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Department of Mechanical Engineering
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NUMERICAL METHODS IN OSCILLATIONS OF MECHANICAL SYSTEMS

KINEMATIC MECHANISM - PART A

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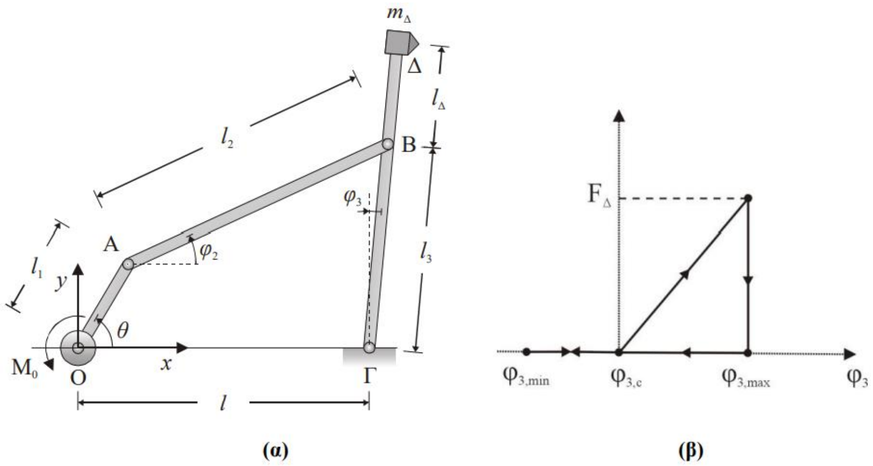
CONTENTS

PROBLEM DESCRIPTION

The present paper is the 2nd computational assignment in the course 'Numerical Methods in Oscillations of Mechanical Systems'. This work consists in total of 2 distinct parts. In this particular file we study and solve Part A.

The problem we are asked to study is presented in Figure 1.



Shape1. Mechanism model, plucker and tip force diagram D as a function of angleφ_3

The above mechanism consists of 3 distinct members and a concentrated mass at the D end of the 3rd member. The force diagram depicted in Figure 1b. refers to the force applied at point D and is always perpendicular to the line segment GBD. All geometric dimensions and mass properties of the mechanism members are known. The analysis that will follow in all queries is initially carried out using variables, without entering numerical values. During the process of extracting the requested diagrams and results, the programming code will first be compiled in a Matlab environment, where the necessary numerical values will be entered. Table 1 shows a summary of all the numerical values that the pronunciation gives us.

Panel1. Prices for Numerical application

$(F_Δ)_{max}$	3 [kN]	$l_Δ$	15 [cm]
ω	240 [rpm]	$\varphi_{3,c}$	-10 [°]
l_1	15 [cm]	m_1	30[kg]
l_2	1[m]	$m_2 = m_3$	60[kg]
l	1 [m]	$m_Δ$	90[kg]
l_3	55 [cm]		

In the analysis that follows each question, all the necessary theoretical knowledge and the calculation process are first given, then the code written to extract the results is presented and the results are presented. Each question is completed with a comment on what is being asked. At the end of this paper in Appendix A, there are all the codes written to solve each query.

1st QUESTION

In the 1st question we are asked to calculate and graphically display the angles as a function of the angle θ of the crank.φ_2, φ_3

The crank is the driving part of the mechanism, which rotates at a constant angular velocity, and the members of the mechanism are undeformed. First, as shown in Figure 1a, we define an inertial coordinate system at the origin O of the crank. As generalized coordinates of the problem we define the angles from which the coordinate θ is considered independent and known while the other 2 are dependent, due to the motion links. To derive the motion bonds, we calculate the vector sum of the sides of the closed polygon OABGO, so we have:$\omega = \dot{\theta}\ \left[\frac{rad}{s}\right] \theta, \varphi_1, \varphi_2.$,

$$\underline{r_{OA}} + \underline{r_{AB}} + \underline{r_{B\Gamma}} + \underline{r_{\Gamma O}} = 0$$

where the line segment vectors are defined with respect to the inertial coordinate system:

$$\underline{r_{OA}} = l_1 \cos(\theta)\, \underline{e_x} + l_1 \sin(\theta)\, \underline{e_y}$$

$$\underline{r_{AB}} = l_2 \cos(\varphi_2)\, \underline{e_x} + l_2 \sin(\varphi_2)\, \underline{e_y}$$

$$\underline{r_{B\Gamma}} = -\underline{r_{\Gamma B}} = -l_3\, sin(\varphi_3)\, \underline{e_x} - l_3 \cos(\varphi_3)\, \underline{e_y}$$

$$\underline{r_{\Gamma O}} = -l\underline{e_x}$$

Substituting these equations into the original gives:

$$l_1 \cos(\theta)\, \underline{e_x} + l_1 \sin(\theta)\, \underline{e_y} + l_2 \cos(\varphi_2)\, \underline{e_x} + l_2 \sin(\varphi_2)\, \underline{e_y} - l_3\, sin(\varphi_3)\, \underline{e_x} - l_2 \cos(\varphi_3)\, \underline{e_y} - l\underline{e_x} = 0$$

then separating the terms in the x and y direction respectively, we get the system of equations:

$$\begin{cases} l_1 \cos(\theta) + l_2 \cos(\varphi_2) - l_3\, sin(\varphi_3) - l = 0 \\ l_1 \sin(\theta) + l_2 \sin(\varphi_2) - l_3 \cos(\varphi_3) = 0 \end{cases} \quad (\boldsymbol{I})$$

System (I) can be written in compact form:

$$\underline{f}\left(\underline{\varphi}\,; \theta\right) = \underline{0}, \qquad \mu\varepsilon\ \underline{\varphi} = \begin{bmatrix} \varphi_2 \\ \varphi_3 \end{bmatrix}$$

With this form we indicate that we treat the angle θ as a variable parameter, since knowing that the angular velocity is constant, by completing it we can calculate the angle θ for any moment in time as long as we know its value at moment 0.

The system (I) is a system of non-linear algebraic equations in terms of the dependent variables and the independent variable i. Because the system is non-linear for its solution we cannot apply any analytical method, therefore it will be solved numerically. You ask us for the graph of the dependent angles as a function of the angle θ. To get these graphs for a range of values we will apply the Sequential Continuation method.φ_2, φ_3

- **Sequential Continuation Method**

According to this method, our goal is to solve the system in a range of values of the parameter i. To solve this, having an initial solution of the system, we define a discretization step for the interval in which we want to calculate our solution (0 to 2π) and we construct a sequence of points . Thus we proceed from one value to the next in the sequence and calculate the new solution for the vector of angles$\underline{f}\left(\underline{\varphi}\,;\theta\right) = \underline{0}\underline{f}\left(\underline{\varphi_o}\,;\theta_o\right) = \underline{0}\,\Delta\theta(\theta_o, \theta_1, \theta_2 \ldots\ldots \theta_m)\theta_{\kappa+1} = \theta_\kappa + \Delta\theta_\kappa \underline{\varphi}_{\kappa+1}$.

For each new value that we introduce into system (I) a numerical solution is required, due to the non-linear equations of which it is composed. For the solution we apply the Newton – Raphson (NR) method each time.$\theta_{\kappa+1}$

To apply the NR method, we give an initial guess to our solution and a positive number to represent our error. For this initial estimate, a zero-order prediction is made = which is equal to the solution in the previous step. Next, we check if the condition is satisfied($\underline{\varphi}_{\kappa+1}^{[1]},\ \theta_{\kappa+1}$)($\underline{\varphi}_\kappa$, $\theta_{\kappa+1}$)θ_κ.

and we distinguish 2 cases.$\left\| \underline{f}(\ \underline{\varphi}_{\kappa+1}^{i},\ \theta_{\kappa+1})\ \right\| < \varepsilon$

i. Satisfaction of this condition makes the solution acceptable ($\underline{\varphi}_{\kappa+1}^{[i]},\ \theta_{\kappa+1}$)

ii. If the condition is not met, we set a correction
($\underline{\varphi}_{\kappa+1}^{i+1},\ \theta_{\kappa+1}$)=$\left(\underline{\varphi}^i{}_{k+1} + \Delta\underline{\varphi}^i{}_{\kappa+1}, \theta_{\kappa+1}\right)$
Substituting this correction into system (I), develop a Taylor series around the point $\left(\underline{\varphi}_{\kappa+1}^{i+1},\ \theta_{\kappa+1}\right)$ and keeping terms up to 1st order gives us the system of equations (written in compact form):

$$\underline{f}\left(\underline{\varphi}_{\kappa+1}^{i+1}, \theta_{\kappa+1}\right) = \underline{f}\left(\left(\underline{\varphi}^i{}_{k+1} + \Delta\underline{\varphi}^i{}_{\kappa+1}, \theta_\kappa + \Delta\theta_\kappa\right)\right)$$
$$= \underline{f}\left(\underline{\varphi}^i{}_{k+1}; \theta_\kappa\right) + J\left(\underline{\varphi}^i{}_{k+1}\,; \theta_\kappa\right)\Delta\underline{\varphi}^i{}_{\kappa+1} + \underline{f_\theta}\left(\underline{\varphi}^i{}_{k+1}\,; \theta_\kappa\right)\Delta\theta_\kappa = 0$$

where the Jacobian matrix of the system and .$J = \frac{\partial \underline{f}}{\partial \underline{\varphi}}\underline{f_\theta} = \frac{\partial \underline{f}}{\partial \theta}$

The process is repeated until the error ε is satisfied or we exceed a certain number of iterations that we define.

With the NR method completed, we now have our solution to the next point$\left(\underline{\varphi}_{\kappa+1}, \theta_{\kappa+1}\right)$, and we continue until the solutions are computed for all values of the sequence of values for θ.$\left(\underline{\varphi}_{\kappa+2}^{[1]}, \theta_{\kappa+2}\right) = \left(\underline{\varphi}_{\kappa+1}, \theta_{\kappa+1}\right)$

Before proceeding with the derivation of the results, let us mention that the sequential continuation method shows problems when the Jacobian matrix is not inverted, which means zero determinant. Calculating this matrix and its determinant we get:

$$J\left(\underline{\varphi}; \theta\right) = \frac{\partial \underline{f}}{\partial \underline{\varphi}}\left(\underline{\varphi}; \theta\right) = \begin{bmatrix} \frac{\partial f_1}{\partial \varphi_2} & \frac{\partial f_1}{\partial \varphi_3} \\ \frac{\partial f_2}{\partial \varphi_2} & \frac{\partial f_2}{\partial \varphi_3} \end{bmatrix}$$

Where the partial derivatives:

$$\frac{\partial f_1}{\partial \varphi_2} = -l_2 \sin(\varphi_2)\ ,$$

$$\frac{\partial f_1}{\partial \varphi_3} = -l_3\, cos(\varphi_3)$$

$$\frac{\partial f_2}{\partial \varphi_3} = l_2 \cos(\varphi_2)\ ,$$

$$\frac{\partial f_2}{\partial \varphi_3} = l_3 \sin(\varphi_3)$$

$$\det J\left(\underline{\varphi};\theta\right) = l_2 l_3[\cos(\varphi_2)\cos(\varphi_3) - \sin(\varphi_2)\sin(\varphi_3)]$$

$$\Rightarrow \det J\left(\underline{\varphi};\theta\right) = l_2 l_3 \cos(\varphi_2 + \varphi_3)$$

From the last relation we see that for the relations of the angles, the determinant becomes zero so we are at a singular point. Therefore, the values of the above cosine will be checked using the graphs that will be presented next.$\begin{cases} \varphi_2 = -\varphi_3 \\ \varphi_2 + \varphi_3 = \pm 90° \end{cases}$

As mentioned at the beginning of the Sequential Continuation method, we need to know some initial solution of the system. For the selection of this point, we consider that the mechanism at time zero has the following set of values for the independent and dependent generalized coordinates:$\left(\underline{\varphi_0}, \theta_0\right)$

$\theta_o = 0°\ , \underline{\varphi} = \begin{bmatrix}\varphi_2 \\ \varphi_3\end{bmatrix} = \begin{bmatrix}33° \\ -2°\end{bmatrix}$

The above initial estimate was selected through trials to produce the angle graphs. In addition, we define as a discretization step for the interval of values of the angle .$\Delta\theta = 0.1°\theta \in (0°, 360°)$

Using these values as an initial prediction we apply the method described above after first defining the calculation error of the NR method, . In addition, due to computational errors and numerical approximations of Matlab that may occur, we also define a maximum number of iterations when applying the NR method for each value of , this number is set to the value$\varepsilon = 10^{-5}\theta_\kappa$

$maxIterations = 1000$. In general, the choice of the maximum number of iterations of the error and the discretization step of the parameter depend on the accuracy we want to achieve and on the characteristics of the system. A system say with very rapidly changing characteristics or complex responses requires more precise values. Another factor that affects these characteristics is the computing power available to the user. For our problem, with the means available, the above numbers give satisfactory accuracy and clear results.

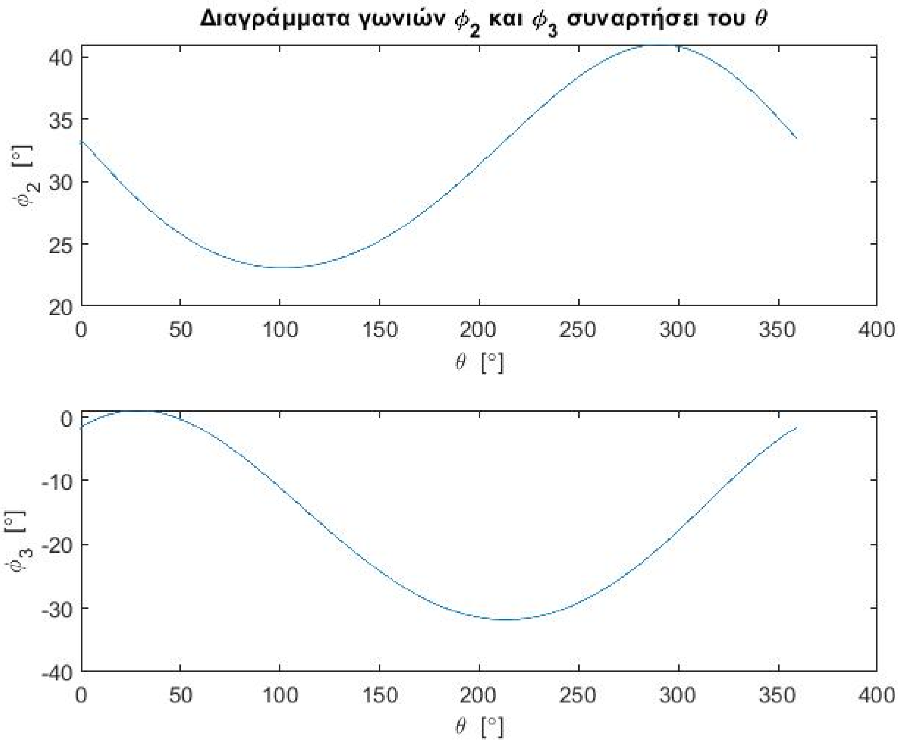
To apply the method we make use of the Jacobian we calculated above and also calculate the derivative with respect to θ of system (I):

$$\underline{f_\theta} = \frac{\partial \underline{f}}{\partial \theta}\left(\underline{\varphi};\theta\right) = \begin{bmatrix}-l_1 \sin(\theta) \\ l_1 \cos(\theta)\end{bmatrix}$$

Having described the course we will follow, the programming code written to extract the graphs is then analyzed. The main file medium in which the initial estimate, the error is defined, the calculations are made and the graphs are produced, for all questions of the work, is 'Solver'.

Through this file we call for each set of values of the three coordinates the files 'funA' , 'jac' , 'Ftheta' , which include the system of equations, the Jacobian and the derivative of the system with respect to θ respectively. For the last 3 files we have as input the values of the angles and as output the vector of its values$\underline{f}\left(\underline{\varphi}\,;\theta\right)$, the matrix and the vector . In the Solver file, after all the values of for each value of θ are approximated with the error we defined, the graphs of φ as a function of θ are produced for the interval of values that we define at the beginning.$J\left(\underline{\varphi};\theta\right)\underline{f_{\theta}\varphi}\theta \in (0°, 360°)$

By entering the numerical values of Table 1. in the process we described we get the graphs of Figure 2. Observing this figure we see that the positions of members AB and GD of the mechanism display a periodic and harmonic behavior as we expected due to the constant angular velocity. The negative values we see for angle , are due to the clockwise positive direction convention for angles throughout the mechanism, for negative the DG member is to the right of the vertical dotted line (Figure 1a). We also distinguish, as a verification of the results, for the angle , observing Figure 2. but also Figure 1b. that the ranges of values shown in Figure 2. on either side of the angle correspond approximately to the intervals defined for that angle in Figure 1b.. The point with the maximum absolute value for the angle can be seen to appear at an angle with value . Finally, we can easily see that the relations which would give us a singular point do not hold for any θ. The 4 files written to extract the diagrams are presented in Appendix A1 at the end of the paper.$\varphi_3\varphi_3\varphi_3\varphi_{3,c} = -10°\ \varphi_3\theta \approx 200°\varphi_{3,max} = -31.82°\begin{cases} \varphi_2 = -\varphi_3 \\ \varphi_2 + \varphi_3 = 90° \end{cases}$

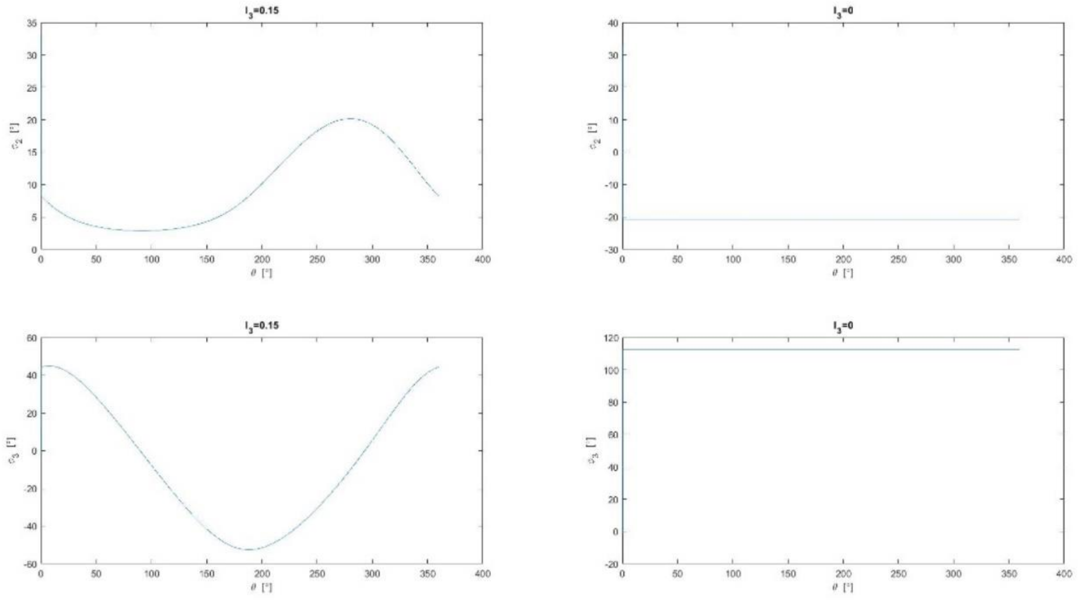


Shape2. Graphs of generalized coordinates as a function of angle $\theta\varphi_2$ και φ_3,

We are then asked for which values of the length , the examined mechanism displays abnormal positions. The anomalous positions of the mechanism appear at the points where the Jacobian matrix is peculiar. A matrix is singular when its determinant equals zero as we mentioned above. The zeroing of the determinant indicates the zeroing of some eigenvalue of it. Therefore as we calculated above:l_3

$$\det J\left(\underline{\varphi};\theta\right) = l_2 l_3 \cos(\varphi_2 + \varphi_3)$$

This determinant becomes zero only for after we have proved that . In essence for this value of the length the mechanism will work for a range of values of θ, until for some value of θ it will get stuck. This is also verified by Figure 3. in which we run 2 times the calculations. As for , we notice in this figure that for the value 0.15 of the length, the mechanism works, of course deleting different angles from the previous one. But on the contrary for length 0, the iterative algorithm points to the continuous finding at a particular point, and the values of the angles remain constant at the initial point we give.$l_3 = 0 \cos(\varphi_2 + \varphi_3) \neq 0l_3 l_3 = 0.15$ $\kappa\alpha\iota$ $l_3 = 0$



Shape3. Diagrams of mechanism angles for different lengthsl_3

2nd QUESTION

In the 2nd question, you request that the angular velocity and acceleration of members 2 and 3 be represented graphically as a function of the angle θ. Initially, with a derivative with respect to the time of the system (I)

$$\begin{cases} l_1\cos(\theta) + l_2\cos(\varphi_2) - l_3\,sin(\varphi_3) - l = 0 \\ l_1\sin(\theta) + l_2\sin(\varphi_2) - l_3\cos(\varphi_3) = 0 \end{cases} \quad \boldsymbol{(I)}$$

the equations for calculating the velocities are derived.

$$\begin{cases} -l_1\dot{\theta}\,sin(\theta) - l_2\dot{\varphi}_2\sin(\varphi_2) - l_3\dot{\varphi}_3\,cos(\varphi_3) = 0 \\ l_1\dot{\theta}\,cos(\theta) + l_2\dot{\varphi}_2\cos(\varphi_2) + \dot{\varphi}_3 l_3\,sin(\varphi_3) = 0 \end{cases} \quad \boldsymbol{(II)}$$

where in compact form it is written:

$$\begin{bmatrix} -l_2\sin(\varphi_2) & -l_3\,cos(\varphi_3) \\ l_2\cos(\varphi_2) & l_3\sin(\varphi_3) \end{bmatrix} \begin{bmatrix} \dot{\varphi}_2 \\ \dot{\varphi}_3 \end{bmatrix} = l_1\dot{\theta}\begin{bmatrix} \sin(\theta) \\ -\cos(\theta) \end{bmatrix} \Rightarrow J\left(\underline{\varphi};\theta\right)\underline{\dot{\varphi}} = -\dot{\theta}\underline{f_\theta}\left(\underline{\varphi};\theta\right) \quad (\mathbf{1})$$

all the quantities in equation (1), are known from the previous question. Furthermore, the angular velocity is given by saying that it is constant. By therefore inserting the values we calculated for the coordinates as a function of θ and inverting the Jacobian, we can calculate the generalized coordinates from the equation:$\dot{\theta} = \omega\varphi_1, \varphi_2$

$$\underline{\dot{\varphi}} = J\left(\underline{\varphi};\theta\right)^{-1}\left(-\dot{\theta}\underline{f_\theta}\left(\underline{\varphi};\theta\right)\right) \ (\mathbf{2})$$

To now express the accelerations, we once again derive the system (II):

$$\begin{cases} -l_1\left(\ddot{\theta}\,sin(\theta) + \dot{\theta}^2\,cos(\theta)\right) - l_2\left(\ddot{\varphi}_2\sin(\varphi_2) + \dot{\varphi}_2^2\cos(\varphi_2)\right) - l_3\left(\ddot{\varphi}_3\,cos(\varphi_3) - \dot{\varphi}_3^2\sin(\varphi_3)\right) = 0 \\ l_1\left(\ddot{\theta}\,cos(\theta) - \dot{\theta}^2\sin(\theta)\right) + l_2\left(\ddot{\varphi}\cos(\varphi_2) - \dot{\varphi}_2^2\sin(\varphi_2)\right) + l_3\left(\ddot{\varphi}_3\,sin(\varphi_3) + \dot{\varphi}_3^2\cos(\varphi_3)\right) = 0 \end{cases} \quad \boldsymbol{(III)}$$

Rearranging the terms, we can write the system in log form:

$$\begin{bmatrix} -l_2\sin(\varphi_2) & -l_3\,cos(\varphi_3) \\ l_2\cos(\varphi_2) & l_3\sin(\varphi_3) \end{bmatrix} \begin{bmatrix} \ddot{\varphi}_2 \\ \ddot{\varphi}_3 \end{bmatrix} = l_1\ddot{\theta}\begin{bmatrix} \sin(\theta) \\ -\cos(\theta) \end{bmatrix} + l_1\dot{\theta}^2\begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} + \begin{bmatrix} l_2\dot{\varphi}_2^2\cos(\varphi_2) - l_3\dot{\varphi}_3^2\sin(\varphi_3) \\ l_2\dot{\varphi}_2^2\sin(\varphi_2) - l_3\dot{\varphi}_3^2\cos(\varphi_3) \end{bmatrix}$$

where since thus:$\dot{\theta} = \omega = \sigma\tau\alpha\theta\varepsilon\rho ό \Rightarrow \ddot{\theta} = 0$

$$\begin{bmatrix} -l_2\sin(\varphi_2) & -l_3\,cos(\varphi_3) \\ l_2\cos(\varphi_2) & l_3\sin(\varphi_3) \end{bmatrix} \begin{bmatrix} \ddot{\varphi}_2 \\ \ddot{\varphi}_3 \end{bmatrix} = l_1\dot{\theta}^2\begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} + \begin{bmatrix} l_2\dot{\varphi}_2^2\cos(\varphi_2) - l_3\dot{\varphi}_3^2\sin(\varphi_3) \\ l_2\dot{\varphi}_2^2\sin(\varphi_2) - l_3\dot{\varphi}_3^2\cos(\varphi_3) \end{bmatrix}$$

and in compact form:

$$J\left(\underline{\varphi};\theta\right)\underline{\ddot{\varphi}} = l_1\dot{\theta}^2\,\psi_1(\theta) + l_2\dot{\varphi}_2^2\psi_1(\theta) + l_3\dot{\varphi}_3^2\psi_2(\theta) \ (\mathbf{3})$$

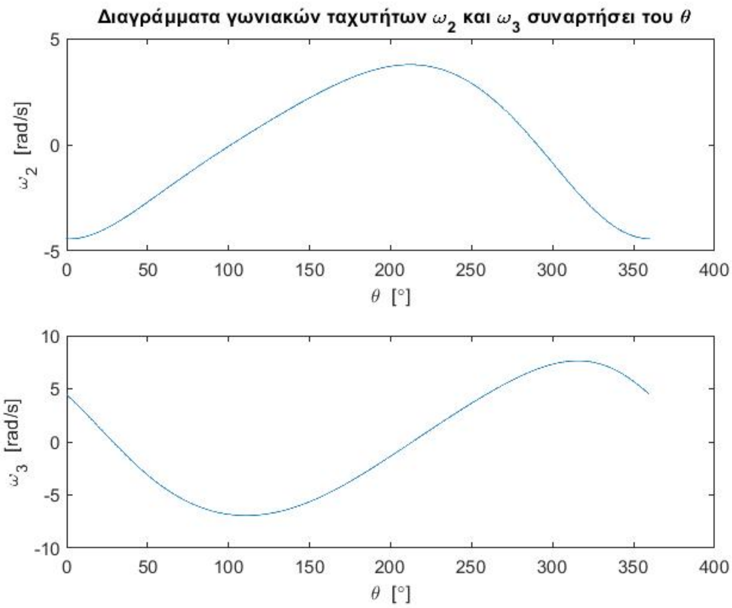
where the registers are:ψ_1, ψ_2 $\psi_1(\theta) = \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix}$, $\psi_2(\theta) = \begin{bmatrix} -\,sin(\theta) \\ -\cos(\theta) \end{bmatrix}$

the values of the positions for the angles in relation (3) have been calculated in the 1st question for the corresponding θ, in addition the values of the angular velocities are calculated from relation (2) as analyzed above. Thus, the values of the accelerations for the corresponding positions of the crank θ are calculated by inverting the Jacobian register from the relation:

$$\underline{\ddot{\varphi}} = J\left(\underline{\varphi};\theta\right)^{-1}\left(l_1\dot{\theta}^2\,\psi_1(\theta) + l_2\dot{\varphi}_2^2\psi_1(\theta) + l_3\dot{\varphi}_3^2\psi_2(\theta)\right) \quad \mathbf{(4)}$$

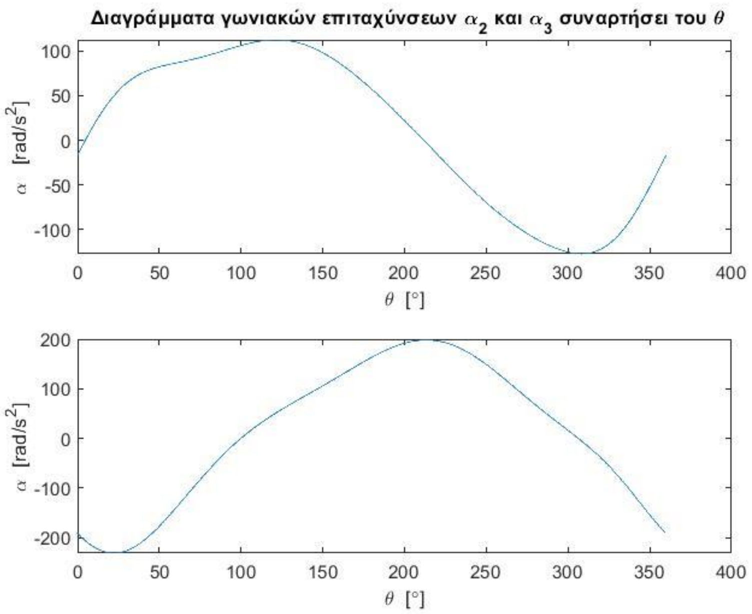
Therefore, through the system equations in compact form (2) and (4) the angular velocities and accelerations of the system coordinates are calculated. To produce these diagrams as a function of the crank angle, as before, through the 'Solver' file, we repeatedly call the functions 'funB1' and 'funB2', where they calculate for each value of the generalized coordinates, the values of the speeds and accelerations . The funB1 file has as input each time the set of values and as output the velocity vector While the funB2 file has as input the vector and as output the acceleration vector. Having now calculated the values of these 2 vectors for the entire range of the discretized θ, we construct the corresponding diagrams. The files of the 2 systems funB1 and funB2 are in Appendix A2. Before proceeding to the numerical implementation, we convert the angular velocity from the equation:$(\dot{\theta},\theta_\kappa,\varphi_{1,\kappa},\varphi_{2,\kappa})\underline{\dot{\varphi_\kappa}}(\dot{\theta},\theta_\kappa,\varphi_{1,\kappa},\varphi_{2,\kappa},\dot{\varphi_{1,\kappa}},\dot{\varphi_{2,\kappa}})\underline{\ddot{\varphi_\kappa}}rpm\ \sigma\varepsilon\frac{rad}{s}\omega = \frac{240\ rpm}{60\frac{s}{minute}}\ 2\pi\frac{rad}{rev} = \ 8\pi\frac{rad}{s}$

Figures 4. and 5. show the diagrams of the angular velocities and accelerations, respectively, as a function of the crank angle θ. First, for the velocities, we distinguish their periodic behavior, which we expected due to the linear relationship (2) and the periodic behavior of angles (Figure 2). We also see that the speeds show maxima in different positions of the crank. We have zeroing of the speeds at the extreme positions of the corners. Of course, at the point of contact, which again from Figure 2. we see is located at an angle, the speed there approximately becomes zero and changes direction.$\varphi_2,\varphi_3\theta\approx 200°$



Shape4.Graphs of generalized speeds and , as a function of the angle $\theta\omega_2\omega_3$

Correspondingly for the accelerations in Figure 5. we have a periodic variation and again, the strange curvatures of these diagrams are due to the now non-linear relationship (4) between the accelerations with the velocities and the positions. Comparing the diagrams of figures 4 and 5, we notice that the maxima of the accelerations are found at the zero points of the velocities, and at the extreme positions of the angles, a fact which, from a physical point of view, due to a change in the direction of movement, confirms our results. Having calculated the values of the dependent generalized coordinates, velocities and accelerations as a function of the angle θ, we have fully expressed the behavior of our mechanism. These results will be used in the next questions to calculate Kinetic quantities of the mechanism.



Shape5Graphs of generalized accelerations, as a function of the angle θα_2 και α_3

3rd QUESTION

In the 3rd question you ask for the graphical representation of the speed and acceleration of the concentrated mass, as a function of the angle θ of the crank.

The concentrated mass performs a rotation about the fixed axis passing through the point C and parallel to the z-axis of the inertial frame of reference. Therefore, using kinematic relations, the velocity at Δ is derived by deriving the equation:m_{Δ}

$$\underline{r_{\Delta}} = l\underline{e_x} + (l_3 + l_{\Delta})\big(\sin(\varphi_3)\,\underline{e_x} + \cos(\varphi_3)\,\underline{e_y}\big)$$

We produce in terms of time:

$$\underline{v_{\Delta}} = 0 + \dot{\varphi}_3(l_3 + l_{\Delta})\big(\cos(\varphi_3)\,\underline{e_x} - \sin(\varphi_3)\,\underline{e_y}\big) \implies$$

$$\implies \underline{v_{\Delta}} \;=\; \dot{\varphi}_3(l_3 + l_{\Delta})\big(\cos(\varphi_3)\,\underline{e_x} - \sin(\varphi_3)\,\underline{e_y}\big) \;\;(\mathbf{5})$$

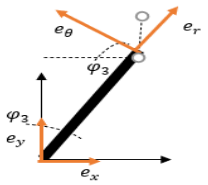
Correspondingly for the acceleration we again derive in terms of time:

$$\underline{a_{\Delta}} = \ddot{\varphi}_3(l_3 + l_{\Delta})\big(\cos(\varphi_3)\,\underline{e_x} - \sin(\varphi_3)\,\underline{e_y}\big) - \dot{\varphi}_3^2(l_3 + l_{\Delta})\big(sin(\varphi_3)\,\underline{e_x} + \cos(\varphi_3)\,\underline{e_y}\big)(\mathbf{6})$$

We express relations (5), (6) in cylindrical coordinates through the transformations,$\underline{e_r} = \sin(\varphi_3)\,\underline{e_x} + \cos(\varphi_3)\,\underline{e_y}$and with the unit vectors having the direction shown in Figure 5. and thus we get:$\underline{e_{\theta}} = \sin(\varphi_3)\,\underline{e_y} - \cos(\varphi_3)\,\underline{e_x}$

$$\underline{v_{\Delta}} \;=-\,\dot{\varphi}_3\,(l_3 + l_{\Delta})\underline{e_{\theta}} \;\;(\mathbf{7})$$

$$\underline{a_{\Delta}} = -(l_3 + l_{\Delta})\ddot{\varphi}_3\,\underline{e_{\theta}} - \dot{\varphi}^2{}_3\,(l_3 + l_{\Delta})\underline{e_r}\;(\mathbf{8})$$



Shape6Coordinate transformation

From relations (7) and (8) we see that the velocity of the mass at D is perpendicular to the straight line segment GBD and tangential to the curve crossing the end D. Correspondingly the acceleration has a tangential component and a radial component representing the centripetal acceleration , towards the center of rotation C. For the graphical illustration of the above we visualize the measures of relations (7), (8). Specifically for the acceleration, the diagram will be made for each component separately as well as for the overall measure of the acceleration. The measures are calculated:

$$\left|\underline{v_{\Delta}}\right| = \dot{\varphi}_3\,(l_3 + l_{\Delta})\;(\mathbf{9})$$

$$Εφαπτομενική\;\left|\underline{a_{\theta}}\right| = (l_3 + l_{\Delta})\ddot{\varphi}_3\;\;(\mathbf{10})$$

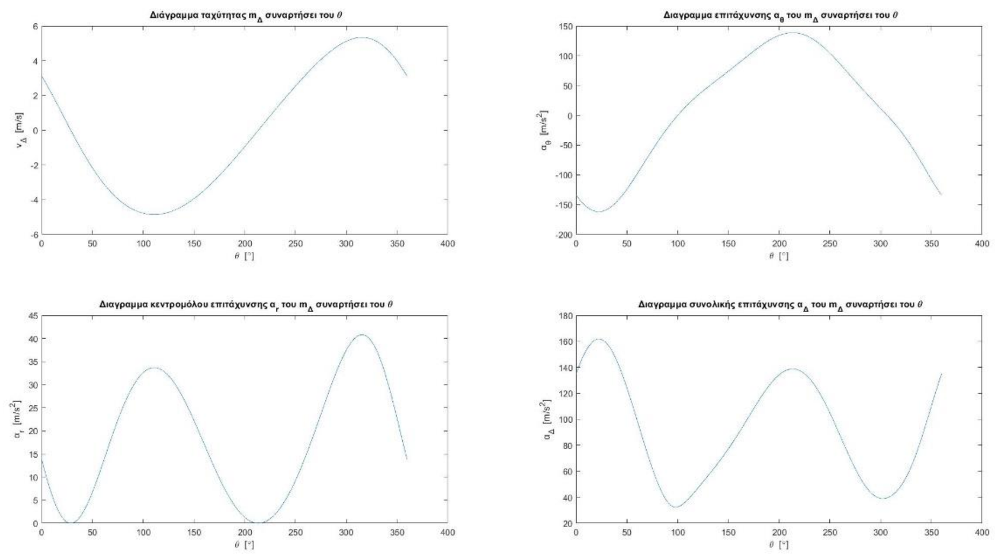
$$Κεντρομόλος\left|\underline{a_r}\right| = \dot{\varphi}^2{}_3\,(l_3 + l_{\Delta})\;\;(\mathbf{11})$$

$$\left|\underline{a_{\Delta}}\right| = \sqrt{\big((l_3 + l_{\Delta})\ddot{\varphi}_3\big)^2 + \big(\dot{\varphi}^2{}_3\,(l_3 + l_{\Delta})\big)^2}\;\;\;(\mathbf{12})$$

In the previous questions we calculated the angles and angular velocities as a function of θ, , therefore from relations (9),(10),(11),(12) we can calculate and generate the graphs for the corresponding functions$\varphi_2(\theta), \varphi_3(\theta), \dot{\varphi}_2(\theta), \dot{\varphi}_3(\theta)$in the period we defined previously.$\theta \in (0°, 360°)$

To produce the above graphs we create the functions in Matlab 'funG1', 'funG2', 'funG3' and 'funG4' in which we define the relations (9), (10), (11), (12) respectively. We call these functions through our main Solver file, with inputs the set of values respectively for the 4 functions. The output of these files is the velocity and acceleration for each position of θ. Then after the value vectors for all θ are produced through an iterative process, we return to the Solver file and construct the graphs. These 4 useless functions are presented in Appendix A3.$(\dot{\varphi}_3)$, $(\ddot{\varphi}_3)$, $(\dot{\varphi}_3)$, $(\dot{\varphi}_3, \ddot{\varphi}_3)$

The diagrams we get are shown in Figure 7.



Shape7. Velocity and acceleration of concentrated mass

All the above charts show periodic behavior. We observe that the velocity and the tangential acceleration follow the periodic behavior of the generalized velocity and acceleration respectively, which is also confirmed by equations (9) and (10) due to the linear relationship that governs them. On the other hand, the centripetal acceleration and, as a result, the total acceleration show the same periodic behavior, with the same period as the previous ones but passing through twice the number of local peaks than the previous ones. In an interval of 360 degrees we have 2 zeros of the centripetal acceleration due to the zeroing of the speed and a change of the direction of the movement (see Figure 2. of generalized coordinates). The tangential acceleration becomes zero in the positions where the centripetal force is maximum, in these positions we have a change in the direction of the tangential acceleration. In addition, the zeroing of the centripetal acceleration goes hand in hand with the change of the direction of the velocity vector for the mass D. Up to this point of the work, the sequence and the connection of the results, which from a physical point of view are well established, confirm the correct operation of the numerical methods that we apply.$\dot{\varphi}_3, \ddot{\varphi}_3\dot{\varphi}_3$

4th QUESTION

In the 4th question you ask for the position, velocity and acceleration of the center of mass of the mechanism during one complete rotation of the crank. In addition, you ask us to consider whether it is possible to immobilize the center of mass by adding masses to the extensions of members 1 and 3. First we calculate the position of the center of mass from the equation:

$$\underline{r}_G = \frac{1}{m}\sum_{i=1}^{4} m_i \underline{r}_{G,i} \;\; (\mathbf{13}) \;\; \mu\varepsilon \;\;\; i = 4 \; \gamma\iota\alpha \; \tau\eta\nu \; \sigma\upsilon\gamma\kappa\varepsilon\nu\tau\rho\omega\mu\acute{\varepsilon}\nu\eta \; \mu\acute{\alpha}\zeta\alpha \; m_\Delta$$

$$\kappa\alpha\iota \; m = \sum_{i=1}^{4} m_i = m_1 + m_2 + m_3 + m_\Delta$$

Then we calculate the position vectors for the centers of mass of the four members and the concentrated mass, assuming a uniform distribution of mass for each member.

$$\underline{r}_{G,1} = \frac{l_1}{2}\left(\cos(\theta)\;\underline{e}_x + \sin(\theta)\,\underline{e}_y\right)$$

$$\underline{r}_{G,2} = \underline{r}_A + \underline{r}_{AG_2} = l_1\left(\cos(\theta)\;\underline{e}_x + \sin(\theta)\,\underline{e}_y\right) + \frac{l_2}{2}\left(\cos(\varphi_2)\underline{e}_x + \sin(\varphi_2)\,\underline{e}_y\right)$$

$$\underline{r}_{G,3} = \underline{r}_{O\Gamma} + \underline{r}_{\Gamma G_3} = l\underline{e}_x + \frac{l_3}{2}\left(sin(\varphi_3)\;\underline{e}_x + \cos(\varphi_3)\,\underline{e}_y\right)$$

$$\underline{r}_{G,4} = \underline{r}_\Delta = l\underline{e}_x + (l_3 + l_\Delta)\left(sin(\varphi_3)\;\underline{e}_x + \cos(\varphi_3)\,\underline{e}_y\right)$$

Substituting the above expressions for the position vectors into equation 13 gives:

$$\underline{r}_G = \frac{1}{m}\Bigg(m_1\frac{l_1}{2}\left(\cos(\theta)\;\underline{e}_x + \sin(\theta)\,\underline{e}_y\right) + m_2\left(l_1\left(\cos(\theta)\;\underline{e}_x + \sin(\theta)\,\underline{e}_y\right) + \frac{l_2}{2}\left(\cos(\varphi_2)\underline{e}_x + \sin(\varphi_2)\,\underline{e}_y\right)\right) + m_3\left(l\underline{e}_x + \frac{l_3}{2}\left(sin(\varphi_3)\;\underline{e}_x + \cos(\varphi_3)\,\underline{e}_y\right)\right) + m_\Delta\left(l\underline{e}_x + (l_3 + l_\Delta)\left(sin(\varphi_3)\;\underline{e}_x + \cos(\varphi_3)\,\underline{e}_y\right)\right)\Bigg)$$

$$\Rightarrow \underline{r_G} = \frac{1}{m}\Bigg(\Bigg(\Bigg(\left(m_1\frac{l_1}{2} + m_2 l_1\right)\cos(\theta) + \frac{l_2}{2}m_2\cos(\varphi_2) + m_3 l + \left(m_3\frac{l_3}{2} + m_\Delta(l_3 + l_\Delta)\right)sin(\varphi_3) + m_\Delta l\Bigg)\underline{e_x} + \left(\left(m_1\frac{l_1}{2} + m_2 l_1\right)\sin(\theta) + \frac{l_2}{2}m_2\sin(\varphi_2) + \left(m_3\frac{l_3}{2} + m_\Delta(l_3 + l_\Delta)\right)cos(\varphi_3)\right)\underline{e_y}\Bigg)$$

Now defining the constant coefficients of the trigonometrics as:

$$A_1 = m_1\frac{l_1}{2} + m_2 l_1 \quad, A_2 = \frac{l_2}{2}m_2 \quad, A_3 = m_3\frac{l_3}{2} + m_\Delta(l_3 + l_\Delta)\ ,\ A_4 = m_1\frac{l_1}{2} + m_2 l_1$$

$$A_5 = \frac{l_2}{2}m_2\ ,\ A_6 = m_3\frac{l_3}{2} + m_\Delta(l_3 + l_\Delta)$$

the equation of the center of mass comes in the form:

$$\underline{r_G} = \frac{1}{m}\Big(\left(A_1\cos(\theta) + A_2\cos(\varphi_2) + m_3 l + A_3\, sin(\varphi_3) + m_\Delta l\right)\underline{e_x} + \left(A_4\sin(\theta) + A_5\sin(\varphi_2) + A_6\, cos(\varphi_3)\right)\underline{e_y}\Big) \quad (\mathbf{14})$$

The velocity of the center of mass is obtained by deriving relation (14) as:

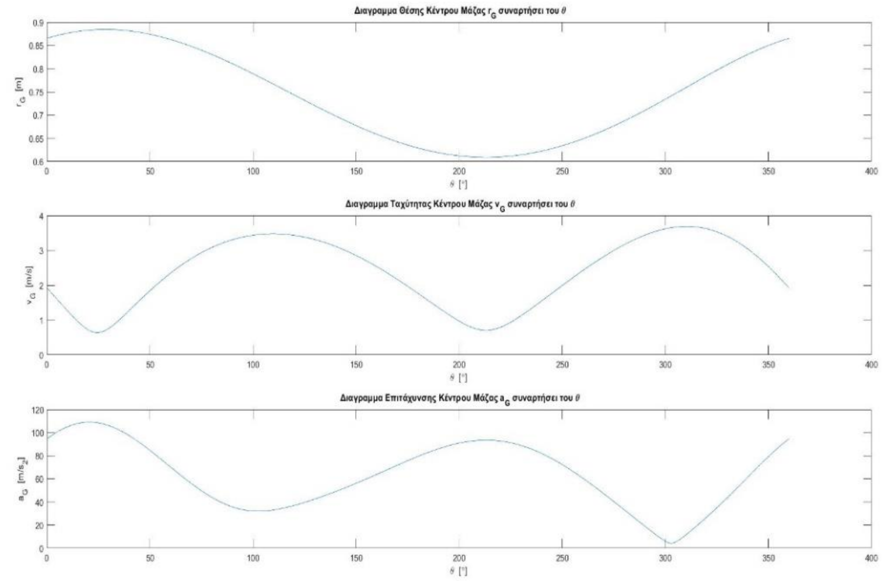
$$\underline{\dot{r_G}} = \frac{1}{m}\Big(\left(-A_1\dot{\theta}\, sin(\theta) - \dot{\varphi_2}A_2\, sin(\varphi_2) + \dot{\varphi_3}A_3\, cos(\varphi_3)\right)\underline{e_x} + \left(A_4\dot{\theta}\, cos(\theta) + \dot{\varphi_2}A_5\, cos(\varphi_2) - \dot{\varphi_3}A_6\, sin(\varphi_3)\right)\underline{e_y}\Big) \quad (\mathbf{15})$$

Accordingly, the acceleration:

$$\underline{\ddot{r_G}} = \frac{1}{m}\Big(\Big(-A_1\dot{\theta}^2\cos(\theta) - \ddot{\varphi_2}A_2\sin(\varphi_2) - \dot{\varphi}_2^2 A_2\, cos(\varphi_2) + \ddot{\varphi_3}A_3\cos(\varphi_3) - \dot{\varphi}_3^2 A_3\, sin(\varphi_3)\Big)\underline{e_x} + \Big(-A_4\dot{\theta}^2\, sin(\theta) + \ddot{\varphi_2}A_5\cos(\varphi_2) - \dot{\varphi}_2^2 A_5\sin(\varphi_2) - \ddot{\varphi_3}A_6\sin(\varphi_3) - \dot{\varphi}_3^2 A_6\, cos(\varphi_3)\Big)\underline{e_y}\Big) \quad (\mathbf{16})$$

From relations (14), (15) and (16) for the various sets of values of the generalized coordinates as a function of the angle θ, we can graphically represent the position, velocity and acceleration of the center of mass of the mechanism. Of course we represent the measures of the vectors of these relations, the measures are obtained as the square root of the sum of the 2 components of each vector.

To produce these graphs we write the corresponding functions in the files 'funD1', 'funD2', 'funD3'. These files have as input the generalized coordinates, velocities and accelerations respectively and as output the position, velocity and acceleration of the center of mass respectively. Through the Solver file we call these functions and after constructing the vectors for the three graphs we visualize the results. In Figure 8. the diagrams are illustrated. Observing this figure, the periodic behavior of all 3 dimensions is also evident here. In addition the center of mass velocity and acceleration compared to the generalized velocities and accelerations, show faster characteristics, they may all have the same period, but in the center of mass diagrams we distinguish more local extrema. This behavior is due to the contribution of all generalized values in equations (14), (15) and (16). From the figure above we can see that the center of mass does not have zero velocity for any position of the crank θ.$\underline{r_G}(\theta)$,$\underline{\dot{r_G}}(\theta)$, $\underline{\ddot{r_G}}(\theta)$



Shape8. Diagrams of Position, Velocity and Acceleration of Center of Mass of Mechanism as a Function of Angle θ

In this question you also ask, as we mentioned above, if it is possible to immobilize the center of mass by adding masses to the extension of members 1 and 3. Suppose we add masses to members 1 and 3 at a distance from points O and C respectively. The terms to be added to the sum of the center of mass will have the form:$m_\kappa\ \kappa\alpha\iota\ m_\lambda\ l_\kappa\ \kappa\alpha\iota\ l_\lambda$

$$\underline{r_\kappa} = l_\kappa\big(\cos(\theta)\ \underline{e_x} + \sin(\theta)\,\underline{e_y}\big)$$

$$\underline{r_\lambda} = l\underline{e_x} + \ l_\lambda\big(sin(\varphi_3)\ \underline{e_x} + \cos(\varphi_3)\,\underline{e_y}\big)$$

Therefore in relation (14) we will have the addition of the above terms multiplied by the corresponding masses:

$$\begin{aligned}\underline{r_G} = \frac{1}{m}\Big(\ &(A_1\cos(\theta)\ + A_2\cos(\varphi_2) + m_3 l + A_3\,sin(\varphi_3) + m_\Delta l + \ m_\lambda l + \ m_\kappa l_\kappa\cos(\theta) \\ &+ m_\lambda l_\lambda\,sin(\varphi_3)\ )\underline{e_x} \\ &+ (\ A_4\sin(\theta)\ + A_5\sin(\varphi_2) \\ &+ A_6\,cos(\varphi_3) + l_\lambda m_\lambda\cos(\varphi_3) + m_\kappa l_\kappa\sin(\theta))\underline{e_y}\Big)\ (\mathbf{17})\end{aligned}$$

From the system (I) we developed in the 1st question we had:

$$\begin{cases} l_1\cos(\theta) + l_2\cos(\varphi_2) - l_3\,sin(\varphi_3) - l = 0 \\ \ l_1\sin(\theta) + l_2\sin(\varphi_2) - l_3\cos(\varphi_3) = 0 \end{cases}\ (\boldsymbol{I})$$

therefore we can write the trigonometric terms of the angle as a function of the other angles, so we have:φ_2

$$\begin{cases} -l_1\cos(\theta) + l_3\,sin(\varphi_3) + l = l_2\cos(\varphi_2) \\ \ -l_1\sin(\theta) + l_3\cos(\varphi_3) = l_2\sin(\varphi_2) \end{cases}\ (\mathbf{18})$$

Expanding the terms in relation (17) and substituting in the same relation the system equations (18) gives:$A_2 \kappa\alpha\iota\, A_5 A_2 = \frac{l_2}{2} m_2 = A_5$

$$\underline{r_G} = \frac{1}{m}\Big(\Big(A_1 \cos(\theta) + \frac{m_2}{2}(-l_1 \cos(\theta) + l_3\, sin(\varphi_3) + l) + m_3 l + A_3\, sin(\varphi_3) + m_\Delta l + m_\lambda l + m_\kappa l_\kappa \cos(\theta) + m_\lambda l_\lambda\, sin(\varphi_3)\Big)\underline{e_x} + \Big(A_4 \sin(\theta) + \frac{m_2}{2}(-l_1 \sin(\theta) + l_3 \cos(\varphi_3)) + A_6\, cos(\varphi_3) + l_\lambda m_\lambda \cos(\varphi_3) + m_\kappa l_\kappa \sin(\theta)\Big)\underline{e_y}\Big)$$

In order for the center of mass of the mechanism to remain constant, the components of the above vector should not depend on changing quantities, so the coefficients of the trigonometric terms should be set to zero. We write the above vector in a system of equations by separating its components and rearranging the terms:

$$\underline{r_G} = \begin{cases} \frac{1}{m}\left(\left(A_1 - \frac{l_1 m_2}{2} + m_\kappa l_\kappa\right)\cos(\theta) + \left(\frac{m_2 l_3}{2} + A_3 + m_\lambda l_\lambda\right)\sin(\varphi_3) + l\left(\frac{m_2}{2} + m_3 + m_\Delta + m_\lambda\right)\right) \\ \frac{1}{m}\left(\left(A_4 - \frac{m_2 l_1}{2} + m_\kappa l_\kappa\right)\sin(\theta) + \left(A_6 + \frac{m_2}{2} l_3 + l_\lambda m_\lambda\right)\cos(\varphi_3)\right) \end{cases}$$

Therefore choosing the values of so that:$l_\lambda, m_\lambda, l_\kappa, m_\kappa$

$$\begin{cases} A_4 - \frac{m_2 l_1}{2} + m_\kappa l_\kappa = 0 \\ A_6 + \frac{m_2}{2} l_3 + l_\lambda m_\lambda = 0 \\ A_1 - \frac{l_1 m_2}{2} + m_\kappa l_\kappa = 0 \\ \frac{m_2 l_3}{2} + A_3 + m_\lambda l_\lambda = 0 \end{cases}$$

We notice that the 1st and 3rd and the 2nd and 4th equations are the same, so by arbitrarily choosing the values for the masses (or the corresponding lengths), we calculate the lengths (or the masses) so that the position of center of mass for all positions θ of the crank. Preferably, we choose the values of the masses, so that the results are physically feasible, because one of the quantities calculated afterwards will come out negative.$m_\kappa, m_\lambda l_\kappa, l_\lambda m_\kappa, m_\lambda$

By adding masses, we calculate$m_\kappa = m_\lambda = 100\ [kg]$

$$l_\kappa = \frac{1}{100}\left(\frac{m_2 l_1}{2} - A_4\right),\ l_\lambda = \frac{1}{100}\left(-\frac{m_2}{2} l_3 - A_6\right)$$

and so finally the position vector of the center of mass comes in the form:

$$\underline{r_G} = l\left(\frac{m_2}{2} + m_3 + m_\Delta + m_\lambda\right)\underline{e_x} \Rightarrow \underline{\dot{r_G}} = \underline{0}$$

So the center remains fixed on the x-axis, at a distance$l\left(\frac{m_2}{2} + m_3 + m_\Delta + m_\lambda\right)$.

5th QUESTION

In the 5th question you ask us to graphically represent the equivalent mass moment of inertia of the mechanism as a function of θ, which is given by the relation$T = \frac{1}{2} I_{eq}(\theta)\omega^2$with T the total kinetic energy. So we need to find an expression for the total mass moment of inertia of the system. We know that the total kinetic energy of the system results from the sum of the kinetic energies of the members of which you therefore make up:

$$T = \sum_{i=1}^{4} T_i \ \mu\varepsilon\ i = 4\ \gamma\iota\alpha\ \tau\eta\nu\ \sigma\upsilon\gamma\kappa\varepsilon\nu\tau\rho\omega\mu\acute{\varepsilon}\nu\eta\ \mu\acute{\alpha}\zeta\alpha$$

Therefore it is sufficient to calculate the individual kinetic energies. The members of the mechanism perform planar motion, therefore their kinetic energy is the sum of the kinetic energy due to the transfer of their center of mass and the kinetic energy due to rotation relative to their center of mass and is given by the equation:

$$T_i = T_G + T_R = \frac{1}{2} m_i \underline{\dot{r}}^2{}_{G,i} + \frac{1}{2} I_{zz}^{G_i} \omega_i^2 \quad (\mathbf{19}) \quad \gamma\iota\alpha\ \tau\alpha\ 3\ \mu\acute{\varepsilon}\lambda\eta\ i = 1,2,3$$

where the velocity of the center of mass of each member and the rotational velocity of each member relative to its center of mass . The mass moment of inertia of each member is obtained for bars from the equation:$\underline{\dot{r}}_{G,i}\omega_i(\dot{\theta}, \dot{\varphi}_2, \dot{\varphi}_3)$

$$I_{zz}^{G_i} = \frac{1}{12} m_i l_i^2$$

For the mass D, the kinetic energy is calculated from the relation:

$$T_4 = T_\Delta = \frac{1}{2} m_\Delta v_\Delta^2$$

where the speed has been calculated in the 3rd question:v_Δ

$$\underline{v_\Delta} = \dot{\varphi}_3\ (l_3 + l_\Delta)\big(\sin(\varphi_3)\,\underline{e_y} - \cos(\varphi_3)\,\underline{e_x}\big)$$

$$\Longrightarrow \underline{v_\Delta^2} = (\dot{\varphi}_3\ (l_3 + l_\Delta)\ )^2$$

we calculate respectively the velocities of the members. In the previous question we defined the position vectors for the centers of mass:

$$\underline{r_{G,1}} = \frac{l_1}{2}\big(\cos(\theta)\ \underline{e_x} + \sin(\theta)\,\underline{e_y}\big)$$

$$\underline{r_{G,2}} = l_1\big(\cos(\theta)\ \underline{e_x} + \sin(\theta)\,\underline{e_y}\big) + \frac{l_2}{2}\big(\cos(\varphi_2)\underline{e_x} + \sin(\varphi_2)\,\underline{e_y}\big)$$

$$\underline{r_{G,3}} = l\underline{e_x} + \frac{l_3}{2}\big(sin(\varphi_3)\ \underline{e_x} + \cos(\varphi_3)\,\underline{e_y}\big)$$

Deriving these relations we get:

$$\underline{\dot{r}_{G,1}} = \frac{l_1}{2}\dot{\theta}\big(-sin(\theta)\ \underline{e_x} + \cos(\theta)\,\underline{e_y}\big)$$

$$\underline{\dot{r}}_{G,1}^2 = \left(\frac{l_1}{2}\dot{\theta}\right)^2$$

$$\underline{\dot{r}}_{G,2} = l_1\dot{\theta}\big(-sin(\theta)\ \underline{e}_x + \cos(\theta)\,\underline{e}_y\big) + \frac{l_2}{2}\dot{\varphi}_2\big(-sin(\varphi_2)\underline{e}_x + \cos(\varphi_2)\,\underline{e}_y\big)$$

$$\Rightarrow \underline{\dot{r}}_{G,2} = -\left(\ l_1\dot{\theta}\,sin(\theta) + \frac{l_2}{2}\dot{\varphi}_2\,sin(\varphi_2)\right)\ \underline{e}_x + \left(l_1\dot{\theta}\cos(\theta) + \frac{l_2}{2}\dot{\varphi}_2\cos(\varphi_2)\right)\underline{e}_y$$

$$\underline{\dot{r}}_{G,2}^2 = \left(\ l_1\dot{\theta}\,sin(\theta) + \frac{l_2}{2}\dot{\varphi}_2\,sin(\varphi_2)\right)^2 + \left(l_1\dot{\theta}\cos(\theta) + \frac{l_2}{2}\dot{\varphi}_2\cos(\varphi_2)\right)^2$$

$$\underline{\dot{r}}_{G,3} = \dot{\varphi}_3\ \frac{l_3}{2}\big(\cos(\varphi_3)\ \underline{e}_x - \sin(\varphi_3)\,\underline{e}_y\big)$$

$$\underline{\dot{r}}_{G,3}^2 = \left(\dot{\varphi}_3\ \frac{l_3}{2}\right)^2$$

Using the measures we calculated above, the kinetic energy for each member results from equation (19):

$$T_1 = \frac{1}{2}m_1\left(\frac{l_1}{2}\dot{\theta}\right)^2 + \frac{1}{2}\left(\frac{1}{12}m_1 l_1^2\right)\dot{\theta}^2$$

$$T_2 = \frac{1}{2}m_2\left(\left(\ l_1\dot{\theta}\,sin(\theta) + \frac{l_2}{2}\dot{\varphi}_2\,sin(\varphi_2)\right)^2 + \left(l_1\dot{\theta}\cos(\theta) + \frac{l_2}{2}\dot{\varphi}_2\cos(\varphi_2)\right)^2\right) + \frac{1}{2}\left(\frac{1}{12}m_2 l_2^2\right)\dot{\varphi}_2{}^2$$

$$T_3 = \frac{1}{2}m_3\left(\dot{\varphi}_3\ \frac{l_3}{2}\right)^2 + \frac{1}{2}\left(\frac{1}{12}m_3 l_3^2\right)\dot{\varphi}_3{}^2$$

In the expressions of the rotational kinetic energies for members 2 and 3 we used the derivative of the angle by which the points of each member rotate relative to its center of mass. Now summing the kinetic energies of the 3rd members and the mass D,$T_\Delta = \frac{1}{2}m_\Delta(\dot{\varphi}_3\ (l_3 + l_\Delta)\ )^2$, we get the total kinetic energy:

$$T_{O\Lambda} = \frac{1}{2}m_1\left(\frac{l_1}{2}\dot{\theta}\right)^2 + \frac{1}{2}\left(\frac{1}{12}m_1 l_1^2\right)\dot{\theta}^2 + \frac{1}{2}m_2\left(\left(\ l_1\dot{\theta}\,sin(\theta) + \frac{l_2}{2}\dot{\varphi}_2\,sin(\varphi_2)\right)^2 + \left(l_1\dot{\theta}\cos(\theta) + \frac{l_2}{2}\dot{\varphi}_2\cos(\varphi_2)\right)^2\right) + \frac{1}{2}\left(\frac{1}{12}m_2 l_2^2\right)\dot{\varphi}_2{}^2 + \frac{1}{2}m_3\left(\dot{\varphi}_3\ \frac{l_3}{2}\right)^2 + \frac{1}{2}\left(\frac{1}{12}m_3 l_3^2\right)\dot{\varphi}_3{}^2 + \frac{1}{2}m_\Delta(\dot{\varphi}_3\ (l_3 + l_\Delta)\ )^2 \quad \mathbf{(20)}$$

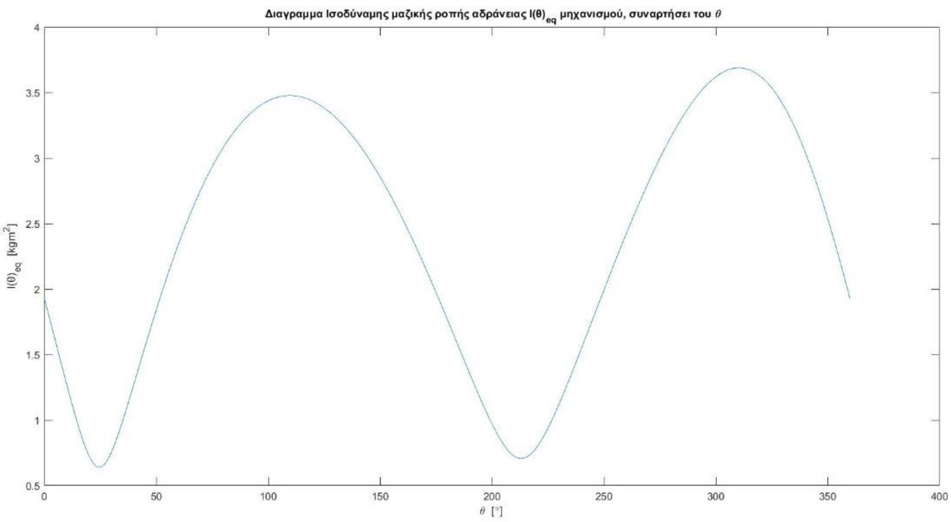
all the quantities in the last relation, are functions of θ, therefore and . Now equating the total kinetic energy with the equivalent gives us:$\varphi_2, \varphi_3, \dot{\varphi}_2, \dot{\varphi}_3 T_{O\Lambda} = T_{O\Lambda}(\theta)$

$$T_{O\Lambda}(\theta) = T_{eq} = \frac{1}{2}I_{eq}(\theta)\omega^2$$

$$\Rightarrow I_{eq}(\theta) = \frac{2T_{O\Lambda}(\theta)}{\omega^2} \quad [kg\ m^2], \acute{o}\pi o\upsilon\ \omega\ \sigma\tau\alpha\theta\varepsilon\rho\acute{o}$$

From this relationship and equation (20) we can graphically represent the equivalent mass moment of inertia of the mechanism as a function of the angle θ of the crank. To do this, the function 'funE1' is constructed with inputs the generalized coordinates and velocities calculated in the previous queries for each position of the crank θ and output the value of the mass moment of inertia for the specific position θ. The diagram is generated in the file SolverA , where after calculating all the values of the mass moment of inertia of the mechanism for all positions of the crank, these values are visualized. The file funE1 is in Appendix A5.

In Figure 9. the function is illustrated$I_{eq}(\theta)$.



Shape9. Diagram of Equivalent mass moment of inertia of a mechanism, as a function of i$I_{eq}(\theta)$

Observing the figure above and comparing it with Figure 2. in which the functions are plotted, we see that the equivalent moment of inertia is maximized at the positions where the angle shows its maximum and minimum value and is minimized at the positions where the angle shows the 2 extreme values . The distribution of the moment of inertia can also be related to the change in kinetic energy as a function of the angle θ, due to the linear relationship that governs these 2 quantities. The kinetic energy is logically maximized and minimized at the points where the angular velocities show their extreme values, which can be easily seen from the graphs of the 2nd question.$\varphi_2(\theta), \varphi_3(\theta)\varphi_2\varphi_3$

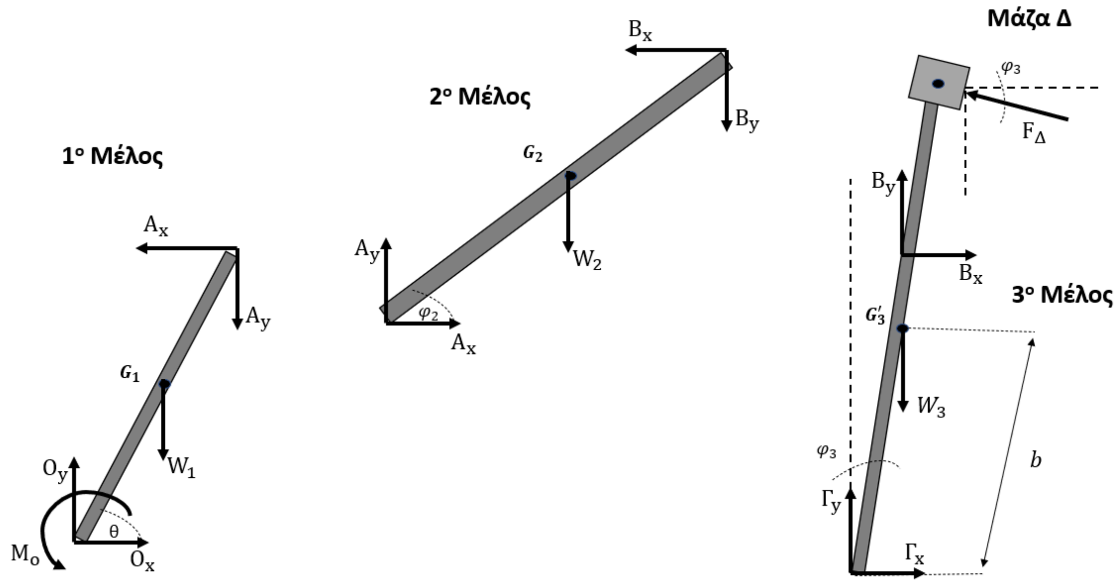
6th QUESTION

In the 6th question, the production of the graphs of the forces in the joints O, A, B, C and of the required driving torque as a function of the angle θ of the crank is requested. In addition, it is requested to calculate the required average power of the engine. To calculate these forces, we apply, for each member separately, Newton's and Euler's laws:M_o

$$\underline{\Sigma F_{G,i}} = m_i \underline{\ddot{r}_{G,i}} \quad \gamma\iota\alpha\ i = 1,2,3 \quad (\mathbf{21})$$

$$\underline{\Sigma M_{G,i}} = I_{zz}^{G,i}\, \underline{\alpha_i} \quad \gamma\iota\alpha\ i = 1,2,3 \quad (\mathbf{22})$$

where the angular velocity of each member with direction on the Oz axis. Member 3 and the concentrated mass Δ are studied as one body (i=3). To express the above equations, the Free Body Diagram (FED) is required for each body separately, these diagrams are presented in Figure 10. In this figure we can see that all the external forces exerted on the mechanism and all the reactions have been entered in vector form in the joints. Before we proceed to construct equations (21) and (22), we calculate the moments of inertia for each member with respect to its center of mass and the acceleration of the center of mass of each body.$\underline{\alpha_i}$



Shape10. Free Body Diagrams for all members of the mechanism

- 1st Member

  Moment of inertia with respect to O :$I_{zz}^{O} = I_{zz}^{G_1} + \frac{m_1 l_1^2}{4} = \frac{4}{12} m_1 l_1^2$

  The center of mass is in the middle of the rod, so as we calculated the speed of the center of mass in the 5th question we have:

  $$\underline{\dot{r}_{G,1}} = \frac{l_1}{2}\dot{\theta}\big(-sin(\theta)\ \underline{e_x} + \cos(\theta)\,\underline{e_y}\big)$$

  By derivation:

  $$\overset{\ddot{\theta}=0}{\Longrightarrow} \underline{\ddot{r}_{G_1}} = -\frac{l_1}{2}\dot{\theta}^2\big(cos(\theta)\ \underline{e_x} + \sin(\theta)\,\underline{e_y}\big)$$

- 2nd Member

  Moment of inertia with respect to A :$I_{zz}^{A} = \frac{1}{12} m_2 l_2^2 + \frac{m_2 l_2^2}{4} = \frac{4}{12} m_2 l_2^2$

  Center of mass:

$$\underline{\dot{r}}_{G,2} = -\left(l_1 \dot{\theta}\, sin(\theta) + \frac{l_2}{2} \dot{\varphi}_2\, sin(\varphi_2)\right) \underline{e}_x + \left(l_1 \dot{\theta} \cos(\theta) + \frac{l_2}{2} \dot{\varphi}_2 \cos(\varphi_2)\right) \underline{e}_y$$

$$\xRightarrow{\ddot{\theta}=0} \underline{\ddot{r}}_{G_2} = -\left(l_1 \dot{\theta}^2 \cos(\theta) + \frac{l_2}{2} \ddot{\varphi}_2\, sin(\varphi_2) + \frac{l_2}{2} \dot{\varphi}_2{}^2 \cos(\varphi_2)\right) \underline{e}_x + \left(-l_1 \dot{\theta}^2 \sin(\theta) + \frac{l_2}{2} \ddot{\varphi}_2 \cos(\varphi_2) - \frac{l_2}{2} \dot{\varphi}_2{}^2 \sin(\varphi_2)\right) \underline{e}_y$$

- 3rd House

  Moment of inertia about C:$I_{zz}^{\Gamma_3} = I_{zz}^{G_3} + \frac{m_3 (l_3 + l_\Delta)^2}{4} + m_\Delta (l_3 + l_\Delta)^2\ \alpha\pi ó\ \Theta.Steiner$

  with$I_{zz}^{G_3} = \frac{1}{12} m_3 (l_3 + l_\Delta)^2$

$$\Rightarrow I_{zz}^{\Gamma_3} = \frac{1}{12} m_3 (l_3 + l_\Delta)^2 + \frac{m_3 (l_3 + l_\Delta)^2}{4} + m_\Delta (l_3 + l_\Delta)^2 = \frac{4}{12} m_3 (l_3 + l_\Delta)^2 + m_\Delta (l_3 + l_\Delta)^2 = (l_3 + l_\Delta)^2 \left(\frac{4}{12} m_3 + m_\Delta \right)$$

Center of mass: $\underline{r}_{G_3'} = \frac{1}{(m_3 + m_\Delta)} \sum_{i=1}^{2} m_i \underline{r}_{G,i} = \frac{m_3 \underline{r}_{G_3} + m_\Delta \underline{r}_\Delta}{(m_3 + m_\Delta)}$

with$\underline{r}_{G_3} = l \underline{e}_x + \frac{l_3 + l_\Delta}{2} \left(\sin(\varphi_3)\, \underline{e}_x + \cos(\varphi_3)\, \underline{e}_y \right)$

$\underline{r}_\Delta = l \underline{e}_x + (l_3 + l_\Delta) \left(\sin(\varphi_3)\, \underline{e}_x + \cos(\varphi_3)\, \underline{e}_y \right)$

Therefore:

$$\underline{r}_{G_3'} = \frac{m_3 \left(l \underline{e}_x + \frac{l_3 + l_\Delta}{2} \left(\sin(\varphi_3)\, \underline{e}_x + \cos(\varphi_3)\, \underline{e}_y \right) \right) + m_\Delta \left(l \underline{e}_x + (l_3 + l_\Delta) \left(\sin(\varphi_3)\, \underline{e}_x + \cos(\varphi_3)\, \underline{e}_y \right) \right)}{m}$$

$$\underline{r}_{G_3'} = \left(\frac{(m_3 + m_\Delta) l + \left(m_3 \frac{l_3 + l_\Delta}{2} + m_\Delta (l_3 + l_\Delta) \right) \sin(\varphi_3)}{m} \right) \underline{e}_x + \left(\frac{\left(m_3 \frac{l_3 + l_\Delta}{2} + m_\Delta (l_3 + l_\Delta) \right) \cos(\varphi_3)}{m} \right) \underline{e}_y$$

$$\Rightarrow \underline{r}_{G_3'} = \left(l + \frac{\left(m_3 \frac{l_3 + l_\Delta}{2} + m_\Delta (l_3 + l_\Delta) \right) \sin(\varphi_3)}{m} \right) \underline{e}_x + \left(\frac{\left(m_3 \frac{l_3 + l_\Delta}{2} + m_\Delta (l_3 + l_\Delta) \right) \cos(\varphi_3)}{m} \right) \underline{e}_y$$

In addition, along the way we will also need the distance of the center of mass for body 3, to find this we subtract from the vector we calculated above the vector so we have:$\underline{d} = l_3 \underline{e_x}$

$$\underline{b} = \underline{r_{G_3'}} - \underline{d} = \left(\frac{\left(m_3 \frac{l_3 + l_\Delta}{2} + m_\Delta (l_3 + l_\Delta) \right) \sin(\varphi_3)}{m} \right) \underline{e_x} + \left(\frac{\left(m_3 \frac{l_3 + l_\Delta}{2} + m_\Delta (l_3 + l_\Delta) \right) \cos(\varphi_3)}{m} \right) \underline{e_y}$$

$$= \frac{\left(m_3 \frac{l_3 + l_\Delta}{2} + m_\Delta (l_3 + l_\Delta) \right)}{m} \left(\sin(\varphi_3)\, \underline{e_x} + \cos(\varphi_3) \underline{e_y} \right)$$

As demonstrated in a previous question$\underline{e_r} = \sin(\varphi_3)\, \underline{e_x} + \cos(\varphi_3) \underline{e_y}$, so the vector b has the same direction as the longitudinal axis of member 3 and measure:

$$b = \frac{\left(m_3 \frac{l_3 + l_\Delta}{2} + m_\Delta (l_3 + l_\Delta) \right)}{m}$$

where is the distance of the center of mass of body 3 (member 3 and mass Δ) from joint C. By deriving the expression of$\underline{r_{G_3'}}$2 times we calculate the acceleration of the center of mass of the body 3 :

$$\underline{\dot{r}_{G_3'}} = \dot{\varphi}_3 \left(\frac{\left(m_3 \frac{l_3 + l_\Delta}{2} + m_\Delta (l_3 + l_\Delta) \right)}{m} \right) \left(\cos(\varphi_3)\, \underline{e_x} - \sin(\varphi_3)\, \underline{e_y} \right)$$

$$\Rightarrow \underline{\ddot{r}_{G_3'}} = \ddot{\varphi}_3 \left(\frac{\left(m_3 \frac{l_3 + l_\Delta}{2} + m_\Delta (l_3 + l_\Delta) \right)}{m} \right) \left(\cos(\varphi_3)\, \underline{e_x} - \sin(\varphi_3)\, \underline{e_y} \right) - \dot{\varphi}_3^2 \left(\frac{\left(m_3 \frac{l_3 + l_\Delta}{2} + m_\Delta (l_3 + l_\Delta) \right)}{m} \right) \left(\sin(\varphi_3)\, \underline{e_x} + \cos(\varphi_3)\, \underline{e_y} \right)$$

$$\Rightarrow \underline{\ddot{r}_{G_3'}} = b \left(\left(\ddot{\varphi}_3 \cos(\varphi_3) - \dot{\varphi}_3^2 \sin(\varphi_3) \right) \underline{e_x} - \left(\ddot{\varphi}_3 \sin(\varphi_3) + \dot{\varphi}_3^2 \cos(\varphi_3) \right) \right) \underline{e_y}$$

Then we construct the equations (21), (22) for each member with the help of figure 10, starting from the 2nd Body:

$$\underline{\Sigma F_{G_2}} = m_2 \underline{\ddot{r}_{G_2}}$$

$$\Longrightarrow (A_x - B_x) \underline{e_x} + \left(A_y - W_2 - B_y \right) \underline{e_y} = m_2 \underline{\ddot{r}_{G_2}}$$

$$\underline{\Sigma M_A} = I_{zz}^{A}\underline{\ddot{\varphi}}_2$$

$$\Longrightarrow B_x \sin(\varphi_2)\, l_2 - B_y \cos(\varphi_2)\ \ l_2 - \frac{\cos(\varphi_2)\, l_2}{2} W_2 = \frac{4}{12} m_2 l_2^2 \underline{\ddot{\varphi}}_2$$

From the above 3 equations, separating the terms in the x and y direction in the first one, we have the system:

$$\begin{cases} A_x - B_x = -m_2\left(l_1\dot{\theta}^2\cos(\theta) + \frac{l_2}{2}\ddot{\varphi}_2\, sin(\varphi_2) + \frac{l_2}{2}\dot{\varphi}_2{}^2\cos(\varphi_2)\right) \\ A_y - W_2 - B_y = m_2\left(-l_1\dot{\theta}^2\sin(\theta) + \frac{l_2}{2}\ddot{\varphi}_2\cos(\varphi_2) - \frac{l_2}{2}\dot{\varphi}_2{}^2\sin(\varphi_2)\right) \\ B_x \sin(\varphi_2) - B_y\cos(\varphi_2)\ - \frac{\cos(\varphi_2)}{2} W_2 = \frac{4}{12} m_2 l_2 \underline{\ddot{\varphi}}_2 \end{cases} \quad (\boldsymbol{IV})$$

where . So we have 3 equations and 4 unknowns.$W_2 = m_2 g,\ g = 9.81\frac{m}{s^2}$

For the 1st Body we now have respectively:

$$\underline{\Sigma F_{G_1}} = m_1 \underline{\ddot{r}}_{G_1}$$

$$\Longrightarrow (O_x - A_x)\underline{e_x} + \left(O_y - A_y - W_1\right)\underline{e_y} = m_1\underline{\ddot{r}}_{G_1}$$

$$\underline{\Sigma M_o} = I_{zz}^{o}\underline{\ddot{\theta}} \overset{\ddot{\theta}=0}{\Longrightarrow} \underline{\Sigma M_{G_1}} = 0$$

$$\Longrightarrow M_o + A_x\sin(\theta)\, l_1 - A_y\cos(\theta)\ \ l_1 - \frac{l_1}{2}\cos(\theta)\, W_1 = 0$$

From the above 3 equations, separating the terms in the x and y direction in the first one, we have the system:

$$\begin{cases} O_x - A_x = -m_1\left(\frac{l_1}{2}\dot{\theta}^2\, cos(\theta)\right) \\ O_y - A_y - W_1 = -m_1\left(\frac{l_1}{2}\dot{\theta}^2\sin(\theta)\right) \\ M_o + A_x\sin(\theta)\, l_1 - A_y\cos(\theta)\ \ l_1 - \frac{l_1}{2}\cos(\theta)\, W_1 = 0 \end{cases} \quad (\boldsymbol{V})$$

The above system has 5 unknowns and 3 equations and .$W_1 = m_1 g$

For the 3rd Body we apply equations (21) and equation (22) with respect to point C:

$$\underline{\Sigma F_{G_3'}} = (m_3 + m_\Delta)\underline{\ddot{r}}_{G_3'}$$

$$(\Gamma_x + B_x - \cos(\varphi_3)\, F_\Delta)\underline{e_x} + \left(\Gamma_y + B_y - W_3 + \sin(\varphi_3)\, F_\Delta\right)\underline{e_y} = (m_3 + m_\Delta)\underline{\ddot{r}}_{G_3'}$$

$$\underline{\Sigma M_\Gamma} = I_{zz}^{\Gamma_3}\underline{\ddot{\varphi}}_3$$

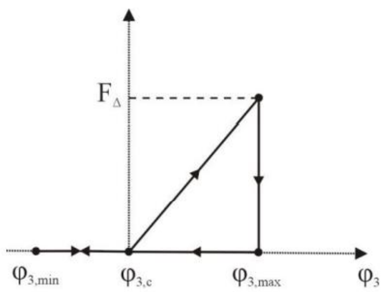
$$-\sin(\varphi_3)\, bW_3 - \cos(\varphi_3)\, l_3 B_x + \sin(\varphi_3)\, l_3 B_y + F_\Delta(l_3 + l_\Delta) = I_{zz}^{\Gamma_3}\underline{\ddot{\varphi}}_3$$

From the above 3 equations, separating the terms in the x and y direction in the first one, we have the system:

$$\begin{cases} \Gamma_x + B_x - \cos(\varphi_3)\, F_\Delta = (m_3 + m_\Delta) b \left(\ddot{\varphi}_3 \cos(\varphi_3) - \dot{\varphi}_3^2 \sin(\varphi_3) \right) \\ \Gamma_y + B_y - W_3 + \sin(\varphi_3)\, F_\Delta = -(m_3 + m_\Delta) b \left(\ddot{\varphi}_3 \sin(\varphi_3) + \dot{\varphi}_3^2 \cos(\varphi_3) \right) \\ -\sin(\varphi_3)\, b W_3 - \cos(\varphi_3)\, l_3 B_x + \sin(\varphi_3)\, l_3 B_y + F_\Delta (l_3 + l_\Delta) = I_{zz}^{\Gamma_3} \underline{\ddot{\varphi}_3} \end{cases} \quad (\boldsymbol{VI})$$

where$W_3 = (m_3 + m_\Delta) g$

From the diagram given to us in the speech about the force, we see that you do not exert on the end D for the entire range of values of the angle, also its linear relationship with the angle and the discontinuity that the force displays when the angle reaches its maximum can be easily distinguished its negative value. Therefore we derive the expression of the force at the end D by the following procedure. First in the diagram we present again in Figure 10. as a reminder, we see that the maximum angle refers to a negative angle according to the convention we have taken for positive angles θ. Therefore from the values we have calculated for the angle , we find the maximum by absolute value, also given to us in the utterance that . So after converting these angles to rad for ease of calculations, we have 2 points for the force diagram and calculate the coefficients in the linear force equation.$F_\Delta \varphi_3 \varphi_3 \varphi_3$ $\max(|\varphi_3|) = 31.82° \varphi_{3,c} = -10° (-0.1745, 0)\ \kappa\alpha\iota\ (-0.5554, 3000) F_\Delta = \alpha \varphi_3 + b\ \ [N]$



Shape11Force diagram at end D

$$\Rightarrow \begin{cases} 0 = -0.1745a + b \Rightarrow b = 0.1745a \ \ (*) \\ 3000 = -0.5554a + b \overset{(*)}{\Rightarrow} a = -7876.06 \end{cases}$$

$\Rightarrow b = -1374.37$

Furthermore, to introduce the discontinuity of the diagram, we know that when the angle terminates to the right, its velocity becomes zero and immediately afterwards becomes positive. Therefore, for the above linear relationship to be valid for the force, it should also be .$\varphi_3 \varphi_3 \in (-31.82, -10) \underline{\omega_3} = \underline{\dot{\varphi}_3} \leq 0$

After all :

$$F_\Delta = \begin{cases} -7876.06\varphi_3 - 1374.37 \quad \gamma\iota\alpha\ \varphi_3 \in \{-31.82, -10\} \cap \{\ \underline{\dot{\varphi}_3} \leq 0\} \\ 0\ \ \gamma\iota\alpha\{\ \varphi_3\ \notin (-31.82, -10)\} \cup\ \{\underline{\dot{\varphi}_3} \geq 0\} \end{cases}$$

Therefore in system (VI) we have 3 equations for 4 unknowns since all the remaining quantities are either data or known functions of θ.

In total from the systems (IV), (V) and (VI) we have 9 equations and 9 unknowns. Having calculated in the previous questions all the values for the positions, velocities and accelerations of the corners as a function of the angle θ of the crank, we can calculate the reactions of the joints and the required external torque of the motor as a function of the angle θ.φ_2, φ_3

We rewrite the systems in a more convenient form:

$$\begin{cases} A_x - B_x = A_1 \\ A_y - B_y = A_2 \\ B_x \sin(\varphi_2) - B_y \cos(\varphi_2) = A_3 \end{cases} \quad (\boldsymbol{III})$$

$$\begin{cases} O_x - A_x = A_4 \\ O_y - A_y = A_5 \\ M_o + A_x \sin(\theta)\, l_1 - A_y \cos(\theta)\ l_1 = A_6 \end{cases} \quad (\boldsymbol{IV})$$

$$\begin{cases} \Gamma_x + B_x = A_7 \\ \Gamma_y + B_y = A_8 \\ -\cos(\varphi_3)\, l_3 B_x + \sin(\varphi_3)\, l_3 B_y = A_9 \end{cases} \quad (\boldsymbol{V})$$

$$A_1 = -m_2 \left(l_1 \dot{\theta}^2 \cos(\theta) + \frac{l_2}{2} \ddot{\varphi}_2\, sin(\varphi_2) + \frac{l_2}{2} \dot{\varphi}_2{}^2 \cos(\varphi_2) \right)$$

$$A_2 = m_2 \left(-l_1 \dot{\theta}^2 \sin(\theta) + \frac{l_2}{2} \ddot{\varphi}_2 \cos(\varphi_2) - \frac{l_2}{2} \dot{\varphi}_2{}^2 \sin(\varphi_2) \right) + W_2$$

$$A_3 = \frac{4}{12} m_2 l_2 \underline{\ddot{\varphi}}_2 + \frac{\cos(\varphi_2)}{2} W_2 \quad , \qquad A_4 = -m_1 \left(\frac{l_1}{2} \dot{\theta}^2\, cos(\theta) \right)$$

$$A_5 = -m_1 \left(\frac{l_1}{2} \dot{\theta}^2 \sin(\theta) \right) + W_1 \ \ , \ \ A_6 = \frac{l_1}{2} \cos(\theta)\, W_1$$

$$A_7 = (m_3 + m_\Delta) b \left(\ddot{\varphi}_3 \cos(\varphi_3) - \dot{\varphi}_3^2 \sin(\varphi_3) \right) + \cos(\varphi_3)\, F_\Delta$$

$$A_8 = -(m_3 + m_\Delta) b \left(\ddot{\varphi}_3 \sin(\varphi_3) + \dot{\varphi}_3^2 \cos(\varphi_3) \right) + W_3 - \sin(\varphi_3)\, F_\Delta$$

$$A_9 = I_{zz}^{\Gamma_3} \underline{\ddot{\varphi}}_3 + \sin(\varphi_3)\, b W_3 - F_\Delta (l_3 + l_\Delta)$$

The above 3 systems consist of 9 unknowns and 9 linear equations, and are written in matrix form:

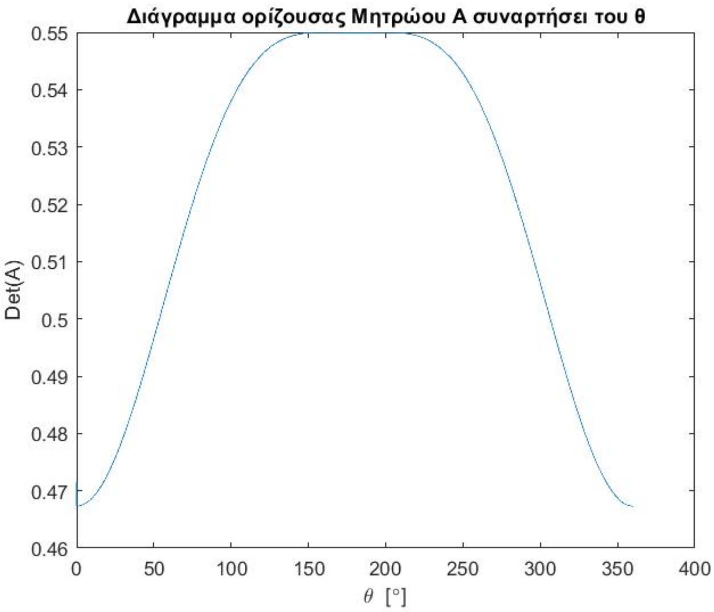
$$\begin{bmatrix} 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sin(\varphi_2) & -\cos(\varphi_2) & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sin(\theta)\, l_1 & -\cos(\theta)\, l_1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -\cos(\varphi_3)\, l_3 & \sin(\varphi_3)\, l_3 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} O_x \\ O_y \\ A_x \\ A_y \\ B_x \\ B_y \\ \Gamma_x \\ \Gamma_y \\ M_o \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \\ A_7 \\ A_8 \\ A_9 \end{bmatrix}$$

Therefore we have a system of the form , where all the elements of the vector b and the register A are known from previous queries and functions of θ. So after first verifying that the register A is inverted for the whole range of values of θ, we will then calculate the values of all the reactions and the required torque for and we will plot them.$A\underline{x} = \underline{b}\theta \in (0,2\pi)$

In order for the A register to be reversed it must . Using Matlab we calculate the determinant of A:$\det(A) \neq 0$

$$\det(A) = \left(\frac{11}{20} \cos(\varphi_2) \cos(\varphi_3) \right) - \left(\frac{11}{20} \sin(\varphi_2) \sin(\varphi_3) \right) = \frac{11}{20} \cos(\varphi_2 - \varphi_3)$$

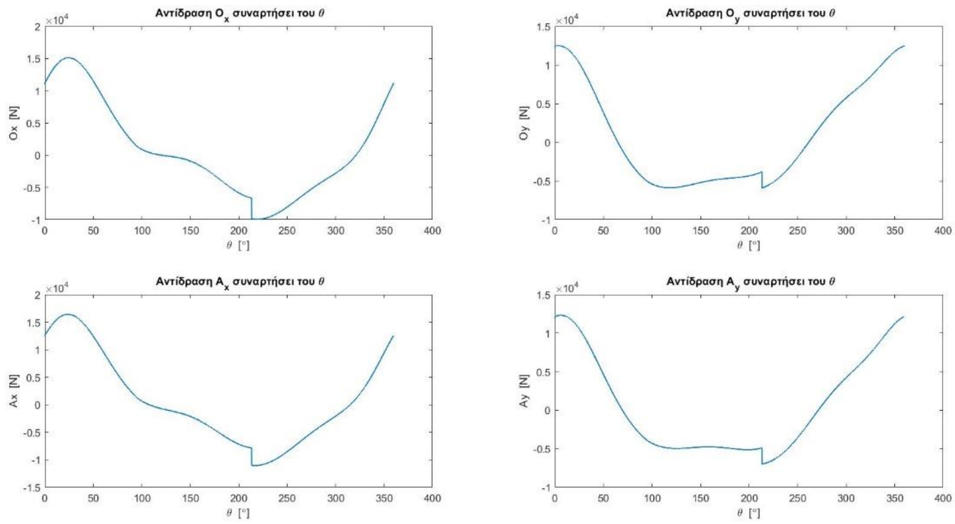
And we produce its graphic representation for all the values of the angles we calculated in the previous questions:$\varphi_2(\theta), \varphi_3(\theta)$



From Figure 12 we see that the determinant does not go to zero for any value of the crank angle so we can invert the A register and calculate the values of the mechanism forces for all positions θ.

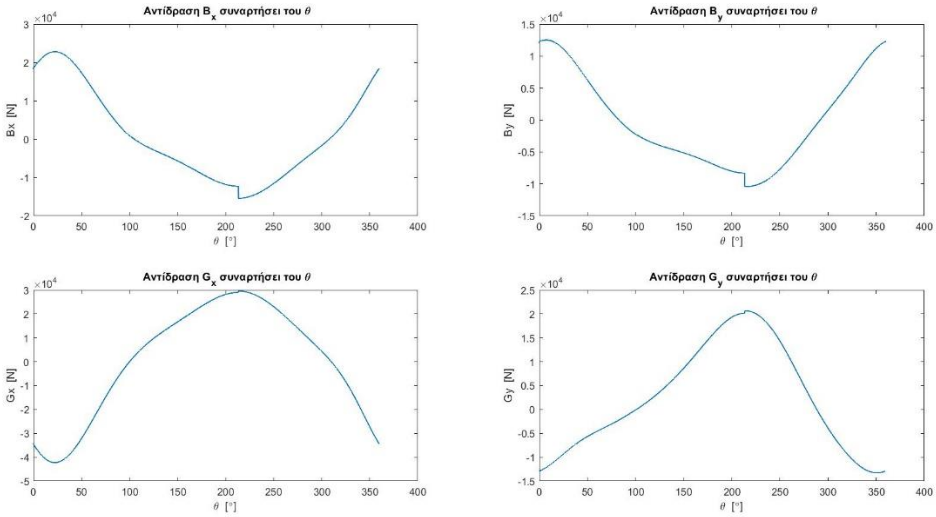
Shape12. Diagram of Determinant Register A as a function of angle θ

In the file 'funZ1' the above system is solved for . This file has as input the values of the variables that we calculated in the previous questions and as output a vector that includes all the forces of the joints of the mechanism at the specific position θ as well as the applied moment at O. The function funZ1 is called through the main file Solver and after constructing the value vectors of the forces for the entire range of values of θ, the corresponding graphical representation is made. Figures 13 to 15 show the corresponding diagrams.$\theta \in (0,2\pi)\theta, \varphi_2, \varphi_3, \dot{\varphi_2}, \dot{\varphi_3}, \ddot{\varphi_2}, \ddot{\varphi_3}$



Shape13. Diagrams of Forces in joints O and A as a function of the angle θ

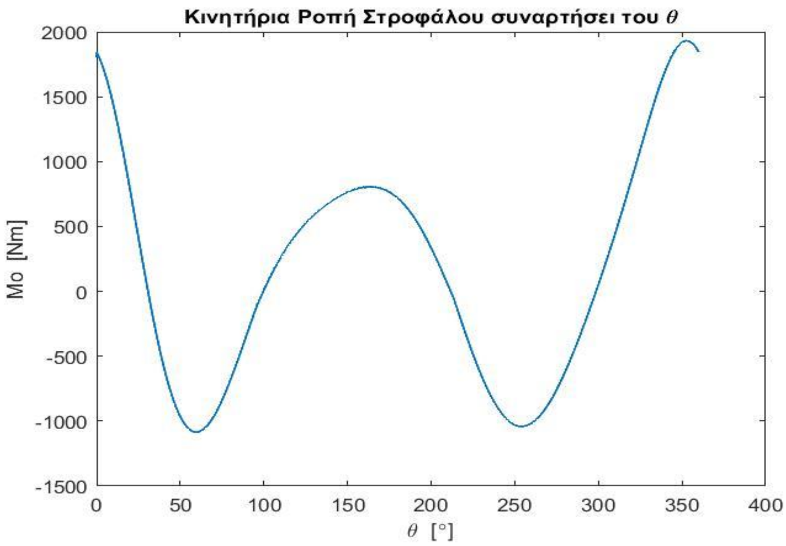
In Figure 13. we distinguish the forces developed in joints O and A. We can see that the appearing forces in these 2 joints have an identical form, which is due to the constant rotational speed. The difference we distinguish in the values is mainly due to the introduced Torque. Figure 14 shows the forces in joints B and C as a function of angle θ. From these 2 figures we can see that the distribution of values for the forces of joints B and A, which belong to the 2nd member, are marginally identical in terms of format, of course they differ in prices.$\dot{\theta}. M_o$



Shape14. Diagrams of Forces in joints B and C as a function of the angle θ

The force diagrams show periodic behavior like all the kinematic quantities we saw in the previous questions. The discontinuity that is easily perceived in all the diagrams is due to the discontinuity of the force at the end D. The extreme values of the forces appear at the extreme positions of member 3 where in one of the 2 we have the discontinuity. We can also see that in all the diagrams the value ranges on the vertical axes extend to both positive and negative values, this behavior is due to the complex movement of the mechanism and the trajectories that its members trace. The magnitudes of the forces are an order of magnitude larger than the maximum value of the applied force at Δ, for this reason when the force is reduced to zero we do not observe a greater change in the graphs.$F_\Delta F_\Delta$

In figure 15 we can see the driving torque of the crank, its maximum negative values appear approximately at the extreme positions of member 3 while its maximum value appears for θ approximately equal to zero. The negative torque values are due to the fact that you maintain a constant angular velocity on the crank, otherwise if we introduce a constant torque to the mechanism, the angular velocity would show a periodic behavior like the rest of the kinematic quantities we studied in the previous questions.



Shape15. Diagram of cranking torque as a function of i

In addition you ask us to calculate the required average engine power. We know that engine power is derived from the equation:

$$P = M_o\dot{\theta}\ [Watt]$$

To calculate the average power, after summing the values for the torque given to the crank, we multiply by the constant angular velocity and divide by the number of points that we discretized in the 1st question, the range, with step. Thus we have:$\theta \in (0{,}360°)\Delta\theta = 0.1°$

$$P_{mean} = \frac{M_o\dot{\theta}}{n} = 0.75\ [kW]$$

To understand the magnitude of this power, let's think of the model engineBMW 3 Series Sedan produces power while correspondingly a two-cylinder aero engine can offer values in the power range. The codes written to produce the force diagrams are presented in appendix A6.$141.74\ [kW] 370W - 3.7\ [kW]$

7th QUESTION

In the 7th question we are asked to calculate and construct the diagram of the force exerted by the mechanism on its support frame as a function of the angle θ. For this calculation we apply Newton's 2nd law to the entire mechanism:

$$\underline{\Sigma F(\theta)} = m_{O\Lambda}\underline{\ddot{r}_G}(\theta)$$

where and the acceleration of the center of mass as a function of the crank angle was calculated in the 4th question from equation (16):$m_{O\Lambda} = m_1 + m_2 + m_3 + m_\Delta$

$$\begin{aligned}\underline{\ddot{r}_G} = \frac{1}{m}\Big(\Big(&-A_1\dot{\theta}^2\cos(\theta) - \ddot{\varphi}_2 A_2\sin(\varphi_2) - \dot{\varphi}_2^2 A_2\, cos(\varphi_2) + \ddot{\varphi}_3 A_3\cos(\varphi_3) \\ &- \dot{\varphi}_3^2 A_3\, sin(\varphi_3)\Big)\underline{e_x} \\ &+ \Big(-A_4\dot{\theta}^2\, sin(\theta) + \ddot{\varphi}_2 A_5\cos(\varphi_2) - \dot{\varphi}_2^2 A_5\sin(\varphi_2) - \ddot{\varphi}_3 A_6\sin(\varphi_3) \\ &- \dot{\varphi}_3^2 A_6\, cos(\varphi_3)\Big)\underline{e_y}\Big)\end{aligned}$$

where:

$$A_1 = m_1\frac{l_1}{2} + m_2 l_1 \quad, A_2 = \frac{l_2}{2}m_2 \quad,$$

$$A_3 = m_3\frac{l_3}{2} + m_\Delta(l_3 + l_\Delta)\ ,\ \ A_4 = m_1\frac{l_1}{2} + m_2 l_1$$

$$A_5 = \frac{l_2}{2}m_2\ ,\ \ A_6 = m_3\frac{l_3}{2} + m_\Delta(l_3 + l_\Delta)$$

Since we are interested in the measure of the force exerted on the frame for the graphical representation we have:

$$\left|\underline{\Sigma F(\theta)}\right| = m_{O\Lambda}\left|\underline{\ddot{r}}_G(\theta)\right|$$

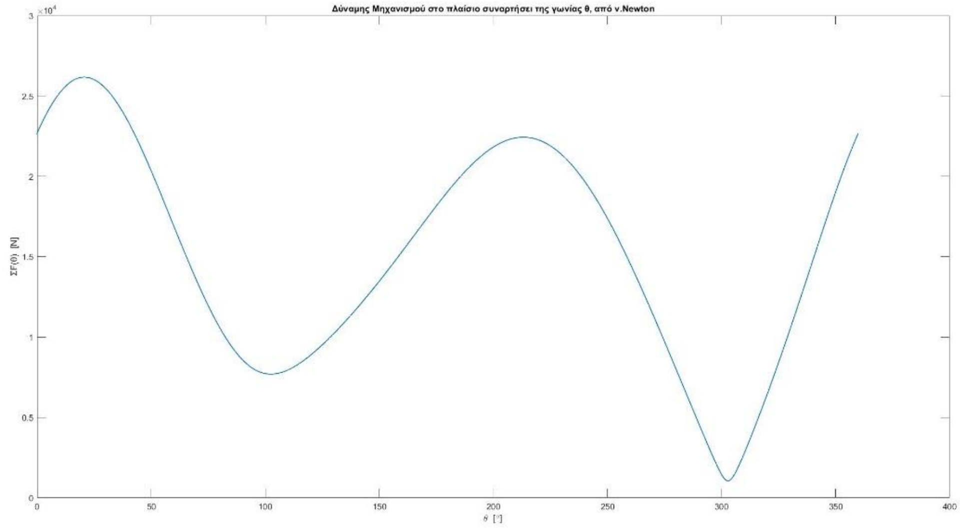
with$\left|\underline{\ddot{r}}_G(\theta)\right| = \frac{1}{m}(\sqrt{A^2 + B^2}$

$$A = \left(-A_1\dot{\theta}^2\cos(\theta) - \ddot{\varphi}_2 A_2 \sin(\varphi_2) - \dot{\varphi}_2^2 A_2\, cos(\varphi_2) + \ddot{\varphi}_3 A_3 \cos(\varphi_3) - \dot{\varphi}_3^2 A_3\, sin(\varphi_3)\right)$$

$$B = \left(-A_4\dot{\theta}^2\, sin(\theta) + \ddot{\varphi}_2 A_5 \cos(\varphi_2) - \dot{\varphi}_2^2 A_5 \sin(\varphi_2) - \ddot{\varphi}_3 A_6 \sin(\varphi_3) - \dot{\varphi}_3^2 A_6\, cos(\varphi_3)\right)$$

The values of the measure of the acceleration of the center of mass have been calculated in the 4th question and we will use them to get the diagram that follows.

The diagram is produced using the 'funH1' file which has as input the acceleration values of the center of mass calculated in the 4th question and as output the inertial force exerted by the whole mechanism. This function is called through the main file Solver, in which after the construction of the vector of values for all θ is done, it is completed by visualizing them. Figure 16 shows the diagram$\Sigma F(\theta)$.



Shape16. Mechanism force in the frame as a function of the angle θ

We observe the periodic behavior of the component force of the mechanism. We also see that the extreme positions appear at the points where the extreme positions of the generalized coordinates also appear.

To verify the results solved in the previous questions we will derive the diagram of the constitutive force in the framework of the mechanism by force analysis and compare it with the diagram shown in Figure 16. To derive this by studying the mechanism as a body and considering we only have the forces that do not cancel each other out:

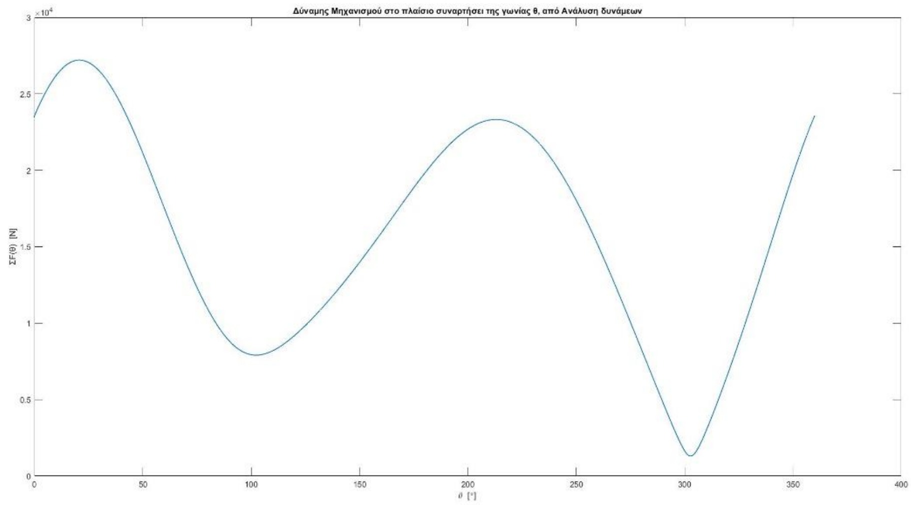
$$\underline{\Sigma F(\theta)} = (O_x(\theta) + \Gamma_x(\theta) - F_\Delta \cos(\varphi_3(\theta)))\underline{e}_x + \left(O_y(\theta) + \Gamma_y(\theta) - F_\Delta \sin(\varphi_3(\theta)) - W\right)\underline{e}_y$$

where the forces at joints O and C were calculated in the previous questions. Also the force was defined in the previous question. Therefore we have for the measure:$W = g(m_1 + m_2 + m_3 + m_\Delta)F_\Delta$

$$\left|\underline{\Sigma F(\theta)}\right| = \left(\sqrt{A^2 + B^2}\right)$$

with$A = \left(O_x(\theta) + \Gamma_x(\theta) - F_\Delta \cos\left(\varphi_3(\theta)\right)\right)^2$, $B = \left(O_y(\theta) + \Gamma_y(\theta) - F_\Delta \sin\left(\varphi_3(\theta)\right) - W\right)^2$

The diagram we get is produced at the end of the Solver file and is shown in Figure 17. From the comparison of Figures 16 and 17 we see that the two diagrams are identical. In reality, of course, the values of one from the other differ by a minimum which is due to calculation errors during the entire work. The codes written for the 7th question are presented in Appendix A7.



Shape17. Mechanism force in the frame as a function of the angle θ