Heuristic Optimization Methods

Tabu Search: Advanced Topics

Agenda

- More about Tabu Search
  - Probabilistic Move Acceptance
  - Strategic Oscillation
  - Exploiting infeasible solutions
  - Path Relinking
  - Ejection Chains

Probabilistic Move Acceptance

- The move evaluation function is very myopic
- The best *global* move might not be the best *local* move
- But, it might be among the top few moves
- PMA – probabilistic move acceptance
  - Select the best (non-tabu) move with probability *p*.
  - If not selected, select to the next best move with probability *p*, etc.

Strategic Oscillation

- Aim: efficient interplay between intensification and diversification
- Some aspect of the search is varied systematically to focus on different aspects of the solutions
  - E.g., oscillate between feasible space and infeasible space
    - Feasible solutions that are good w.r.t. the constraints
    - Infeasible solutions that are good w.r.t. the objective function
  - Example: 0/1 MKP – Glover and Kochenberger

Exploiting Infeasible Solutions

- Why should the search be allowed to visit infeasible solutions?
  - They cannot be good, since they violate some constraints
- The feasible space can be fragmented
  - Given a neighborhood operator: there is no path between feasible solutions S_A and S_B
- There are infeasible solutions for which the optimal solution is a neighbor
  - Knapsack: removing one item can change infeasible to feasible

Candidate List Strategies

- Strategies to limit the exploration of the neighborhood
- Limited list of moves that look promising
  - Random choice
  - Partitioning
- The Candidate list can be extended if no good moves are found
- Reuse of information (and thus reducing the computational effort)
  - *Elite* candidate list with potentially good moves
  - Found in previous iterations, but not executed yet
- Parallell processing – each processor processes a part of the search neighborhood

Tabu Search: Trade-offs

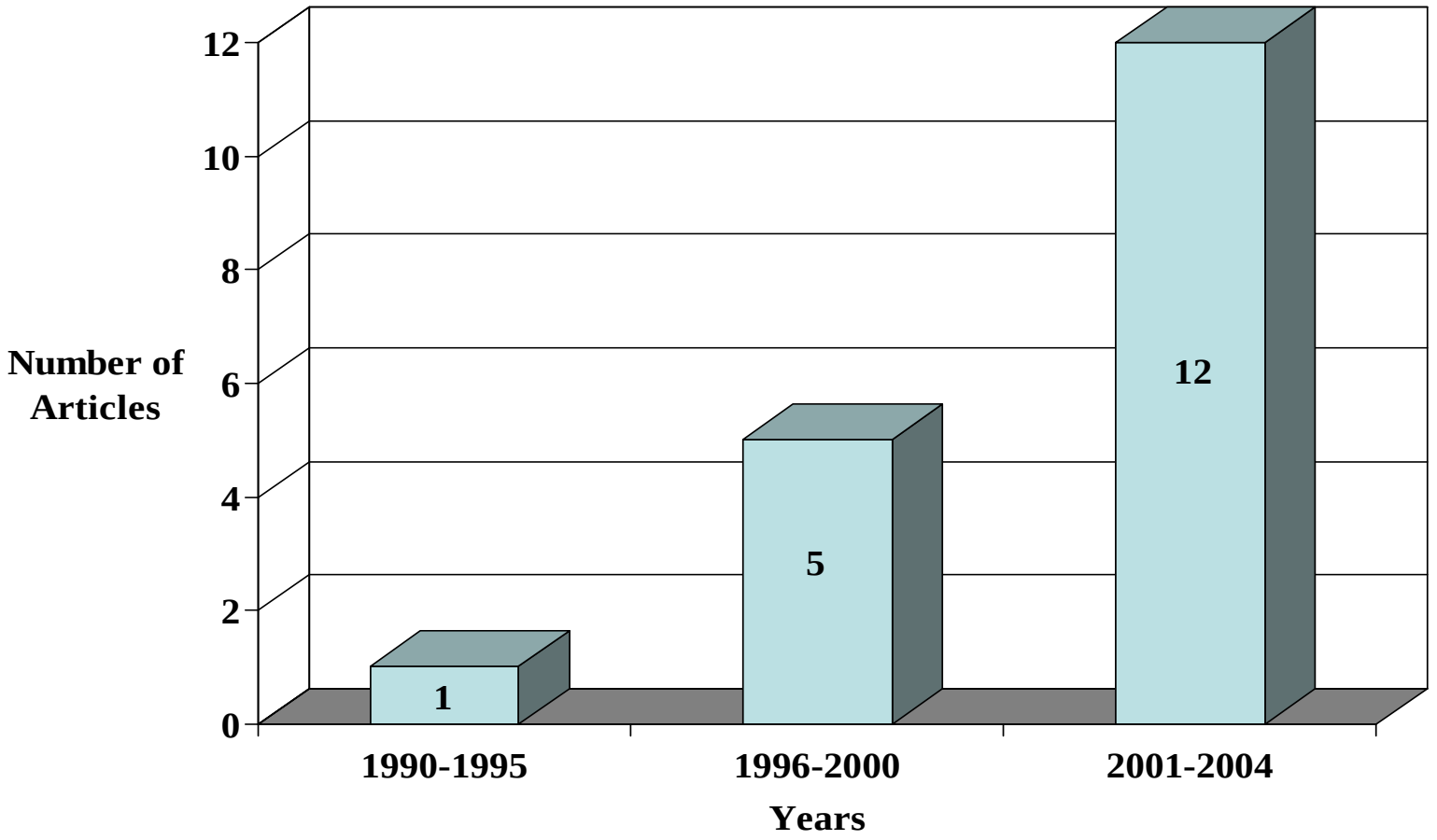
- Collecting and using tabu status and frequency based information takes cpu time
- The local search is more focused when the extra information is used
  - I.e., each iteration accomplishes more
- There is a trade-off between when the cost of using the extra information and the added search time per iteration

Path Relinking

- Seminal ideas originated in connection with tabu search …
  - Glover, F. and M. Laguna (1993) “Tabu Search,” in Modern Heuristic Techniques for Combinatorial Problems, C. Reeves (ed.) *Blackwell Scientific Publications*, pp. 70-150.

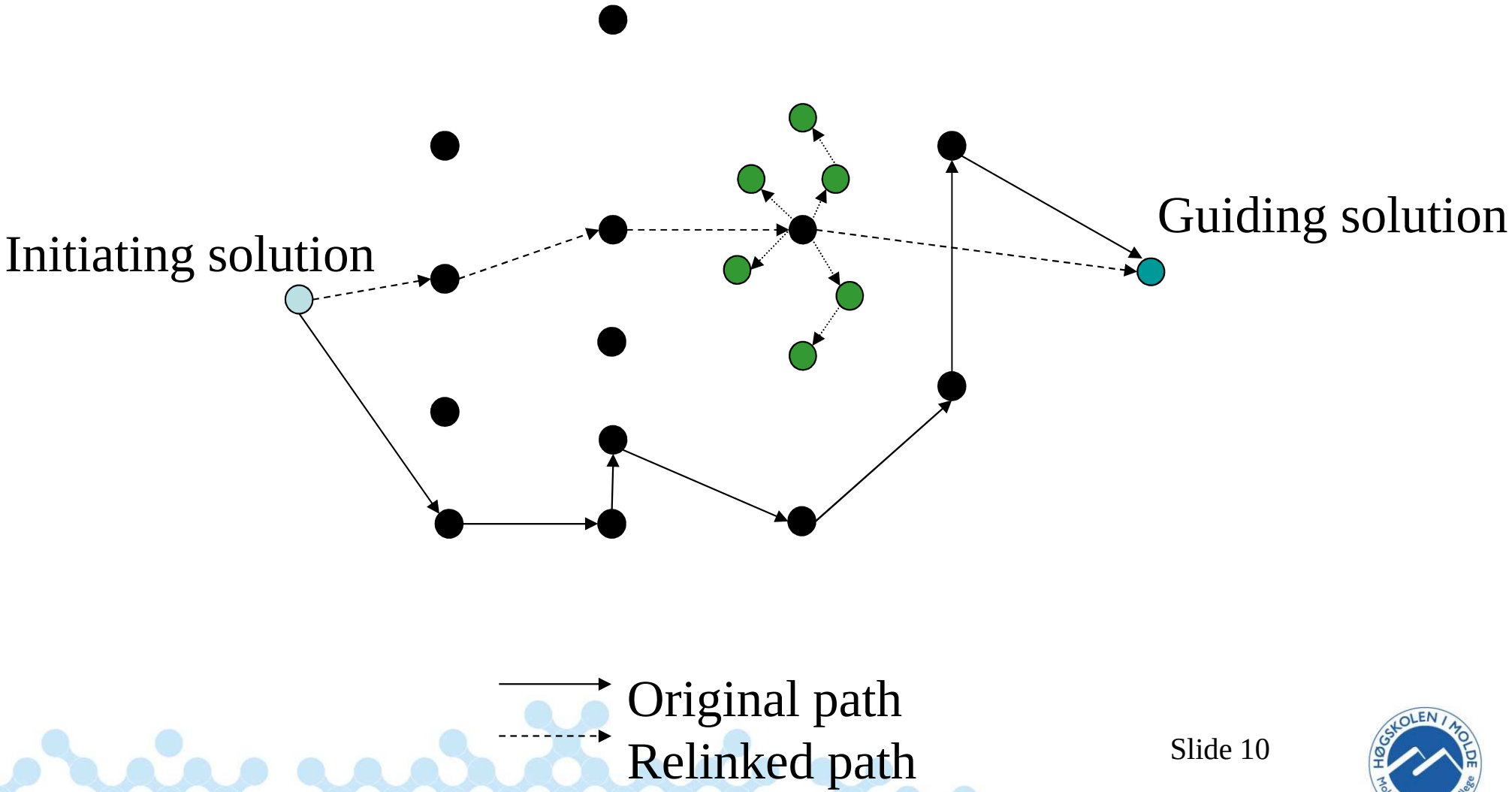
- Modern versions have been applied as a combination method within scatter search and in the improvement phase of GRASP

Path Relinking Research

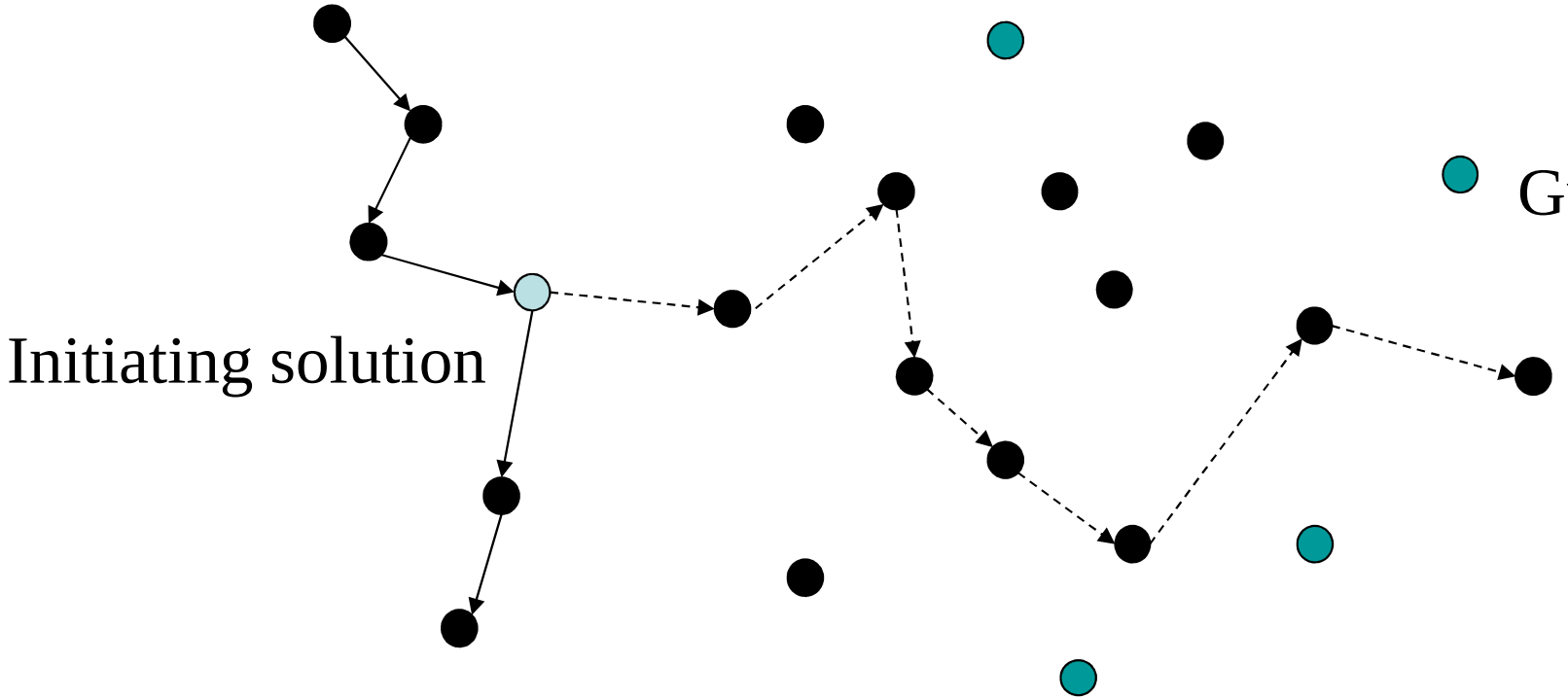


Source: Web of Science
24 Articles found on 3/11/2004

Relinking Solutions



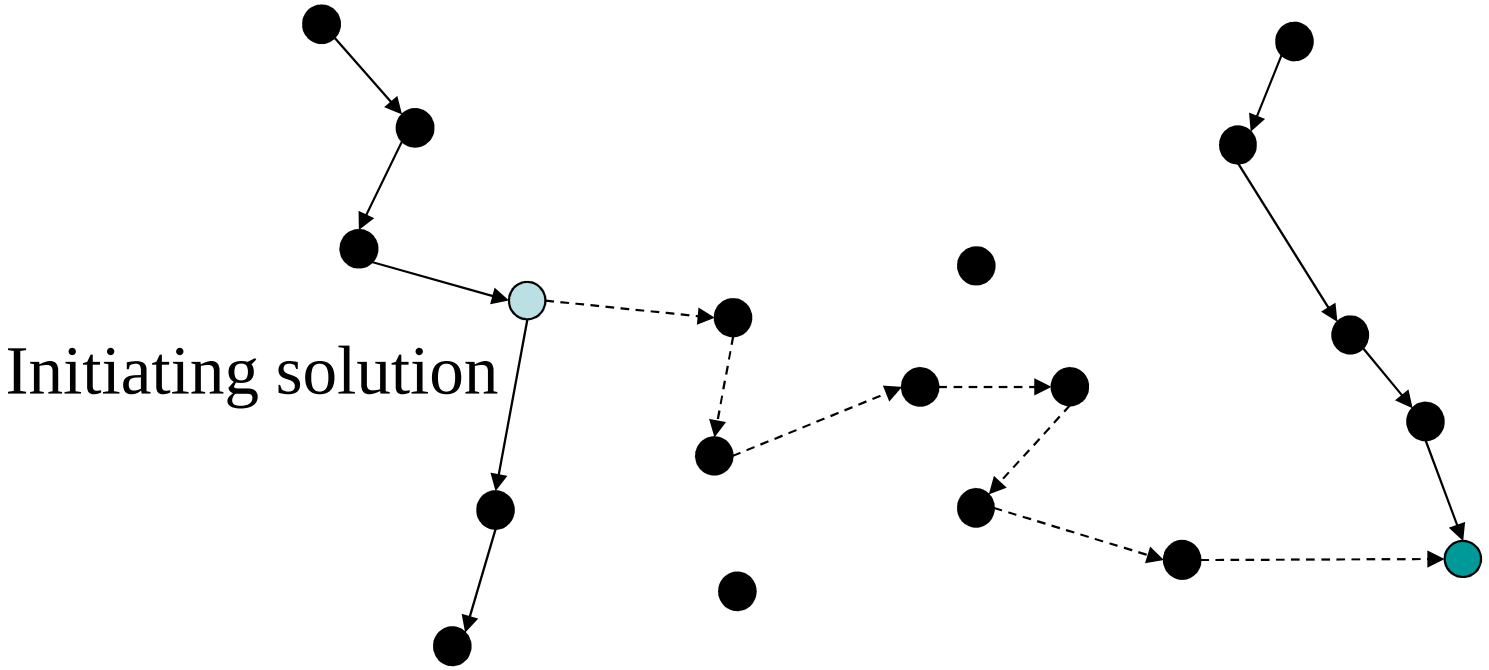
Multiple Guiding Solutions



Original path

Relinked path

Linking Solutions



Original path

Relinked path

GRASP (Greedy Randomized Adaptive Search Procedure)

- Multi-start and local search procedure introduced by Feo and Resende (1989)

do

{

$x \leftarrow$ RandomizedGreedyConstruction(α)

$x \leftarrow$ LocalSearch(x)

$x^* \leftarrow$ UpdateBest(x)

} **while** (termination criterion not met)

GRASP with Path Relinking

- Originally suggested in the context of Graph Drawing by Laguna and Marti (1999)
  - A guiding solution is selected from a small set (size 3) of elite solutions
  - Initiating solutions are the result of GRASP iterations
  - The number of relinking steps is the number of vertices in the graph
  - Local search is applied every $\beta \sim |E|$ steps

- Extensions and comprehensive review are due to Resende and Riberio (2003) “GRASP with Path Relinking: Recent Advances and Applications” http://www.research.att.com/~mgcr/doc/sgrasppr.pdf

Applications

- three index assignment problem [1, 3]
- job-shop scheduling problem [1, 2]
- prize-collecting Steiner tree problem [9]
- MAX-CUT problem [17]
- quadratic assignment problem [31]
- routing private circuits in telecommunication networks [40]
- *p*-median problem [43]
- 2-path network design problem [47]
- Steiner problem in graphs [49]
- capacitated minimum spanning tree problem [53]

Relinking Strategies

- *Periodical relinking* — not systematically applied to all solutions
- *Forward relinking* — worst solution is the initiating solution
- *Backward relinking* — best solution is the initiating solution
- *Backward and forward relinking* — both directions are explored
- *Mixed relinking* — relinking starts at both ends
- *Randomized relinking* — stochastic selection of moves
- *Truncated relinking* — the guiding solution is not reached

Issues for Future PR Research

- Selection of initiating and guiding solutions
- Application of local search to intermediate solutions
- Testing of standalone PR procedures

Ejection Chains

- Ejection chain procedures are based on the notion of generating compound sequences of moves, leading from one solution to another, by linked steps in which changes in selected elements cause other elements to be "ejected from" their current state, position or value assignment
- There are several types of ejection chains, some structured to induce successive changes in problem variables and others structured to induce changes in particular types of model components (such as nodes and edges of a graph).

Ejection Chains

- An ejection chain of L levels consists of a succession of operations performed on a given set of elements,
- The k-th operation changes the state of one or more elements which are said to be ejected in the k+1th operation.
- This ejection thereby changes the state of other elements, which then lead to further ejections, until no more operations can be made according to some pre-defined conditions.
- State-change steps and ejection steps typically alternate, and the options for each depend on the cumulative effect of previous steps

Ejection Chains - Example

- In a CVRP (Capacitated Vehicle Routing Problem), feasibility with respect to weight constraints is a concern
- Let a move be to insert a customer into a different tour.
  - The basic neighborhood is based on this
- If a move leads to an infeasible solution, can start an ejection chain.
  - From the violated tour, make another move, moving another customer to a different tour
  - Repeat this until feasibility is obtained, or enough moves made.
  - See the results of potential ejection chains before actually executing them
- This gives rise to sets of compound moves, linked through the ejection chains.
- These can be very powerful operators

Summary

- More about Tabu Search
  - Probabilistic Move Acceptance
  - Strategic Oscillation
  - Exploiting infeasible solutions
  - Path Relinking
  - Ejection Chains