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A critical decision facing firms across industries is the selection of a mix of products to offer in the marketplace. Both in practice and in the academic literature, the product line design problem has typically been considered from a marketing perspective, with a focus on how alternative sets of products interact and compete in the marketplace. The operational implications of product line decisions have been largely ignored, even while the importance and complexity of interactions among products in the manufacturing environment increase with broadening product lines. Furthermore, consideration of manufacturing synergies among products in product line design is increasingly beneficial given efforts in many industries to improve co-ordination of manufacturing activities across products. In this work we examine the benefits of integrating marketing implications of product mix with more detailed manufacturing cost implications. Traditional product line models are extended to capture both individual product costs and relevant cost interactions among products. The relevant marketing and manufacturing elements are considered in a mathematical programming formulation that identifies a profit maximizing mix of products. The resulting normative model of the product line design problem is used to generate insights into important cross-functional issues in product line management. Specifically, we examine the impact of alternative manufacturing environment characteristics on the composition of the optimal product line.

1. Introduction

Product line design involves determining a mix of products that can compete in the marketplace and earn profits for the firm. Both in practice and in the academic literature, emphasis has been placed on the marketing implications of product line decisions. In practice, marketing managers are responsible for identifying widely diverse customer preferences, and developing marketing strategies for satisfying demands from increasingly differentiated customer segments. This responsiveness to detailed customer preferences often leads firms to overextend their product lines without considering the resulting impact on manufacturing effectiveness. Traditional incentive schemes that link rewards to increases in earnings, market share, or sales from new products also encourage marketing managers to extend existing product lines in order to boost short-term performance. Quelch and Kenny (1994) discuss problems associated with unchecked product line expansion, including hidden cost increases and poor manufacturing performance. These inefficiencies can be avoided, and firm profitability enhanced, when product line decisions are based on a detailed understanding of how products interact, both in the marketplace and in the firm's manufacturing environment. In the consumer products industry, where the past decade has been marked by aggressive proliferation aimed at luring more customers with new packaging, scents, and other aesthetic attributes for the same functional products, Proctor & Gamble has led the way with initiatives to trim their product lines in order to cut expenditures in manufacturing, distribution, and promotion (Schiller *et al.*, 1996). In addition to the expected cost savings, P&G has actually realized market share gains in some categories where lines have been significantly reduced and simplified.

Firms across industries increasingly recognize the importance of providing variety to the marketplace while simultaneously identifying and exploiting synergies among products in manufacturing. The recent focus on product modularity observed in many industries is driven in part by the resulting increased synergies in manufacturing. In an article that details the success of modular product design strategies in the computer industry, Baldwin and Clark (1997) cite increased flexibility and the opportunity to design families of products that have similar manufacturing requirements as two key advantages of modularity. These developments increase the potential benefits manufacturers can realize by making intelligent product line decisions that exploit opportunities for production efficiencies.

The product line design problem lies at the marketing/manufacturing interface (Damon and Schramm, 1972; Shapiro, 1977; Montgomery and Hausman, 1986; de Groote, 1994; Eliashberg and Steinberg, 1987; Porteus and Whang, 1991). Yano and Dobson (1998a) provide a comprehensive review of the profit-maximizing product line design literature, and develop a classification scheme that categorizes contributions according to their treatment of product interactions in the marketplace and production costs in the manufacturing environment. Monroe *et al.* (1976) introduce a model that captures pairwise interactions among products in the marketplace. Green and Krieger (1985) and McBride and Zufryden (1988) build on this work with formulations of the product line design problem that identify a profit maximizing mix of k products, out of a set of n available products, while considering interactions among all selected products in the marketplace. Nair *et al.* (1995) also consider product interactions, but focus on feasible combinations of product attributes, and present a model for determining the mix that maximizes total return. Yano and Dobson (1998a) identify two additional classifications of product line models in the literature that consider production costs beyond the unit variable cost. The first category includes models that introduce a fixed cost in addition to the unit variable cost for each selected product (Shugan and Balachandran, 1977). Also in this category, Dobson and Kalish (1988, 1993) select products and their prices while considering both fixed and variable costs, and develop heuristic solution approaches. A final category identified in Yano and Dobson (1998a) includes models that consider shared manufacturing resources. Models that integrate product and process selection are developed by Raman and Chhajed (1995) and Yano and Dobson (1998b). The model developed in this paper fits into this category as well, but extends the literature by explicitly considering manufacturing cost interactions among candidate product profiles.

The normative model of the product line design problem presented in this paper accounts for diverse customer preferences while capturing a more meaningful set of fundamental manufacturing implications. The market portion of the model is kept simple to capture the crucial implications of customer preferences in the marketplace while allowing for a more complex model of the internal firm. Specifically, appropriate marketing inputs consistent with those found in existing product line models are integrated with detailed cost implications including variable costs, holding costs, and two set-up cost elements that together capture the costs associated with broader product lines and the benefits that result from selecting a product mix that exploits any available synergies among products. The resulting profit maximizing product line design model more completely captures the trade-off between the benefits derived from providing variety to the marketplace, and the cost savings that can be realized by selecting a mix of products that can be produced efficiently within a firm's manufacturing environment. We use the model to examine how the optimal product mix is affected by a firm's cost structure, as dictated by the relevant manufacturing capabilities and the degree to which a firm successfully designs sets of products that can be offered with minimum impact on manufacturing requirements. Product line characteristics studied include profitability, breadth, diversity of manufacturing requirements associated with the selected mix, and market share captured.

The paper is organized as follows. In Section 2, we introduce the marketing and manufacturing parameters relevant to the product line design problem, and formulate a decision model that identifies the profit maximizing product mix. A small example is developed in Section 3 to illustrate how the optimal mix of products and associated product line profitability can change when manufacturing synergies among products are explicitly considered. In Section 4 we report computational experience over a large set of randomly-generated problem instances, and discuss general insights derived from the experimental results. Extensions to the basic model that incorporate additional complexities, including capacity constraints in the production environment, dynamic demand or cost considerations that require multi-period analysis, and demand uncertainty, are addressed in Section 5. Section 6 concludes with a summary and suggestions for further research.

2. Problem setting

Consider a firm that selects a line of products to produce for a given product category from a set $N = \{1, 2, \ldots, n\}$ of n potential product profiles, where $j, j' \in N$ implies that products j and j' share common manufacturing resources (e.g., manpower, equipment, or facilities) and are considered substitutes by at least some customers. Let $L = \{1, 2, \ldots, l\}$ represent the set of l product families, or manufacturing classes, to which the n candidate products belong, with

$$q_{jk} = \begin{cases} 1 & \text{if product } j \in N \text{ is a member of} \\ & \quad \text{manufacturing class } k \in L, \\ 0 & \text{otherwise} \end{cases}$$

Thus, $j, j' \in k$ $(q_{jk} = q_{j'k} = 1)$ indicates that products j and j' are in the same manufacturing class and thus have similar manufacturing requirements in terms of common parts, set-ups, or systems.

The potential product set N can be generated by a number of procedures, including those of Green and Krieger (1985), McBride and Zufryden (1988), Dobson and Kalish (1988, 1993), and with modification according to managerial insight. Consistent with much of the literature (including, for example Green and Krieger (1985), McBride and Zufryden (1988) and Dobson and Kalish (1988, 1993), we assume that each customer's utility for each candidate product can be constructed based on part-worths, and that each customer will choose the product for which he has the greatest utility, among those offered. For a review of the two main approaches used to model customer preference in product line research, including that used in our model, the reader is referred to Yano and Dobson (1998a). In our model, D represents total customer demand forecast for the product category over a given time horizon, H. Let $N'' = \{n+1, \dots, n+n'\}$ be the set of competitive products in the market during the time horizon H; thus, the total potential set of products available during time horizon H is $N' = \{1, \dots, n, n+1, \dots, n+n'\}$. Also, $M = \{1, \dots, m\}$ is the set of customer segments in the firm's target market. The part-worth utility that the ith segment ($i \in M$) has for each level of each product attribute, including price, can be estimated by conjoint analysis. While we associate part-worths with each segment, it should be noted that each segment could conceivably consist of a single person. Part-worths forecast the utility that each potential product would provide each customer segment, U_{ij} ($i \in M, j \in N'$), and each customer then chooses the product with maximum utility. This slightly restrictive assumption is made for tractability of the ensuing optimization model; however, note that Moore *et al.* (1998) find that the predictive power of the maximum utility model is comparable to that of the rescaled logit model, the hierarchical Bayes model, and the individual-level conjoint model.

Thus, we make several simplifying assumptions about the set of candidate products considered in the model, customers' preferences among those products, and the resulting demand for products in the selected product line. Specifically, we assume perfect information about customers' purchase decisions given estimates of their utilities for alternative product profiles, and a static competitive product offering over an appropriately selected planning horizon. These assumptions are consistent with the majority of approaches used to address product line design issues in practice and in the literature (Yano and Dobson, 1998a), and result in a model that is attractive for both its simplicity and its ability to capture complex trade-offs that yield important conceptual insights. However, note that the model may be useful as a decision support tool in environments that require a more complex representation of these elements due to, e.g., extended planning horizons that encompass changes in costs or customer preferences, or significant levels of uncertainty surrounding the input parameters. We discuss extensions to the basic model that capture these added complexities in Section 5.

Let p_j denote the unit price charged for product $j \in N$. Manufacturing cost elements are incorporated to reflect the impact of manufacturing cost structure on product line profitability. Let c_j represent the unit variable production cost, and h_j the unit inventory holding cost over the planning horizon. S_k denotes the manufacturing class set-up cost incurred if any product j for which $q_{jk} = 1$ is assigned to the product line. In addition, let s_j represent the individual product set-up cost, incurred in addition to the class set-up cost if product $j \in N$ is included in the product line. Including individual product set-up costs in the model allows the trade-off between focus and diversity, and thus the impact of product line breadth on profitability, to be examined. In contrast, manufacturing class set-up costs capture any manufacturing synergies, e.g., common parts, systems, or set-ups, that may exist among products in the same manufacturing class. The set-up cost structure used in the model is also useful in capturing any revenue implications of constrained capacity, and is sufficient for generating conceptual insights into the product line design problem.

The frequency with which products in the line are manufactured also affects product line profitability. In order to capture the relevant effect on costs, and still maintain tractability, we assume that every product included in the line is produced in each of T production cycles over the given planning horizon, H. Total manufacturing class set-up costs are then minimized by producing products from the same manufacturing class consecutively. The production frequency, T, is included in the model as a decision variable; in practice, an appropriate production frequency may be influenced by accepted industry standards, such as periodic delivery requirements imposed by either suppliers or distributors. Bounds on the production frequency can be accommodated by simply adding a constraint that restricts the solution set for T. The problem formulation can also be adapted to relax the common cycle scheduling assumption, but the resulting model is significantly more complex, as discussed in Section 5. The problem of selecting the subset of products that maximizes the total profits earned by the product line over the planning horizon can then be formulated as the following constrained optimization problem:

(PLDP)

$$\text{Maximize} \quad \sum_{j \in N} (p_j - c_j) \sum_{i \in M} x_{ij} d_i - \left(\frac{1}{2T}\right) \sum_{j \in N} \sum_{i \in M} x_{ij} d_i h_j - T \sum_{k \in L} r_k S_k - T \sum_{j \in N} y_j s_j, \tag{1}$$

subject to

$$\sum_{j \in N'} x_{ij} = 1 \quad i \in M, \tag{2}$$

$$\sum_{i \in M} x_{ij} \le My_j \quad j \in N, \tag{3}$$

$$\sum_{h \in N'} U_{ih}x_{ih} \ge U_{ij}y_j \quad i \in M, j \in N, \tag{4}$$

$$r_k \ge \frac{1}{N}\sum_{j \in N} q_{jk}y_j \quad k \in L, \tag{5}$$

$$T > 0; \quad r_k, y_j, x_{ij} \in \{0,1\} \quad k \in L, i \in M, j \in N, \tag{6}$$

where the decision variables are defined as:

$$y_j = \begin{cases} 1, & \text{if product } j \text{ is offered,} \\ 0, & \text{otherwise.} \end{cases}$$

$$x_{ij} = \begin{cases} 1, & \text{if product } j \text{ is chosen by segment } i, \\ 0, & \text{otherwise.} \end{cases}$$

$$r_k = \begin{cases} 1, & \text{if class } k \text{ set-up will be incurred,} \\ 0, & \text{otherwise.} \end{cases}$$

T = number of production cycles in the planning horizon.

Objective function (1) represents the total contribution to profits from the products included in the product line by subtracting associated variable costs, holding costs, and individual product and manufacturing class set-up costs from the total revenue earned satisfying demands from the targeted customer segments. Constraints (2) ensure that each customer segment chooses exactly one product. Constraints (3) prevent products not included in the product line from satisfying demands from any customer segment. Constraints (4) guarantee that customers choose their most preferred available product. Constraints (5) require that the common manufacturing class set-up cost be incurred for a family of products if any member of the product family is included in the product line. Non-negativity of the selected production frequency and integrality requirements for the other decision variables are enforced by constraints (6).

Optimal solutions for problems with up to $n = 20$ products and $m = 40$ segments can be obtained with reasonable computational effort using straightforward implicit enumeration. However, problem (PLDP) is clearly NP-hard, since it contains the Uncapacitated Plant Location problem, a well-known NP-hard problem (Mirchandani and Francis, 1990), as its subproblem.

3. An illustrative example

A small example will clarify the input requirements of the model, and illustrate the impact of incorporating detailed manufacturing cost implications on the composition of the optimal product line. For this purpose, consider an automobile manufacturer that must determine the mix of vehicles it will offer to a target market comprised of $m = 9$ customer segments. Three levels of three attributes are used to describe vehicle profiles that are desirable to the target market. Through market research activities and elimination of infeasible product profiles, $n = 6$ candidate vehicle profiles are identified. The three attributes relevant to the target market are vehicle class (sedan, station wagon, or SUV), engine size (120, 140, 160 HP), and option package (OP0: stereo with AM/FM cassette, courtesy lamps, and rear window defog, OP1: content in OP0 plus air conditioning, cruise control, power locks, and power windows, OP2: content in OP1 plus power seats, power mirrors, and express down windows). Table 1 defines the firm's six candidate product profiles by attribute level using dummy variable coding. Product $j = 1$, for example, is a sedan equipped with a 120 HP engine and option package OP0.

Conjoint analysis is used to estimate part-worths for each customer segment. These part-worths, provided in Table 2 for this example, are used to forecast the utility, U_{ij}, provided by each product $j \in \{1, 2, \ldots, 7\}$ to each segment (where product 7 represents the competing product most preferred by segment i). Using the part-worth utility weights for segment 1 from Table 2, for example, the forecast utility for product $j = 3$ is $U_{13} = 2.68 + 1.92 - 0.38 + 2.68 = 6.90$. Product utilities are listed in Table 3 along with information on the demand, d_i, associated with each segment $i \in M$ over a planning horizon of $H = 90$ days. Table 3 indicates that segment 1 will purchase 190 units from the product category during the planning horizon. Product 6 provides the highest forecast utility, $U_{16} = 8.05$.

Observe in Table 4 that products 1, 3, and 5 belong to manufacturing class 1 and products 2, 4, and 6 to manufacturing class 2. In Tables 4 and 5, both S_k, associated with class $k = 1$, 2, and s_j, associated with product $j = 1, \ldots, 6$, vary relatively with what is defined below as the Synergy Ratio, *SR*. Given a set of products from the same manufacturing class, the synergy ratio reflects the percentage of the average total set-up cost for any product in the class that is captured by a common class set-up cost. For example, a synergy ratio of 0.4 for a class of products indicates that, on average, 40% of a member product's set-up is in common with the set-ups required

Table 1. Candidate product attribute profiles (X_{jl})

Product	*Sedan*	*Wagon*	*140 HP*	*160 HP*	*OP1*	*OP2*
$j = 1$	1	0	0	0	0	0
$j = 2$	0	0	0	0	0	1
$j = 3$	1	0	0	1	1	0
$j = 4$	0	0	0	1	0	0
$j = 5$	1	0	1	0	0	1
$j = 6$	0	0	1	0	1	0

Table 2. Attribute part-worth utilities (β_{fi}) for customer segment

Attribute	*i = 1*	*i = 2*	*i = 3*	*i = 4*	*i = 5*	*i = 6*	*i = 7*	*i = 8*	*i = 9*
Intercept	2.68	2.61	5.33	5.16	3.00	3.96	5.00	3.41	2.03
Sedan	1.92	1.87	1.00	−0.40	2.40	3.96	0.33	−0.62	−0.81
Wagon	1.15	2.24	4.00	−2.38	1.80	1.32	−2.00	1.86	−3.66
140 HP	2.68	−0.75	−4.00	−5.55	3.00	3.96	−2.33	1.55	−1.63
160 HP	−0.38	2.99	−4.00	0.79	−1.20	1.32	−1.67	−1.55	4.07
OP1	2.68	−0.75	1.00	1.59	−1.50	−5.28	3.67	−0.31	3.25
OP2	−1.53	−0.37	2.00	3.17	−2.10	−2.64	−0.67	2.17	5.29

Table 3. Segment demand and utility information

Segment	*di*	*j = 1*	*j = 2*	*j = 3*	*j = 4*	*j = 5*	*j = 6*	*j = 7*
$i = 1$	190	4.60	1.15	6.90	2.30	5.75	8.05	3.45
$i = 2$	205	4.48	2.24	6.72	5.60	3.36	1.12	7.84
$i = 3$	140	6.33	7.33	3.33	1.33	4.33	2.33	5.33
$i = 4$	210	4.76	8.33	7.14	5.95	2.38	1.19	3.57
$i = 5$	256	5.40	0.90	2.70	1.80	6.30	4.50	3.60
$i = 6$	221	7.92	1.32	3.96	5.28	9.24	2.64	6.60
$i = 7$	297	5.33	4.33	7.33	3.33	2.33	6.33	1.33
$i = 8$	231	2.79	5.58	0.93	1.86	6.51	4.65	3.72
$i = 9$	250	1.22	7.32	8.54	6.10	4.88	3.66	2.44

Table 4. Manufacturing class set-up cost information

Class (k)	*Products*	S_k *(SR = 0.0)*	S_k *(SR = 0.4)*	S_k *(SR = 0.8)*
1	1,3,5	0.0	254.8	509.6
2	2,4,6	0.0	243.0	485.9

for other products in the class (note: if synergies are not recognized and exploited in production, each product incurs its total set-up cost). As the level of synergies designed into products increases (i.e., *SR* increases to reflect greater commonality in manufacturing requirements across products), the individual product set-up cost for each product decreases, since a greater portion of the total set-up expenditure required for the product is captured by the manufacturing class set-up cost incurred once in each production cycle for all products in the class. The *SR* construct allows explicit evaluation of the impact on profitability of recognizing different levels of synergies across products when selecting the product mix.

To illustrate, Fig. 1 shows the impact on set-up cost expenditures of recognizing increasing design commonalities (reflected by the increasing values of $SR \in \{0.0, 0.4, 0.8\}$) across products in class $k = 1$, assuming a single production cycle and production of all three products (P1, P3, and P5). The first three entries reflect individual product set-up costs (s_1, s_3, s_5), the fourth reflects the common class set-up cost S_1, and the last depicts total expenditures on set-up costs for all three products (individual and class set-up costs) for each level of the synergy ratio considered.

Table 5 also provides data on the unit margin, $(p_j - c_j)$, associated with each product $j \in N$. Given this information, the holding cost of product j is computed as $h_j = 2.7 \times (p_j - c_j)$, and the total set-up cost of product j is estimated as $TS_j = 2 \times (p_j - c_j) \times (D/H)$, where

Table 5. Product margin, holding cost, and individual set-up cost information

Product (j)	$(p_j - c_j)$	h_j	TS_j	s_j *(SR = 0.0)*	s_j *(SR = 0.4)*	s_j *(SR = 0.8)*
1	13.0	35.1	577.8	577.8	323.0	68.1
2	16.0	43.2	711.1	711.1	468.1	225.2
3	19.0	51.3	844.5	844.5	589.6	334.8
4	15.0	40.5	666.7	666.7	423.7	180.7
5	11.0	29.7	488.9	488.9	234.1	0.0
6	10.0	27.0	444.5	444.5	201.5	0.0

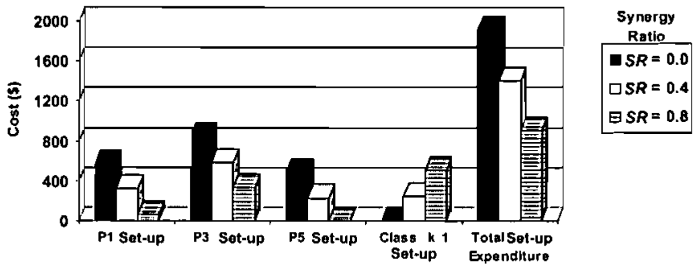

Fig. 1. Set-up cost expenditures for products P1, P3 and P5 with varying synergy levels.

$$D = \sum_{i \in M} d_i = 2000.$$

The individual product set-up cost associated with product j is then determined by subtracting the appropriate manufacturing class set-up cost from the total set-up cost, i.e., $s_j = TS_j - r_k S_k$, with $s_j = 0$ if

$$TS_j < \sum_{k \in L} q_{jk} S_k.$$

Optimal solutions for the example problem with $SR = 0.0$, 0.4, and 0.8 are given in Table 6, which shows the composition of the optimal product line and the corresponding profit over the planning horizon. Table 6 also provides information on the number of products in the optimal product line (Product Line Breadth, *PLB*), the number of manufacturing classes represented in the product mix (Manufacturing Class Breadth, *MCB*), the number of customer segments whose demands are satisfied by the product line (Customer Segments, *CS*), the proportion of total targeted demand captured by the product line (Target Market Share, *TMS*), and the optimal production frequency (T^*).

The profit maximizing mix of products (shown in Table 6) varies significantly when alternative levels of manufacturing synergies are relevant in the manufacturing environment. When opportunities for synergies among products don't exist or are ignored ($SR = 0.0$), the optimal product line consists of two products (products 1 and 2) from different manufacturing classes, and yields profits of \$12 420. If the manufacturing environment is characterized by a moderate level of manufacturing synergies among products ($SR = 0.4$), *and* if the associated cost implications are incorporated into the product selection process, then the optimal mix shifts to include two alternative products (products 3 and 5) from the same manufacturing class, yielding a 9% increase in profits to \$13 527. As the level of relevant manufacturing synergies increases ($SR = 0.8$), the optimal product line expands to include all of the products in manufacturing class 1 (products 1, 3, and 5), raising profits an additional 18% to \$15 938. Environments with greater levels of manufacturing synergies across products incur smaller total set-up cost expenditures; as a result, the optimal production frequency increases from $T^* = 5.2$ (for $SR = 0.0$) to $T^* = 5.7$ (for $SR = 0.4$) and $T^* = 6.4$ (for $SR = 0.8$).

We next compare the optimal solutions in Table 6 with solutions derived from existing models in the product line literature. Recall that the earlier models accounted only for variable costs (Green and Krieger, 1985; McBride and Zufryden, 1988) or for a combination of variable and fixed costs (Dobson and Kalish, 1988, 1993) for each product. We specify two corresponding models, ($f_j = 0$) and ($f_j > 0$). In order to specify a comparable fixed cost for the ($f_j > 0$) model, we use information about the optimal product line breadth and associated market share (as identified by PLDP) and assume that the total demand to be satisfied by any individual product is equal to the average demand, $\bar{D}$, satisfied by individual products in the optimal product line, i.e.,

$$\bar{D} = \frac{1}{n_{P^*}} \sum_{j \in P^*} D_j,$$

where n_{P^*} denotes the number of products in the optimal product line and D_j is the resulting demand for product j. To illustrate, consider the example problem for $SR = 0.0$. The optimal product line consists of two products, and captures 1795 units of demand over the planning horizon; thus, $\bar{D} = 1795/2 = 897.5$. Since earlier models did not address product cost interactions, we estimate a fixed cost of including product $j \in N$ in the product line by ignoring available manufacturing synergies and calculating the minimum total holding plus set-up cost incurred by replenishing product $j \in N$ in economic lot sizes, using $\bar{D}$ as estimated demand for each product. For details on the calculation of fixed costs, the reader is referred to Morgan (1996).

Table 7 shows that when manufacturing costs are ignored ($f_j = 0$ for all $j \in N$), products 1, 2, and 3 are selected for the product line, with profits that are 11% ($SR = 0.0$), 10% ($SR = 0.4$), and 16% ($SR = 0.8$) lower than the profits earned by the optimal product mix from

Table 6. Optimal solutions for example problem

SR	*Optimal product line*	*Optimal profit* \$	*PLB*	*MCB*	*CS*	*TMS* (%)	T^*
0.0	1,2	12 420	2	2	8	89.8	5.2
0.4	3,5	13 527	2	1	7	82.8	5.7
0.8	1,3,5	15 938	3	1	8	89.8	6.4

the (PLDP) model, thus indicating that product line profitability can be enhanced significantly by explicitly considering manufacturing costs in selecting the appropriate product mix (note that synergies among *selected* products are assumed to be exploited, hence the increasing profits as *SR* increases). The ($f_j > 0$) model selects a product line consisting of products 1 and 2 for all three values of *SR*. Since products 1 and 2 belong to different manufacturing classes, total profits for this product line remain constant at \$12 420, a profit level that is 0, 8, and 22% below the profit from the corresponding optimal product line for $SR = 0.0$, 0.4, and 0.8, respectively. This indicates that while the fixed cost included in the ($f_j > 0$) model acts to limit product line breadth, this cost does not adequately reflect interactions among products in manufacturing, resulting in a less-profitable product mix.

4. Insights for product line management from numerical results

In this section we discuss the results of computational experimentation, highlighting the impact of cost structure and manufacturing synergies on the breadth, composition, and profit performance of the optimal product mix. Specifically, we use the (PLDP) model to generate solutions and report information about optimal profits (*PROFIT*), optimal Product Line Breadth (*PLB*), optimal Manufacturing Class Breadth (*MCB*), and total Target Market Share (*TMS*) captured by the optimal product mix. For comparison purposes, we generate similar results for two alternative product line design models with zero fixed costs ($f_j = 0$) and positive fixed costs ($f_j > 0$). As described in Section 3, profit calculations assume that sequencing decisions in the manufacturing environment are done intelligently so as to exploit any synergies among products selected by the alternative models, even though these cost implications are not considered in selection of the product mix.

4.1. *Experimental design*

Our experimental design is outlined in Table 8. We vary *n*, *m*, and *l* to reflect alternative settings for the number of candidate products considered by the firm, the degree of segmentation in the firm's target market, and the number of distinct manufacturing classes that include products with similar processing requirements. Since a principal objective of our analysis is to gain insight into the composition and profitability of an optimal product mix in manufacturing environments characterized by alternative cost structures, we also vary the Synergy Ratio (*SR*). Using margins as a surrogate for relative costs across products, the holding cost and total set-up cost for product j are calculated as $h_j = \alpha_1 \times (p_j - c_j)$ and $TS_j = \alpha_2 \times (p_j - c_j) \times (D/H)$. In the analysis two cost levels are evaluated for each parameter ($\alpha_1 = 1, 3$ and $\alpha_2 = 1, 3$). As in Section 3, the manufacturing class set-up cost, S_k, associated with each class $k \in L$ is set proportional to the synergy ratio. Individual product set-up costs are then determined by setting

$$s_j = \max\left\{0, TS_j - \sum_{k \in L} q_{jk} S_k\right\}.$$

Within each of the 324 problem settings, 10 test problems were generated. Equal-sized customer segments were constructed by setting $d_i = D/m$ for all segments $i \in M$

Table 7. Alternative solutions for example problem

Model	*SR*	*Product line*	*Profit* \$	*PLB*	*MCB*	*CS*	*TMS* (%)	*T**
$f_j = 0$	0.0	1,2,3	11 065	3	2	8	89.8	4.3
	0.4	1,2,3	12 201	3	2	8	89.8	4.6
	0.8	1,2,3	13 417	3	2	8	89.8	5.0
$f_j > 0$	0.0	1,2	12 420	2	2	8	89.8	5.2
	0.4	1,2	12 420	2	2	8	89.8	5.2
	0.8	1,2	12 420	2	2	8	89.8	5.2

Table 8. Experimental design

Parameter		*Values*		
n	Number of candidate product profiles	10	15	20
m	Number of unique customer segments in target market	10	20	40
l	Number of manufacturing classes to which products are assigned	2	5	8
SR	Synergy Ratio reflecting level of synergies among products in a class	0.0	0.4	0.8
α_1	Multiplier used for calculating unit holding cost	1	3	
α_2	Multiplier used for calculating total product set-up cost	1	3	

(note that Morgan (1996) also examined unequal segment sizes; the distribution of demand among segments had no systematic effect on the results). The unit margin, $p_j - c_j$, for each candidate product $j \in N$ was randomly drawn from a uniform distribution on the interval [10,20]. Utilities were randomly generated for each customer segment $i \in M$ and each product $j \in N'$. (While it is possible to expand the design to include variations in the way that utilities and unit margins are generated, the nature of the problem is such that the results are not likely to be sensitive to these characteristics of the design.) Finally, product profiles were randomly assigned to each of the l manufacturing classes to determine q_{jk} values $\forall\, j, k$.

The experimental design includes 3240 problem instances. The optimal product line for each test problem was identified using an implicit enumeration scheme coded in FORTRAN and implemented on a Pentium II Personal Computer. Raw data on the optimal profit, product line breadth, manufacturing class breadth, and total target market share captured is included in Morgan *et al.* (1999). Results associated with the two earlier models (with fixed costs $f_j = 0$ and $f_j > 0$) are also reported.

4.2. *Analysis of results for model (PLDP)*

We used regression analysis to examine the main effects of the parameters defining the decision environment (outlined in Table 8) as well as three relevant interaction effects ($\alpha_1 \times \alpha_2$, $\alpha_s \times SR$, $l \times SR$) on the four dependent variables outlined above: *PROFIT*, *PLB*, *MCB*, and *TMS*. In Table 9, we report standardized coefficients and associated significance levels to indicate the relative importance of each predictor. We first summarize our findings related to the optimal product mix.

4.2.1. *PROFIT (Regression 1)*

Holding and set-up costs (captured by α_1, α_2, and $\alpha_1 \times \alpha_2$) have the biggest impact on profit, with higher cost manufacturing environments earning less profit. Markets characterized by more diversity in customer preferences (m) require that the firm either incur greater costs to produce and sell a broader mix of products that will satisfy the same share of the market, or satisfy a smaller share of the market and earn less revenue. As the number of candidate products (n) increases, profits increase since there are more alternative product lines from which the optimal mix can be selected to most profitably satisfy customers. As the degree of synergies among products in each manufacturing class (SR) increases, the shift in set-up costs from individual products to the common class set-up improves profitability. This positive impact on profits is amplified for environments with higher set-up costs, as reflected by the interaction term $\alpha_2 \times SR$. Higher synergies together with an increasing number of manufacturing classes across which a given number of candidate products are distributed ($l \times SR$) make it more difficult to find a focused product mix that includes products from a small number of classes and exploits increasing synergies, thus increasing costs and reducing product line profitability.

4.2.2. *PLB (Regression 2)*

Higher holding and set-up costs have the greatest limiting impact on Product Line Breadth (PLB) since the cost impact of each additional product is increased. Alternatively, a higher level of synergies across products acts to broaden product lines since the cost of including more products from any given class is reduced (captured by SR and $\alpha_2 \times SR$), though the impact of SR on PLB is diminished as products are distributed across more manufac-

Table 9. Regressions for model (PLDP)

	Standardized coefficients for regressions			
	Regression 1 *PROFIT*	*Regression 2* *PLB*	*Regression 3* *MCB*	*Regression 4* *TMS*
Constant	0	0	0	0
N	0.131	0.095	0.013[a]	0.177
M	−0.193	0.103	−0.002[a]	−0.303
l	0.010[a]	0.039[b]	0.323	0.016[a]
SR	0.178	0.406	−0.267	0.094[b]
α_1	−0.418	−0.355	−0.391	−0.099
α_2	−0.424	−0.433	−0.508	−0.247
$\alpha_1 \times \alpha_2$	0.260	−0.057[a]	0.040[a]	−0.337
$\alpha_2 \times$ SR	0.049[b]	0.181	0.182	0.348
$l \times$ SR	−0.101	−0.244	−0.091[c]	−0.178
Observations	3240	3240	3240	3240
R^2	0.763	0.468	0.412	0.413

[a] Not significant. [b] Significant at ≤ 0.1. [c] Significant at ≤ 0.01.
All other factors significant at ≤ 0.001.

turing classes, as reflected by the interaction term $l \times SR$. As the number of candidate products (n) grows, the breadth of the optimal product line grows slightly as more cost effective combinations of products can be identified to capture more profits. As the number of customer segments (m) grows, a broader mix of products is typically identified to satisfy the more diverse mix of customers in the target market.

4.2.3. *MCB (Regression 3)*

As the level of synergies (SR) across products in each manufacturing class increases, the optimal product mix is focused on fewer manufacturing classes in order to exploit those synergies. Not surprisingly, increasing set-up costs (α_2) also decrease *MCB* considerably. However, when both synergies and set-up costs are high ($\alpha_2 \times SR$), manufacturing class breadth recovers as profits are maximized by a somewhat less focused product mix that captures a greater share of the market to earn more revenues for the firm. Higher holding costs (α_1) also drive a narrower product mix, which will on average reduce the number of manufacturing classes represented in the optimal mix. Alternatively, the number of classes represented in the optimal product mix increases as the candidate products are distributed across an increasing number of manufacturing classes (l).

4.2.4. *TMS (Regression 4)*

For *TMS*, all coefficients had the same signs as for optimal profits. Many of the same relationships outlined for the *PROFIT* regression apply here. When we compare results from the *TMS* regression with those obtained for *PLB*, it is interesting to note that the impact of greater diversity of preferences in the market place (i.e., increasing m) is significant in opposite directions for these two dependent variables. For *PLB*, we saw that increasing m increases product line breadth. Here, despite the increasing product line breadth, increases in m yield profit-maximizing product lines that capture less of the market. Also, the considerable positive influence of $\alpha_2 \times SR$ on market share reflects the extent to which an environment with high set-up costs can be overcome if manufacturing synergies are designed into classes of products and explicitly considered in determination of the optimal product mix.

4.3. *Comparative analysis – PLDP versus alternative models*

In this section, we contrast the results generated by model (PLDP) with those generated by the alternative models in the literature. We report mean comparisons of optimal product line characteristics versus alternative models in Table 10. To gain additional insight into the impact of using a more robust product line decision model like (PLDP), we ran regressions to determine the impact of the parameters defining the decision environment (n, m, l, SR, CL) on the differences between optimal and alternative model characteristics ($PROFIT_{\text{OPT-F0}}$, $PLB_{\text{OPT-F0}}$, $MCB_{\text{OPT-F0}}$, and $TMS_{\text{OPT-F0}}$ for model ($f_j = 0$), and $PROFIT_{\text{OPT-F+}}$, $PLB_{\text{OPT-F+}}$, $MCB_{\text{OPT-F+}}$, and $TMS_{\text{OPT-F+}}$ for model ($f_j > 0$)). The standardized coefficients and associated significance levels are reported in Table 11.

4.3.1. *PROFIT (Regressions 5 and 6)*

Based on the results of the paired sample t-tests reported in Table 10, we observe that optimal profits (PLDP) are substantially greater than those obtained with either of the alternative models. On average, optimal profits exceed those obtained with the ($f_j = 0$) model by 32%. From Regression 5 ($PROFIT_{\text{OPT-F0}}$), we see that as the environment becomes more complex or costly through increases in n, m, $l \times SR$ (reflecting more classes with greater commonality across products in the same class), α_1, and/or α_2, the gap between optimal profits and those obtained with the zero fixed cost model grows. The advantages of the (PLDP) model are particularly notable in higher cost environments, with average profits associated with the optimal product mix 475% greater than those earned with the ($f_j = 0$) model when α_1 and α_2 are at their highest levels. The profit gap shrinks as SR increases due to the broader product lines selected by the zero fixed cost model, which yield greater opportunities to benefit from synergies across products.

In contrast, the gap between optimal profits and those obtained with the positive fixed cost model ($f_j > 0$) is

Table 10. Paired sample t-tests

Pair tested	*Mean*	t	*Significance P ≤*
$PROFIT_{\text{OPT}} - PROFIT_{\text{F0}}$	10 637.99	61.98	0.001
$PROFIT_{\text{OPT}} - PROFIT_{\text{F+}}$	1563.80	36.67	0.001
$PLB_{\text{OPT}} - PLB_{\text{F0}}$	−2.65	−82.25	0.001
$PLB_{\text{OPT}} - PLB_{\text{F+}}$	−0.06	−4.41	0.001
$MCB_{\text{OPT}} - MCB_{\text{F0}}$	−1.44	−69.04	0.001
$MCB_{\text{OPT}} - MCB_{\text{F+}}$	−0.43	−41.41	0.001
$TMS_{\text{OPT}} - TMS_{\text{F0}}$	−0.09	−53.24	0.001
$TMS_{\text{OPT}} - TMS_{\text{F+}}$	−0.02	−14.97	0.001

Table 11. Regressions comparing model (PLDP) to alternative models

	Standardized coefficients for regressions							
	Regressions 5 and 6 PROFIT		*Regressions 7 and 8* PLB		*Regressions 9 and 10* MCB		*Regressions 11 and 12* TMS	
	OPT-F0	OPT-F+	OPT-F0	OPT-F+	OPT-F0	OPT-F+	OPT-F0	OPT-F+
Constant	0	0	0	0	0	0	0	0
n	0.171	0.148	−0.219	−0.136	−0.107	−0.188	−0.012[a]	−0.139
m	0.195	0.072	−0.418	0.049	−0.216	−0.071	−0.269	0.021[a]
l	0.021[a]	0.003[a]	−0.002[a]	0.041[b]	−0.476	−0.132	−0.003[a]	0.031[a]
SR	−0.185	0.181	0.209	0.516	−0.134	−0.293	0.079[b]	0.169
α_1	0.053[b]	0.019[a]	−0.220	−0.093[c]	−0.222	−0.270	−0.103	−0.004[a]
α_2	0.125	−0.048[a]	−0.256	−0.205	−0.253	−0.318	−0.300	−0.256
$\alpha_1 \times \alpha_2$	0.649	0.456	0.016[a]	−0.055[a]	0.016[a]	0.158	−0.423	−0.285
$\alpha_2 \times SR$	−0.141	0.123	0.060[b]	0.261	0.076[b]	0.158	0.400	0.526
$l \times SR$	0.191	0.022[a]	−0.082[b]	−0.308	−0.065[b]	−0.041[a]	−0.162	−0.281
Obs	3240	3240	3240	3240	3240	3240	3240	3240
R^2	0.631	0.344	0.353	0.314	0.419	0.157	0.497	0.298

[a] Not significant. [b] Significant at ≤ 0.1. [c] Significant at ≤ 0.01.
All other factors significant at ≤ 0.001.

greater in environments where there are higher levels of synergies (SR) designed into products, especially in combination with high set-up costs ($\alpha_2 \times SR$). We also see that model ($f_j > 0$) becomes increasingly suboptimal as the level of costs (α_1, α_2) in the manufacturing environment increase, due to the more naïve mechanism used to capture cost implications (i.e., without considering cost interactions across products). In fact, when α_1, α_2, and SR are all at their highest levels, optimal profits are 20% greater than those earned by the ($f_j > 0$) model, as compared with a gap of 4% across all problems in the experimental design. The gap between optimal profits and ($f_j > 0$) profits also grows as the decision environment becomes more complex with increases in with n and m.

4.3.2. *PLB (Regressions 7 and 8)*

Not surprisingly, the product lines identified by the ($f_j = 0$) model are broader than those obtained with model (PLDP). Furthermore, the breadth of the product line identified by the zero fixed cost model increases at a much faster rate with increases in the number of candidate products (n) and in the degree of diversity in the marketplace (m) as it attempts to maximize profits without consideration of fixed costs incurred in the manufacturing environment. The growing profit gap with increases in α_1, α_2, and $l \times SR$ (noted above) is due to the superior ability of model (PLDP) to capture the impact of these manufacturing environment characteristics on the inherent trade-off between breadth in the marketplace and focus in the manufacturing environment. The difference in resulting decisions about the selected product mix is reflected in Regression 7 by the relative decrease in the optimal PLB (as compared to PLB ($f_j = 0$)) with increases α_1, α_2, and $l \times SR$, and relative increase in optimal PLB with higher levels of synergies among products (SR).

Across all problems, the average breadth of the product lines identified by the ($f_j > 0$) model is greater than that for model (PLDP). Due to the inability of the ($f_j > 0$) model to account for synergies in production, this model is also less responsive than model (PLDP) to changes in the manufacturing environment, as evidenced by the coefficients for SR, $l \times SR$, α_1, and α_2 in Regression 8. In fact, when we break down the data according to the level of synergies across products, we observe that while $\overline{PLB}$ ($f_j > 0$) remains constant at 2.5 across all three levels of SR, optimal $\overline{PLB}$ increases from 2.1 for $SR = 0.0$, to 2.3 for $SR = 0.4$, to 3.0 for $SR = 0.8$, with a paired sample t-test on results for $SR = 0.8$ showing optimal product line breadth significantly greater than that identified by the ($f_j > 0$) model ($P < 0.001$).

4.3.3. *MCB (Regressions 9 and 10)*

The product lines identified by the alternative models are less focused in terms of the number of manufacturing classes (MCB) represented in the product mix due to their inability to recognize and exploit opportunities to cut costs in the manufacturing environment. As the number of different classes to which products are assigned (l) increases, the optimal Manufacturing Class Breadth (MCB) grows by less than that associated with the product lines identified by either of the alternative models, since model (PLDP) attempts to find a mix of products that earns revenues but also exploits synergies in the manufacturing environment by limiting the number of different class set-ups. Similarly, MCB for each of the alternative models decreases less than the optimal MCB with increases in SR, α_1, and α_2, again reflecting their inability to exploit available synergies and limit the impact of higher costs in the manufacturing environment, respectively.

4.3.4. *TMS (Regressions 11 and 12)*

Based on the paired sample t-tests, the total market share captured by the product lines identified by the either of the alternative models is greater than that captured by the optimal product mix. This is not surprising since the optimal model generally identifies narrower product lines and is more sensitive to cost implications and other complexities of the manufacturing environment, focusing more on profits and less on market share. The regression coefficients support this observation, with *TMS* captured by the optimal product mix decreasing more (versus alternative models) as it identifies a more focused and less costly product mix when there are higher cost levels (α_2, $\alpha_1 \times \alpha_2$) or more classes with considerable synergies in a class ($l \times SR$), and increasing more to exploit synergies and cost-effectively capture more customers with increasing SR and $SR \times \alpha_2$.

4.4. *Summary of insights from results*

The computational results indicate that product line profitability is significantly enhanced when synergies among products are explicitly recognized in the product selection process, especially when these synergies are pronounced and manufacturing costs are high. Increased synergies, which in this study act to reduce individual product set-up costs, also lead to broader product lines, though Morgan *et al.* (1999) note that optimal product line breadth can remain unchanged or even decrease for a given problem instance over a wide range of SR. In contrast, increased synergies lead to a decrease in the manufacturing class breadth of the profit maximizing product mix, reflecting the strong incentives synergies provide to focus on a set of products that share commonalities in manufacturing. The results also demonstrate that product line decisions that attempt to maximize market share without considering the associated impact on manufacturing costs are significantly flawed. Even models that attempt to account for manufacturing costs in product selection can be improved by distinguishing between set-up costs attributable to the individual product and those associated with the manufacturing class.

5. Modeling and algorithmic extensions

The model presented in Section 2 provides a representation of the manufacturing environment that includes key cost elements, such as variable production costs, inventory holding costs, and individual product and manufacturing class set-up costs. In this section, we discuss how other operational factors, which may be important in some manufacturing environments, can be incorporated into the (PLDP) model.

5.1. *Capacity constraints and lotsizing issues*

In the (PLDP) model, capacity constraints in the firm's production facilities are not modeled explicitly, but rather indirectly with the individual product and manufacturing class set-up costs. In addition, the model addresses the issue of production frequency for each product (and the resulting lot size) by using a common cycle scheduling approach, whereby each product is produced in each production cycle, and the number of production cycles is optimized as part of the model.

Capacity constraints can be easily appended to the set of constraints that define model (PLDP). Let:
$t_j = 1/P_j$ = time required to process one unit of product j, where P_j is the production rate (in units/H);
$\bar{T}_k$ = time required to set-up for the production of manufacturing class k;
$\bar{t}_j$ = time required to set-up for production of product j, assuming class set-up already incurred;
C_T = time available in each production cycle for set-up and processing, allowing for scheduled downtime.

The value of C_T is clearly driven by the number of production cycles, T. Given these parameters, a limit on the productive capacity of a facility can be formulated as:

$$\sum_{j\in N}\left(\sum_{i\in M} x_{ij}d_i\right) t_j + \sum_{k\in L} r_k \bar{T}_k + \sum_{j\in N} y_j \bar{t}_j \le C_T.$$

The addition of this capacity constraint to model (PLDP) still results in a straightforward linear integer programming problem that can be easily solved. In the original model, set-up costs capture whatever appropriate penalty is incurred for production revenue lost due to capacity limitations. In contrast, the inclusion of explicit set-up times and capacity considerations in the modified model implies that set-up costs reflect only those variable costs incurred by the resources (e.g., labor) used in set-up activities.

Allowing different production frequencies for both individual products and product classes, and thus explicitly accounting for lotsizing considerations, complicates the problem formulation considerably. Let:
T = number of production cycles in the planning horizon;
$T'_k = (1/\gamma_k)T$ = number of production cycles in which product class k is produced (γ_k a positive integer);
$\tau_j = (1/m_j)T'_k$ = number of production cycles in which product j ($q_{jk} = 1$) is produced (m_j a positive integer).
The objective function then becomes:

$$\textbf{(NL-PLDP)}\ \text{Maximize} \sum_{j\in N}(p_j - c_j)\sum_{i\in M} x_{ij}d_i - \frac{1}{2T}\sum_{j\in N}\sum_{k\in L} h_j m_j \gamma_k \left(\sum_{i\in M} x_{ij}d_i\right) - \sum_{k\in L}\frac{T}{\gamma_k} r_k S_k - \sum_{j\in N}\sum_{k\in L} q_{jk}\frac{T}{m_j\gamma_k} y_j s_j.$$

The constraint set (2)–(5) remains unchanged (adding capacity constraint if so desired), with additional definitional constraints m_j, γ_k = integer ≥ 1, $\forall\, j, k$. The modified objective function is clearly non-linear, so that non-linear solution approaches would need to be incorporated within a custom branch-and-bound procedure in order to solve problem (NL-PLDP). In such an approach, upper bounds can be obtained from the solution to the non-linear programming problem that results when the integrality requirements on the m_j and γ_k are relaxed. Similarly, lower bounds are easily generated by heuristically rounding fractional solutions to integer values.

5.2. *Multi-period extensions*

Most firms address product line design issues by estimating demand and cost data over an entire planning horizon (the typical duration of which is from 1 to 5 years, depending on the time between new product introductions or major redesigns of existing models). A single-period formulation like the (PLDP) model, where the period chosen is the length of the planning horizon, is an appropriate framework for situations such as these. However, for environments with extended planning horizons, and in particular with cost and/or demand data that change significantly within the planning horizon, the (PLDP) model can serve as the basic building block on which to develop a dynamic model.

To update the notation, subscript t, which indexes the period (with $t \in \Im$), must be added to all input data and decision variables in the model. We assume that each period in the planning horizon contains an equal number of production cycles, i.e., T is not time dependent. Then, let $\phi_t(\tilde{X}_t, \tilde{Y}_t)$ be the single-period objective function of (PLDP) using the input data from period t, with $\tilde{X}_t = (x_{ijt}, i \in M, j \in N, t \in \Im)$ and $\tilde{Y}_t = (y_{jt}, j \in N, t \in \Im)$. If desired, the set of products can be restricted to time-dependent product subsets N_t for each period. Let $(\tilde{X}_t^*, \tilde{Y}_t^*)$ denote the optimal solution to (PLDP) for period t, and define $F_t(\tilde{X}_{t-1}, \tilde{Y}_{t-1})$ as the optimal profit function from period t through the end of the planning horizon (period $\eta = |\Im|$), given that $(\tilde{X}_{t-1}, \tilde{Y}_{t-1})$ is the product line configuration in period $t-1$. Note that the product line configuration in a period represents not only the product offerings in that period, but also the choices of customers in each market segment given these product offerings. A dynamic framework for making product line decision can then be formulated as:

$$\textbf{(MP-PLDP)}\ F_t(\tilde{X}_{t-1}, \tilde{Y}_{t-1}) = \max_{\tilde{X}_t, \tilde{Y}_t} \big[\phi_t(\tilde{X}_t, \tilde{Y}_t) - \Delta_t(\tilde{X}_{t-1}, \tilde{Y}_{t-1}, \tilde{X}_t, \tilde{Y}_t) + \theta F_{t+1}(\tilde{X}_t, \tilde{Y}_t)\big], \quad \text{for } t = 1, 2, \ldots, \eta$$

$$F_{\eta+1}(\cdot, \cdot) = 0,$$

where θ is an appropriate discount factor, and $\Delta_t(\tilde{X}_{t-1}, \tilde{Y}_{t-1}, \tilde{X}_t, \tilde{Y}_t)$ is the cost of changing the product line configuration from period $t-1$ to period t, including redesign expenses and promotional and advertising expenditures incurred as a result of a change in the product mix.

The computational tractability of dynamic programming formulation (MP-PLDP) is dependent on both the number of periods in the planning horizon and the number of potential product line configurations considered in each period. The computational effort required to solve (MP-PLDP) increases linearly with the number of periods, and exponentially with the number of product line configurations. Therefore, the number of alternative solutions considered in each period is the most critical factor in determining the computational efficiency of the dynamic programming approach. The efficiency of the solution process can be further improved by reducing the state space, as discussed in Morgan *et al.* (1999).

Morgan *et al.* (1999) also discuss how the basic (PLDP) model can be modified to incorporate uncertainty with respect to the demand associated with each customer segment. In all of these extensions, we observe that more precise, yet of course more complex, representations of the manufacturing environment can be developed from the basic building blocks of the (PLDP) model, if the desired result is a decision support tool tailored to the specific realities of an individual firm. We also note that the relevant trade-offs discussed in Section 4 do not change appreciably as additional detail is built into the model. This underscores the main appeal of model (PLDP) – that despite its simplicity, we generate conceptual insights into the relevant trade-offs in product line design, insights that do not fundamentally change as we build in additional complexity.

6. Conclusions

This paper presents a model for product line selection (PLDP) that accounts for both diverse customer preferences across market segments, and manufacturing costs that vary with the composition of the product line. By integrating marketing inputs with more detailed manufacturing cost information, including manufacturing synergies attained through co-ordinated product design efforts, the model captures the trade-off between the benefits derived by providing variety to the marketplace, and the cost savings that can be realized by selecting a mix of products that can be produced efficiently within a firm's manufacturing environment. The model is capable of determining the optimal product mix for problem instances involving 20 products and 40 customer segments; for larger problems, the model becomes intractable and heuristic solution approaches are necessary.

We utilize two constructs that together allow us to better characterize product interactions in the manufacturing environment. The synergy ratio is used conceptually to capture the level of manufacturing commonality designed into products, and the manufacturing class breadth of a product line is used as a measure of the diversity of manufacturing requirements associated with the optimal product mix. These complement notions of product line breadth, as together they facilitate understanding of the manufacturing implications that are critical to good decision making in product line management. Based on regression analysis of results from our extensive computational experiments, it is apparent that product line profitability can be significantly improved through the use of decision models that account for more detailed manufacturing implications, including product cost interactions in the manufacturing environment. Our results also provide insight into the types of manufacturing environments that reap the greatest benefits from efforts to attain and exploit manufacturing synergies among products.

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