

## Aprendizagem 2021/22

### Homework IV

Deadline 14/11/2021 23:59 via Fenix as PDF

- Homework limited to 6 pages according to the provided template
- Include your programming code as an Appendix (no page limits)
- Submission Gxxx.PDF in Fenix where xxx is your group number. Please note that it is possible to submit several times on Fenix to prevent last-minute problems. Yet, only the last submission is considered valid
- Exchange of ideas is encouraged. Yet, if copy is detected after automatic/manual clearance, homework is nullified and IST guidelines apply for content sharers and consumers, irrespectively of the underlying intent
- Please consult the FAQ before posting questions to your faculty hosts

### I. Pen-and-paper [12v]

Consider the following bivariate observations in a Euclidean space:

	$y_1$	$y_2$
$\mathbf{x}_1$	2	4
$\mathbf{x}_2$	-1	-4
$\mathbf{x}_3$	-1	2
$\mathbf{x}_4$	4	0

- 1) [6v] Compute and sketch the clustering solution given by EM assuming considering  $\mathbf{x}_1$  and  $\mathbf{x}_2$  to be the centroid means and:

$$\Sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \Sigma_2 = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}, \pi_1 = p(c_1 = 1) = 0.7, \pi_2 = p(c_2 = 1) = 0.3$$

- 2) [3v] Compare the quality of the produced clustering solutions using silhouette.
- 3) [3v] Identify the VC dimension of the following two-class/binary classifiers: **i)** MLP with three hidden layers with as much nodes as the number of input variables; **ii)** decision tree assuming input variables are discretized using three bins; and **iii)** Bayesian classifier with a multivariate Gaussian likelihood.
  - (a) Assume the data dimensionality is five.
  - (b) Plot in a single chart how the VC dimension varies with data dimensionality for  $m \in \{2, 5, 10, 12, 13\}$ . What can you conclude (one sentence, English or Portuguese)?
  - (c) Plot in a single chart how the VC dimension of **i)** and **iii)** with data dimensionality for  $m \in \{2, 5, 10, 30, 100, 300, 1000\}$ . What can you conclude (one sentence, English or Portuguese)?

### II. Programming and critical analysis [8v]

Recall the `breast.w.arff` dataset from previous homeworks.

- 4) [4v] Apply  $k$ -means clustering unsupervised on the original data with  $k = 2$  and  $k = 3$ .
  - a. Compare the produced solutions against the ECR (external measure)
  - b. Compare the produced solutions against the Silhouette coefficient (internal measure).
- 5) [2v] Visually plot the  $k = 3$  clustering solution using the top-2 features with higher mutual information.
- 6) [2v] Using empirical results from (5), comment on the quality of the produced clustering solution.

END