11.3 & 11.4 : Integral Tests & comparison

38. a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4}$

$S_{10} = \frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \frac{1}{4^4} + \frac{1}{5^4} + \frac{1}{6^4} + \frac{1}{7^4} + \frac{1}{8^4} + \frac{1}{9^4} + \frac{1}{10^4} \approx \boxed{1.0820}$

$R_n \leq \displaystyle\int_{10}^{\infty} f(x)\, dx$

$R_n \leq \displaystyle\int_{10}^{\infty} \frac{1}{x^4}\, dx = \lim_{t \to \infty} \frac{1}{3x^3}\Big|_{10}^{t} = 0 + \frac{1}{3000} = 0.0003$

$\therefore \boxed{R_n \leq 0.0003}$

d)

$R_n \leq \displaystyle\int_{n}^{\infty} f(x)\, dx = \int_{n}^{\infty} \frac{1}{x^4}\, dx$

$\displaystyle\int_{n}^{\infty} \frac{1}{x^4}\, dx \leq 0.00001$

$- \lim_{t \to \infty} \frac{1}{3x^3}\Big|_{n}^{t} \leq 0.00001$

$- \frac{1}{3t^3} + \frac{1}{3n^3} \leq 0.00001$

$\boxed{n \geq 33}$

39. a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$

$S_{10} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \frac{1}{7^2} + \frac{1}{8^2} + \frac{1}{9^2} + \frac{1}{10^2} \approx \boxed{1.5498}$

$R_n \leq \displaystyle\int_{10}^{\infty} f(x)\, dx = \int_{n}^{\infty} \frac{1}{x^2}\, dx$

$R_n \leq \displaystyle\int_{10}^{\infty} \frac{1}{x^2}\, dx = -\lim_{t \to \infty} \frac{1}{x}\Big|_{10}^{\infty} = 0 + \frac{1}{10} = \boxed{0.1}$

d) $R_n \in \int_n^\infty \frac{1}{x^2} \, dx \le 0.001$

$-\lim\limits_{t \to \infty} \frac{1}{x} \Big|_r^t = -\frac{1}{t} + \frac{1}{n} \le 0.001$

$\frac{1}{n} \le 0.001$

$\boxed{n \ge 1000}$

41. $\sum\limits_{n=1}^\infty (2n+1)^{-6}$

$S_n + \int_{n+1}^\infty f(x) \, dx \le S \le S_n + \int_n^\infty f(x) \, dx$

$\int_{n+1}^\infty f(x) \, dx \le R_n \le \int_n^\infty f(x) \, dx$

$\int_n^\infty f(x) \, dx < 0.00001$

$\left[\frac{1}{2} \cdot \frac{(2x+1)^{-5}}{5} \right]_n^\infty < 0.0001$

$\frac{(2n+1)^{-5}}{10} \Big|_n^\infty < 0.00001$

$0 + \frac{(2n+1)^{-5}}{10} < 0.00001$

$\frac{1}{10(2n+1)^5} < \frac{1}{100000}$

$10(2n+1)^5 > 100000$

$(2n+1)^5 > 10000$

$2n+1 > 6.3096$

$2n > 5.3096$

$n > 2.6548$

$n = 3$

$S_3 = (2(1)+1)^{-6} + (2(2)+1)^{-6} + (2(3)+1)^{-6}$

$= \boxed{0.00144}$

11.4 : comparison (written)

5. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^3+1}$ [converges]

use : c) $\dfrac{n}{n^3+1} \leq \dfrac{1}{n^2}$

6. $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ [diverges]

use : c) $\dfrac{n}{n^2+1} \geq \dfrac{1}{2n}$