A Rationale for Preference Reversal

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Ever since the formulation by John von Neumann and Oskar Morgenstern of a set of axioms of rational choice under uncertainty, a number of situations have been identified in which there are significant and repeated violations of one or more of those basic axioms. A comprehensive survey of relevant evidence has been provided by Paul Schoemaker (1982), together with an outline of various attempts that have been made to explain the observed behavior.

One such attempt was our 1982 article, and was given the name *regret theory*. We showed that several behavioral patterns which contradict standard expected utility theory (the "common consequences effect" or Allais' paradox, the "common ratio effect," and the "isolation effect" in two-stage gambles) are predicted by regret theory; and other patterns, such as the "reflection effect" and simultaneous gambling and insurance, if not firmly predicted, are at least consistent with regret theory.

However, a number of problematical cases were not explicitly treated in our earlier paper. One such case, the phenomenon of preference reversal, was the subject of an article by David Grether and Charles Plott (1979). This phenomenon, first reported by Sarah Lichtenstein and Paul Slovic (1971) and by Harold Lindman (1971), takes the following general form: asked to make a direct choice between gamble A and gamble B, an individual states a preference for A; but asked to consider the two gambles separately, he places a higher certainty equivalent value, or reservation price, on B.

This phenomenon is particularly interesting because, as Grether and Plott pointed out, with such reversals it appeared that "...no optimization principles of any sort lie behind even the simplest of human choices" (p. 623). The purpose of this paper is to suggest that this is not so, and that regret theory may provide the optimization principles which explain the asymmetric pattern of choice observed by Grether and Plott, and others.

I. An Outline of Regret Theory

The essential notion underlying regret theory is that people tend to compare their actual situations with the ones they would have been in, had they made different choices in the past. If they realize that a different choice would have led to a better outcome, people may experience the painful sensation of regret; if the alternative would have led to a worse outcome, they may experience a pleasurable sensation we call "rejoicing." When faced with new choice situations, people remember their previous experiences and form expectations about the rejoicing and regret that the present alternatives might entail. They then take these expectations into account when making their decisions. We model all this as follows.

In any pairwise choice, individuals are regarded as choosing between pairs of *actions* denoted A_i and A_k, where each action is an n-tuple of state-contingent *consequences*. The consequence of choosing A_i if the jth state occurs is denoted x_{ij}; and the probability of the jth state occurring is given as p_j.

It is proposed that, for any individual, there exists a *choiceless* utility function, $C(.)$, unique up to an increasing linear transformation, which assigns a real-valued utility index number to every conceivable consequence. The value $C(x_{ij})$ is written as c_{ij}, and represents the utility an individual would derive if the consequence were experienced other than as a result of choice, for example, if the consequence were imposed in some way.

The choice an individual faces is either to accept A_i and simultaneously reject A_k, or else to accept A_k and simultaneously reject A_i. Suppose he contemplates accepting A_i

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and rejecting A_k. Should the jth state occur, he will experience x_{ij} but simultaneously miss out on x_{kj}. So in addition to c_{ij}, he may also derive some increment or decrement of utility due to rejoicing or regret: an increment if $c_{ij} > c_{kj}$; a decrement if $c_{ij} < c_{kj}$.

In the simplest restricted form of the model, it is proposed that regret-rejoicing be represented by the function $R(.)$, where the degree of regret-rejoicing depends only on the difference $(c_{ij} - c_{kj})$. We assume that $R(.)$ is strictly increasing and three times differentiable, and that $R(0)=0$. On this basis, the *modified utility* that results if A_i is chosen and A_k is rejected and the jth state occurs can be written as m_{ij}^k where

$$(1) \qquad m_{ij}^k = c_{ij} + R(c_{ij} - c_{kj}).$$

We can then define the expected modified utility of accepting A_i and rejecting A_k as E_i^k where

$$(2) \qquad E_i^k = \sum_{j=1}^{n} p_j m_{ij}^k.$$

The proposed choice rule that the individual will prefer A_i, prefer A_k, or be indifferent between them according to whether E_i^k is greater than, less than, or equal to E_k^i, generates the proposition that

$$(3) \quad A_i \succsim A_k \quad \text{iff} \quad \sum_{j=1}^{n} p_j\big[c_{ij} - c_{kj} + R(c_{ij} - c_{kj}) - R(c_{kj} - c_{ij})\big] \gtreqless 0.$$

For compactness, we define a function $Q(.)$ such that for all ξ, $Q(\xi) = \xi + R(\xi) - R(-\xi)$. (Note that for all ξ, $Q(\xi) = -Q(-\xi)$.) Thus

$$(4) \quad A_i \succsim A_k \quad \text{iff} \quad \sum_{j=1}^{n} p_j Q(c_{ij} - c_{kj}) \gtreqless 0.$$

If $Q(.)$ is linear, regret theory predicts choices that are consistent with expected utility theory, but if the function is nonlinear, the predictions of the two theories diverge. If $Q(\xi)$ is assumed to be convex for all $\xi > 0$, regret theory predicts a number of patterns of behavior which, although inconsistent with expected utility theory, have been observed in repeated experiments, including those referred to in the introduction to this paper. We shall now show that under the same assumptions, regret theory can explain the main kind of preference reversal observed by Grether and Plott.

II. Preference Reversal and Regret Theory

Grether and Plott asked individuals to consider pairs of bets where the "P bet" offered a high probability of a relatively small money gain, while the "\$ bet" offered a lower probability of a larger gain. Both bets had the same mean value,[1] but the variance was greater for the \$ bet. All pairs of bets can be described with the following notation.

The P bet offered the dollar consequences y_P and x_P with probabilities Π_P and $1-\Pi_P$; $y_P > 0 > x_P$. The \$ bet offered the consequences $y_\$$ and $x_\$$ with probabilities $\Pi_\$$ and $1-\Pi_\$$; $y_\$ > 0 > x_\$$. In all Grether and Plott's experiments, $\Pi_P > 0.5 \geqslant \Pi_\$$ and $y_\$ > y_P$. In some pairs of bets $x_P > x_\$$, while in others $x_P < x_\$$, but in all cases the difference between the two negative consequences was relatively small; in particular:

$$|x_P - x_\$| < |y_P - y_\$|$$

and

$$|x_P - x_\$| < |y_P - x_\$|.$$

Each individual was asked to state the lowest price at which he would sell each of the bets; we shall denote these two reservation prices by r_P and $r_\$$. He was also at a different time asked to make a direct choice between the P bet and the \$ bet. In 561 cases (74.4 percent) out of 754,[2] individuals placed

[1] In Grether and Plott's experiments, the two bets did not have exactly the same expected money values, but only because consequences were rounded to the nearest dollar and probabilities to the nearest one-thirty-sixth.

[2] These figures are based on Grether and Plott, Tables 5, 6, 8 and 9 (pp. 632–33), but omitting 14 cases where $P \sim \$$, since Grether and Plott gave no information about reservation prices in these cases.

a higher reservation price on the \$ bet, despite the greater variance of those bets. Moreover Grether and Plott reported (p. 632) that the reservation prices for \$ bets were frequently greater than their expected values. These observations can be squared with expected utility theory, but only by dropping the usual assumption of risk aversion.

Much more serious for standard theory, however, was the incidence of preference reversal. In 76 cases (10.1 percent) $P \prec \$$ was combined with $r_P \geqslant r_\$$ and in 205 cases (27.2 percent) $P \succ \$$ was combined with $r_P \leqslant r_\$$.

These observations are completely inconsistent with conventional theory. But now consider the implications of regret theory. Tables 1–3 portray three matrices of state-contingent consequences corresponding to the three choice problems facing an individual for any pair of bets. (The columns in each table represent states of the world, with the probability of each state given above.)

Table 1 represents the choice between retaining the P bet and selling it for the sum r_P (an action we denote by P^*). If r_P is the reservation price, then $P \sim P^*$. Similarly in Table 2, \$* is the action of selling the \$ bet, and if $r_\$$ is the reservation price, then $\$ \sim \$^*$. Table 3 represents the choice between the two bets. This matrix is constructed to correspond with the random device used in Grether and Plott's experiments (see p. 629).

To produce firm predictions, and to highlight the role of regret and rejoicing in our theory, we shall make the additional simplifying assumption that the choiceless utility function $C(.)$ is linear over the relevant range. On that basis, we obtain the following general result.

Let A_i be an action that gives the consequence w with certainty while A_k is an actuarially fair gamble offering $w+u$ with probability Π, and $w-\Pi u/(1-\Pi)$ with probability $1-\Pi$; $1>\Pi>0$ and $u>0$. To simplify notation, we choose a transformation of the linear function $C(.)$ such that $C(\xi)=\xi$ for all ξ. Applying expression (4) and rearranging, we have

$$(5)\quad A_i \succsim A_k \quad \text{iff} \quad Q\left(\frac{\Pi u}{1-\Pi}\right)-\frac{\Pi}{1-\Pi}Q(u) \gtreqless 0.$$

TABLE 1

	Π_P	$1-\Pi_P$
P	y_P	x_P
P^*	r_P	r_P

TABLE 2

	$\Pi_\$$	$1-\Pi_\$$
\$	$y_\$$	$x_\$$
\$*	$r_\$$	$r_\$$

TABLE 3

	$\Pi_\$$	$\Pi_P-\Pi_\$$	$1-\Pi_P$
P	y_P	y_P	x_P
\$	$y_\$$	$x_\$$	$x_\$$

With $Q(\xi)$ convex for all $\xi>0$, it follows that $A_i \succsim A_k$ iff $\Pi \gtreqless 0.5$. Thus, under the special assumption that $C(.)$ is linear, regret theory predicts that individuals will accept small-stake large-prize fair gambles ($\Pi<0.5$) but reject large-stake small-prize fair gambles ($\Pi>0.5$).

In the light of this result, consider Table 1. If r_P were equal to the expected dollar value of the P bet (which we shall write as $V(P)$) then P (seen in relation to P^*) would be a large-stake small-prize fair gamble (recall that $\Pi_P>0.5$). Thus $P \prec P^*$. So to achieve $P \sim P^*$, which must be the case if r_P is a reservation price, it is necessary that $r_P<V(P)$. Now consider Table 2. In this case $\Pi_\$ \leqslant 0.5$, so to achieve $\$ \sim \$^*$ it is necessary that $r_\$ \geqslant V(\$)$. But $V(P)=V(\$)$. Combining these results, $r_p<r_\$$. Thus, under the special assumption of linearity, regret theory explains both the tendency for individuals to place a higher reservation price on the \$ bet, and the tendency for that price to exceed the expected value of the bet.

Now consider the choice between P and \$ represented in Table 3. Applying expression

(4) again, and retaining our assumption about $C(.)$, we have

$$(6)\qquad P \gtreqless \$ \text{ iff } \Pi_\$ Q(y_P - y_\$) + (\Pi_p - \Pi_\$)Q(y_P - x_\$) + (1-\Pi_P)Q(x_P - x_\$) \gtreqless 0.$$

Because both bets have the same expected dollar value, we know that

$$(7)\quad \Pi_\$(y_P - y_\$) + (\Pi_P - \Pi_\$)(y_P - x_\$) + (1-\Pi_P)(x_P - x_\$) = 0.$$

We also know that

(i) $1 > \Pi_\$,\ \Pi_P - \Pi_\$,\ 1-\Pi_P > 0$;
(ii) $y_P - y_\$ < 0$;
(iii) $y_P - x_\$ > 0$;
(iv) $|x_P - x_\$| < |y_P - y_\$|, |y_P - x_\$|$.

Given all this, the assumption that $Q(\xi)$ is convex for all $\xi > 0$ entails the following results (see the Appendix for proof):

Case I: $P \prec \$$ if $x_P - x_\$ > 0$ and $y_\$ - y_P > y_P - x_\$$
Case II: $P \prec \$$ if $x_P - x_\$ < 0$ and $\Pi_P - \Pi_\$ \geqslant 0.5$
Case III: $P \succ \$$ if $x_P - x_\$ < 0$ and $y_\$ - y_P < y_P - x_\$$
Case IV: $P \succ \$$ if $x_P - x_\$ > 0$ and $\Pi_\$ \geqslant 0.5$

These four cases are not exhaustive—there are other cases in which no firm prediction can be made—but they cover all six pairs of gambles in Grether and Plott's experiments; four of those pairs were cases for which $P \prec \$$ is predicted, and two were cases where the prediction is $P \succ \$$.[3] But the most significant feature is that in cases III and IV, preference reversal is predicted, and the form of reversal—$P \succ \$$ and $r_P < r_\$$—is the one most frequently observed. Of course, if we relax the assumption that $C(.)$ is linear, we can no longer make such clear-cut predictions. Nevertheless, the common form of preference reversal remains consistent with regret theory and with its presumption that individuals are rational, optimizing agents.

[3]See their Table 2. Case I covers pairs of gambles numbers 1, 4 and 5; Case II covers pair number 2; Case III covers pairs numbers 3 and 6.

Appendix

To simplify notation, let:

$$\Pi_1 \equiv \Pi_\$ \qquad a_1 \equiv y_P - y_\$$$
$$\Pi_2 \equiv \Pi_P - \Pi_\$ \qquad a_2 \equiv y_P - x_\$$$
$$\Pi_3 \equiv 1 - \Pi_P \qquad a_3 \equiv x_P - x_\$.$$

Then the problem is to determine the sign of

$$(\text{A1})\quad \Pi_1 Q(a_1) + \Pi_2 Q(a_2) + \Pi_3 Q(a_3),$$

given that

$$(\text{A2})\qquad \Pi_1 a_1 + \Pi_2 a_2 + \Pi_3 a_3 = 0$$

$$(\text{A3})\qquad 1 > \Pi_1, \Pi_2, \Pi_3 > 0$$

$$(\text{A4})\qquad a_2 > 0 > a_1$$

$$(\text{A5})\qquad |a_1|, |a_2| > |a_3|$$

$$(\text{A6})\qquad Q(\xi) = -Q(-\xi),$$

and $Q(\xi)$ is convex for all $\xi > 0$.

Case I: $-a_1 > a_2 > a_3 > 0$

In this case, a_1 is the only a_i to be negative, and is also the a_i with the largest absolute value. Thus, because of (A2) and (A6), (A1) must be negative, that is, $P \prec \$$.

Case III: $-a_2 < a_1 < a_3 < 0$

In this case, a_2 is the only positive a_i, and is the a_i with the largest absolute value. Thus (A1) must be positive, that is, $P \succ \$$.

Case II: $a_1, a_3 < 0 < a_2;\ \Pi_2 \geqslant 0.5$

since $Q(\xi)$ is concave for all $\xi < 0$:

$$(\text{A7})\quad \Pi_1 Q(a_1) + \Pi_3 Q(a_3) < (\Pi_1 + \Pi_3) Q\left(\frac{\Pi_1 a_1 + \Pi_3 a_3}{\Pi_1 + \Pi_3}\right).$$

Consider a choice between (i) the certainty of a consequence of zero and (ii) the consequences a_2, $(\Pi_1 a_1 + \Pi_3 a_3)/(\Pi_1 + \Pi_3)$ with probabilities Π_2, $\Pi_1 + \Pi_3$. Since (ii) is an actuarially fair gamble with the probability of winning, Π_2, $\geqslant 0.5$, we know from the result concerning fair gambles given in the paper that

$$\text{(A8)}\quad \Pi_2 Q(a_2) + (\Pi_1 + \Pi_3) Q\left(\frac{\Pi_1 a_1 + \Pi_3 a_3}{\Pi_1 + \Pi_3}\right) \leqslant 0.$$

Combining (A7) and (A8):

$$\text{(A9)}\quad \Pi_1 Q(a_1) + \Pi_2 Q(a_2) + \Pi_3 Q(a_3) < 0,$$

that is, $P \prec \$$.

Case IV: $a_2, a_3 > 0 > a_1;\ \Pi_1 \geqslant 0.5$

A similar argument to that given for Case II above establishes that (A1) is positive, that is, $P \succ \$$.

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