Preference Reversal and Temporal Discounting by Optimizing Growth Rates

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Abstract

An important question in economics is how people evaluate payments in the future. The standard phrasing of the problem is in part psychological: the value we attach to a future payment is the dollar value of the payment discounted by a factor whose functional form is determined by our subjective psychology and whose (objective) argument is how long we have to wait for the payment. The functional form is called the "discounting function", in practice commonly exponential or hyperbolic. Here we present an interpretation of these forms in terms of growth rates. A payment in the future, we posit, is often viewed as a growth rate of wealth averaged over the time until the payment. Choosing the greatest multiplicative growth rate is mathematically equivalent to exponential discounting, while maximising the additive growth rate is equivalent to hyperbolic discounting. Multiplicative and additive processes are important models of wealth evolution, corresponding approximately to unearned and earned income. Other growth processes result in different discounting functions.

Keywords: Decision theory, Hyperbolic discounting, Ergodicity economics

1 Introduction

Preference reversal (PR) is a behavioral phenomenon documented during the past half a century in many studies in economics and psychology (Lichtenstein and Slovic, 1971; Lindman, 1971; Grether and Plott, 1979; Loomes and Sugden, 1983; Tversky, Slovic and Kahneman, 1990; Ainslie, 1992; Laibson, 1997). It takes various forms in different contexts. In its original psychological context (Tversky, 1969; Lichtenstein and Slovic, 1971) it refers to the intransitivity in decision making under uncertainty. It also refers to the phenomenon in which a decision maker changes mind between two options in time.

Observed PR phenomena puzzled economics, leading to various explanations and theories. One theory that gives rise to PR is hyperbolic discounting (Ainslie, 1992; Sozou, 1998; Laibson, 1997), suggesting that the valuation of choices falls hyperbolically in time. This means that decision makers may favor earlier guaranteed reward over higher later reward, in a way that is inconsistent with standard economic theory. Hyperbolic discounting has been established as a plausible explanation for PR. Yet, the dynamically inconsistent preferences it induces had challenged standard economic theory (Laibson, 1997; Starmer, 2000; Thaler, 2016). This questions some of the basic axioms of expected utility theory, which predicts exponential discounting, *i.e.* valuation of choices that falls exponentially in time.

Rubinstein (2003) suggested that the same experiments used as evidence for supporting the validity of hyperbolic discounting, can also be used to reject it under different axioms. In addition, various behavioral explanations for hyperbolic discounting have been given in the economic literature. One approach is to place the conditions on the information of decision makers. Sozou (1998); Dasgupta and Maskin (2005) suggested that a decision maker is learning over time, which allows for PR. This approach implicitly assumes constructivist rationality similar to that of Smith (2003). In the most basic sense, the methodological approach is to posit the cognitive situation of the agent and to deduce his discounting rule.

The reasoning behind the model of Sozou (1998), for example, is that agents do not know the hazard rate of an event and learn about the hazard rate over time. The logic behind this is that the agents use Bayesian updating to gradually learn over time, the longer an event does not occur, the more likely it is that it will not occur (decreasing hazard rate). Using this approach, it is shown that an exponential distribution yields hyperbolic discounting.

On the other hand, Dasgupta and Maskin (2005) assume that the agent knows that an event will occur for certain but it is unclear when. The mechanics behind the model are that since an event will occur at some future time, the closer we are to that future time the more certain one of the events will occur very soon. This is because it is initially assumed that the probability of early realizations is the same for both gambles. This, in turn, means that the chance for early realization is more valuable for larger payouts than smaller payouts. They later extend their result to say that the density is not the same but that the probability of one event increases relatively more over time which allows for a wider class of hazard functions. This provides a model for how hyperbolic discounting can describe PR under uncertainty.

This paper takes a different approach. Our model consists of a decision maker, who chooses between two known and different payoffs to be received at known and different times by comparing the growth rates of total wealth associated with each option. The model is further specified by assumptions about the wealth dynamics of the decision maker and the time frame of the decision. In some specifications, the model produces forms of discounting – including hyperbolic – which predict PR. In another specification, standard exponential discounting under multiplicative growth is recovered, which does not predict PR. Thus, we propose that a model that assumes a growth rate maximizing decision maker under various assumptions is consistent with a wide range of experimental evidence.

This approach is similar to Radner (1998) – he assumed that firms maximize their survival rate, which is a special case of maximizing the growth rate. To see why we note that if a strategy results in non-survival, then the growth rate for it will be zero. XXX OP: zero growth rate means maintaining status quo, not death... XXX He further shows that firms that maximize the probability of survival, will outcompete those which maximize profits and hence, in the long run there will only be survival maximizing firms. XXX OP: not sure this is true, but I don't know what the terms mean. I can easily imagine a situation where I can guarantee survival at zero growth, and a little risk gets a big reward, which may well lead to outperformance despite the risk of non-survival.XXX

The main contribution of this paper is to shed new light on the possible explanations for PR and hyperbolic discounting, while demonstrating this can be achieved without being inconsistent with the standard exponential discounting. The importance of these findings lies in the absence of rationality criteria – the same model and the same criteria can produce different types of discounting. We stress that the importance lies in specifying the dynamics of the problem in question.

The paper also contributes to the growing branch of ergodicity economics (Peters and Gell-Mann, 2016; Berman, Peters and Adamou, 2017; Peters and Adamou, 2018), which proposes an alternative to mainstream decision theories, such as expected utility theory and prospect theory, namely that agents maximize the growth of their resources averaged over time. This joins recent evidence on the effect changes in the dynamics of wealth have on decision makers under uncertainty (Meder et al., 2019).

The paper is organized as follows. Section 2 lays out our model and the basic setup of the problem we are addressing. In Section 3 we present different specifications for the problem in question. We describe how a decision maker will discount payoffs in each specification under our model, giving rise to preference reversal. We conclude in Section 4.

2 Model

Our model consists of a decision maker at time t_0 choosing between two future cash payments, one earlier than the other, whose amounts and payment times are known with certainty. The two options are:

a) an earlier payment of Δx_a at time t_a; and

b) a later payment of $\Delta x_b > \Delta x_a$ at time $t_b > t_a$.

We have confined our attention to $\Delta x_b > \Delta x_a$ because we assume a larger and earlier payment is trivially always preferred. We also assume that the decision maker knows his net wealth at time t_0, which we denote by $x(t_0)$. In general, $x(t)$ denotes the net wealth of the decision maker at time t. This setup is illustrated in Fig. 1.

We note that in this setup there is no uncertainty in the payoffs or in the times in which they are realized. Thus, there is no risk.

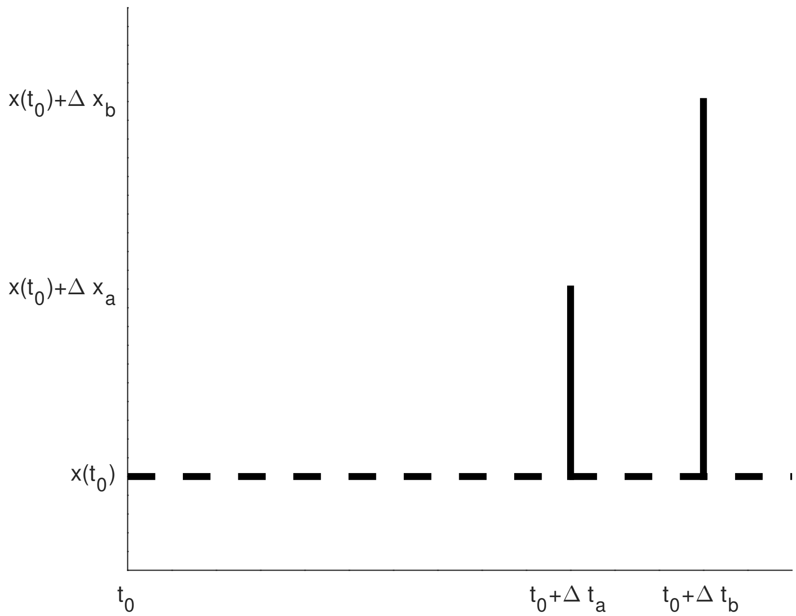

Figure 1: The basic setup of the model. A decision maker faces a choice at time t_0 between option a, which guarantees a payment of Δx_a at time t_a, and option b, which guarantees a payment of $\Delta x_b > \Delta x_a$ at time $t_b > t_a$.

This setup corresponds to a standard question that arises in the context of temporal discounting, *e.g.* "would you prefer to receive \$100 tomorrow or \$200 in a month's time?" Despite its apparent simplicity, answering this question requires additional assumptions. Or, put another way, the problem is underspecified. One extra assumption needed concerns the dynamics under which the decision-maker's wealth grows. Often it is assumed that wealth grows exponentially, compounding continuously at a constant riskless rate like funds in a savings account. Another assumption concerns the time frame of the decision, specifically whether a decision-maker accepting the earlier payment at t_a is free immediately to make his next decision, or whether he must wait until the later time t_b (or, indeed, some other time) before the decision can be repeated. Such assumptions are needed to permit computation of the decision maker's maximand – in our model, the growth rate of his wealth – so that the options can be compared quantitatively.

We will describe four different specifications of this basic setup. In each we will calculate the growth rates, g_a and g_b, of wealth associated with options a and b. The decision maker prefers the option whose growth rate is larger.

We will also infer the discount factor (DF) from this analysis. This is the multiplicative factor, δ, by which the later payout, Δx_b, must be multiplied to equal the earlier payout, Δx_a, when the payout amounts and times are such that the decision maker is indifferent between the two options. In symbols,

$$\delta \equiv \left.\frac{\Delta x_a}{\Delta x_b}\right|_{g_a = g_b}. \tag{2.1}$$

As we show below, this setup predicts decisions equivalent to hyperbolic and exponential discounting under different specifications. Some specifications of the model predict preference reversal. Our model differs from many standard models in the literature by assuming that decision makers maximize the growth rate of their wealth, rather than the expected change in their utility.

3 Results

3.1 Specification

We begin by describing four different specifications for our basic setup. Each specifies two aspects necessary to quantify the growth rate of wealth: the time frame of the decision; and the dynamics under which wealth evolves.

3.1.1 Time frame

The time frame is a key aspect, often left unspecified in similar setups in the literature. Consider the following scenarios:

1. Dana, the real estate developer, loves to work and always wants to keep busy with her building projects, she always gets paid at their completion. Dana has a choice between a project that lasts three months and a project that lasts six months.
2. Every year, Nate the Naval officer must go for either a three month long mission or a six month long mission. He is given the choice at the beginning

of every year(both missions finish before the end of the year). He is paid right after his mission is completed.

In the first scenario, the time frame depends on the choice made. We call this the *elastic* time frame because Dana is more flexible to pursue other opportunities if she chooses the shorter project. On the other hand, if she chooses the longer project, it locks her in a for a longer time period, which means it also changes when she will have another choice.

In the second scenario, the important element to note is that no matter which choice is made, it will not affect future choices, said otherwise, the time frame is independent of the choice, it is *fixed*. That is, the timing of Nate's next choice is not affected by his decision.

In our model, we must choose the time period over which the growth rates of wealth in each option are computed. We can choose it to be the time period associated with each payment, *i.e.* $t_a - t_0$ for option a and $t_b - t_0$ for option b. This specification corresponds to Dana's situation, the elastic time frame specification. Or we can choose it always to be the longer time period, $t_b - t_0$, resembling Nate's dilemma, the fixed time specification.

For convenience, we denote the two periods between the decision time and the known future payment times as $\Delta t_a \equiv t_a - t_0$ and $\Delta t_b \equiv t_b - t_0$. Additionally, we denote the known delay between the two payments as $D \equiv t_b - t_a$.

3.1.2 Wealth dynamics

The wealth dynamics can also take different forms. A standard assumption would be that wealth grows exponentially in time, assuming a risless rate r. We label this dynamic as multiplicative. This dynamic corresponds to investing wealth in income-generating assets, in which the income is proportional to the amount invested. This is the dynamic traditionally assumed in temporal discounting, and when present values are calculated for future expected payouts. In this case it is also assumed that the payout itself is re-invested at the risk-free rate.

Another possible form is additive dynamics. Under this dynamic wealth grows linearly in time, with a rate k, and it is not invested in income-generating assets. It is equivalent to assuming a flow of wealth at some rate, *i.e.* labor income. In this case, there is essentially no re-investment of the payout – the income generated by this dynamic is not proportional to wealth as in the multiplicative dynamics.

The definition of the wealth growth rate differs for the different dynamics. The growth rate between time $t+\Delta t$ and t under additive dynamics is $\frac{x(t+\Delta t)-x(t)}{\Delta t}$ and under multiplicative dynamics it is $\frac{\log x(t+\Delta t)-\log x(t)}{\Delta t}$, see also (Peters and Gell-Mann, 2016; Peters and Adamou, 2018).

We will discuss the four specifications, as illustrated in Fig. 2. In each case we will: compute the growth rates g_a and g_b associated with each option; compare them to determine the conditions under which each option is preferred; elicit the form of temporal discounting equivalent to our decision model; and, finally, determine whether PR is predicted.

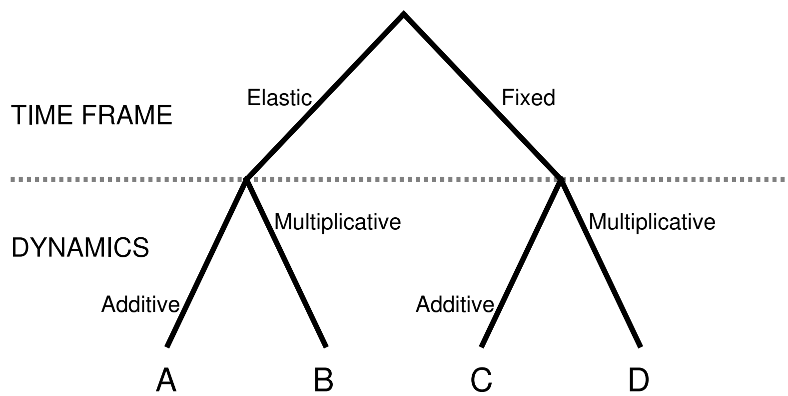

Figure 2: The four model specifications, determined by specifying a time frame and wealth dynamics. The labels A, B, C, and D, are used for the different cases.

3.2 Case A – Elastic time frame with additive dynamics

Specification: the time frame for computing the growth rate is time to the chosen payout; and the wealth dynamics are additive (with growth rate k). We begin by writing down the final wealth under the two options, evaluated at $t_0+\Delta t_a$ and $t_0+\Delta t_b$ respectively:

$$x(t_0+\Delta t_a) \quad = x(t_0)+\Delta x_a+k\Delta t_a\,; \tag{3.1}$$

$$x(t_0+\Delta t_b) \quad = x(t_0)+\Delta x_b+k\Delta t_b\,. \tag{3.2}$$

The growth rates are:

$$g_a = \frac{x(t_0 + \Delta t_a) - x(t_0)}{\Delta t_a} = \frac{\Delta x_a}{\Delta t_a} + k\,; \tag{3.3}$$

$$g_b = \frac{x(t_0 + \Delta t_b) - x(t_0)}{\Delta t_b} = \frac{\Delta x_b}{\Delta t_b} + k\,. \tag{3.4}$$

It follows that the criterion $g_a > g_b$ is

$$\frac{\Delta x_a}{\Delta t_a} > \frac{\Delta x_b}{\Delta t_b}\,. \tag{3.5}$$

This criterion suggests that under this specification, the only thing that matters to the decision maker is the payout rate of each option with respect to the reference point in time, t_0. The same payouts would translate into different payout rates for each option with a different reference point in time. This, in turn, would allow for preference reversal. This is illustrated in Fig. 3.

It shows that the payout rate of option b is higher than that of option a, which makes it preferable. This holds for some time. However, at some point in time, this changes and the payout rate of option a would be higher. Assuming that initially option $g_b > g_a$, there will always be a point in time in which PR will occur. If time τ elapsed from t_0, than the updated payout rates of options a and b are $\frac{\Delta x_a}{\Delta t_a - \tau}$ and $\frac{\Delta x_b}{\Delta t_b - \tau}$, respectively. The reversal will occur when these are equal, or

$$\tau_{\text{reversal}} = \frac{\Delta t_a \Delta x_b - \Delta t_b \Delta x_a}{\Delta x_b - \Delta x_a}\,. \tag{3.6}$$

This case not only demonstrates PR, but also hyperbolic discounting, *i.e.* that the discount factor changes hyperbolically in time.

The discount factor, δ, is the factor by which the later payout, Δx_b, must be multiplied to equal the earlier payout, Δx_a, under indifference between options a and b. Practically, it can be found by considering the equality of growth rates g_a and g_b, which is the condition for indifference. δ is typically presented as a function of the time difference between options. In our setup this would be $\mathcal{E} \equiv \Delta t_b - \Delta t_a$.

Now that we have the hyperbolic DF in terms of the delay, $\Delta t_b - \Delta t_a$, and the time

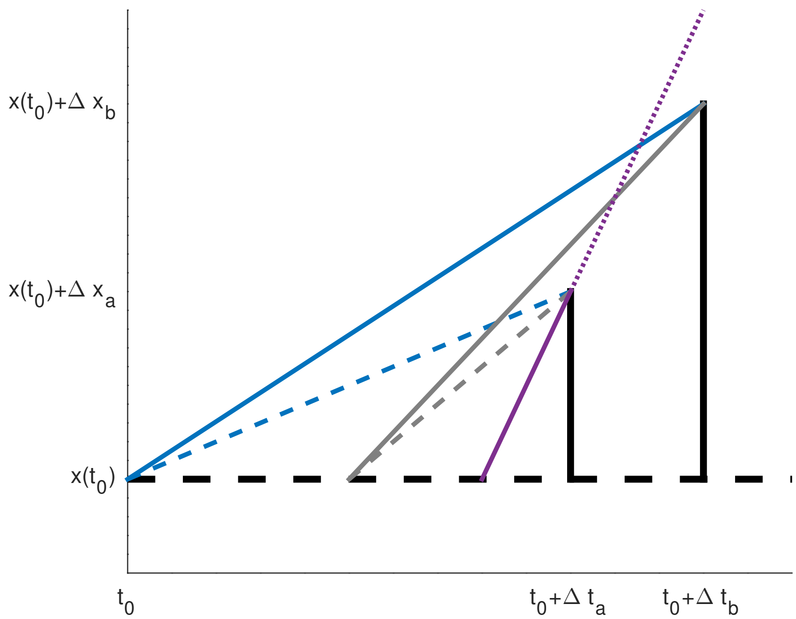

Figure 3: An illustration of preference reversal in case A. Initially, option b is preferable. It is reflected in the slopes of the blue lines. The solid blue line shows the payout rate of option b and the dashed blue line shows the payout rate of option a. Even after some time, option b is still preferable, as reflected by the difference in the slopes of the gray lines (solid for option b, dashed for a). At some point option a becomes preferable, as the payout rate surpasses that of option b. This is demonstrated in the purple line. XXXOP something is not right with this figure. I like the different colors for different times. Perhaps let's have one case preferences 1, one case equanimity, one case preferences 2.XXX

to the first payout, Δt_a, as

$$\delta = \frac{1}{1+\frac{\Delta t_b - \Delta t_a}{\Delta t_a}} = \frac{1}{1+kD}, \tag{3.7}$$

it would be much clearer to present PR as a consequence of changing the time to the first payout rather than the reference time. Then $t_0 \to t_0 + \Delta t_a$ is simply $\Delta t_a \to 0$, which is also expressible as $k \to \infty$. This minor reframing will require some changes below, including to the figure, because we would treat t_0 as fixed and Δt_a as movable. Do we need $\mathcal{E}$? The letter has no intuitive meaning (to me) and perhaps we can get away without relabelling $\Delta t_b - \Delta t_a$. If not, D for delay or W for wait might be better. XXXOP: not sure – I like the physical time t_0 changing. What happens in real life is that time ticks on and we get swept towards t_a, it's not that we sit still in time, and someone tunes t_a. Agree about $\mathcal{E}$ – better letter?XXX

In case A we therefore get

$$\delta_A = \frac{\Delta x_a}{\Delta x_b} = \frac{\Delta t_a}{\Delta t_b} = \frac{1}{1 + \frac{1}{\Delta t_a}\mathcal{E}} \,. \tag{3.8}$$

This is exactly hyperbolic discounting with degree of discounting $\kappa = \frac{1}{\Delta t_a}$.

We note also that, under additive dynamics, the background growth rate, k, of the decision maker's wealth does not appear in the decision criterion. This is because wealth growth under additive dynamics is not affected by exogenous cash flows: the gain $k\Delta t$ over period Δt occurs regardless of what else happens. This contrasts with multiplicative dynamics, where early cash flows can be reinvested to increase gains from growth.

3.3 Case B – Elastic time frame with multiplicative dynamics

Specification: the time frame for computing the growth rate is time to the chosen payout; and the wealth dynamics are multiplicative (with growth rate r). We follow the same steps as in case A. Wealth evolves to:

$$x\left(t_0 + \Delta t_a\right) = x\left(t_0\right) e^{r\Delta t_a} + \Delta x_a \tag{3.9}$$

$$x\left(t_0 + \Delta t_b\right) = x\left(t_0\right) e^{r\Delta t_b} + \Delta x_b \,. \tag{3.10}$$

The corresponding growth rates are:

$$g_a = \frac{1}{\Delta t_a} \log\left(\frac{x(t_0+\Delta t_a)}{x(t_0)}\right) = \frac{1}{\Delta t_a} \log\left(e^{r\Delta t_a} + \frac{\Delta x_a}{x(t_0)}\right) \tag{3.11}$$

$$g_b = \frac{1}{\Delta t_b} \log\left(\frac{x(t_0+\Delta t_b)}{x(t_0)}\right) = \frac{1}{\Delta t_b} \log\left(e^{r\Delta t_b} + \frac{\Delta x_b}{x(t_0)}\right) . \tag{3.12}$$

This setting also displays PR, although no closed-form expression for τ_{reversal} can be derived like in case A. It is clear that PR occurs, since when τ approaches $t_0 + \Delta t_a$, g_a will increase indefinitely, while $\log\left(e^{r\Delta t_a} + \frac{\Delta x_a}{x(t_0)}\right)$ remains positive.

Similarly, the discount factor δ_B cannot be derived explicitly without approximations. Yet, we could assume that $\Delta x_a \ll x\left(t_0\right) e^{r\Delta t_a}$ and $\Delta x_b \ll x\left(t_0\right) e^{r\Delta t_b}$, *i.e.*

that the payouts are much smaller than the wealth at the time the payouts are realized. Then, using the first order expansion $\log(x+\delta) \approx \log(x) + \delta/x$, we obtain

$$\delta_B = \frac{\Delta x_a}{\Delta x_b} \approx \frac{\Delta t_a}{\Delta t_b} \cdot e^{-r(\Delta t_b - \Delta t_a)} = \frac{1}{1 + \frac{1}{\Delta t_a}\mathcal{E}} e^{-r\mathcal{E}} \,, \tag{3.13}$$

which is a mixed case of hyperbolic and exponential discounting.

3.4 Case C – Fixed time frame with additive dynamics

Now we assume additive dynamics again, but with a fixed time frame – the outcomes of both choices are compared at Δt_b. The growth rate of option b would be the same as in case A, since it was already evaluated at $t_0 + \Delta t_b$. In this case, option a leads to

$$x(t_0 + \Delta t_b) = x(t_0) + \Delta x_a + k\Delta t_b \,, \tag{3.14}$$

and the corresponding growth rate would be

$$g_a = \frac{1}{\Delta t_b}\left(x(t_0 + \Delta t_b) - x(t_0)\right) = \frac{\Delta x_a}{\Delta t_b} + k \,. \tag{3.15}$$

Recall that $g_b = \frac{\Delta x_b}{\Delta t_b} + k$. Since we have assumed $\Delta x_b > \Delta x_a$, it is clear that option b would always be preferable to option a. This is a trivial case – assuming additive dynamics (or no dynamics) the only thing that matters is the payout size, if we compare the options at the same time, or if they repeat at fixed time windows. In this case, the discount factor δ_C cannot be defined, since the larger final payout is always preferred and the indifference condition cannot be satisfied.

3.5 Case D – Fixed time frame with multiplicative dynamics

Finally, we assume multiplicative dynamics and a fixed time frame. This is the specification that corresponds to the standard assumptions usually considered in temporal discounting – that wealth is continuously compounding at the risk-free rate and that payouts are re-invested at this rate.

The growth rate of option b would be the same as in case B, since it was already evaluated at $t_0+\Delta t_b$ ($g_b=\frac{1}{\Delta t_b}\log\left(e^{r\Delta t_b}+\frac{\Delta x_b}{x(t_0)}\right)$). In this case, option a leads to

$$x\left(t_0+\Delta t_b\right)=x\left(t_0\right)e^{r\Delta t_b}+\Delta x_a e^{r\left(\Delta t_b-\Delta t_a\right)}\,, \tag{3.16}$$

and the corresponding growth rate would be (recall that $\mathcal{E}\equiv\Delta t_b-\Delta t_a$)

$$g_a=\frac{1}{\Delta t_b}\log\left(\frac{x\left(t_0+\Delta t_b\right)}{x\left(t_0\right)}\right)=\frac{1}{\Delta t_b}\log\left(e^{r\Delta t_b}+\frac{\Delta x_a e^{r\mathcal{E}}}{x\left(t_0\right)}\right). \tag{3.17}$$

It follows that the criterion $g_a>g_b$ is

$$\Delta x_a e^{r\mathcal{E}}>\Delta x_b\,. \tag{3.18}$$

The discount factor can be then easily calculated by considering the equality of the growth rates. In this case we get $\Delta x_a e^{r\mathcal{E}}=\Delta x_b$ and therefore

$$\delta_D=\frac{\Delta x_a}{\Delta x_b}=e^{-r\mathcal{E}}\,, \tag{3.19}$$

which is the standard exponential discounting result. In other words, if it is possible to re-invest the reward given at time Δt_a so that it surpasses the alternative reward given at time Δt_b, option a is preferable over option b, and vice versa.

4 Discussion

This paper describes a model in which a decision maker chooses between two payoffs realized at different points in time by comparing the growth rate of wealth associated with each option.

The main finding is that discounting can be interpreted as growth rate optimization. We find that depending on the wealth dynamics assumed by the decision maker, growth rate optimization can be equivalent to hyperbolic discounting, in which case it predicts preference reversal. It can also be equivalent to a mixed case of hyperbolic and exponential discounting, which also implies preference reversal. Under multiplicative dynamics, we find that growth-rate optimization reproduces standard exponential discounting. This reveals the standard form of discounting as just one of many possible forms of discounting, each of which is optimal under a different type of wealth growth.

A fundamental question about the model is why would someone maximize growth instead of expectation value of utility? This, of course, differs from the traditional framework for analyzing decisions in economics, usually taking it as axiomatic that people maximize utility. Yet, this is only one way of studying optimal choice under different conditions. At the same time it is important to question where does the utility optimizing mechanism come from?

The framework we used here has an answer – evolutionary mechanisms. That is, some utility functions survive better than others. The utility functions that will survive over the long run are the growth utility functions.

This paper discusses discounting from a theoretical perspective. An important complementary step of this research would be comparing the theoretical predictions of the results to empirical and experimental results. In particular, the predicted discount factors and discount rates can be compared to results from controlled experiments. This is planned for future work.

An additional extension is the inclusion of risk. The standard explanations to hyperbolic discounting consist of a behavioral response to risk (Sozou, 1998; Dasgupta and Maskin, 2005), while here we showed that hyperbolic discounting can be observed even in the absence of uncertainties in the payouts or in their timing. Adding uncertainty to our model might create additional forms of temporal discounting, which might be more realistic and closer to empirical evidence.

Given the framework here, risk can be interpreted in two ways. Either it represents the actual probability of an arrival of a choice or it represents the belief about the environment. The corresponding discount rate then be some weighted average between the discount rates presented here.

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