01 - GARCH Models Estimation

January 31, 2020

[1]:
```python
# Here I define the stock I want to work on.
# If you simply change the name and re-run the whole notebook the whole process␣
↪will be done automatically.
# For example if I want to load Google.csv I type in 'Google'.
# This is NOT case sensitive.
stock = 'Google'
```

0.1 Loading Data

[2]:
```python
import numpy as np
import pandas as pd
df1 = pd.read_csv(f'{stock}.csv')
df1.head()
```

[2]:

	Date	Open	High	Low	Close	Adj Close	Volume
0	2004-08-19	49.813286	51.835709	47.800831	49.982655	49.982655	44871300
1	2004-08-20	50.316402	54.336334	50.062355	53.952770	53.952770	22942800
2	2004-08-23	55.168217	56.528118	54.321388	54.495735	54.495735	18342800
3	2004-08-24	55.412300	55.591629	51.591621	52.239193	52.239193	15319700
4	2004-08-25	52.284027	53.798351	51.746044	52.802086	52.802086	9232100

[3]:
```python
# I use the daily adjusted close prices. These prices are adjusted for possible␣
↪dividends and/or splits.
df = pd.DataFrame()
df[['Date', f'{stock}']] = df1[['Date', 'Adj Close']]
df.head()
```

[3]:

	Date	Google
0	2004-08-19	49.982655
1	2004-08-20	53.952770
2	2004-08-23	54.495735
3	2004-08-24	52.239193
4	2004-08-25	52.802086

[4]:
```python
df.dropna(inplace=True)
df.info()
```

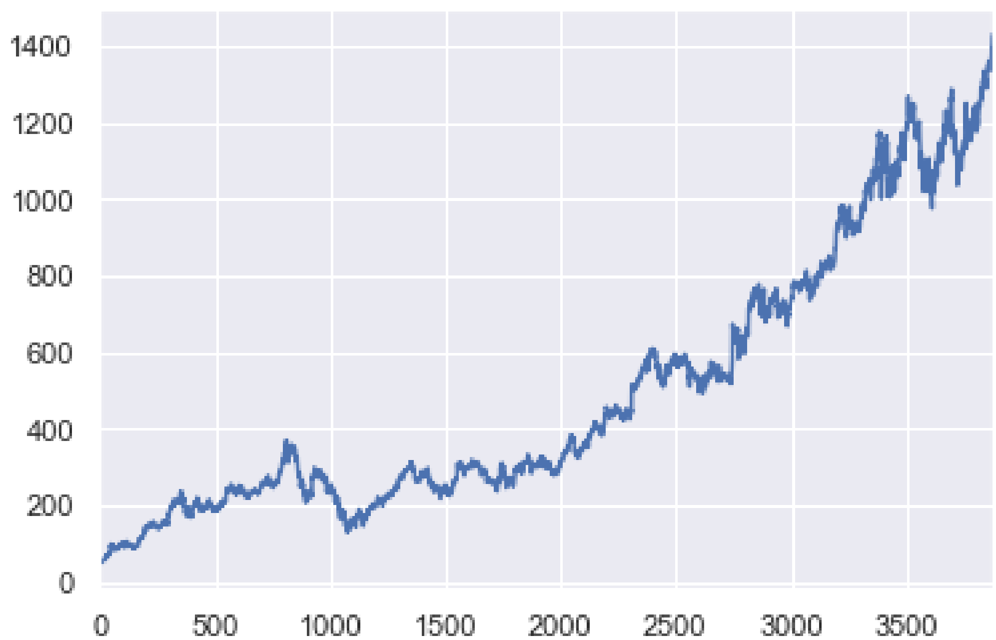
```
<class 'pandas.core.frame.DataFrame'>
Int64Index: 3876 entries, 0 to 3875
Data columns (total 2 columns):
Date      3876 non-null object
Google    3876 non-null float64
dtypes: float64(1), object(1)
memory usage: 90.8+ KB
```

[5]:

```
import matplotlib as plt
import seaborn as sns
sns.set()
df[f'{stock}'].plot()
```

[5]:

```
<matplotlib.axes._subplots.AxesSubplot at 0x2360c2cca88>
```



0.2 Checking for unit root

Before estimating any model, we have to check if the time series is stationary over time. If not then we take the first differences and check again. If not the second differences and so on. If the p-value of the test is lower than 0.05, then we reject the null hypothesis of non-stationarity.

[6]:

```
from statsmodels.tsa.stattools import adfuller
adf = adfuller(df[f'{stock}'])
print("The p-value of the Augmented Dickey-Fuller test is " + str(round(adf[1], 3)))
```

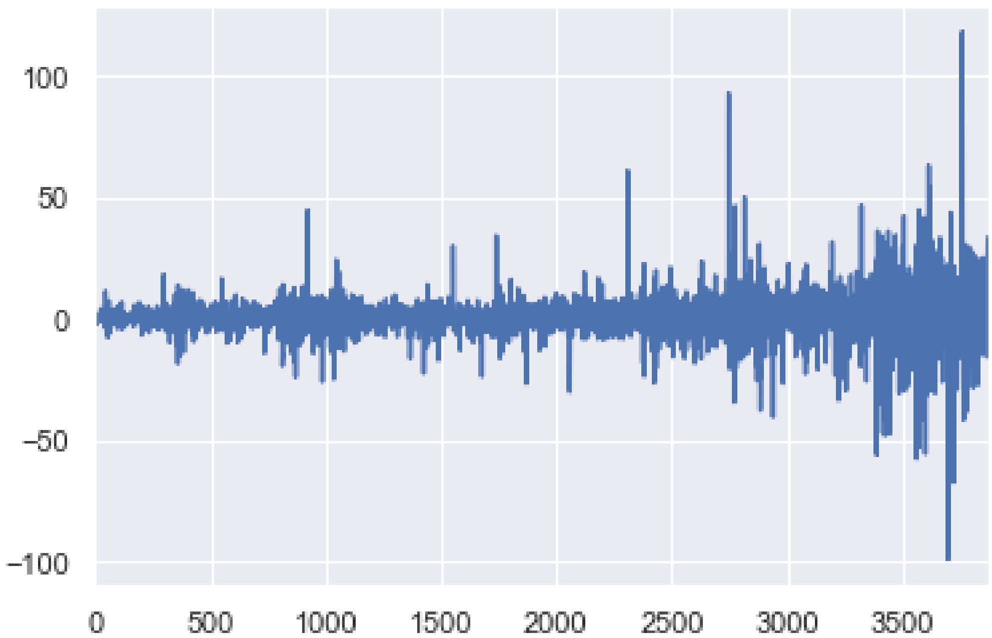
The p-value of the Augmented Dickey-Fuller test is 0.997

[7]:
```
df[f'D{stock}'] = df[f'{stock}'].diff()
df.head()
```

[7]:
```
         Date     Google   DGoogle
0  2004-08-19  49.982655       NaN
1  2004-08-20  53.952770  3.970115
2  2004-08-23  54.495735  0.542965
3  2004-08-24  52.239193 -2.256542
4  2004-08-25  52.802086  0.562893
```

[8]:
```
df[f'D{stock}'].plot()
```

[8]: <matplotlib.axes._subplots.AxesSubplot at 0x2360f3aa588>



[9]:
```
df.dropna(inplace=True)
df.reset_index(inplace=True, drop=True)
adf = adfuller(df[f'D{stock}'])
print("The p-value of the Augmented Dickey-Fuller test is " + str(round(adf[1],␣
↪3)))
```

The p-value of the Augmented Dickey-Fuller test is 0.0

0.3 Estimating the best ARIMA model for the mean

The first step before estimating the GARCH models is to estimate an ARIMA model for the mean. I estimate all models for orders p and q from 0 to 10 and take the model with the smallest AIC. All of the estimated coefficients should also be statistically significant.

[11]:
```
# I am making use of the multiprocessing tool for faster estimation of the models.
# The map function takes 2 arguments. The first is the function I want to pool.
# The second is a list of the arguments for each process. Here I am creating that list.
# Each item of the list contains a list with 2 items.
# The first is the series I want to estimate the ARIMA model on.
# The second is the order q of the ARIMA model the process is going to estimate.
# So, the first process will estimate all ARIMA models for order p from 0 to 10 and q = 0 and return the best of them.
# The second process for p from 0 to 10 and q = 1 etc.
f = []
for i in range(11):
    f.append((df[f'{stock}'][:-3], i))
```

[12]:
```
# For some reason the multiprocessing doesn't work if I define the function inside the Jupyter Notebook.
# So, I am calling the function from a separate file.
# Check gbam.py
# WARNING!!! This uses a lot of the CPU.
from gbam import gbam
import multiprocessing
if __name__ == '__main__':
    p = multiprocessing.Pool()
    results = p.map(gbam, f)
    p.close()
```

[13]:
```
# Here I make a dataframe with the results from the pool.
# They are sorted by AIC.
aic1 = pd.DataFrame(results, columns=['AIC', 'Order'])
aic1.sort_values('AIC', inplace=True)
aic1.reset_index(inplace=True, drop=True)
#It is possible that some processes didn't find any models with the desired results so I delete them.
aic1.replace([np.inf, -np.inf], np.nan, inplace=True)
aic1.dropna(inplace=True)
aic1
```

[13]:

	AIC	Order
0	27981.1457	(10, 8)
1	28004.6839	(6, 2)
2	28029.6657	(4, 1)

```
3  28047.0913   (0, 0)
```

[14]:
```
# The first row gives me the order of p and q with the lowest AIC.
p1 = aic1.iloc[0][1][0]
q1 = aic1.iloc[0][1][1]
```

[15]:
```
# Saving the best order in a dictionary for later use.
pq = {}
pq.update({'ARIMA' : (p1 , q1)})
```

[16]:
```
arima = ARIMA(df[f'{stock}'][:-3], order=(p1,1,q1)).fit()
print(arima.summary())
```

```
                              ARIMA Model Results
==============================================================================
Dep. Variable:               D.Google   No. Observations:                 3871
Model:                ARIMA(10, 1, 8)   Log Likelihood              -13970.573
Method:                       css-mle   S.D. of innovations              8.934
Date:                Mon, 27 Jan 2020   AIC                          27981.146
Time:                        20:16:34   BIC                          28106.371
Sample:                             1   HQIC                         28025.609

================================================================================
===
                      coef    std err          z      P>|z|      [0.025
0.975]
--------------------------------------------------------------------------------
---
const               0.3441      0.131      2.619      0.009       0.087
0.602
ar.L1.D.Google     -0.4194      0.034    -12.448      0.000      -0.485
-0.353
ar.L2.D.Google      0.1082      0.044      2.452      0.014       0.022
0.195
ar.L3.D.Google      0.3096      0.039      7.851      0.000       0.232
0.387
ar.L4.D.Google      0.1084      0.031      3.466      0.001       0.047
0.170
ar.L5.D.Google      0.3724      0.034     11.019      0.000       0.306
0.439
ar.L6.D.Google      0.0664      0.033      2.014      0.044       0.002
0.131
ar.L7.D.Google     -0.3402      0.029    -11.763      0.000      -0.397
-0.283
ar.L8.D.Google     -0.8705      0.035    -25.228      0.000      -0.938
-0.803
ar.L9.D.Google     -0.0366      0.021     -1.760      0.078      -0.077
```

```
0.004
ar.L10.D.Google    -0.0373      0.020     -1.880      0.060      -0.076
0.002
ma.L1.D.Google      0.4305      0.030     14.519      0.000       0.372
0.489
ma.L2.D.Google     -0.1621      0.041     -3.974      0.000      -0.242
-0.082
ma.L3.D.Google     -0.3289      0.040     -8.252      0.000      -0.407
-0.251
ma.L4.D.Google     -0.0915      0.032     -2.847      0.004      -0.154
-0.028
ma.L5.D.Google     -0.4030      0.034    -11.928      0.000      -0.469
-0.337
ma.L6.D.Google     -0.1189      0.030     -3.902      0.000      -0.179
-0.059
ma.L7.D.Google      0.3823      0.028     13.557      0.000       0.327
0.438
ma.L8.D.Google      0.8822      0.035     25.035      0.000       0.813
0.951
                                    Roots
=============================================================================
                  Real          Imaginary           Modulus         Frequency
-----------------------------------------------------------------------------
AR.1            0.9592           -0.3248j            1.0127           -0.0520
AR.2            0.9592           +0.3248j            1.0127            0.0520
AR.3            0.2976           -0.9751j            1.0195           -0.2029
AR.4            0.2976           +0.9751j            1.0195            0.2029
AR.5           -0.9183           -0.3997j            1.0015           -0.4347
AR.6           -0.9183           +0.3997j            1.0015            0.4347
AR.7           -0.5450           -0.8863j            1.0404           -0.3378
AR.8           -0.5450           +0.8863j            1.0404            0.3378
AR.9           -0.2845           -4.8048j            4.8132           -0.2594
AR.10          -0.2845           +4.8048j            4.8132            0.2594
MA.1            0.9661           -0.3192j            1.0175           -0.0508
MA.2            0.9661           +0.3192j            1.0175            0.0508
MA.3            0.2952           -0.9657j            1.0098           -0.2028
MA.4            0.2952           +0.9657j            1.0098            0.2028
MA.5           -0.5578           -0.8705j            1.0339           -0.3407
MA.6           -0.5578           +0.8705j            1.0339            0.3407
MA.7           -0.9202           -0.3971j            1.0023           -0.4352
MA.8           -0.9202           +0.3971j            1.0023            0.4352
-----------------------------------------------------------------------------
```

0.4 Estimating best GARCH model

[17]:
```python
# The same logic as the ARIMA model.
# This time the first item of each list is the residuals from the best ARIMA␣
↪model.
# I also added a third item which is the method for estimating volatility.
# Check gbgm.py
f = []
for i in range(11):
    f.append((arima.resid, i, 'GARCH'))
```

[18]:
```python
# WARNING!!! This uses a lot of the CPU.
from gbgm import gbgm
if __name__ == '__main__':
    p = multiprocessing.Pool()
    results = p.map(gbgm, f)
    p.close()
```

[19]:
```python
aic2 = pd.DataFrame(results, columns=['AIC', 'Order'])
aic2.sort_values('AIC', inplace=True)
aic2.reset_index(inplace=True, drop=True)
aic2.replace([np.inf, -np.inf], np.nan, inplace=True)
aic2.dropna(inplace=True)
aic2
```

[19]:

	AIC	Order
0	26038.5153	(7, 0)

[20]:
```python
p2 = aic2.iloc[0][1][0]
q2 = aic2.iloc[0][1][1]
pq.update({'GARCH' : (p2 , q2)})
```

[21]:
```python
from arch import arch_model
arma_garch = arch_model(arima.resid, p=p2, q=q2, mean='Zero', rescale=True).
↪fit(disp='off')
print(arma_garch.summary())
```

```
                       Zero Mean - ARCH Model Results
==============================================================================
Dep. Variable:                   None   R-squared:                       0.000
Mean Model:                 Zero Mean   Adj. R-squared:                  0.000
Vol Model:                       ARCH   Log-Likelihood:               -13011.3
Distribution:                  Normal   AIC:                           26038.5
Method:            Maximum Likelihood   BIC:                           26088.6
                                        No. Observations:                 3871
Date:                Mon, Jan 27 2020   Df Residuals:                     3863
Time:                        20:17:07   Df Model:                            8
                              Volatility Model
```

```
==============================================================================
                 coef    std err          t      P>|t|     95.0% Conf. Int.
------------------------------------------------------------------------------
omega         11.0765      2.312      4.791  1.658e-06    [  6.545, 15.608]
alpha[1]       0.1192  5.360e-02      2.224  2.615e-02  [1.415e-02,  0.224]
alpha[2]       0.1637  7.804e-02      2.098  3.590e-02  [1.078e-02,  0.317]
alpha[3]       0.1821  5.918e-02      3.077  2.093e-03  [6.608e-02,  0.298]
alpha[4]       0.1774  5.107e-02      3.475  5.114e-04  [7.735e-02,  0.278]
alpha[5]       0.2193  9.205e-02      2.382  1.722e-02  [3.885e-02,  0.400]
alpha[6]       0.0797  2.593e-02      3.074  2.112e-03  [2.889e-02,  0.131]
alpha[7]       0.0586  3.130e-02      1.871  6.133e-02 [-2.781e-03,  0.120]
==============================================================================

Covariance estimator: robust
```

0.5 Estimating best EGARCH model

[22]:
```python
f = []
for i in range(11):
    f.append((arima.resid, i, 'EGARCH'))
```

[23]:
```python
# WARNING!!! This uses a lot of the CPU.
if __name__ == '__main__':
    p = multiprocessing.Pool()
    results = p.map(gbgm, f)
    p.close()
```

[24]:
```python
aic3 = pd.DataFrame(results, columns=['AIC', 'Order'])
aic3.sort_values('AIC', inplace=True)
aic3.reset_index(inplace=True, drop=True)
aic3.replace([np.inf, -np.inf], np.nan, inplace=True)
aic3.dropna(inplace=True)
aic3
```

[24]:

	AIC	Order
0	25589.4342	(2, 1)
1	26191.6798	(10, 0)

[25]:
```python
p3 = aic3.iloc[0][1][0]
q3 = aic3.iloc[0][1][1]
pq.update({'EGARCH' : (p3 , q3)})
```

[26]:
```python
from arch import arch_model
arma_egarch = arch_model(arima.resid, p=p3, q=q3, vol='EGARCH', mean='Zero',␣
 ↪rescale=True).fit(disp='off')
print(arma_egarch.summary())
```

```
                     Zero Mean - EGARCH Model Results
==============================================================================
Dep. Variable:                   None   R-squared:                       0.000
Mean Model:                 Zero Mean   Adj. R-squared:                  0.000
Vol Model:                     EGARCH   Log-Likelihood:               -12790.7
Distribution:                  Normal   AIC:                           25589.4
Method:            Maximum Likelihood   BIC:                           25614.5
                                        No. Observations:                 3871
Date:                Mon, Jan 27 2020   Df Residuals:                     3867
Time:                        20:17:26   Df Model:                            4
                               Volatility Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
omega          0.0144  7.856e-03      1.830  6.718e-02 [-1.017e-03,2.978e-02]
alpha[1]       0.3182      0.111      2.869  4.112e-03      [  0.101,  0.535]
alpha[2]      -0.2488      0.111     -2.246  2.473e-02   [ -0.466,-3.165e-02]
beta[1]        0.9983  1.728e-03    577.771      0.000      [  0.995,  1.002]
=============================================================================

Covariance estimator: robust
```

0.6 Estimating best FIGARCH model

[27]:
```python
f = []
for i in range(11):
    f.append((arima.resid, i, 'FIGARCH'))
```

[28]:
```python
# WARNING!!! This uses a lot of the CPU.
if __name__ == '__main__':
    p = multiprocessing.Pool()
    results = p.map(gbgm, f)
    p.close()
```

[29]:
```python
aic4 = pd.DataFrame(results, columns=['AIC', 'Order'])
aic4.sort_values('AIC', inplace=True)
aic4.reset_index(inplace=True, drop=True)
aic4.replace([np.inf, -np.inf], np.nan, inplace=True)
aic4.dropna(inplace=True)
aic4
```

[29]:

	AIC	Order
0	25711.9405	(0, 1)
1	25787.8595	(0, 0)

[30]:
```python
p4 = aic4.iloc[0][1][0]
q4 = aic4.iloc[0][1][1]
```

```
pq.update({'FIGARCH' : (p4 , q4)})
```

[31]:
```
from arch import arch_model
arma_figarch = arch_model(arima.resid, p=p4, q=q4, vol='FIGARCH', mean='Zero',␣
↪rescale=True).fit(disp='off')
print(arma_figarch.summary())
```

```
                        Zero Mean - FIGARCH Model Results
==============================================================================
Dep. Variable:                   None   R-squared:                       0.000
Mean Model:                 Zero Mean   Adj. R-squared:                  0.000
Vol Model:                    FIGARCH   Log-Likelihood:               -12853.0
Distribution:                  Normal   AIC:                           25711.9
Method:            Maximum Likelihood   BIC:                           25730.7
                                        No. Observations:                 3871
Date:                Mon, Jan 27 2020   Df Residuals:                     3868
Time:                        20:17:34   Df Model:                            3
                              Volatility Model
========================================================================
                 coef    std err          t      P>|t|  95.0% Conf. Int.
------------------------------------------------------------------------
omega          4.1541      1.225      3.390  6.987e-04 [  1.752,  6.556]
d              0.3392  3.338e-02     10.162  2.932e-24 [  0.274,  0.405]
beta           0.2654  5.283e-02      5.024  5.056e-07 [  0.162,  0.369]
========================================================================

Covariance estimator: robust
```

[32]:
```
df['Variance'] = (df[f'D{stock}']-df[f'D{stock}'].mean())**2
fdf = pd.DataFrame()
fdf = df.tail(3).copy()
fdf.set_index('Date', drop=True, inplace=True)
fdf.head()
```

[32]:

	Google	DGoogle	Variance
Date			
2020-01-08	1404.319946	10.979980	112.867595
2020-01-09	1419.829956	15.510010	229.642085
2020-01-10	1429.729980	9.900024	91.087176

[33]:
```
df.reset_index(inplace=True, drop=True)
```

0.7 Making 3 steps ahead forecast for each model

[34]:
```
for i in range(1, len(pq)):
    g = []
    for j in range(1,4):
        farima = ARIMA(df[f'{stock}'][:-j], order=(p1,1,q2)).fit()
        fgarch = arch_model(farima.resid, p=list(pq.values())[i][0], q=list(pq.
↪values())[i][1], vol=f'{list(pq.keys())[i]}', mean='Zero', rescale=True).
↪fit(disp='off')
        garch = fgarch.forecast(horizon=1)
        g.append(garch.variance.tail(1)['h.1'].iloc[0])

    g.reverse()

    fdf[f'{list(pq.keys())[i]}'] = g

fdf.tail()
```

[34]:

	Google	DGoogle	Variance	GARCH	EGARCH	\
Date						
2020-01-08	1404.319946	10.979980	112.867595	357.645955	177.663147	
2020-01-09	1419.829956	15.510010	229.642085	491.215491	226.273557	
2020-01-10	1429.729980	9.900024	91.087176	357.633741	233.373077	

	FIGARCH
Date	
2020-01-08	315.114559
2020-01-09	264.276024
2020-01-10	243.135037

[35]:
```
fdf.dropna(inplace=True)
```

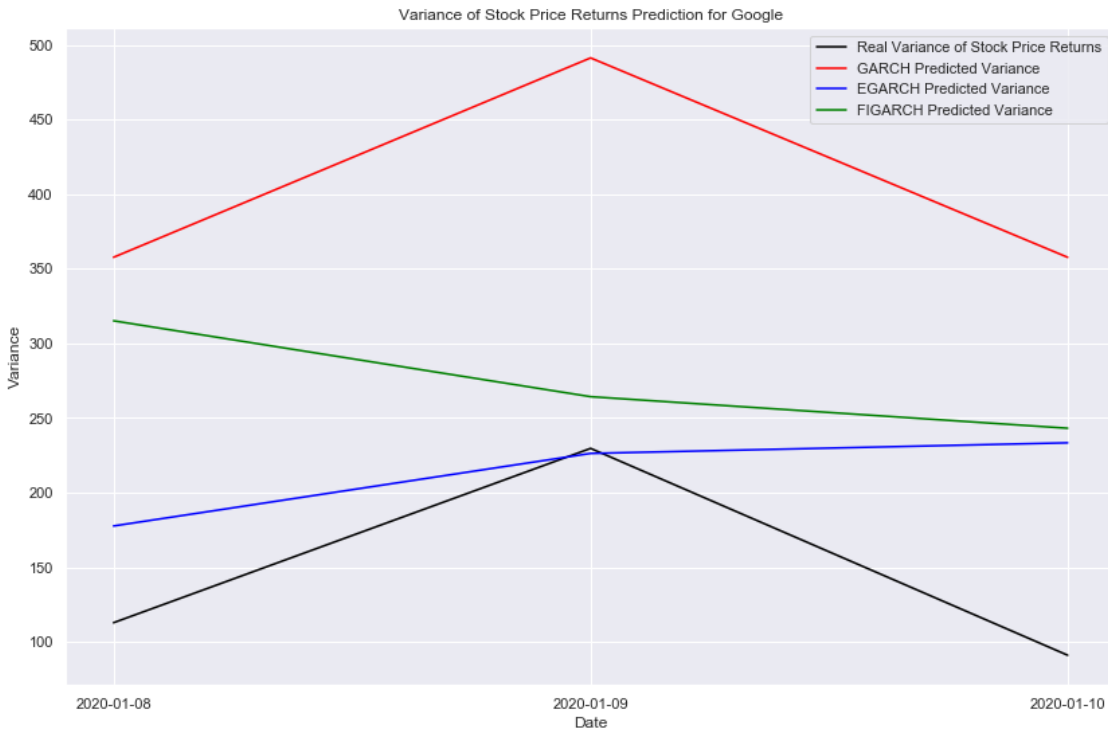
0.8 Evaluation of the models

[36]:
```
from sklearn.metrics import mean_absolute_error
for i in range(1, len(pq)):
    print(f'The MAE for the {list(pq.keys())[i]} model is: ' +␣
↪str(mean_absolute_error(fdf['Variance'], fdf[f'{list(pq.keys())[i]}'])))
```

```
The MAE for the GARCH model is: 257.6327772799213
The MAE for the EGARCH model is: 70.14999386076713
The MAE for the FIGARCH model is: 129.64292170642602
```

0.9 Visualization

[37]:
```
import matplotlib.pyplot as plt
plt.figure(figsize=(14,9))
plt.plot(fdf['Variance'], color = 'black', label = 'Real Variance of Stock␣
↪Price Returns')
plt.plot(fdf['GARCH'], color = 'red', label = 'GARCH Predicted Variance')
plt.plot(fdf['EGARCH'], color = 'blue', label = 'EGARCH Predicted Variance')
plt.plot(fdf['FIGARCH'], color = 'green', label = 'FIGARCH Predicted Variance')
plt.title(f'Variance of Stock Price Returns Prediction for {stock}')
plt.xlabel('Date')
plt.ylabel('Variance')
plt.legend()
plt.show()
```



[38]:
```
fdf.to_excel(f"{stock} Forecasts.xlsx")
```