

## Lista 7

1) Indique a regra de inferência que justifica a validade de:

$$a) \{ (P \rightarrow Q) \} \vdash (P \rightarrow Q) \vee \sim P$$

$$(P \rightarrow Q)$$

Adição

$$(P \rightarrow Q) \vee \sim P$$

$$b) \{ \sim P \wedge (Q \rightarrow R) \} \vdash \sim P$$

$$1. \sim P \wedge (Q \rightarrow R) \vdash \sim P$$

Simplificação

$$c) \{ (P \rightarrow Q), (Q \rightarrow \sim R) \} \vdash (P \rightarrow \sim R)$$

$$1. P \rightarrow Q$$

Silogismo Hipotético

$$2. \frac{Q \rightarrow \sim R}{P \rightarrow \sim R}$$

$$d) \{ P \rightarrow (Q \rightarrow R), P \} \vdash (Q \rightarrow R)$$

$$1. P$$

Modus Ponens

$$2. \frac{P \rightarrow (Q \rightarrow R)}{(Q \rightarrow R)}$$

$$e) \{ (Q \vee R) \rightarrow \sim P, \sim \sim P \} \vdash \sim (Q \vee R)$$

$$1. (Q \vee R) \rightarrow \sim P$$

Modus tollens

$$\sim P$$

$$f) \{ (P \rightarrow Q), (R \rightarrow \sim R) \} \vdash (P \rightarrow Q) \wedge (R \rightarrow \sim R)$$

$$1. (P \rightarrow Q)$$

Conjunção

$$2. (R \rightarrow \sim R)$$

$$(P \rightarrow Q) \wedge (R \rightarrow \sim R)$$

$$9) \{ (p \wedge q) \vee (\sim p \wedge r), \sim(\sim p \wedge r) \} \models p \wedge q$$

$$1. (p \wedge q) \vee (\sim p \wedge r)$$

Silogismo disjuntivo

$$2. \sim(\sim p \wedge r)$$

$$p \wedge q$$

2) Indique uma possível conclusão para:

$$a) \{ (s \vee t) \rightarrow (n \wedge q), (n \wedge q) \rightarrow \sim p \}$$

$$1. (s \vee t) \rightarrow (n \wedge q)$$

$$2. (n \wedge q) \rightarrow \sim p$$

$$(s \vee t) \rightarrow (\sim p)$$

$$b) \{ (p \leftrightarrow q) \rightarrow \sim(n \wedge r), \sim\sim(n \wedge r) \}$$

$$1. (p \leftrightarrow q) \rightarrow \sim(n \wedge r)$$

$$2. \sim(n \wedge r)$$

$$\sim(p \leftrightarrow q)$$

$$c) \{ \sim i \vee (n \wedge t), i \}$$

$$\sim i$$

$$(n \wedge t)$$

$$d) \{ p \rightarrow (n \vee \sim r), (n \vee \sim r) \rightarrow t \}$$

$$1. p \rightarrow (n \vee \sim r)$$

$$2. (n \vee \sim r) \rightarrow t$$

$$p \rightarrow t$$

$$e) \{ p \rightarrow n, \sim q \rightarrow \sim r, p \vee \sim q \}$$

$$1. p \rightarrow n$$

$$2. \sim q \rightarrow \sim r$$

$$3. p \vee \sim q$$

$$1) \{ \overset{A}{\sim P} \vee \overset{B}{\sim Q}, \overset{B}{\sim \sim Q} \}$$

$$\sim P \vee Q$$

$$\sim Q$$

Solugismo Diferencia

$$\sim P$$

$$2) \{ \overset{A}{P} \rightarrow (\overset{B}{\sim P} \wedge Q), \sim (\overset{B}{\sim P} \wedge Q) \vee \overset{D}{\sim P} \rightarrow \overset{D}{\sim Q} \}$$

$$1. P \rightarrow (\sim P \wedge Q)$$

Dilema Destrutivo.

$$\sim Q \rightarrow 1$$

$$\sim (\sim P \wedge Q) \vee \sim 1$$

$$\sim P \vee \sim \sim Q$$

3. Construa as deduções

$$a) \{ (P \wedge Q) \rightarrow R, P, Q \} \vdash R$$

$$1. (P \wedge Q) \rightarrow R$$

$$2. P \quad | \text{Condição}$$

$$3. Q \quad |$$

$$4. (P \wedge Q) \quad \text{Modus Ponens}$$

$$5. AP \wedge Q$$

$$6. (P \wedge Q) \rightarrow R$$

$$b) \{ \overset{A}{P} \rightarrow \overset{B}{Q}, \sim \overset{B}{P} \rightarrow \overset{C}{R}, \sim \overset{C}{Q} \} \vdash R$$

$$1. P \rightarrow Q \quad (\text{Modus tollens}) 1, 3$$

$$2. \sim P \rightarrow R \quad \text{Modus Ponens } 1, 2$$

$$3. \sim Q$$

$$4. P \rightarrow Q$$

$$5. \sim Q$$

$$6. P$$

$$7. \sim P \rightarrow R$$

$$R$$

c)  $\begin{matrix} A & B & C & D & E & A \\ P & \rightarrow & Q & , & Q & \rightarrow & \sim \sim R & , & \sim R & \rightarrow & \sim \sim P \end{matrix}$

1.  $P \rightarrow Q$  Silogismo Hipotético (1,2)

2.  $Q \rightarrow \sim \sim R$

3.  $S \rightarrow \sim R$  Modus Ponens (4,5)

4.  $P$

5.  $P \rightarrow \sim \sim R$  Modus tollens (3,6)

6.  $\sim \sim R$

7.  $\sim S$

d)  $\begin{matrix} A & B \\ P & \wedge & Q & , & P & \rightarrow & R & , & Q & \rightarrow & \sim R \end{matrix} \models R \wedge S$

1.  $P \wedge Q$  Simplificação (1)

2.  $P \rightarrow R$  Modus Ponens (2,4)

3.  $Q \rightarrow \sim R$  Simplificação (1)

4.  $P$  Modus Ponens (2,4)

5.  $Q$  M.P (3,5)

6.  $R \wedge S$

7.  $\sim R$

8.  $R \wedge S$  (conjunção (6,7))

e)  $P \rightarrow (\sim Q \wedge R), P, S \rightarrow Q, S \vee \sim T \models T$

1.  $P \rightarrow (\sim Q \wedge R)$

2.  $P$

3.  $S \rightarrow Q$

4.  $S \vee \sim T$

5.  $\sim Q \wedge R$  (Ponência 1,2)

6.  $\sim Q$  (Simplificação)

7.  $\sim S$  tollens (3,6)

8.  $T$  Distinção (4,7)



$$d) (p \vee q) \rightarrow (p \rightarrow (s \wedge t)), \{ \wedge \wedge \} \vdash t \vee r$$

$$1. (p \vee q) \rightarrow (p \rightarrow (s \wedge t))$$

$$2. p \wedge r$$

$$3. p \quad (\text{Simp. } 2)$$

$$4. p \vee q \quad (\text{Adição } 3)$$

$$5. p \rightarrow (s \wedge t) \quad (\text{Ponência } 4, 1)$$

$$6. s \wedge t \quad (\text{Ponência } 5, 5)$$

$$7. t \quad (\text{Simplificação } 6)$$

$$8. t \vee r \quad (\text{Adição } 7)$$

$$g) p \rightarrow q, \sim q, (\sim p \vee \sim r) \rightarrow s \vdash s$$

$$1. p \rightarrow q$$

$$2. \sim q$$

$$3. (\sim p \vee \sim r) \rightarrow s$$

$$4. \sim p \quad \text{M.t. } (1, 2)$$

$$5. \sim p \vee \sim r \quad (\text{Ad. } 4)$$

$$6. s \quad (\text{M.P. } 3, 5)$$

$$h) p \rightarrow \sim r, p, s \rightarrow r \vdash \sim s$$

$$1. p \rightarrow \sim r$$

$$2. p$$

$$3. s \rightarrow r$$

$$4. \sim r \quad (\text{MP } 1, 2)$$

$$5. \sim s \quad \text{M.t. } (3, 4)$$

$$i) p \rightarrow q, p \rightarrow \sim r, p \vdash q \wedge \sim r$$

$$1. p \rightarrow q$$

$$5. \sim r \quad (\text{MP } 2, 3)$$

$$2. p \rightarrow \sim r$$

$$6. q \wedge \sim r \quad (\text{Conj. } 5, 6)$$

$$3. p$$

$$4. q \quad \text{MP } (1, 3)$$

$$j) \sim P \vee \sim Q, \sim \sim P, \sim R \rightarrow \sim Q \models \sim \wedge R$$

$$1. \sim P \vee \sim \sim Q$$

$$2. \sim \sim P$$

$$3. \sim R \rightarrow \sim Q$$

$$4. \sim \sim Q \quad S.D. (1, 2)$$

$$5. \sim \sim R \quad M.t. (3, 4)$$

$$k) P \wedge \sim Q, Q \vee \sim R, S \rightarrow R \models P \wedge \sim S$$

$$1. P \wedge \sim Q$$

$$2. Q \vee \sim R$$

$$3. S \rightarrow R$$

$$4. \sim Q \quad \text{Simp}(1)$$

$$5. \sim R \quad S.D. (2, 4)$$

$$6. \sim S \quad M.t. (3, 5)$$

$$7. P \quad \text{Simp}(1)$$

$$8. P \wedge \sim S \quad \text{Conjunç\u00e3o} (6, 7)$$