Lista 8

1) Construa as deduções

a) {(P → Q), (P ∧ R)} ⊨ Q

1. P → Q
2. P ∧ R
3. P (Simplificação / L. 2)
4. Q (Modus Ponens 3, 2)

b) {(P ∧ Q), ((P ∨ R) → R)} ⊨ (P ∧ R)

1. P ∧ Q
2. ((P ∨ R) → R
3. S (Modus Ponens 2, 5)
4. P
5. P ∨ R (Adição)
6. P ∧ S (Conjunção 4, 3)

c) {(P → (Q → R)), (P → Q), P} ⊨ R

1. (P → (Q → R))
2. P → Q
3. P
4. Q → R (Modus Ponens (3, 4)
5. P → R (Silogismo Hipo (2, 5)
6. R (Modus Ponens (3, 5))

d) {((P ∨ R) → R), ((R ∨ Q) → (P → ((S ↔ T)))), (P ∧ S)} ⊨ (S ↔ T)

1. (P ∨ R) → R
2. ((R ∨ Q → P → (S ↔ T)
3. P ∧ S
4. P (Simp - 3)
5. P ∨ R (Adição 4)
6. R (Ponens 5, 1)
7. R ∨ Q (Adição 6)
8. P → (S ↔ T)
9. S ↔ T (Ponens 8, 4)

e) $(P \to \sim q), (\sim P \to (R \to \sim q)), ((\sim s \vee \sim R) \to \sim\sim q) \therefore \sim R$

1. $P \to q$
2. $\sim P \to (R \to \sim q)$
3. $(\sim S \vee \sim R) \to \sim\sim q))$
4. $\sim S$
5. $\sim S \vee \sim R$ Adição (4)
6. q M.P (3,5)
7. $\sim P$ M.t (1,6)
8. $R \to \sim q$ M.P (2,7)
9. $\sim R$ M.t (6,8)

f) $(p \wedge q) \to R, (R \to S), (t \to \sim v), t, (\sim s \vee v) \therefore \sim (p \wedge q)$

1. $((p \wedge q) \to R$
2. $R \wedge S$
3. $t \to \sim v$
4. t
5. $\sim S \vee v$
6. $\sim v$ M.P (3,4)
7. $\sim s$ S.D (5,7)
8. $\sim R$ M.t (2,7)
9. $\sim (p \wedge q)$ M.t (1,8)

g) $(P \to q), (q \to R), (S \to t), (P \vee S) \therefore R \vee t$

1. $P \to q$
2. $q \to R$
3. $S \to t$
4. $P \vee S$
5. $P \to R$ S H (1,2)
6. $R \vee t$ D.C (3,4,5)

h) $(p \to q), \sim R \to (s \to t), R \vee (p \vee s), \sim R \models q \vee t$

1. $p \to q$
2. $(\sim R \to (s \to t))$
3. $(R \vee (p \vee s))$
4. $\sim R$
5. $s \to t$ M.P (2,4)
6. $p \vee s$ S.D (3,4)
7. $q \vee t$ D.C (1, 5,6)

i) $p \to R, q \to s, \sim R, (p \vee q) \wedge (R \vee s) \models s$

1. $p \to R$
2. $q \to S$
3. $\sim R$
4. $(p \vee q) \wedge (R \vee S)$
5. $R \vee S$ Simplificação (4)
6. S S.D (3,5)

j) $(p \to q), (q \to R), (R \to s), \sim s, (p \vee t) \models t$

1. $p \to q$
2. $p \to R$
3. $R \to S$
4. $\sim S$
5. $p \vee t$
6. $\sim R$ M.t (3,4)
7. $\sim q$ M.t (2,6)
8. $\sim p$ M.t (1,7)
9. t S.D (5,8)

k) $(P \to q) \wedge (R \to S), (t \to U), (U \to V), (\sim q \vee \sim V) \models \sim P \vee \sim t$

1. $(P \to q) \wedge (R \to S)$
2. $t \to U$
3. $U \to V$
4. $\sim q \vee \sim V$
5. $P \to q$ (Simp. 1)
6. $t \to V$ (S.D (2,3))
7. $\sim P \vee \sim t$ D.D (4,5,6)

l) $(P \wedge q), (P \to R) \models (P \wedge R)$

1. $P \wedge q$
2. $(P \to R)$
3. P Simplificação (1)
4. R M.P (2,3)
5. $P \wedge R$ Conj (3,4)

m) $(\sim P \wedge q), (R \to P) \models (\sim P \wedge \sim R)$

1. $(\sim P \wedge q)$
2. $R \to P$
3. $\sim P$ (Simplificação (1))
4. $\sim R$ M.t (2,3)
5. $\sim P \wedge \sim R$ Conjunção (4,5)

N) $(\sim P \to q), \sim (R \wedge S), (P \to (R \wedge S)) \models \sim P \wedge q$

1. $(\sim P \to q)$
2. $\sim (R \wedge S)$
3. $(P \to (R \wedge S))$
4. $\sim P$ M.tollens (2,3)
5. q M. Ponens (1,4)
6. $\sim P \wedge q$ Conjunção (4,5)

O) $(P \vee Q), \sim R, (Q \rightarrow R) \models P$

1. $P \vee Q$
2. $\sim R$
3. $Q \rightarrow R$
4. $\sim Q$ M.T (2,3)
5. P S.D (1,3)

P) $(P \wedge Q), (R \vee S), (P \rightarrow \sim S) \models R$

1. $P \wedge Q$
2. $R \vee S$
3. $P \rightarrow \sim S$
4. P Simplificação (1)
5. $\sim S$ M.P (3,4)
6. R S.D (2,5)

Q) $P, (P \rightarrow \sim Q), Q \vee R \models P \wedge R$

1. P
2. $P \rightarrow \sim Q$
3. $Q \vee R$
4. $\sim Q$ M.P (1,2)
5. R S.D (3,4)
6. $P \wedge R$ Conjunção (1,5)

R) $\sim P, (P \vee (Q \vee R)), \sim R \models Q$

1. $\sim P$
2. $(P \vee (Q \vee R))$
3. $\sim R$
4. $(Q \vee R)$ S.D (1,2)
5. Q S.D (3,4)

1. $P \vee \sim q$
2. $\sim\sim q$
3. $P \rightarrow (R \wedge S))$
4. q Equivalencia (2)
5. P S.D(1,4)
6. $(R \wedge S)$ M.P(3,5)
7. S Simplificação (6)

+1) $(P \rightarrow q), \sim q, (P \vee R) \models R$
1. $P \rightarrow q$
2. $\sim q$
3. $P \vee R$
4. $\sim P$ M.t. (1,2)
5. R S.D(3,4)

V) $(P \vee \sim q), (R \rightarrow \sim P), R \models \sim q$
1. $P \vee \sim q$
2. $R \rightarrow \sim q$
3. R
4. $\sim P$ (M.P(2,3))
5. $\sim q$ (S.D (1,4))

V) $(\sim P \vee \sim q), \sim\sim q, (R \rightarrow P) \models \sim R$
1. $\sim P \vee \sim q$
2. $\sim\sim q$
3. $R \rightarrow P$
4. q Equivalencia (2)
5. $\sim P$ S.D(1,4)
6. $\sim R$ M.t (3,5)

W) $(p \to \sim q), \sim\sim q, (\sim p \to (R \vee S)) \models R \vee S$

1. $p \to \sim q$
2. $\sim\sim q$
3. $(\sim p \to (R \vee S)$
4. q Equivalencia (2)
5. $\sim p$ M.t (1, 4)
6. $(R \vee S)$ M.P (3, 5)

X) $(p \wedge q), (p \to R), (R \wedge S) \to \sim t, (q \to S) \models \sim t$

1. $p \wedge q$
2. $p \to R$
3. $(R \wedge S) \to t$
4. $q \to S$
5. p Simplificação (1)
6. q Simplificação (1)
7. R M.P (2, 5)
8. S M.P (4, 6)
9. $R \wedge S$ Conjunção (7, 8)
10. $\sim t$ M.P (3, 9)

Y) $\sim p, (q \to p), ((\sim q \vee R) \to S) \models S$

1. $\sim p$
2. $q \to p$
3. $(\sim q \vee R) \to S$
4. $\sim q$ M.t (1, 2)
5. $\sim q \vee R$ Adição (4)
6. S M.P (3, 5)

3) $((P \wedge Q) \rightarrow S), R, (R \rightarrow (P \wedge Q)) \models S \vee Q$

1. $((P \wedge Q \rightarrow S)$
2. R
3. $R \rightarrow (P \wedge Q)$
4. $P \wedge Q$ M.P (2, 3)
5. S M.P (1, 4)
6. $S \vee Q$ Adição (5)

a a) $(P \wedge \sim Q), (R \rightarrow Q), (R \vee S), (P \vee S) \rightarrow t \models t$

1. $P \wedge \sim Q$
2. $R \rightarrow Q$
3. $R \vee S$
4. $(P \vee S) \rightarrow t$
5. $\sim Q$ Simplificação (1)
6. $\sim R$ M.t (2, 5)
7. S S.D (3, 6)
8. $P \vee S$ Adição (7)
9. t M.P (4, 8)

b b) $(P \vee \sim Q), (\sim Q \rightarrow R), (P \rightarrow S), \sim R \models S$

1. $P \vee \sim Q$
2. $\sim Q \rightarrow R$
3. $P \rightarrow S$
4. $\sim R$
5. Q M.t (2, 4)
6. P S.D (1, 5)
7. S M.P (3, 6)

cc) $(P \to Q), (Q \to \sim R), \sim\sim R, (P \vee (S \wedge T)) \models S$

1. $P \to Q$
2. $Q \to \sim R$
3. $\sim\sim R$
4. $P \vee (S \wedge T)$
5. R Equivalencia (3)
6. $\sim Q$ M.T (2, 4)
7. $\sim P$ M.T (1, 6)
8. $(S \wedge T)$ Silogismo Dis. (4, 7)
9. S Simplificação (8)

dd) $(P \vee Q), (Q \to R), (P \to S), \sim S \models (R \wedge (P \vee Q))$

1. $P \vee Q$
2. $Q \to R$
3. $P \to S$
4. $\sim S$
5. $\sim P$ M.T (3, 4)
6. Q S.D (1, 5)
7. R M.P (2, 6)
8. $R \wedge (P \vee Q)$ Conjunção (1, 7)

ee) $(\sim P \vee \sim Q), (\sim Q \to \sim R), (\sim P \to T), T \models \sim R \wedge \sim T$

1. $\sim P \vee \sim Q$
2. $\sim Q \to \sim R$
3. $\sim P \to T$
4. $\sim T$
5. P M.T (3, 4)
6. $\sim Q$ S.D (1, 5)
7. $\sim R$ M.P (2, 6)
8. $\sim R \wedge \sim T$ Conjunção (7, 4)

ff) $(R \to t), (S \to q), (t \vee q) \to \sim p, (R \vee S) \models p$

1. $(R \to t)$
2. $(S \to q)$
3. $(t \vee q) \to \sim q$
4. $(R \vee S)$
5. $(t \vee q)$ D.C (1, 2, 4)
6. $\sim p$ M.P (3, 5)

gg) $(p \to \sim q), (\sim q \to \sim s), ((p \to \sim s) \to \sim t), R \to t \models \sim R$

1. $p \to \sim q$
2. $\sim q \to \sim s$
3. $(p \to \sim s) \to t$
4. $R \to t$
5. $p \to \sim s$ (S. H (1, 2)
6. $\sim t$ M.P (3, 5)
7. $\sim R$ M.T (4, 6)

HH) $((p \vee q) \to \sim R), (S \to p), (t \to q), (S \vee t) \models \vee \vee \sim R$

1. $((p \vee q) \to \sim R$
2. $S \to p$
3. $t \to q$
4. $S \vee t$
5. $P \vee q$ (S. H (2, 3, 4)
6. $\sim R$ M.P (1, 5)
7. $\vee \vee \sim R$ Adição (6)