0% at 60% of the base level to 6.3% at 110%. More so, this value increases with the service level requirements from 7.5% at 90% of the base level to 11.7% at 98%.

5. Application to real-world instances

Our computational tests made evident the conditions in which our proposed model provides its utmost utility. Still, the results from its application to instances generated by uniform distributions with parameters based on a real case do not accurately quantify the benefits that would transfer a real production line. For this reason, we conduct the tests on two real production lines from the beverages industry. In both cases, the planning horizon comprises 12 periods, and the manufacturing categories differ from the consumer perceived ones. We estimate attractiveness values using point-of-sale data. Setup costs are null, and fixed costs were neglected as they were not made available by the firm. Nevertheless, the firm provided setup and production times, which we account for. In this section, we refer to the real production line with 21 potential products as instance A, and to the one with 13 as instance B. Again, we start by assessing the value of an integrated optimization model by comparing its solutions with those heuristics that the industry frequently uses.

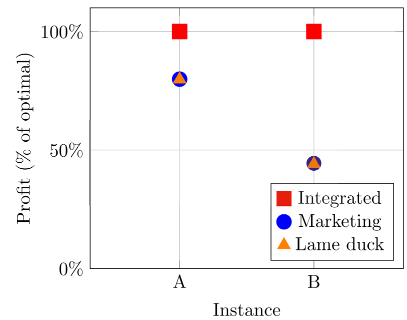

Fig. 7. Results for the integrated approach versus gross-profit optimization and iterative removal (trimming the lame duck) for the real production lines.

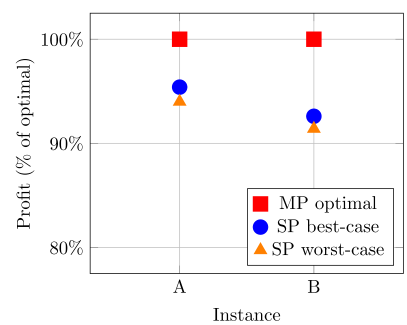

Fig. 8. Results for the multi-period approach versus the single period approaches with the best and worst production frequencies for the real production lines.

Fig. 7 shows the profit of the heuristics as a percentage of the profit of the integrated optimal, for the real instances. The absence of fixed costs makes it easy for both heuristics to converge to the same solution, that being the product line that generates the most revenue. Notwithstanding, the solutions could diverge if several high-margin low-attractiveness products options, which generate little revenue, were available. The lame-duck heuristic could remove these products, and it would stray from the parsimonious approach exposed in Subsection 3.1. The benefits of the integrated approach are a 25.3% increase in profit for instance A and a 125.3% increase for instance B. This demonstrates how removal strategies that are used in practice can be sub-optimal. Furthermore, the disparity between the benefits achieved in both cases sheds light on the risks of using myopic heuristics with no performance guarantee. We have shown the potential increases in profit relative to the practices of the industry. We now follow up with a comparison of our approach with the current state-of-the-art in production modelling for PLS, resulting in an assessment of the value of considering multiple periods.

Fig. 8 presents the profit of the single-period solutions compared to those obtained by our multi-period formulation. The figure contrasts with the results obtained for the virtual instances,

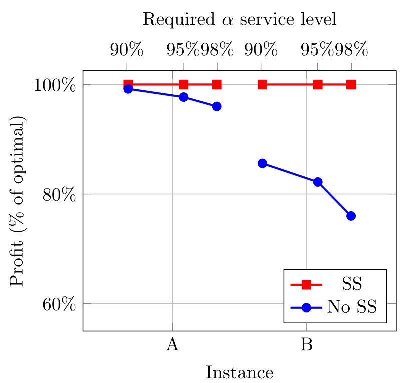

Fig. 9. Results for the integrated approach with and without accounting for safety-stock (SS) for the real production lines.

Table 5
Summary results of the computational experiments.

Comparison	Profit	Size	Parameter	Profit (trend)	Size (trend)
Marketing	+	+	Capacity	↑	↓
Lame-duck	+	+	Priority	↑	↓
	+	+	Dynamicity	↑	↓
Integrated	+	-	Capacity	↓	↓
Marketing	+	-	Priority	↑	↑
	+	-	Dynamicity	↓	↓
Integrated	+	-	Capacity	↓	↓
Lame-duck	+	-	Priority	↑	↑
	+	-	Dynamicity	↓	-
SP best-case	+	+	Capacity	↓	-
SP worst-case	+	+	Seasonality	↑	-
MP optimal	+	-	Capacity	↓	↓
SP best-case	+	-	Seasonality	↑	↑
MP optimal	+	+	Capacity	↓	↓
SP worst-case	+	+	Seasonality	↑	↓
Including SS	+	-	Competitiveness	↓	↓
Excluding SS	+	-	Service level	↑	↑

where the benefits of the multi-period model seem slim. This is a consequence of market size, in the templates, being generated according to a uniform distribution, thus exhibiting no intentional seasonality and of, in this case, holding costs representing a larger portion of revenue. Nevertheless, the results for the real cases reflect the stark seasonality that the market for beverages presents. For instance A, the increase in profit resulting from the use of the multi-period ranges from 4.9% to 6.4%. For instance B, the increase is situated in the 7.9% to 9.4% range. Lastly, after estimating the benefits of our model relative to the state-of-the-art in PLS, we evaluate the potential of accounting for the costs of safety stock when making the selection.

Fig. 9 shows the difference in the performance of the choices made by the integrated model with and without the safety stock extension, in a scenario where, by policy, safety stock is required. While, for instance A, the benefits increase from 0.9% to 4.1% higher profits with the increase in the required service level, for instance B, the benefits range from a 16.8% to a 31.6% profit increase. Although the product lines that both solution methods chose will differ depending on the market share of each product and the inherent variability of that category, the results emphasize the desirability of accounting for safety stock in the decision, especially when high service levels are required.

6. Managerial insights

The insights derived from the computational experiment are summarized in Table 5. The first column presents the coupling of approaches in each comparison. The second and third columns show if the first approach leads to higher profits and larger lines than the second, respectively (plus sign for higher and larger, minus sign for lower and smaller). As the approaches in each couple are ordered by the profit they generate, the first column is populated with plus signs. The third column states the independent variable of the sensitivity analysis, the fourth and fifth display its results. While the second column states if the profit of the first approach is higher or lower than the one of the second, the fourth column shows an arrow pointing up if this difference increases with an increase in the parameter. The fifth column is analogous to the fourth but respecting size. In these last two columns, a flat line represents that there is no clear trend. The table shows that the value of our approach is highest when capacity is tight, consumers prioritize quality, the company has weaker competition and high service level requirements. As an example of interpretation, in the comparison between marketing-driven optimization and the iterative lame-duck removal heuristics, marketing-driven optimization is generally the best performer (plus sign) with its profit advantage increasing with capacity (arrow pointing up). Furthermore, the lines that this approach chooses are usually longer than the ones chosen by its counterpart (plus sign), with this effect diminishing as capacity increases (arrow pointing down). On another note, although no clear trend was found for the difference in line size throughout the different capacity levels when comparing the single-period approaches, the lines that each selects become more similar as capacity increases.

A FMCG producer can benefit from an integrated multi-period approach for PLS. In the virtual instances, the integrated approach leads to profits up to 19.3% higher than those of the current industry practices. The improvements increase with capacity tightness and with the quality sensitiveness of the consumers. We have seen that dynamic categories can allow for capacity to be better utilized, even for sub-optimal selections. Additionally, we exhibited that optimizing for gross profit leads to higher profits than iteratively removing the worst performing product unless the plant is under very tight capacity constrictions. Lastly, the results from the real cases show that PLS should not be left to myopic heuristics with no performance guarantee, as the performance of their solutions obtained can be very disparate from the optimal.

Furthermore, we demonstrate the potential of a multi-period approach. The advantages over the single-period approach, which materialize in a difference of up to 4.7% in average profit, as well as the variation in performance between the best and worst production frequencies, increase with capacity tightness. If a planner manages to foretell the best production frequency, the difference between the single-period approach the multi-period optimal was only of up to 1.4% throughout the tests. We also show that the benefits of considering multiple periods increase with the number of seasonal product categories. The real cases demonstrate the full potential of modelling production as a capacitated lot-sizing problem with shared setups, validating the need for this contribution in cases with predictable demand variations. Especially in FMCG, the inventories needed to handle seasonality and holiday peaks can influence the optimal selection.

When safety stock is a requirement, we showed its inclusion into the decision can lead to significant increases in the profitability of the resulting product lines. These increases become more prominent in markets where competition is weaker, with differences of up to 4.0% in average profit, and in cases with more demanding service levels, these differences can reach 4.2%. The extension leads to different decisions as, besides accounting for demand variability, it accounts for the additional inventory needed to fulfil the policy. This additional component in inventory costs is influenced by the competitive position of the products of the firm and the variability of the category and is particularly relevant when high service levels are required.

The inclusion of the results for the real production lines confirms the existence of the untapped benefits from an approach to PLS that accounts for the specificities of FMCG. In sum, tight capacity, low price sensitivity, and pronounced seasonality in demand maximize the need for our integrated multi-period approach. Moreover, the safety-stock extension is most required when the firm has products that serve close to 50% of the market for a whole category with a lot of variability and high demanded service levels. Optimizing PLS can effectively lead to additional profits relative to those of optimizing production planning. Nevertheless, the high stakes involved in the decision make the approach vulnerable to the risks of improper parametrization.

7. Conclusions

In this paper, we deliver a mixed-integer programming model for PLS to be used by FMCG producers. Adding to the literature, the share of each product is obtained from a multi-category attraction model. Capacity is explicitly modelled, and the approach is multi-period, which allows variations in demand to be managed by the trade-off between stocking and setting. Furthermore, we incorporate safety stock into the mixed-integer programming model in a new fashion. Employing a set of instances generated with real-world parameters and two real cases, we assessed the value of our research by making three performance comparisons. We contrast (1) the heuristics used in practice with the integrated approach, (2) single-period planning with the integrated approach, and (3) the integrated model with the integrated model considering safety stock. To strengthen our conclusions and derive managerial insights, we vary the conditions under which each comparison is made. The results of our computational experiments in virtual instances identify the conditions in which the integrated multi-period model is most beneficial. Furthermore, the results obtained from applying the approach to the real production lines make the potential profit increases appreciable and provided some key insights. They make evident that the decision should not be left to heuristics, and that a multi-period approach is needed when the demand has predictable variations (e.g., seasonality).

Three main streams of future research follow from this work. Although we showed the potential benefits of the approach using a real-world inspired instance set and validated the existence of these benefits with real cases, we are still unable to solve larger real-world instances. With safety stock, our approach took two days to reach the optimum of the larger instance. The magnitude of this solution time is impractical for a decision-maker to iterate over different parameter settings. Firstly, solution methods for this problem need to be further researched. Specifically, this problem integrates the attraction revenue management with fixed costs and capacitated lot-sizing problems which have been extensively studied. Therefore, looking into heuristics combining the solution methods for these problems is a promising direction. Safety stock was modelled as a linear approximation, avoiding the computational stress of uncertainty modelling. Secondly, with better methods, this approximation should be possible to overcome. Furthermore, providing robust product line selections can hedge against poor parametrization, which is much needed for such a high stakes decision. Lastly, demand interactions between the products are limited, and brand loyalty is still not accounted for. For a more realistic consumer choice model, fully incorporating the effects of substitutability and complementarity is vital.

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Appendix A

We hereby present the single-period model that can be used to reproduce the experiment in Subsection 4.2 with changes in the objective function and capacity constraints as postulated.

$$\max_{S,I,Y,X} \sum_{m} \sum_{j \in \mathcal{N}^m} p_j D_j - 2h_j D_j T/\omega - (f_j + \omega T \theta_j) W_j - \omega T \theta^m Y^m \quad \text{(A.1)}$$

$$\text{s.t.} \quad D_j \le d^k S_j \quad \forall k, j \in \mathcal{N}^k \quad \text{(A.2)}$$

$$a_0^k S_j = a_j S_0^k - F_j \quad \forall k, j \in \mathcal{N}^k \quad \text{(A.3)}$$

$$F_j \le a_j S_0^k \quad \forall k, j \in \mathcal{N}^k \quad \text{(A.4)}$$

$$S_0^k + \sum_{j \in \mathcal{N}^k} S_j = 1 \quad \forall k \quad \text{(A.5)}$$

$$S_j \le W_j \quad \forall j \quad \text{(A.6)}$$

$$S_j \le Y^m \quad \forall m, j \in \mathcal{N}^m \quad \text{(A.7)}$$

$$\sum_{m} \omega \tau^m T Y^m + \sum_{j \in \mathcal{N}^m} \omega \tau_j T W_j + r_j D_j \le c \quad \text{(A.8)}$$

$$W_j, Y^m \in \{1, 0\} \quad \forall j, m \quad \text{(A.9)}$$

$$D_j, S_j, S_0^k \in \mathbb{R}_0^+ \quad \forall j, k \quad \text{(A.10)}$$

Hence, expression A.1 and the capacity constraints (A.8) accommodate the assumption that every production cycle setups are incurred. Again, constraints (A.2)-(A.5) ensure the market behaves as desired. Constraints (A.6) and (A.7) activate the binary variables relative to product and manufacturing category setups. We note that, in this single-period model, W_j is used both for product setups and the presence of the product in the assortment. Constraints (A.3) limit the production to the demand, noting that here we use D_j both as production quantity and sales. Lastly the domain of the variables is defined in expressions (A.9) and (A.10).

Appendix B

Table B1
Table of notation.

Indexes and sets	
$j \in \{1, .., J\}$	Product index.
$k \in \{1, .., K\}$	Category index.
$m \in \{1, .., M\}$	Family index.
$t \in \{1, .., T\}$	Planning period index.
$n \in \{1, .., N\}$	Piece index.
$\mathcal{N}^k$	Set of products belonging to category k.
$\mathcal{N}_m$	Set of products belonging to family m.
Parameters	
d_t^k	Demand for category k in period t.
σ_t^k	Standard deviation of category k in period t.
p_j	Margin of product j
h_j	Cost of holding a unit of j.
f_j	Fixed administrative cost for producing j.
θ_j	Setup cost for product j.
θ^m	Setup cost for family m.
τ_j	Setup time for product j.
τ^m	Setup time for family m.
a_j	Attractiveness of product j.
r_j	Time that a unit of j takes to produce.
a_0^k	Attractiveness of the outside competition in category k.
K_{jt}	Maximum production of j in period t.
x_0^n	Beginning of piece n.
y_{0t}^{kn}	Intersection for the straight line of piece n.
δy_{0t}^{kn}	Slope for the straight line of piece n.
M_t^{kn}	Maximum value that linear piece n can take.
Variables	
W	Set of the selected products.
W_j	1 if product j is selected, 0 otherwise.
D_{jt}	Sales of product j in period t.
X_{jt}	Production of product j in period t.
I_{jt}	Inventory of product j in period t to be carried over.
Y_t	1 if machines are setup in period t, 0 otherwise.
S_j	Share of product j.
S_0^k	Share of the outside competition in category k.
F_j	Slack for the ratios between attraction and shares for j.
Z_j^n	1 if share of j is on piece n, 0 otherwise.
Functions	
R(W)	Gross profit generated by product line W.
C(W)	Indirect costs of supplying product line W.

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