Mobile Communications

Wireless Transmission -

Modulation, Coding, OFDM, Spread Spectrum, Diversity

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- *How to transmit bits using a carrier?*
- *What are the modulations commonly used in wireless communications?*

- *How does the BER depend on the modulation and SNR?*
- *What is a code?*
- *What are the benefits of using codes in wireless communications?*
- *Why is interleaving combined with coding?*
- *Why is adaptive modulation/coding used?*

- *What is multicarrier modulation?*
- *What is OFDM?*
- *Why is it so important?*

- *What is spread spectrum?*

- *What is diversity? How to benefit from it?*

Digital Modulation/Demodulation

- Modulation: maps information bits into an analogue signal (carrier)
- Demodulation: determines the bit sequence based on received signal

- Two categories of digital modulation
  - » **Amplitude modulation - *α(t)* / Phase modulation - *θ(t)***
  - » Frequency modulation - *f(t)*

- Modulated signal ***s(t)***

$$\begin{aligned} s(t) &= \Re\left\{u(t)e^{j2\pi f_c t}\right\} \\ &= \Re\{u(t)\}\cos(2\pi f_c t) - \Im\{u(t)\}\sin(2\pi f_c t) \\ &= x(t)\cos(2\pi f_c t) - y(t)\sin(2\pi f_c t), \\ &= s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t) \end{aligned}$$

$$s(t) = \underline{\alpha(t)}\cos[2\pi(f_c + \underline{f(t)})t + \underline{\theta(t)} + \phi_0] = \alpha(t)\cos(2\pi f_c t + \phi(t)\ )$$

$$s(t) = \underline{\alpha(t)\cos\phi(t)}\cos(2\pi f_c t) - \underline{\alpha(t)\sin\phi(t)}\sin(2\pi f_c t) = \boxed{s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)}$$

$$\boxed{u(t) = s_I(t) + j s_Q(t)}$$

- Signal transmitted for symbol ***i*** → ***$s_i(t)$***

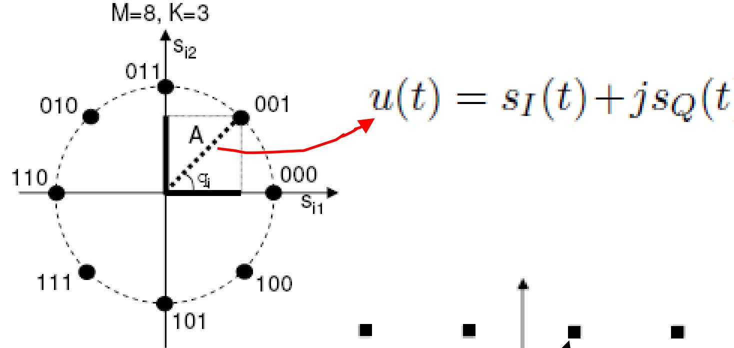
Amplitude and Phase Modulation

- $K = log_2 M$ bits sent in a time symbol interval
- Amplitude/phase modulation can be:

  » Phase Shift Keying (MPSK)

  information coded in phase

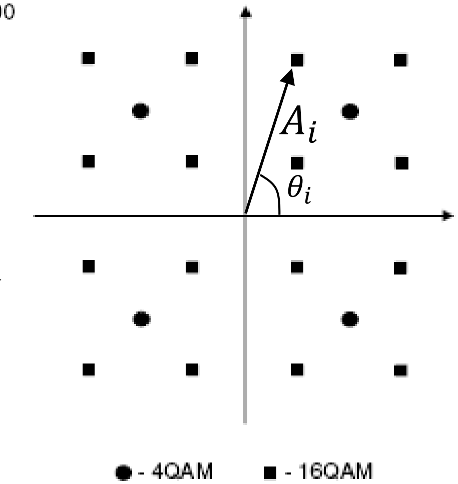
$$\text{MPSK} - s_i(t) = Re\left\{A e^{j\theta_i} e^{j2\pi f_c t}\right\}$$



  » Quadrature Amplitude Modulation (MQAM)

  information coded both in amplitude and phase

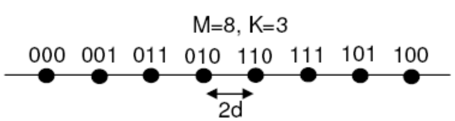
$$\text{MPSK} - s_i(t) = Re\left\{A_i e^{j\theta_i} e^{j2\pi f_c t}\right\}$$



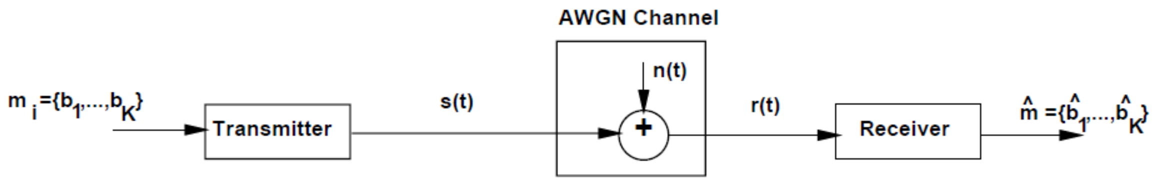
  » Pulse Amplitude Modulation (MPAM)

  information coded in amplitude

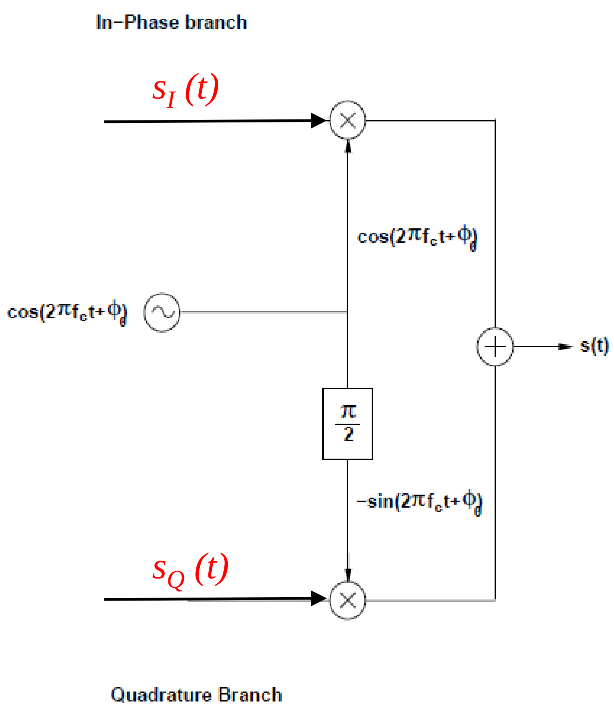
$$\text{MPAM} - s_i(t) = Re\left\{A_i e^{j2\pi f_c t}\right\}$$



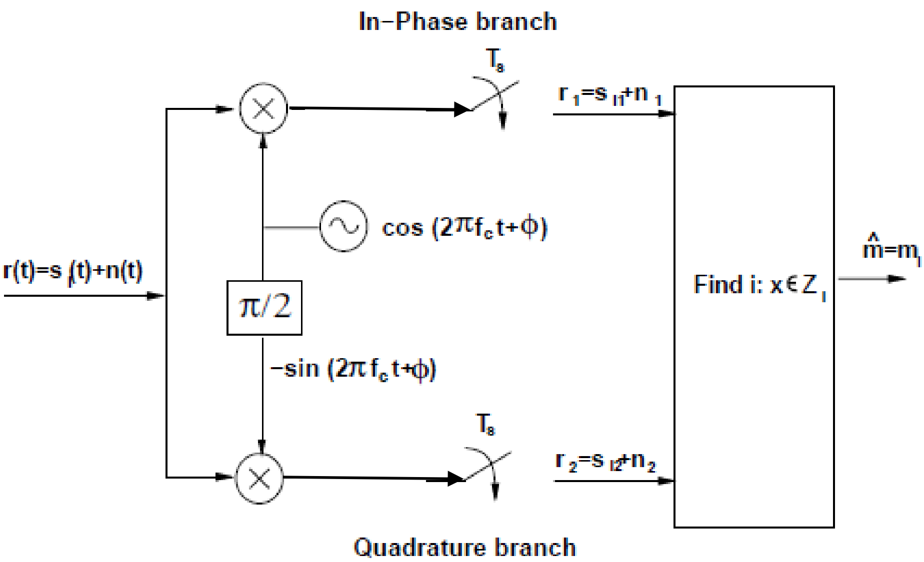
Amplitude/Phase Modulator/Demodulator



Communication System Model (no path loss)



Amplitude/Phase Modulator

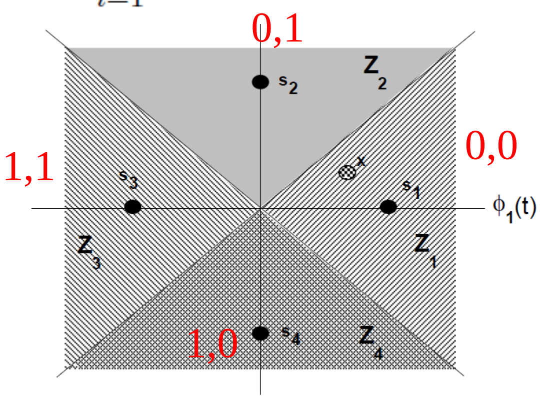


Coherent Amplitude/Phase Demodulator

Estimating BER – Nearest Neighbor Approximation

P_s – Probability of a symbol being received in error

$$P_s = \sum_{i=1}^{M} p(\mathbf{r} \notin Z_i | m_i \text{ sent}) p(m_i \text{ sent}) \longrightarrow P_s \approx M_{d_{min}} Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$$



d_{min} – minimum distance between constellation points

M_{dmin} – number of constellation points at distance d_{min}

$$Q(z) = \frac{1}{2}\text{erfc}\left(\frac{z}{\sqrt{2}}\right) \leq \frac{1}{z\sqrt{2\pi}} e^{-z^2/2},$$

Example

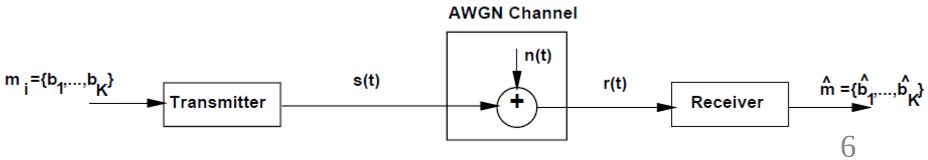
$s_1 = (A, 0)$, $s_2 = (0, A)$, $s_3 = (-A, 0)$ and $s_4 = (0, -A)$

Assume $A/\sqrt{N_0} = 4$.

$d_{min} = d_{12} = d_{23} = d_{34} = d_{14} = \sqrt{A^2 + A^2} = \sqrt{2A^2}$.

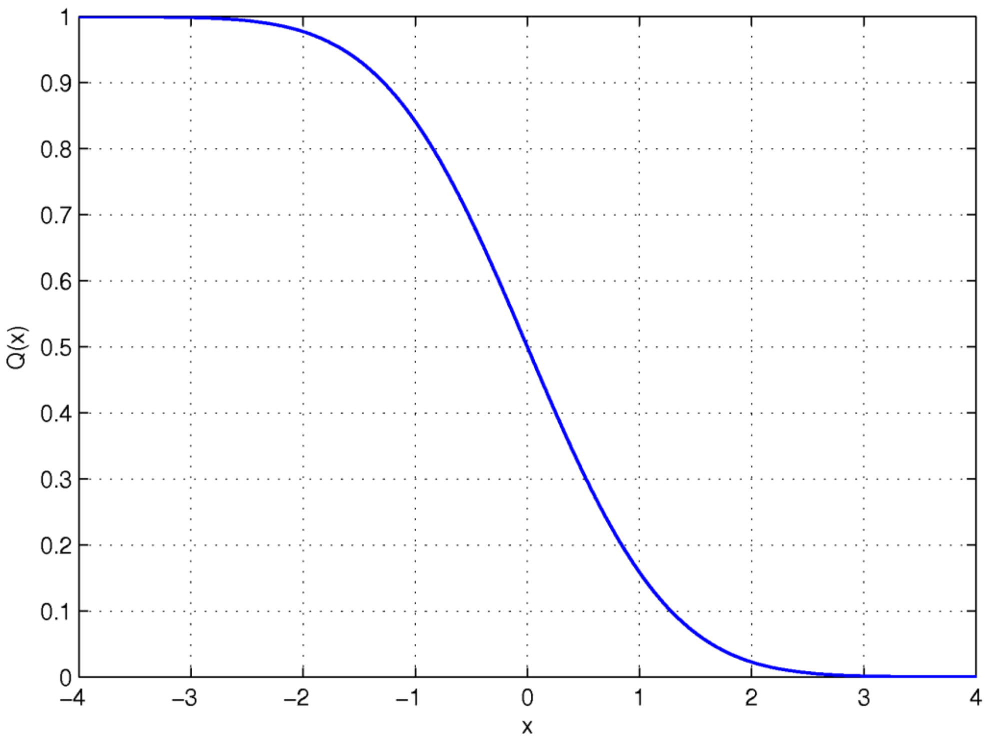
$M_{dmin} = 2$

$$P_s \approx 2Q(4) = 3.1671 * 10^{-5}$$



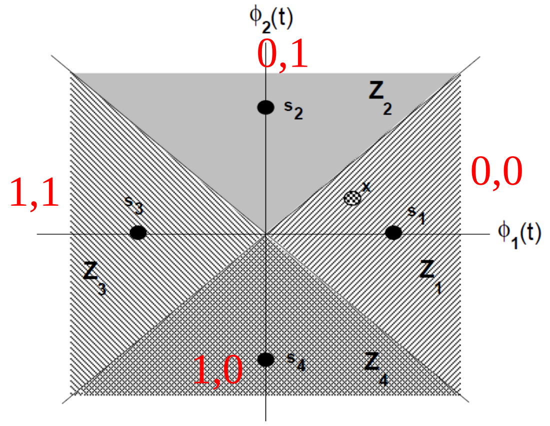
Q function

- The tail probability of the standard normal distribution
- Q(x) is the probability that a standard normal random variable will obtain a value larger than x



Let us assume $P_s = 4 * 10^{-4}$

What is the BER associated to this modulation?



Estimating BER – Nearest Neighbor Approximation

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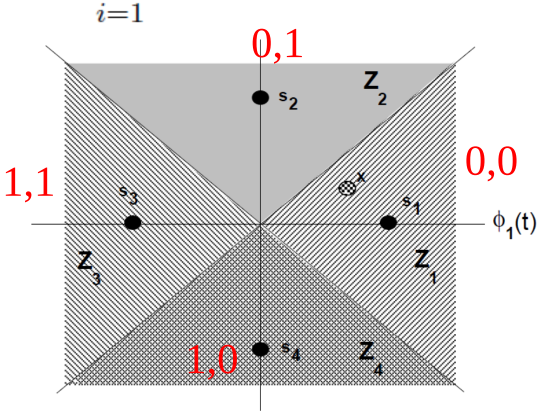

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M_{dmin} – number of constellation points at distance d_{min}

$$Q(z) = \frac{1}{2}\text{erfc}\left(\frac{z}{\sqrt{2}}\right) \leq \frac{1}{z\sqrt{2\pi}} e^{-z^2/2},$$

$$P_b = BER \approx \frac{P_s}{\log_2 M}$$

A symbol error associated with an adjacent decision region corresponds to only one bit error

Example

$s_1 = (A, 0)$, $s_2 = (0, A)$, $s_3 = (-A, 0)$ and $s_4 = (0, -A)$

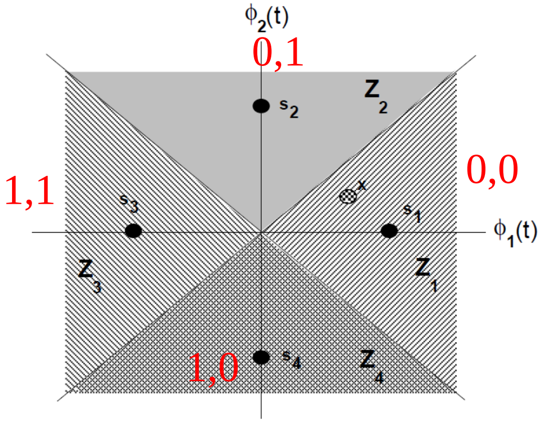
Assume $A/\sqrt{N_0} = 4$.

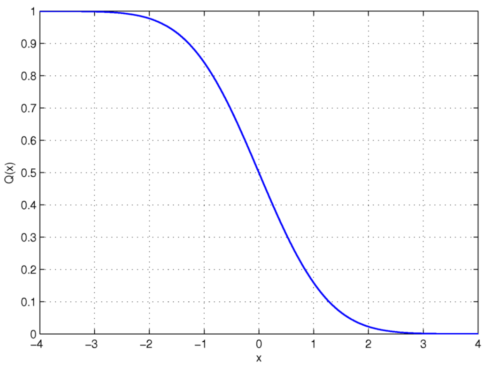
$d_{min} = d_{12} = d_{23} = d_{34} = d_{14} = \sqrt{A^2 + A^2} = \sqrt{2A^2}$.

$M_{dmin} = 2$

$$\boldsymbol{P_s} \approx 2Q(4) = 3.1671 * 10^{-5}$$

$$BER \approx \frac{P_s}{\log_2 M} = \frac{3.17 * 10^{-5}}{2} = 1.58 * 10^{-5}$$





How does P_s depend on the SNR?

$$\boldsymbol{P_s} \approx M_{d_{min}} Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$$

$SNR, \gamma_s, \gamma_b = E_b/N_0$

$$SNR = \frac{P_r}{N_0 B} = \frac{E_s}{N_0 B T_s} = \frac{E_b}{N_0 B T_b}$$

$$T_s \approx \frac{1}{B} \to SNR = \frac{E_s}{N_0} = \gamma_s$$

$$T_b \approx \frac{1}{R} \to SNR = \frac{E_b}{N_0} \cdot \frac{R}{B} \to \gamma_b = \frac{E_b}{N_0} = SNR.\frac{B}{R}$$

SNR: Signal to noise ratio

B: Bandwidth [Hz] (~ equal to baudrate, symbol/s)

T_s: Symbol duration [s]

T_b: Bit duration [s]

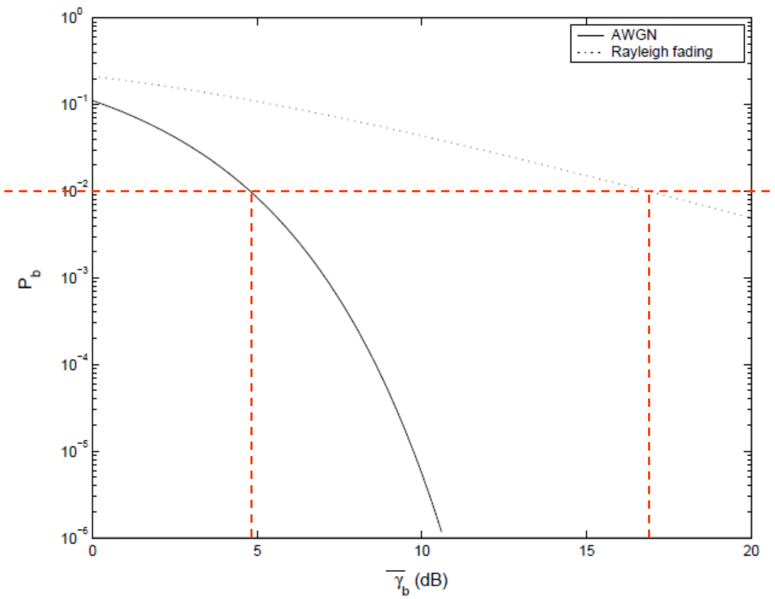
R: Signal bitrate [bit/s] (= B * $\log_2 M$)

E_b/N_0: signal energy per bit to noise power spectral density ratio

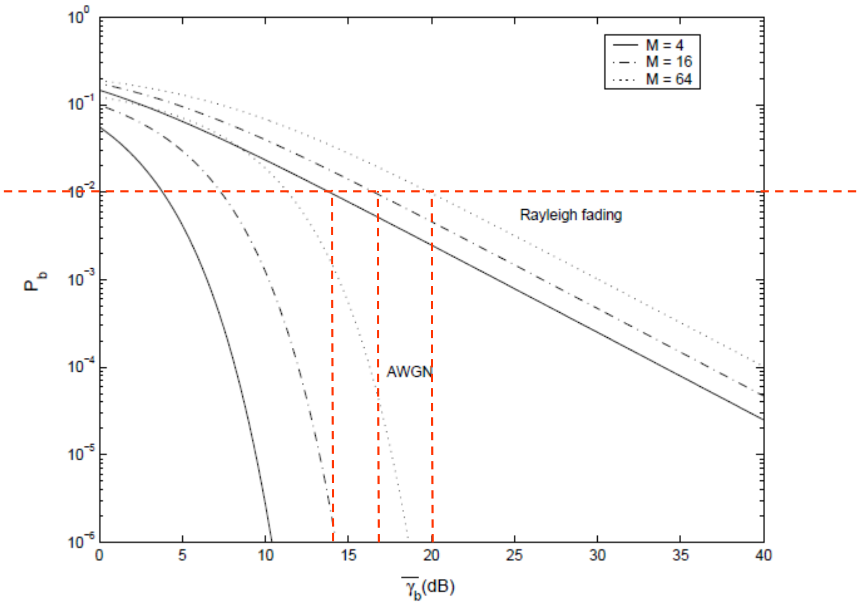
Digital Modulation – BER and E_b/N_0

Modulation	$P_s(\gamma_s)$	$P_b(\gamma_b)$
BFSK:		$P_b = Q\left(\sqrt{\gamma_b}\right)$
BPSK:		$P_b = Q\left(\sqrt{2\gamma_b}\right)$
QPSK,4QAM:	$P_s \approx 2\,Q\left(\sqrt{\gamma_s}\right)$	$P_b \approx Q\left(\sqrt{2\gamma_b}\right)$
MPAM:	$P_s \approx \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6\overline{\gamma}_s}{M^2-1}}\right)$	$P_b \approx \frac{2(M-1)}{M\log_2 M} Q\left(\sqrt{\frac{6\overline{\gamma}_b \log_2 M}{(M^2-1)}}\right)$
MPSK:	$P_s \approx 2Q\left(\sqrt{2\gamma_s}\sin(\pi/M)\right)$	$P_b \approx \frac{2}{\log_2 M} Q\left(\sqrt{2\gamma_b \log_2 M}\sin(\pi/M)\right)$
Rectangular MQAM:	$P_s \approx \frac{4(\sqrt{M}-1)}{\sqrt{M}} Q\left(\sqrt{\frac{3\overline{\gamma}_s}{M-1}}\right)$	$P_b \approx \frac{4(\sqrt{M}-1)}{\sqrt{M}\log_2 M} Q\left(\sqrt{\frac{3\overline{\gamma}_b \log_2 M}{(M-1)}}\right)$
Nonrectangular MQAM:	$P_s \approx 4Q\left(\sqrt{\frac{3\overline{\gamma}_s}{M-1}}\right)$	$P_b \approx \frac{4}{\log_2 M} Q\left(\sqrt{\frac{3\overline{\gamma}_b \log_2 M}{(M-1)}}\right)$

$$P_b = BER$$



Average P_b for BPSK in Rayleigh Fading and AWGN.



Average P_b for MQAM in Rayleigh Fading and AWGN.

Thermal Noise Power

» $P_N = k_B.T.B = N_0.B$
- k_B – Boltzmann's constant
- T – Temperature in Kelvin (e.g. 300K))
- B - Bandwidth
- N_0 – Noise power spectral density

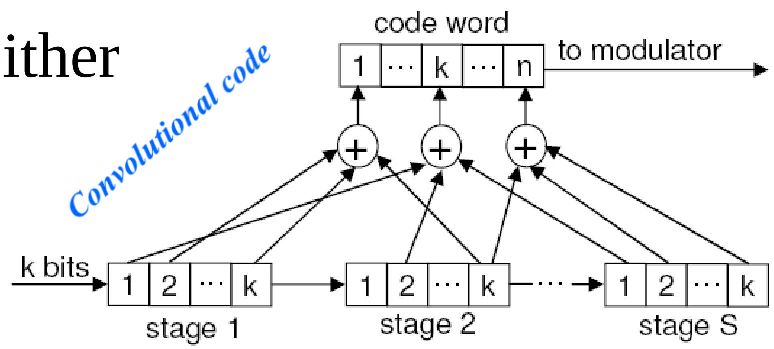
» $P_{N_{dBm}} = 10.\log(k_B.T.B.10^3)$
$= 10.\log(k_B.T.10^3) + 10.\log(B)$
$= -174 + 10.\log(B)$

$$\boxed{SNR_{dB} = P_{r_{dBm}} - P_{N_{dBm}}}$$

Bandwidth (B)	Thermal noise power P_N	Notes
1 Hz	−174 dBm	
10 Hz	−164 dBm	
100 Hz	−154 dBm	
1 kHz	−144 dBm	
10 kHz	−134 dBm	FM channel of 2-way radio
15 kHz	−132.24 dBm	One LTE subcarrier
100 kHz	−124 dBm	
180 kHz	−121.45 dBm	One LTE resource block
200 kHz	−121 dBm	GSM channel
1 MHz	−114 dBm	Bluetooth channel
2 MHz	−111 dBm	Commercial GPS channel
3.84 MHz	−108 dBm	UMTS channel
6 MHz	−106 dBm	Analog television channel
20 MHz	−101 dBm	WLAN 802.11 channel
40 MHz	−98 dBm	WLAN 802.11n 40 MHz channel
80 MHz	−95 dBm	WLAN 802.11ac 80 MHz channel
160 MHz	−92 dBm	WLAN 802.11ac 160 MHz channel
1 GHz	−84 dBm	UWB channel

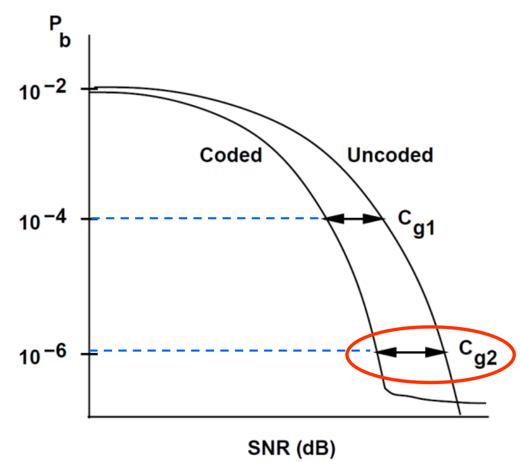
Coding

- Coding enables bits in error to be either
  detected or corrected by receiver



- Coding rate = ***k/n***
  - » Code generates ***n*** coded bits for every ***k*** uncoded bits
  - » If channel+modulation enables the transmission of *R bit/s*
  - » Information rate = *R * k/n bit/s*

- Coding gain = $C_g(P_b)$
  the amount of SNR that can be reduced for a given P_b
  by the introduction of a code



Coding in Wireless Channels

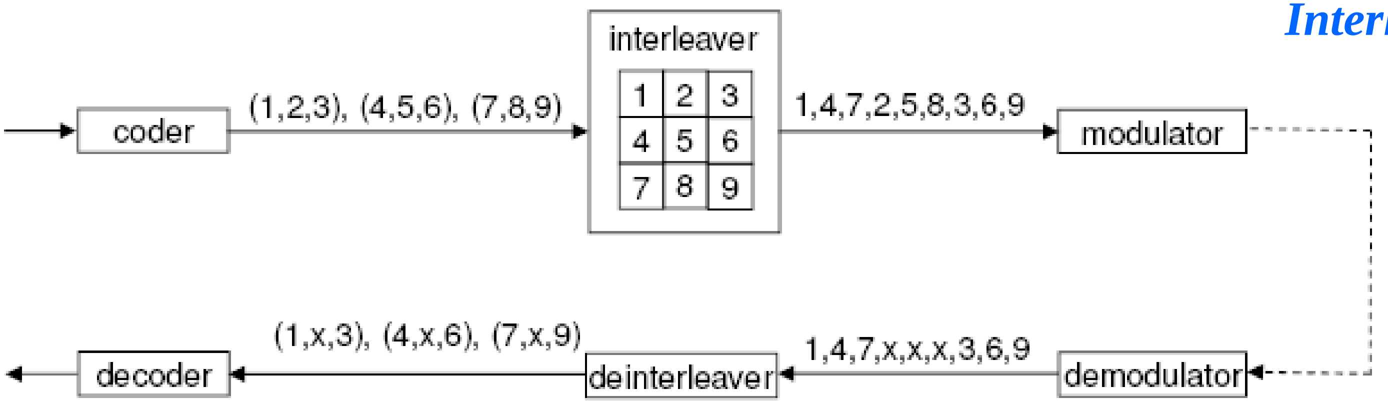
- Codes designed for AWGN channels
  - » do not work well on fading channels
  - » cannot correct the long error bursts that may occur in fading

Rayleigh

$$p_{Z^2}(x) = \frac{1}{P_r} e^{-x/P_r}$$

- Codes for fading channels are usually
  - » based on a code designed for an AWGN channel
  - » combined with interleaving
  - » objective → spread error bursts over multiple codewords

Interleaving



Why does your WLAN interface change dynamically its working bitrate?

What happens, from the modulation and coding points of view, when the WLAN interface changes from 54 Mbit/s to 6 Mbit/s?

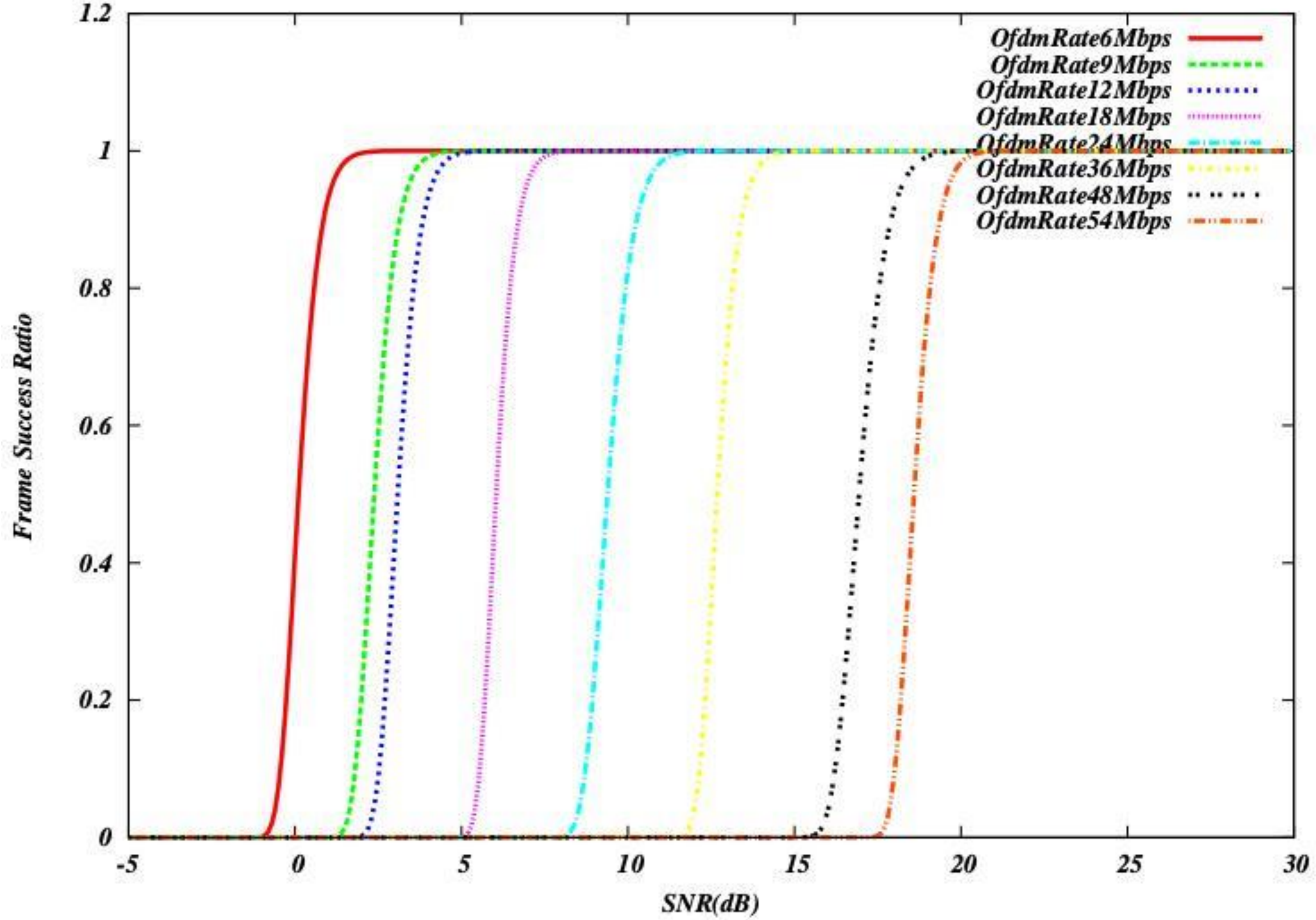
802.11a – Rate Dependent Parameters

Data rate (Mbits/s)	Modulation	Coding rate (R)	Coded bits per subcarrier (N_{BPSC})	Coded bits per OFDM symbol (N_{CBPS})	Data bits per OFDM symbol (N_{DBPS})
6	BPSK	1/2	1	48	24
9	BPSK	3/4	1	48	36
12	QPSK	1/2	2	96	48
18	QPSK	3/4	2	96	72
24	16-QAM	1/2	4	192	96
36	16-QAM	3/4	4	192	144
48	64-QAM	2/3	6	288	192
54	64-QAM	3/4	6 x 48 =	288 x R =	216

% of useful information

250 kSymbol/s (per sub-carrier),
48 sub-carriers

Frame Success Ratio vs SNR for 802.11a (obtained using the ns-3 network simulator)



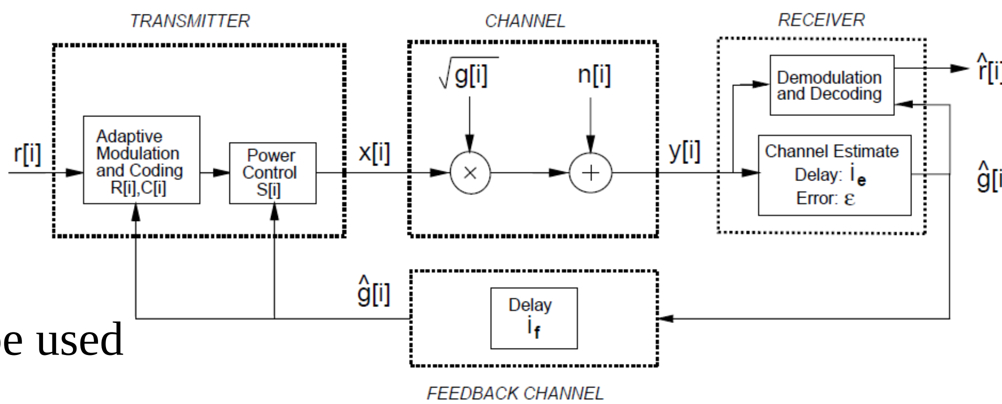
Adaptive Modulation/Coding

- Adaptive transmission techniques
  - aim at maintaining the quality; for data networks → **low and stable BER**
  - work by varying: data rate, power transmitted, codes

$$\gamma = \frac{P_r}{N_0 B}$$

- Adapting the data rate
  - symbol rate kept constant
  - modulation schemes / constellation sizes depend on γ ➔ multiple data rates

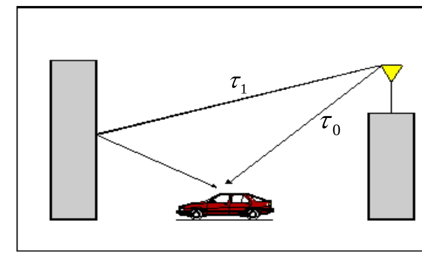
- Adapting the transmit power
  - compensate γ variation due to fading by adapting the power
  - maintain a constant received γ



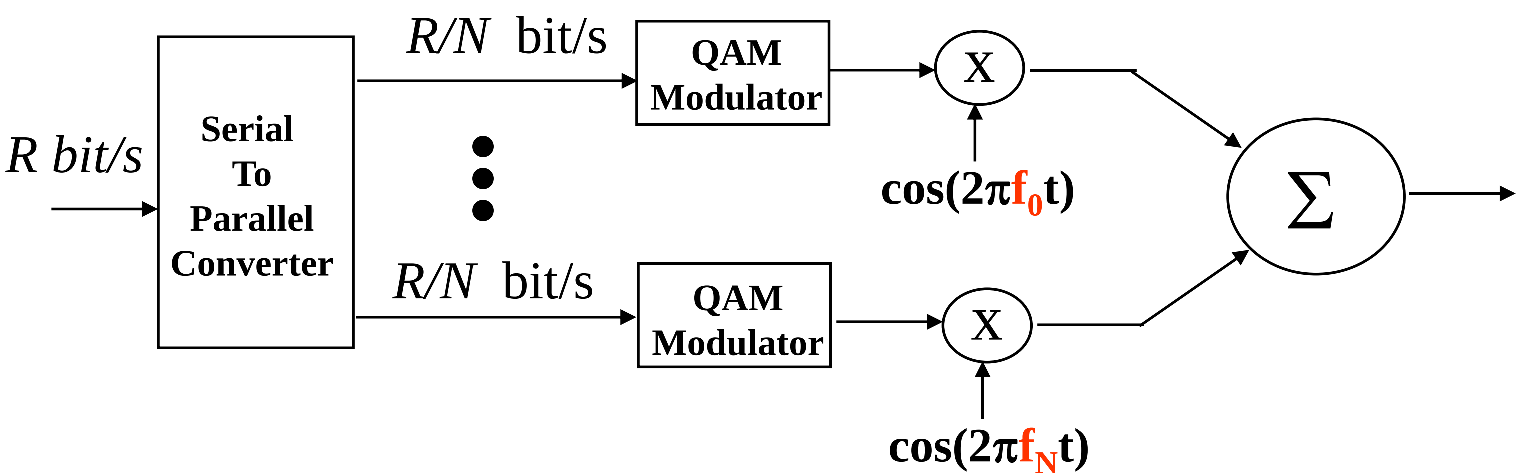
- Adapting the codes
  - γ large → weaker or no codes
  - γ small → stronger code may be used

Multicarrier Modulation

- Divides a bitstream into ***N*** low rate sub-streams
- Sends sub-streams simultaneously over narrowband sub-channels
- Sub-channel
  - » has bandwidth $\boldsymbol{B_N = B/N}$
  - » provides a symbol rate $\boldsymbol{R_N \approx R/N}$
  - » For N large, $\boldsymbol{B_N = B/N << 1/T_m} \Rightarrow \boldsymbol{T_s >> T_m}$
    - ➔ flat fading (narrowband like effects) on each sub-channel, no ISI

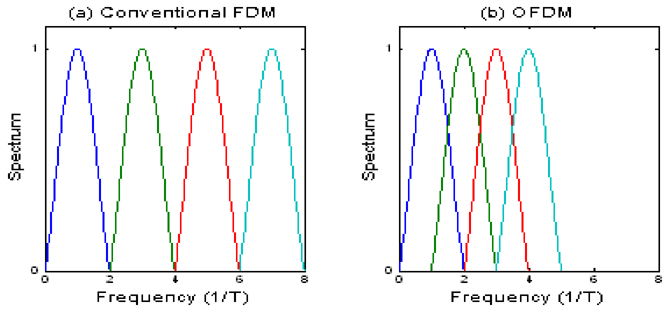


$$T_m = max_n|\tau_n - \tau_0|$$

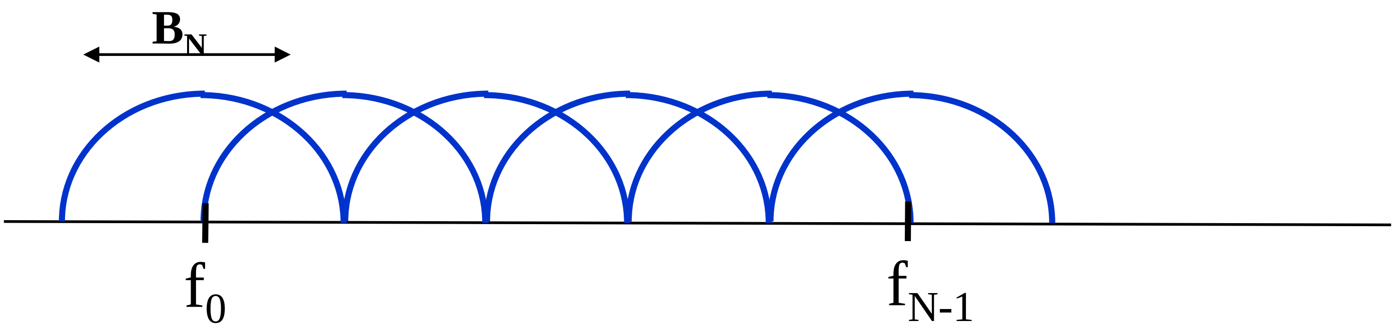


Overlapping Sub-streams

- Separate sub-channels could be used, but
  - » required passband bandwidth B = $N*B_N$



- OFDM uses overlapping sub-streams
  - » Sub-stream bandwidth is B_N
  - » Total required bandwidth is ~$B/2$, for $T_N=1/B_N$



OFDM uses Discrete Fourier Transforms

- Discrete Fourier transforms given by

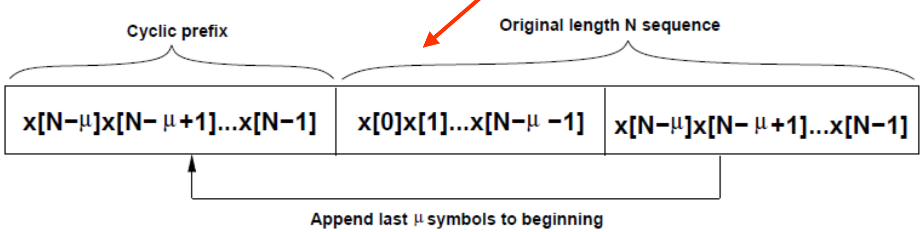
$$\boxed{\mathrm{DFT}\{x[n]\} = X[i] \triangleq \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi ni}{N}},\ \ 0 \le i \le N-1}$$

$$\boxed{\mathrm{IDFT}\{X[i]\} = x[n] \triangleq \frac{1}{\sqrt{N}}\sum_{i=0}^{N-1} X[i]e^{j\frac{2\pi ni}{N}}, 0 \le n \le N-1}$$

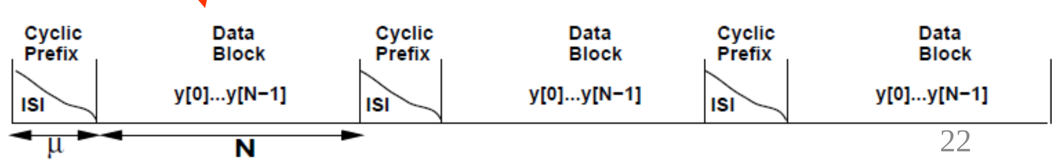
- Circular convolution $\otimes$, discrete-time convolution *

$$\mathrm{DFT}\{y[n] = x[n] \otimes h[n]\} = X[i]H[i],\ 0 \le i \le N-1.$$

$$x[n] \otimes h[n] = \ \tilde{x}[n] * h[n] = \ y[n] \ = \sum_{k=0}^{\mu-1} h[k]\tilde{x}[n-k],\ \ -\mu \le n \le N-1$$



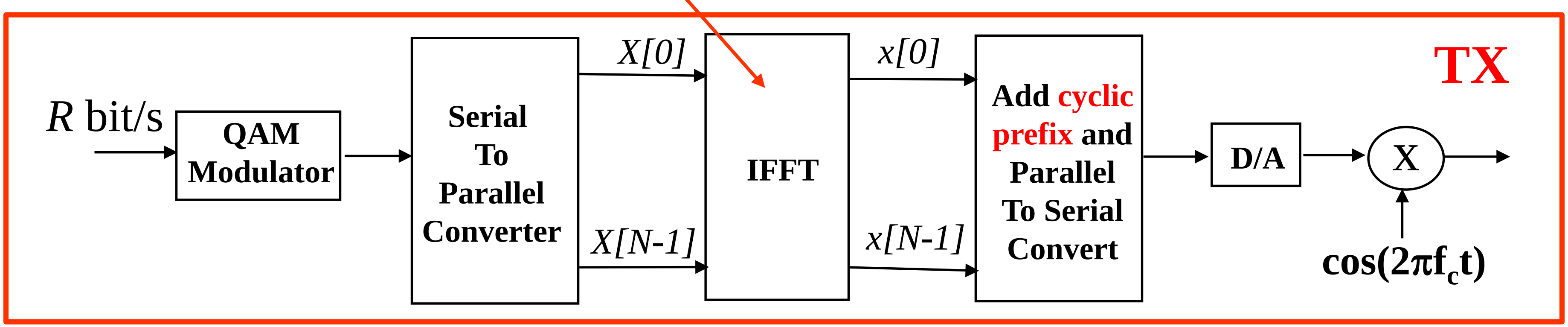
$$\boxed{\mu \approx T_m}$$



FFT Implementation of OFDM - TX

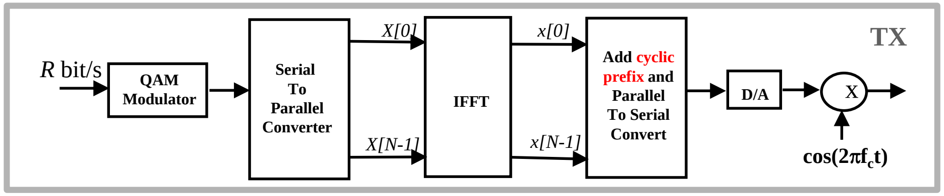
- Uses IFFT at TX to modulate symbols on each subcarrier
- Cyclic prefix makes circular channel convolution
  - ➔ no interference between FFT blocks in RX processing

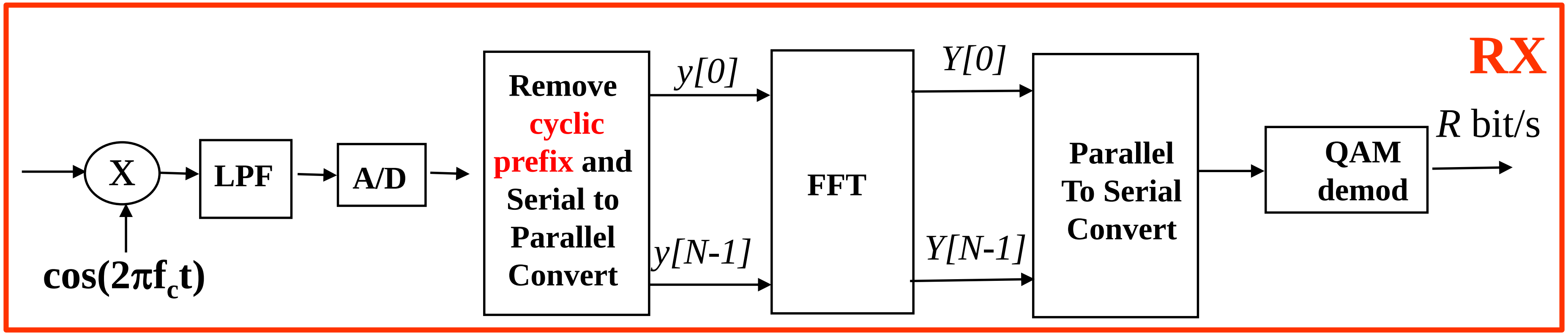
$$x[n] = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} X[i] e^{j2\pi ni/N}, \;\; 0 \leq n \leq N-1.$$



FFT Implementation of OFDM - RX

Reverse structure at RX



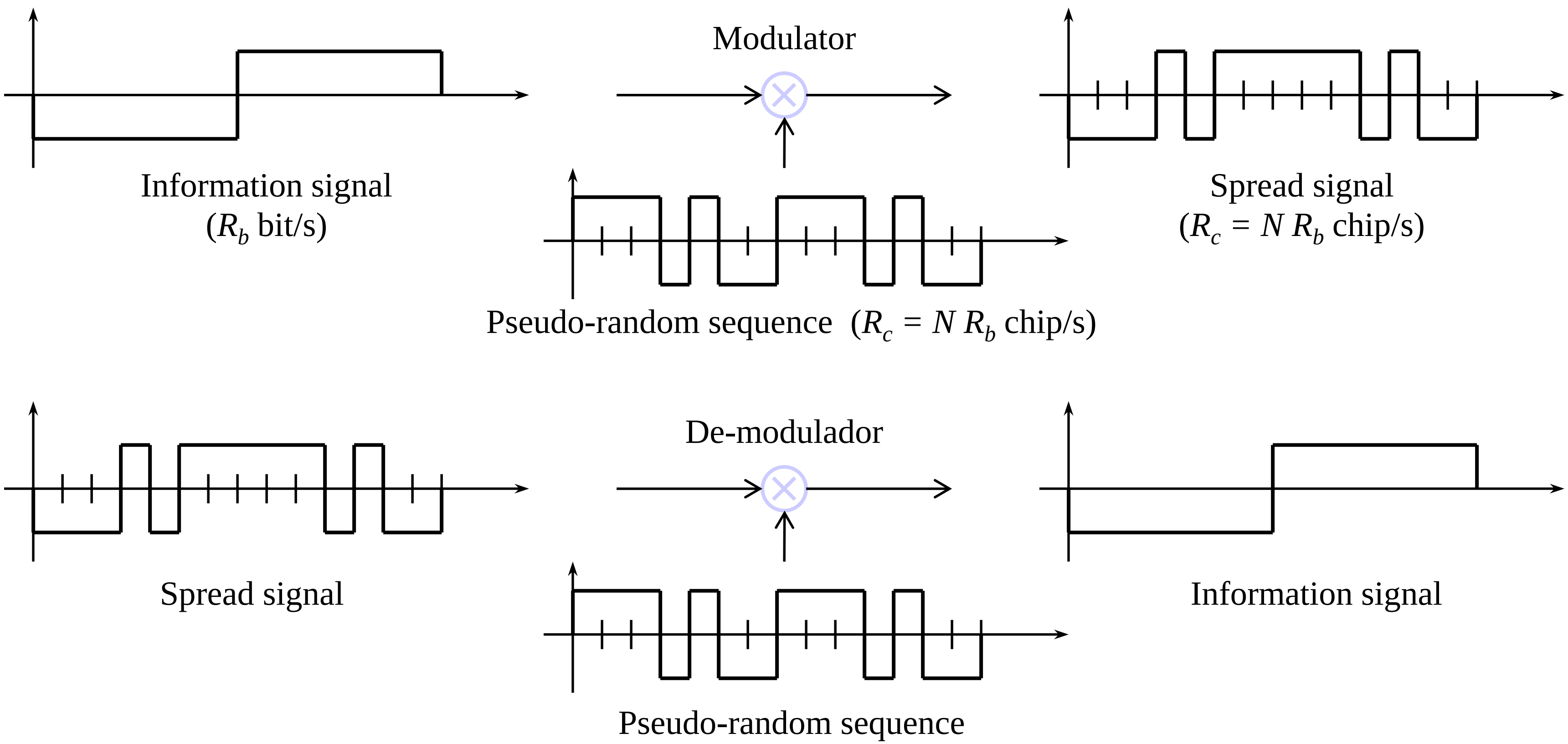


Spread Spectrum

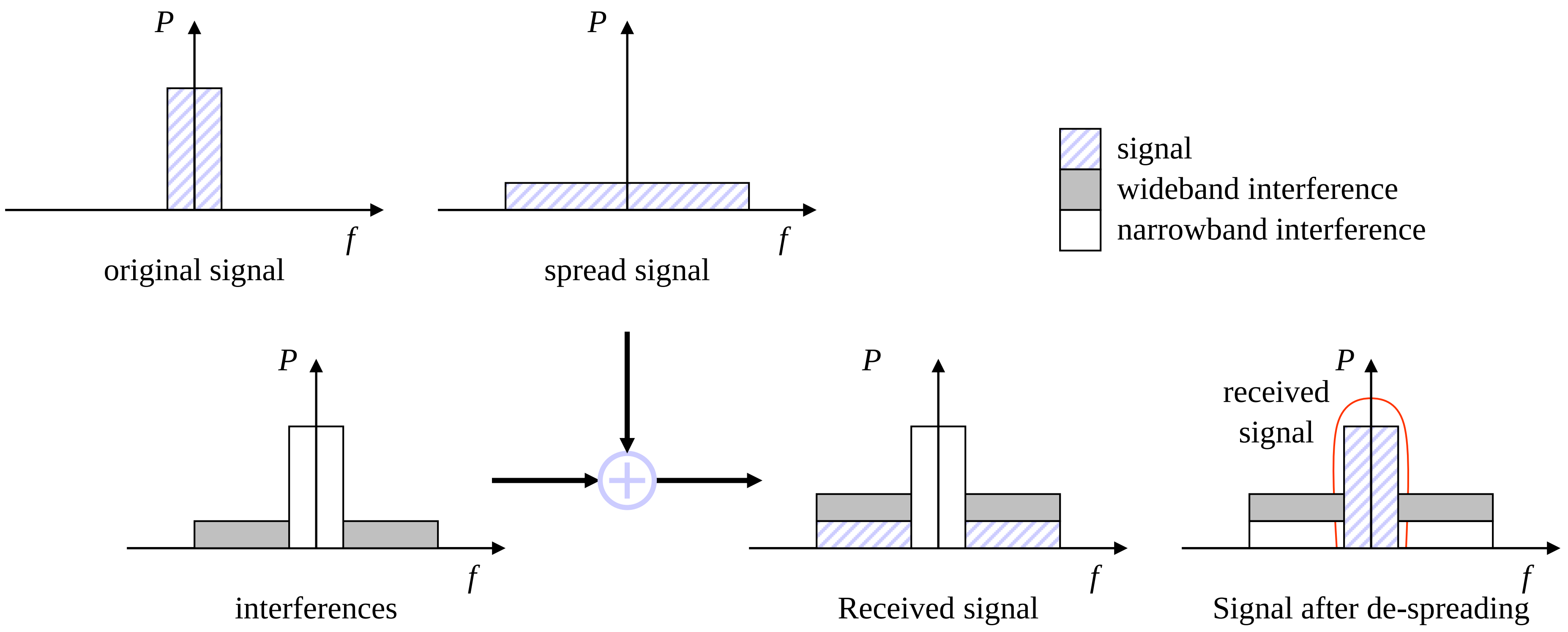
- Spread spectrum techniques
  - hide the information signal below the noise floor
  - mitigate inter-symbol interferences
  - combine multipath components

- The spread spectrum techniques
  - multiply the information signal by a spreading code

Spread Spectrum – Direct Sequence



Direct Sequence Spread Spectrum (DSSS) – Immunity to Interferences

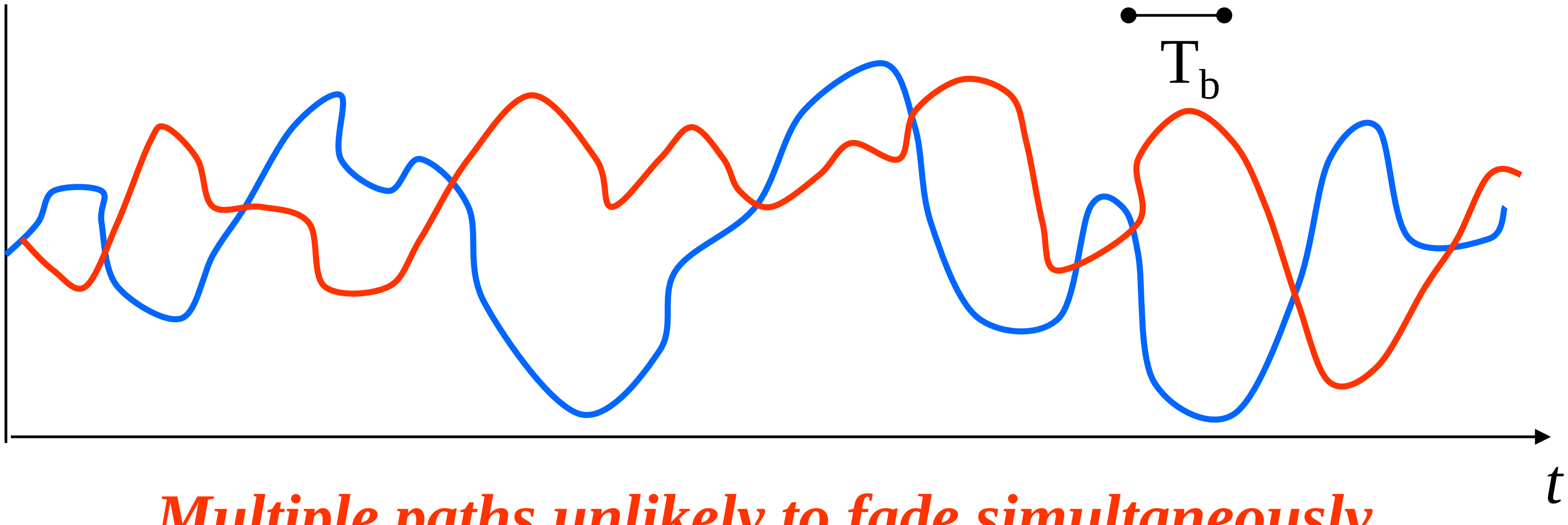


Diversity

Main idea

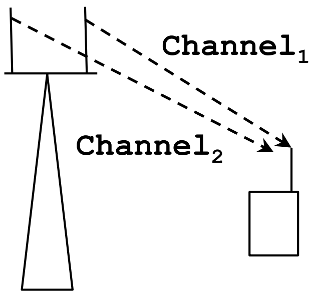
» Send same bits by independent paths

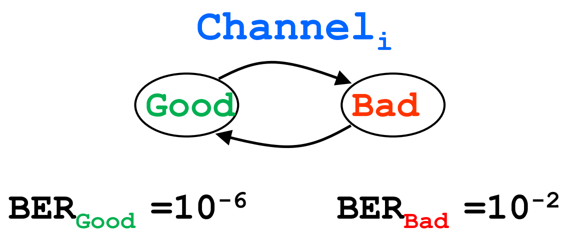
» Combine paths to mitigate fading effects



Multiple paths unlikely to fade simultaneously

Diversity - Example



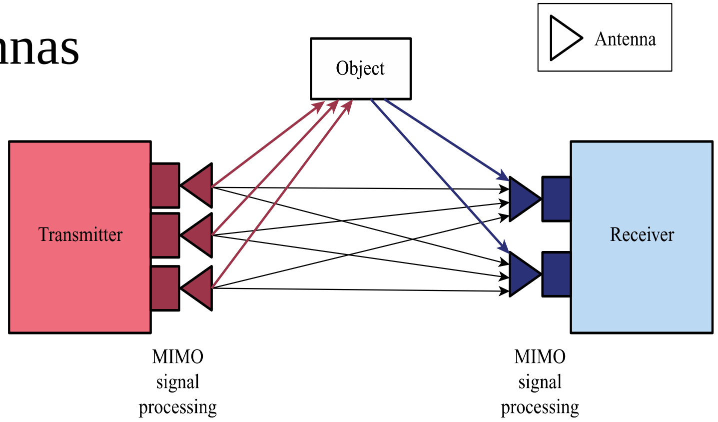


- Let us assume
  - » Channel$_1$ and Channel$_2$ are (fading) independent
  - » P[Channel$_i$=Good] = 0.8 | P[Channel$_i$=Bad] = 0.2
  - » **E[BER$_i$] =** $0.8*10^{-6} + 0.2*10^{-2}$ = **$2*10^{-3}$** (using a single Tx antenna)

- If we take advantage of diversity (using 2 Tx antenna)
  - » P[Channel$_{joint}$=Bad] = P[Channel$_1$=Bad **and** Channel$_2$=Bad] = 0.2*0.2 = 0.04
  - » P[Channel$_{joint}$=Good] = 1 - P[Channel$_{joint}$=Bad] = 1-0.04 = 0.96
  - » **E[BER$_{joint}$]=** $0.96*10^{-6}+0.04*10^{-2}$ = **$0.4*10^{-3}$**
  - » **E[BER$_i$] / E[BER$_{joint}$] = 5**

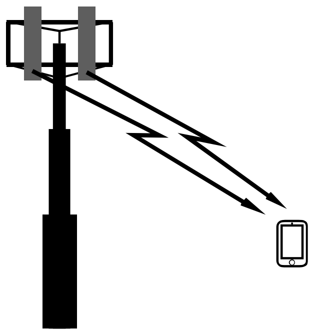
Multiple Antennas and Space-Time Communications

- Multiple Input Multiple Output (MIMO) combines
  - » signals generated by multiple transmit antennas
  - » signals received by multiple receive antennas

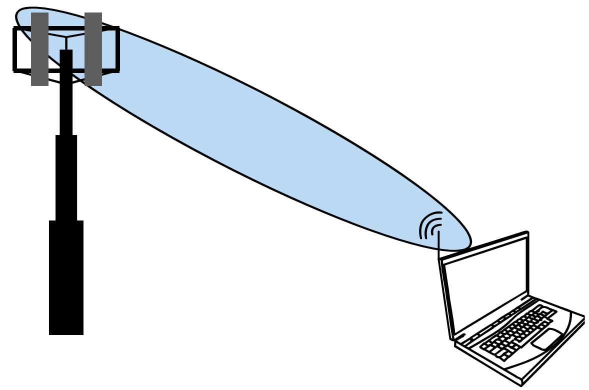


- Antenna arrays used for
  - » Diversity: different signals, same data, from different Tx antennas
  - » Multiple streams: parallel data streams (Single-user MIMO)
  - » Beamforming: directional antennas
  - » Multi-user MIMO: directional beams to multiple simultaneous users

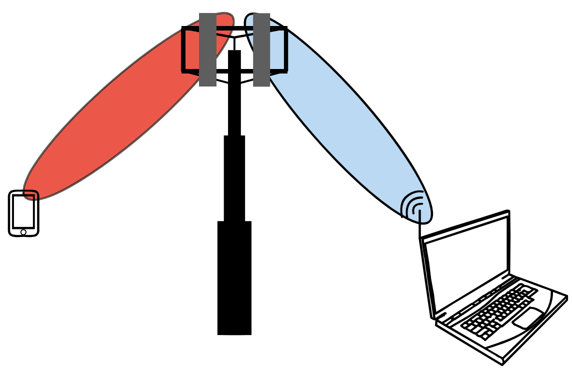
Four Uses of MIMO



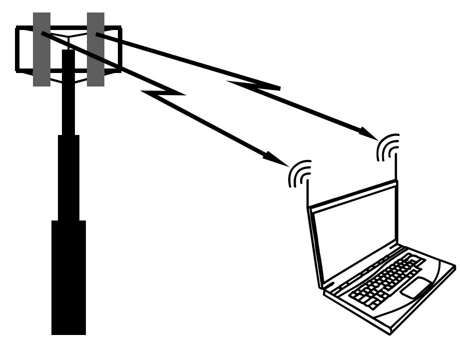
Diversity for improved
system performance



Beam-forming for improved coverage
(less cells to cover a given area)



Spatial division multiple access
("MU-MIMO") for improved capacity
(more user per cell)



Multi layer transmission
("SU-MIMO") for higher data rates
in a given bandwidth

Homework

1. Review slides

2. Answer questions at moodle

Detailed information about these topics can found at the Goldsmith's book

- » Chap. 5 (sections 5.1, 5.2, 5.3, 5.5)
- » Chap. 6 (sections 6.1, 6.3)
- » Chap. 7 (sections 7.1, 7.2)
- » Chap. 8 (section 8.1)
- » Chap. 9 (section 9.1)
- » Chap. 12 (sections 12.1, 12.2, 12.4)
- » Chap. 13 (sections 13.1, 13.2, 13.3)