Mobile Communications

Principles of Cellular Networks

Manuel P. Ricardo

Faculdade de Engenharia da Universidade do Porto

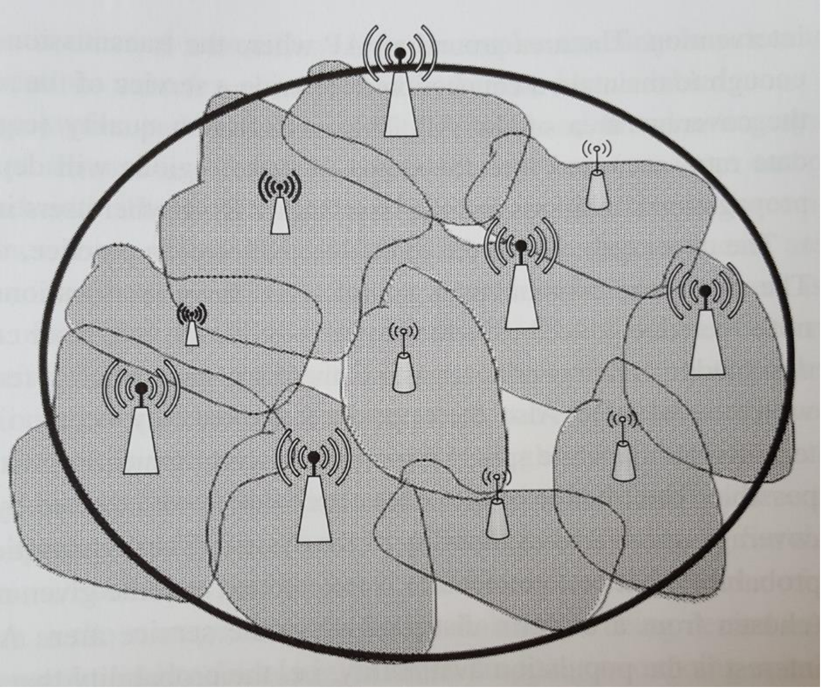
This set of slides is based on the book

G. Miao et al. - Fundamentals of Mobile Data Networks

Wireless Network Models and Capacity

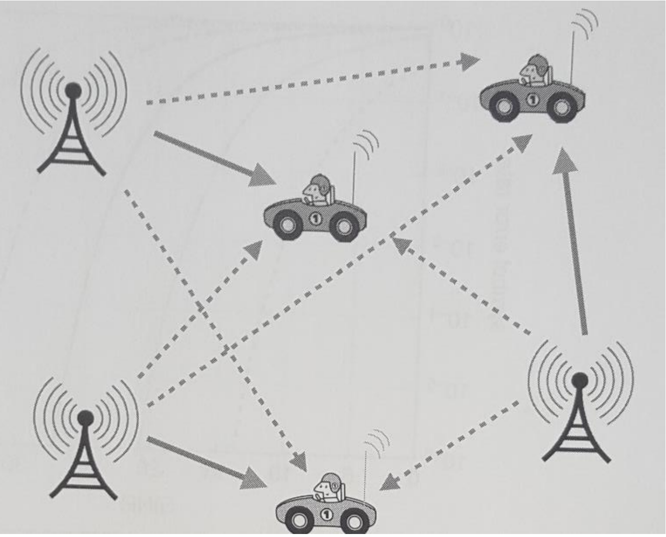
Wireless Cellular Network

- Infrastructure
  - Base stations (APs)
  - Fixed



- Terminals
  - Coverage requirements
  - Service requirements

Wireless Network Analysis



- Multiple transmitters and receivers
  - » Complex propagation scenario
  - » Interference from neighbor transmitters
- Two step analysis
  - » Characterize interference
    - – Compute Carrier to Inference ratio (C/I ~ SNIR) in individual links
  - » Characterize received quality for the given interference
    - – Map C/I into Quality (Q)

Services

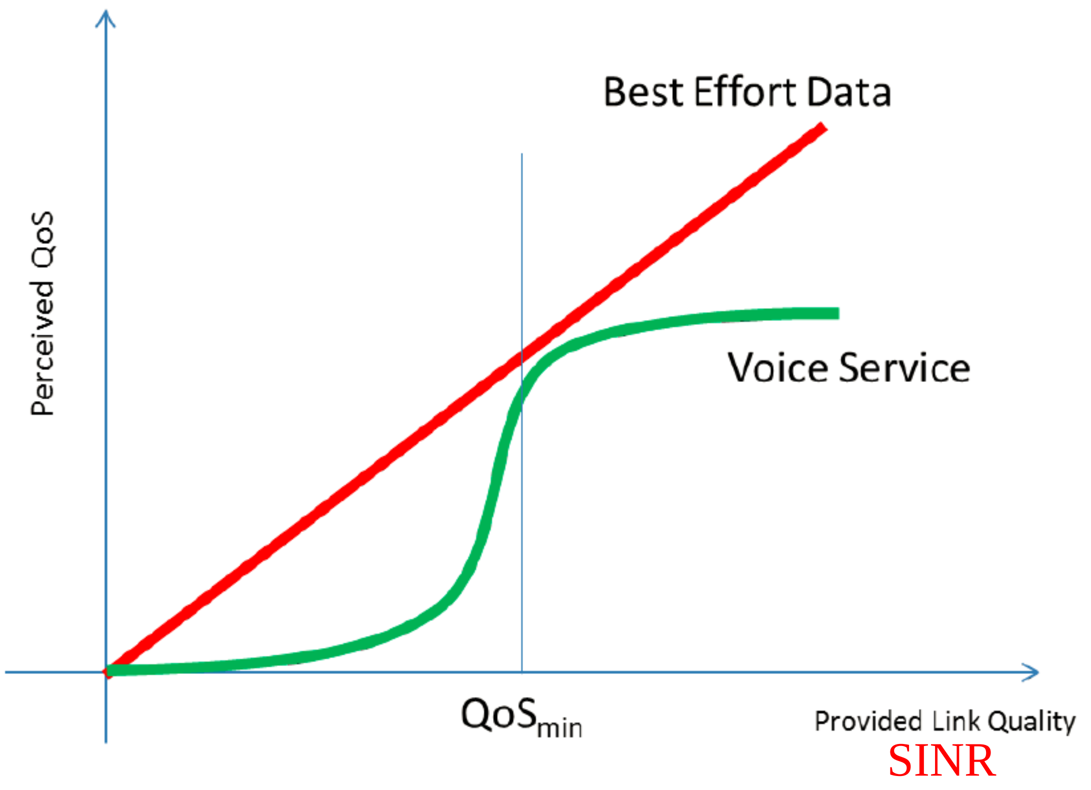
- Let's assume 2 types of service provided by the wireless network
  - » Guaranteed - blocking
  - » Best effort - non-blocking

- Guaranteed service - blocking
  - » Guaranteed bitrate (bit/s) provided to a terminal
  - » If terminal requests service and network is unable to provide it
    ➔ request is blocked (not admitted)
  - » Example: voice call
  - » Measure of service quality: call blocking probability

- Best effort – non blocking
  - » The network provides service to all the terminals
  - » No terminal is blocked
  - » Example: mobile data
  - » Measure of service quality: average bitrate received by the terminal

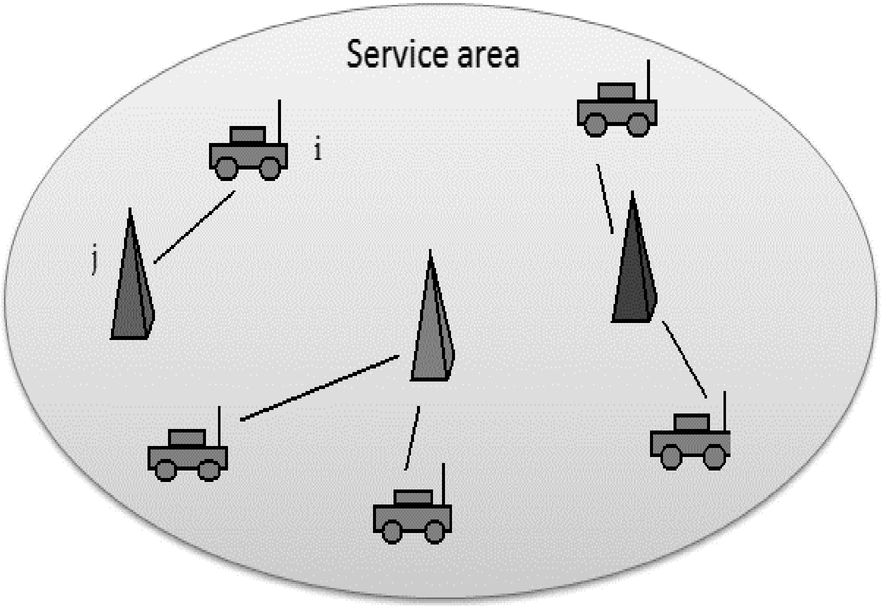
User's Perceived Quality of service (QoS)

For simplicity, let's assume Link Quality = SINR



Mobile Scenario - Problem

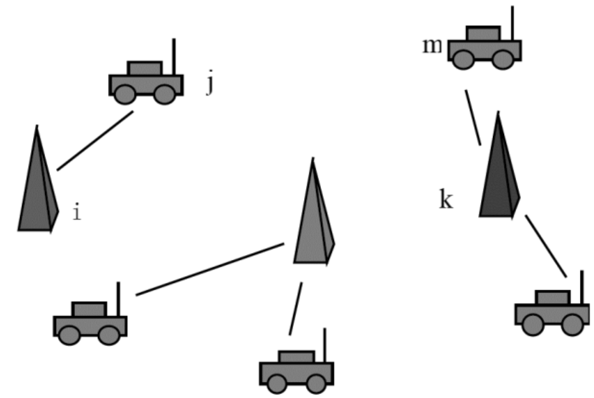
- Assign to each active terminal
  - Base station
  - Channel
  - Transmitter power



such that Link Quality is satisfied for as many terminals as possible

Interference Model

- Collection of wireless links
- $g_{i,j}$: instantaneous gain of link (i, j)
- $P_{rx,j} = P_{tx,i} * g_{i,j}$



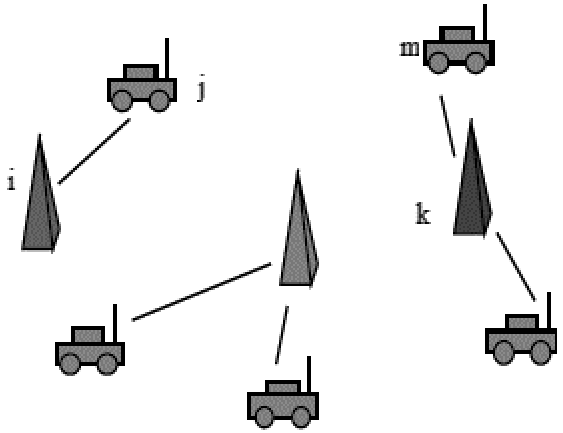
where

- » G: link gain matrix
- » B: number of base stations
- » M: number of terminals

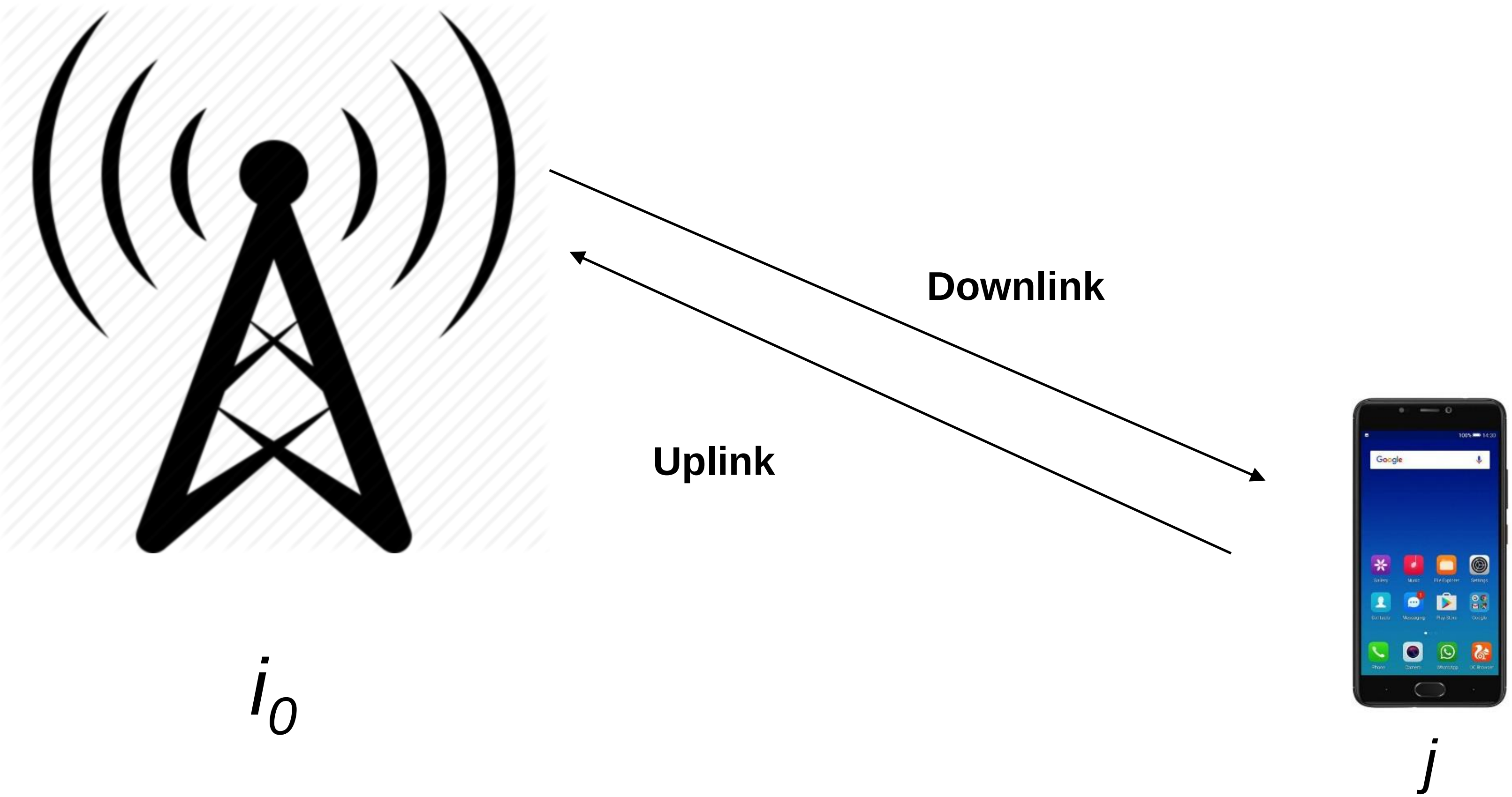
$$G = \begin{pmatrix} g_{11} & g_{12} & .. & .. & g_{1M} \\ g_{21} & g_{22} & .. & .. & g_{2M} \\ .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. \\ g_{B1} & g_{B2} & .. & .. & g_{BM} \end{pmatrix}$$

Traffic Model

- $M = \omega A$ active terminals deployed
  - » uniformly distributed by space
  - » ω active terminals per area unit (terminal/m^2)
  - » A is the deployment area (m^2)



Downlink and Uplink !



Interference and Quality Model

- SINR in downlink (d) $$\Gamma^{d}_{i0,j} = \frac{P_{i0}\, g_{i0,j}}{\sum_{b \neq i0, b \in B} P_b\, g_{b,j} + N_j}$$

- SINR in uplink (u) $$\Gamma^{u}_{i0,j} = \frac{P_j\, g_{i0,j}}{\sum_{m \neq j, m \in M} P_m g_{i0,m} + N_{i0}}$$

(assuming gain is equal in both directions, $g_{i0,j} = g_{j,i0}$)

Guaranteed Service Quality – Blocking: Wireless Network Capacity → max ω

- Terminal is admitted (not blocked) if

$$\Gamma^u_{i0,j} = \frac{P_j g_{i0,j}}{\sum_{m\neq j, m\in M} P_m g_{i0,m} + N_{i0}} \geq \gamma^u_j \quad \textbf{and} \quad \Gamma^d_{i0,j} = \frac{P_{i0} g_{i0,j}}{\sum_{b\neq i0, b\in B} P_b g_{b,j} + N_j} \geq \gamma^d_j$$

- Assume

(Y, Z, M are random variables)

  - » Y active terminals are admitted: Y= #{ j ∈ M : $\Gamma^u_{i0,j} \geq \gamma^u_j$ and $\Gamma^d_{i0,j} \geq \gamma^d_j$}
  - » Z=M-Y terminals not admitted
  - » Admission failure ratio $\nu = \frac{E[Z]}{E[M]} = \frac{E[Z]}{\omega A} \quad \rightarrow \quad \nu \leq \nu_0$

- Wireless network Capacity [active terminal/m²] : **max ω: $\frac{E[Z]}{\omega A} \leq \nu_0$**

  the maximum density of active terminals max ω

  in order to keep the admission failure ratio below the threshold level ν_0

Best effort – non-blocking: Wireless Network Capacity → $\bar{R}$

- R_i : data rate of each terminal

- $R_i = f(\Gamma_i) = \min[\ R_{max},\ \ c\,W log_2(1+\Gamma_i)\ ]$

max bitrate of the hardware

Shannon limit;
constant c <1 represents an implementable % of it

- Wireless network capacity [bit/s]
  - » The rate (bit/s) that can be served by the network in the service area *A*

$$\bar{\boldsymbol{R}} = \boldsymbol{E}\left[\sum_{i=1}^{M} \boldsymbol{R}_i\right]$$

Wireless cellular systems

Multiple Access Cellular Systems

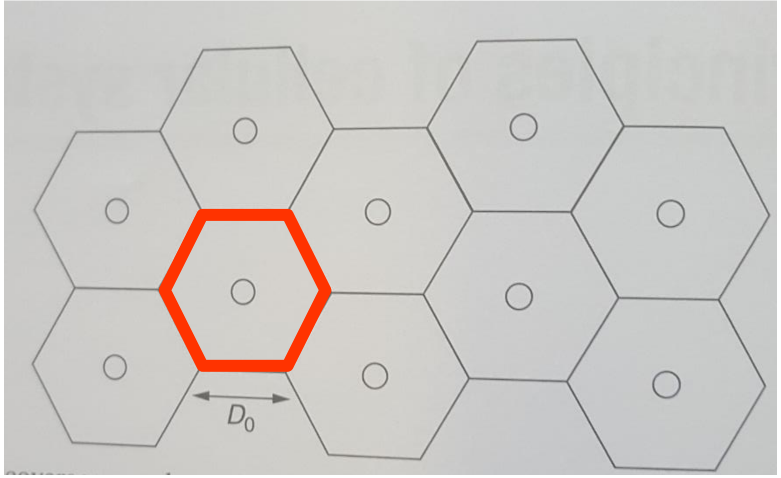
- Design cellular networks made in 2 steps:
  - Coverage planning
  - Frequency allocation

Coverage Planning

- Cell defined as hexagonal-shaped area

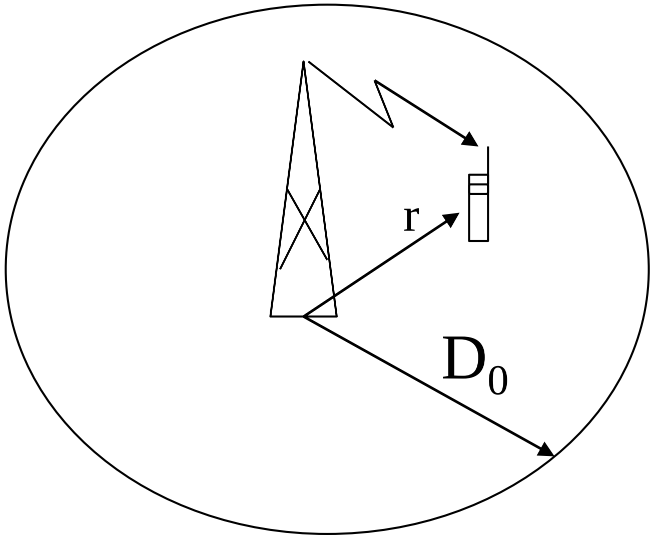
- Cell radius D_0

- Cell area $A_{cell} = 1.5\, D_0^2\sqrt{3}$



Area Coverage Planning

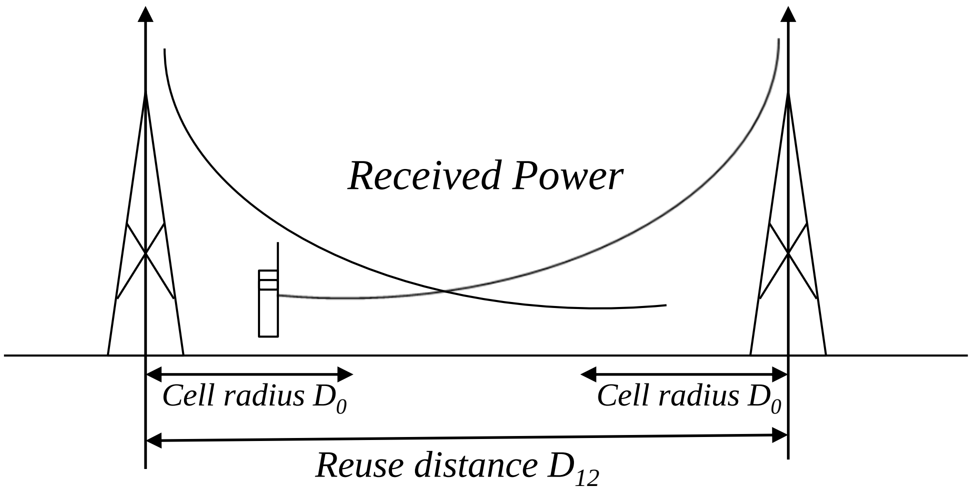
- Received $SNR = \frac{P_r}{N} = \frac{c_t P_t}{r^\alpha N}$
  - » P_t – transmitted power
  - » c_t – antenna gain
  - » r – distance from receiver to base station
  - » α – attenuation constant
  - » N – noise power



- If minimum SNR γ_0 is required in all the cell, then $\frac{c_t P_t}{D_0^\alpha N} \geq \gamma_0$
- And the radius of the cell $D_0 \leq \left(\frac{c_t P_t}{\gamma_0 N}\right)^{1/\alpha} \rightarrow A_{cell} = 1.5\, D_0^2 \sqrt{3}$
- $\textit{Number of cells} = \frac{\textit{total area}}{\textit{cell area}} = \frac{A}{A_{cell}}$

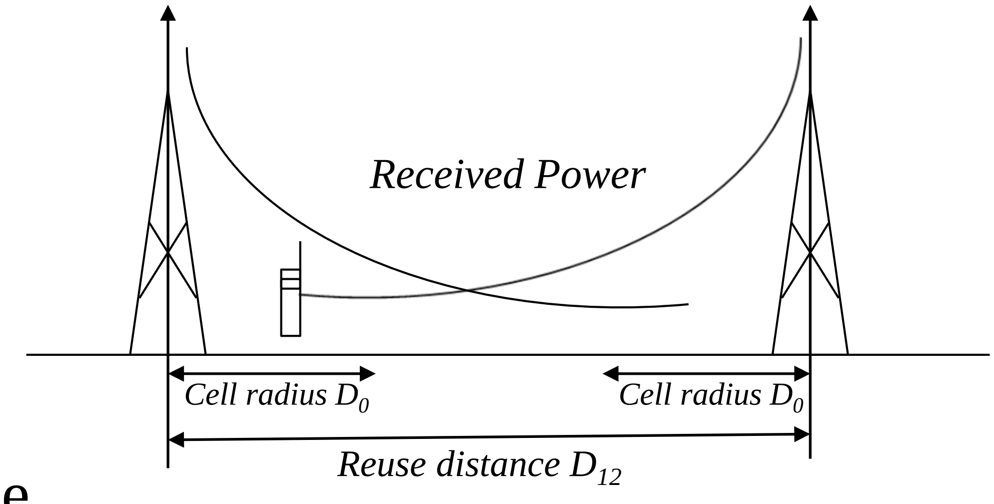
Frequency Planning – Frequency Reuse

- Reuse the same frequency as often as possible



- D_{12} is the reuse distance

Frequency Reuse:2 Base Stations



- Define $\gamma_0 = \frac{c_t P_t}{D_0^\alpha N}$
- The worst SNIR is for a terminal located at cell edge
  - » $\Gamma_1 = \frac{\frac{c_t P_t}{D_0^\alpha}}{\frac{c_t P_t}{(D_{12}-D_0)^\alpha}+N} = \frac{\gamma_0}{1+\frac{\gamma_0}{\left(\frac{D_{12}}{D_0}-1\right)^\alpha}}$
- If a minimum SNIR is required $\Gamma_1 \geq \gamma_t$, assuming $\gamma_0 > \gamma_t$
  - » $D_{12} \geq D_0 \left[1 + \left(\frac{\gamma_0 \gamma_t}{\gamma_0 - \gamma_t}\right)^{1/\alpha}\right]$

*Frequency Reuse: **B** Base Stations*

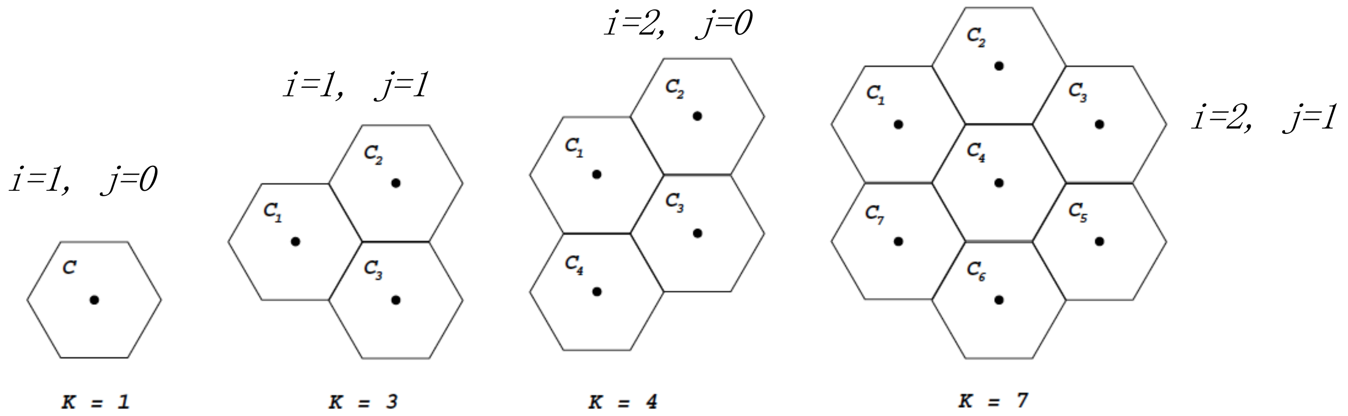
- The same frequency can be used at B different cells

- The SINR at any receiver k (located in cell k)
  - » $\Gamma_k = \dfrac{\frac{cP_k}{r^\alpha}}{\sum_{\substack{i=1\ldots B \\ i\neq k}} \frac{cP_i}{D_{i,k}^\alpha} + N} \geq \gamma_t$
    - – $D_{i,k}$: the distance from receiver k to base station i
    - – r : the distance from receiver k to base station k
    - – γ_t : the minimum required threshold of SNIR
    - – N : noise power

- With a good choice of $D_{i,k}$,
  - » the minimum desired link quality γ_t can be obtained in all the links

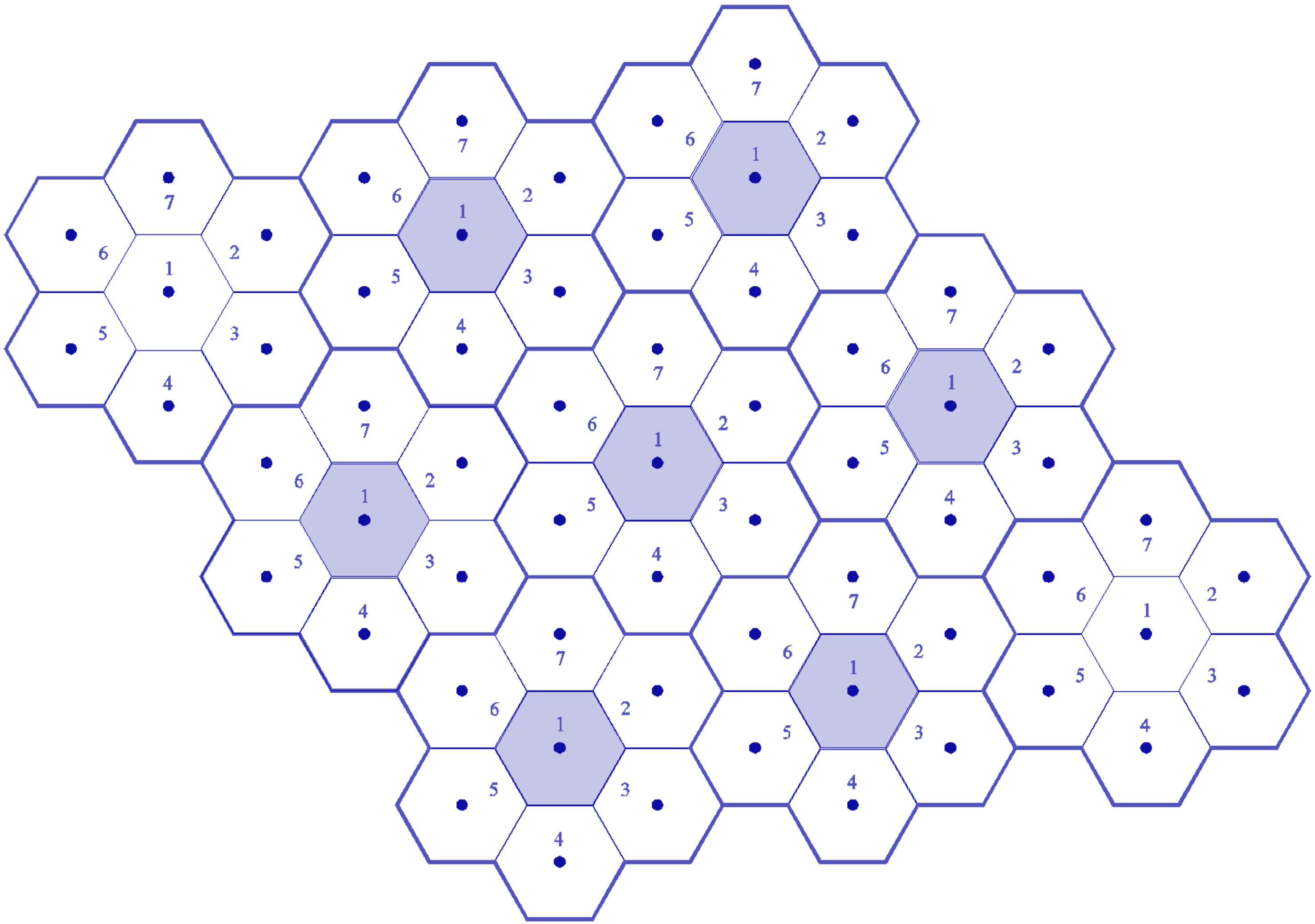
Channel Allocation for Frequency Reuse

- Objective: design a network with the maximum possible capacity
  - Hexagonal cells grouped in clusters of K cells (K = reuse factor)
  - $K = \frac{1}{3}\left(\frac{D}{D_0}\right)^2 \rightarrow \boldsymbol{D = D_0\sqrt{3K}}$, $K = (i+j)^2 - ij$, $i, j = 0,1,2,3\ldots$
  - D : distance between 2 Base Stations using same channel | D_0 : radius of the cell



- Let C be the total number of *channels* available $C = \{ch_1, ch_2, \ldots, ch_C\}$
  - Divide C into K disjoint groups $\{C_1, C_2, \ldots C_K\}$
  - Assign each group C_i to a different cell of the cluster
  - Repeat the same distribution in the other clusters

Channel Allocation for Frequency Reuse: K=7

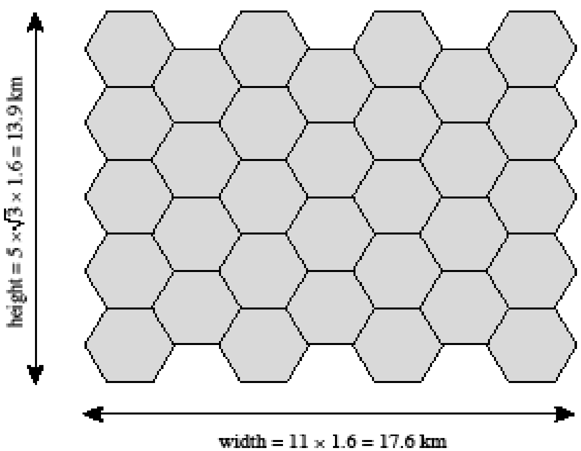


Cells with the same number use the same channel(s)

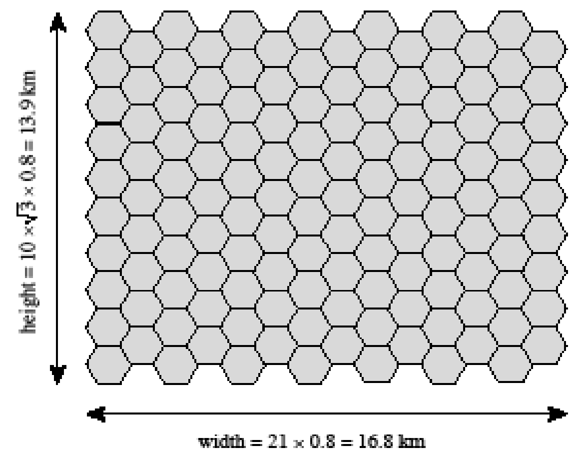
Examples

- Example
  - Deployment of **32 cells**, cell radius $D_0 = 1.6\ km$
  - Total frequency bandwidth equivalent to 336 traffic channels
  - Reuse factor, K=7
  - What geographic area is covered?
  - Total number of channels available?
- Solution
  - Cell area, $A_{cell} = 1.5\ D_0^2\sqrt{3} = 6.65\ km^2$
  - Covered area, Covered $area,\ A = 32 * 6.65\ km^2 = \mathbf{213\ km^2}$
  - Channels/cell = 336/7 = 48
  - **Total channel capacity: 32 cells*48 channel/cell=1536 channels**

- Same example, but
  - Deployment of **128 cells**, cell radius $D_0 = 0{,}8\ km$
  - 336 traffic channels
  - Reuse factor, K=7
  - Same questions ….
- Solution
  - $A_{cell} = 1.66\ km^2$
  - Covered $area,\ A = 128 * 1{,}66\ km^2 = \mathbf{213\ km^2}$
  - **Total channel capacity: 128 * 48 = 6144 channels**



(a) Cell radius = 1.6 km



(b) Cell radius = 0.8 km

$$\frac{0.8\ km}{1.6\ km} = \frac{1}{2}$$

$$\frac{6144\ channels}{1536\ channels} = 4$$

Capacity of a Wireless Network

- Maximum number of simultaneous calls supported by network

- $\eta = \left\lfloor \frac{C}{K} \right\rfloor \; calls/cell$

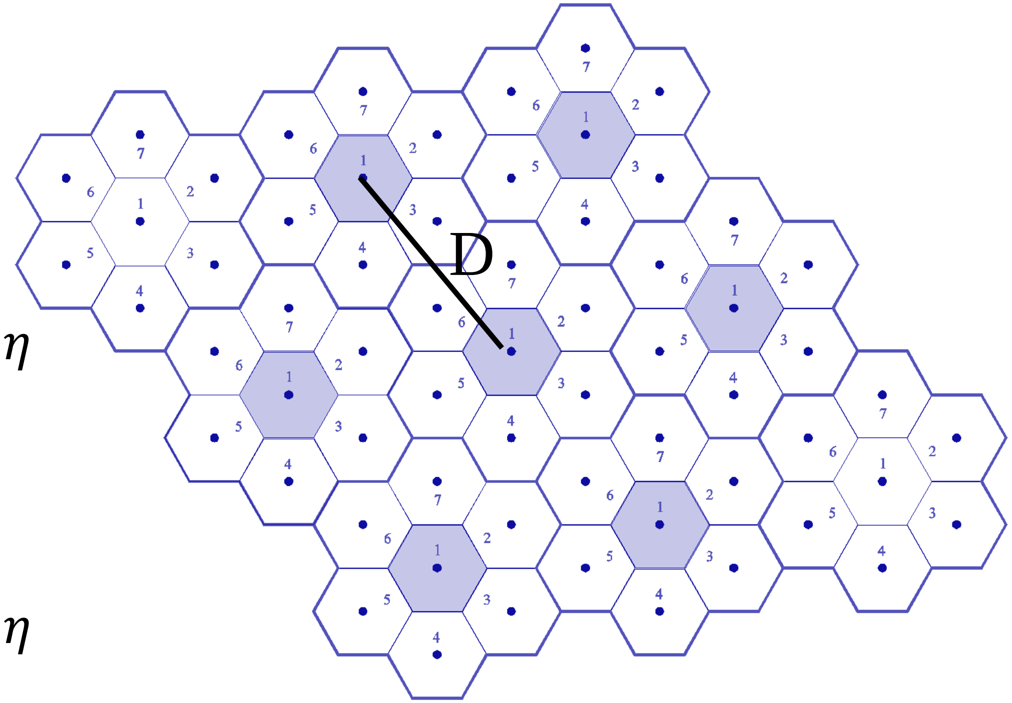
  - η : maximum number of calls in each cell
  - C : total number of channels available
  - K : reuse factor

- Area capacity $= \frac{\eta}{A_{cell}} \; calls/m^2$

Quality (SINR) versus Capacity (η)

- Ideal case: High SINR , High η
- But Interference caused by frequency reuse

  depends on reuse distance D (distance between cells using same channel)



- Large K (reuse factor)

  ➔ Large D ➔ High SNR ➔ Low η

  ➔ High Quality, Low Capacity

- Small K

  ➔ Small D ➔ Low SNR ➔ High η

  ➔ Low Quality, High Capacity

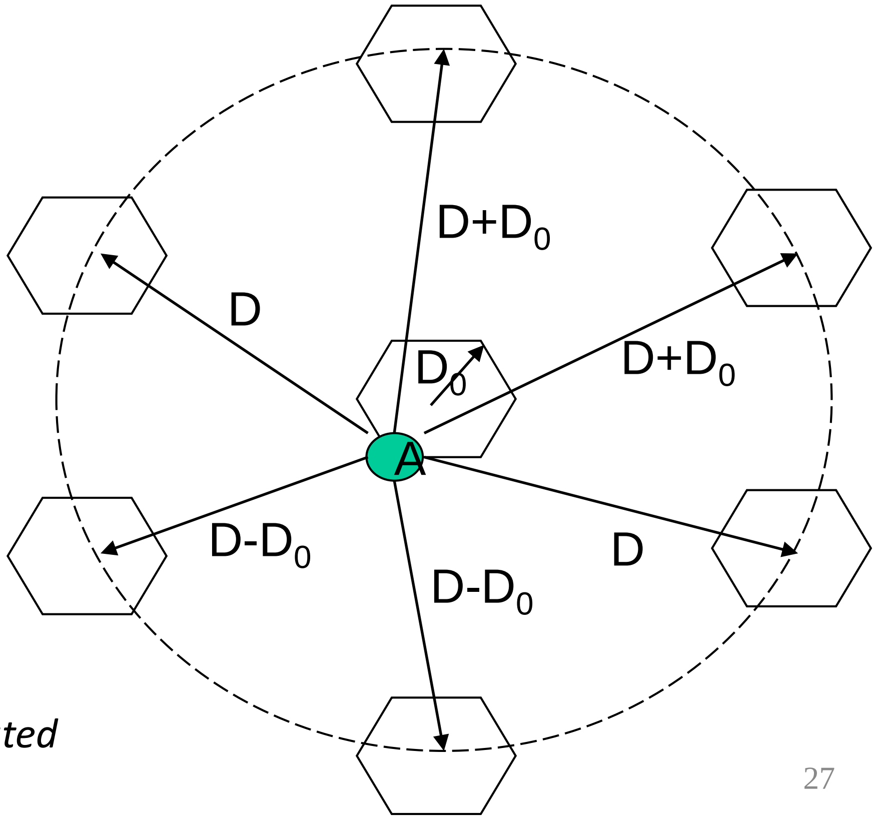
Co-channel Interference

- If transmit power of base stations and α are constant
- SIR is point A, $\Gamma_A \sim \dfrac{D_0^{-\alpha}}{\sum_{i=1}^{i_0} D_i^{-\alpha}} \approx \dfrac{D_0^{-\alpha}}{2(D-D_0)^{-\alpha}+2D^{-\alpha}+2(D+D_0)^{-\alpha}}$

i_0 - number of co-channel interfering cells

D_i - distance from corner A to center of the interfering co-channel cell i

D_0 - radius of a cell



Co-channel cells for a cluster K=7
Indicated distances are approximated

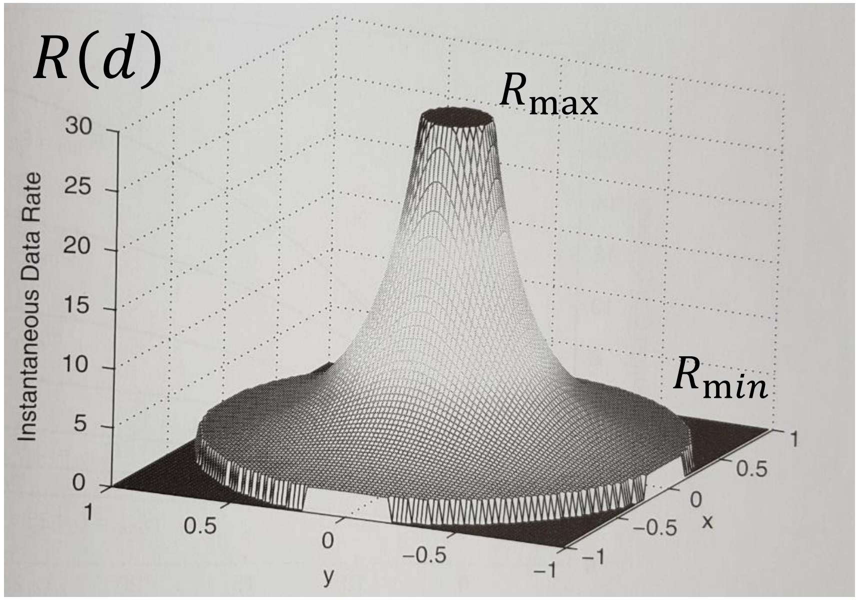
Best effort – non-blocking

- Assume: cellular system with
  - total bandwidth: W_s Hz | cluster size: K | Bandwidth for each cell : $W = \frac{W_s}{K}$
  - $\Gamma_{DL}(D) = \frac{\frac{c_t P_t}{D^\alpha}}{\sum_k \frac{c_t P_t}{D_k^\alpha} + N}$, D: distance between terminal to Base Station
- For the normalized distance $d = D/D_0$, $0 < d < 1$
  - $\Gamma_{DL}(\mathrm{d}) = \frac{1}{d^\alpha} \frac{\frac{c_t P_t}{D_0^\alpha}}{\sum_k \frac{c_t P_t}{D_k^\alpha} + N} = \frac{\Gamma_{DL}(D_0)}{d^\alpha}$,
- Bitrate R(d) of an active terminal within the cell
  - $\boldsymbol{R(d)} = min\{ R_{\max}, c\, W\, log_2(1 + \Gamma_{DL}(\mathrm{d})) \} = \boldsymbol{min\left\{R_{max}, c\, W\, log_2\left(1 + \frac{\Gamma_{DL}(D_0)}{d^\alpha}\right)\right\}}$

    0 < c < 1: constant representing implementable % of Shannon capacity

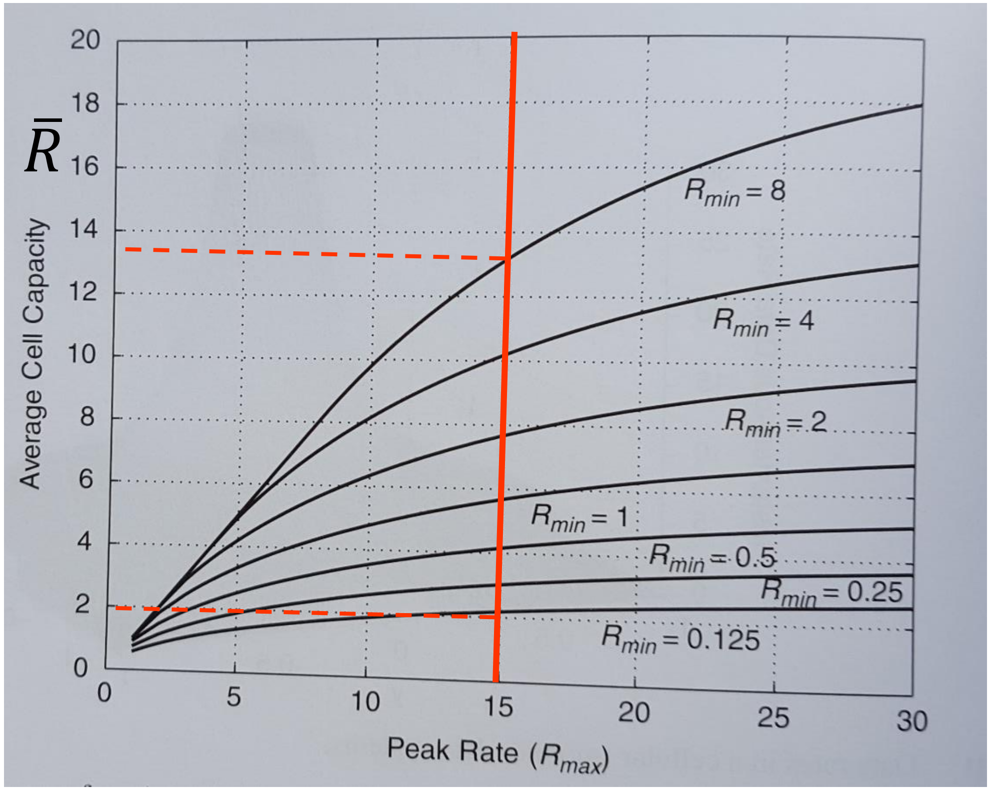
Best effort – non-blocking

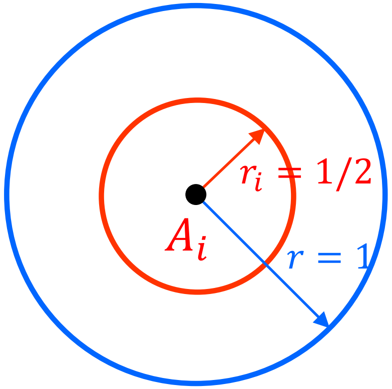
- Assuming circular cell
- Bitrate at the center of the cell, $R(d=0) = R_{\max}$
- Bitrate at the edge of de cell $R(d=1) = R_{min} = c\,W\,log_2\big(1 + \Gamma_{DL}(D_0)\big)$
- $R(d) = \quad min\left\{R_{max}, c\,W\,log_2\left(1 + \frac{2^{R_{min}/cW} - 1}{d^{\alpha}}\right)\right\}$



Best Effort Data Service - Example

- Terminals uniformly distributed over the cell
- $K = 3,\ W_s = 15\ MHz,\ \ \alpha = 4,\ \ \ c = 0.5$
- $Average\ terminal\ rate\ \bar{R} = E[R] = \int_0^{D_0} R(r)\,\frac{2\pi r\,dr}{\pi D_0^2} = \int_0^1 R(x)\,2x\,dx\ ,\ \ x = {}^{r}/_{D_0}$





$$\frac{A_i}{A} = \frac{\pi\left(\frac{1}{2}\right)^2}{\pi} = 25\%$$

Guaranteed Service Quality – Blocking

- Assume calls arrive according to a Poisson process ➔ $Pr\{M_c = k\} = \frac{(\omega A_c)^k}{k!} e^{-\omega A_c}$
  - » M_c : number of offered calls (active terminals)
  - » ω : average number of offered call$/m^2$ | A_c : $cell\ area$
  - » Call inter-arrival time A becomes exponentially distributed with pdf $f_A(t) = \lambda e^{-\lambda t}$ (RCOM)
  - » With call arrival rate **λ call/s** where $\boldsymbol{\lambda\, \Delta t = \omega A_c}$
- If call duration is also exponentially distributed ➔ E[call duration] = $\frac{1}{\mu}$ [s]
- Traffic load $\rho = \frac{\lambda}{\mu}$ $[Erlangs]$
- Example
  - » Call arrival rate $\lambda = 360\, call/hour = 0.1\, call/s$
  - » Average call duration $\frac{1}{\mu} = 20\ s$
  - » Traffic load $\rho = 0.1\ * 20 = 2$ Erlang
  - » In average we need 2 circuits (2 channels) to support the offered traffic

Guaranteed Service Quality – Blocking: Queueing Theory (Erlang-B)

- A cellular network has η channel/cell
  - » May be modeled as a birth-death process
  - » $Queue \quad M/M/\eta$ → η servers , no waiting space

- The blocking probability given by *Erlang-B* formula
  - » $P_{block} = \frac{\rho^{\eta}/\eta!}{\sum_{k=0}^{\eta} \rho^{\eta}/k!}$ $\quad , where \quad \rho = \frac{\lambda}{\mu}$

- $Capacity = \rho_{max} = \max\{\rho \mid P_{block} < p_0\} \;\; [Erlang]$

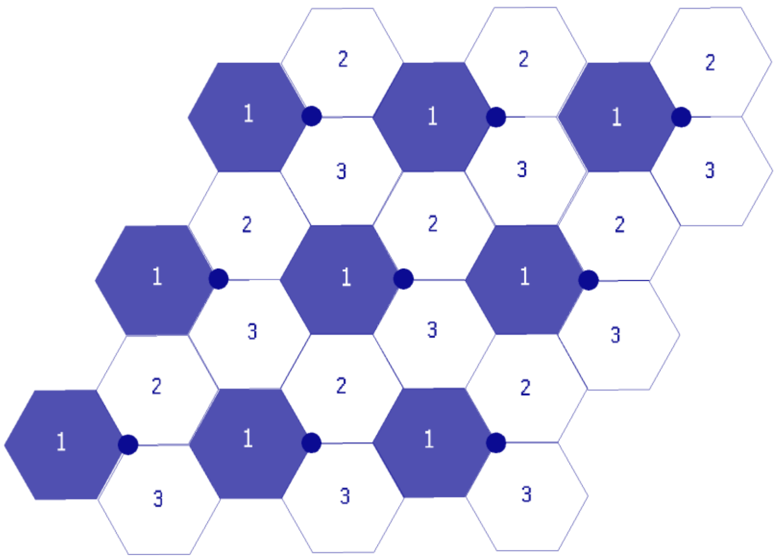
  p_0 : the required blocking probability

- The area capacity is obtained as $\rho_A = \frac{\rho_{max}}{A_c}, \;\; (Erlang/m^2)$

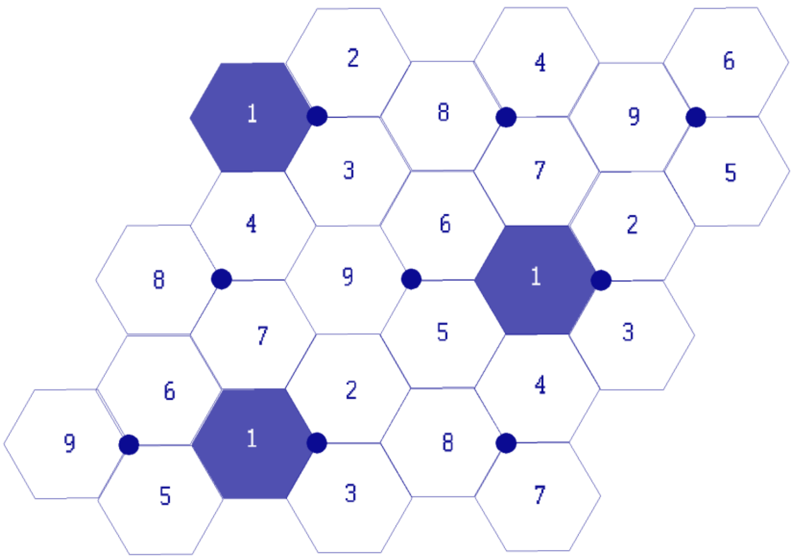
  A_c : the cell area

Directional Antennas in Wireless Networks

Directional antennas (sectors) reduce the number of sites



120 degree site patterns for 1/3 reuse



120 degree site patterns for 3/9 reuse

Homework

1. Review slides
   » use them to guide you through the recommended book

2. Read from *G. Miao - Fundamentals of Mobile Data Networks*
   » Chap. 2 - Wireless network models
   » Chap. 5 – Principles of cellular systems

3. Answer questions at moodle