Preference Reversal and Temporal Discounting by Optimizing Growth Rates

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Abstract

An important question in economics is how people evaluate payments in the future. The standard phrasing of the problem is in part psychological: the value we attach to a future payment is the dollar value of the payment discounted by a factor whose functional form is determined by our subjective psychology and whose (objective) argument is how long we have to wait for the payment. The functional form is called the "discounting function", in practice commonly exponential or hyperbolic. Here we present an interpretation of these forms in terms of growth rates. A payment in the future, we posit, is often viewed as a growth rate of wealth averaged over the time until the payment. Choosing the greatest multiplicative growth rate is mathematically equivalent to exponential discounting, while maximising the additive growth rate is equivalent to hyperbolic discounting. Multiplicative and additive processes are important models of wealth evolution, corresponding approximately to unearned and earned income. Other growth processes result in different discounting functions.

Keywords: Decision theory, Hyperbolic discounting, Ergodicity economics

1 Introduction

Preference reversal (PR) is a behavioral phenomenon documented during the past half a century in many studies in economics and psychology (Lichtenstein and Slovic, 1971; Lindman, 1971; Grether and Plott, 1979; Loomes and Sugden, 1983; Tversky, Slovic and Kahneman, 1990; Ainslie, 1992; Laibson, 1997). It takes various forms in different contexts. In its original psychological context (Tversky, 1969; Lichtenstein and Slovic, 1971) it refers to the intransitivity in decision making under uncertainty. It also refers to the phenomenon in which a decision maker changes his mind between two options as time passes.

Observed PR phenomena puzzled economics, leading to various explanations and theories. One theory that gives rise to PR is hyperbolic discounting (Ainslie, 1992; Sozou, 1998; Laibson, 1997), suggesting that the valuation of choices falls hyperbolically in time. This is in contrast to the standard assumption of exponential discounting in economic theory, where no such reversal occurs. Hyperbolic discounting has been established as a plausible explanation for PR. Yet, the dynamically inconsistent preferences it induces have challenged standard economic theory (Laibson, 1997; Starmer, 2000; Thaler, 2016).

Rubinstein (2003) suggested that the same experiments supporting hyperbolic discounting, can also be used to reject it under different axioms. In addition, various behavioral explanations for hyperbolic discounting have been given in the economic literature. One approach is to place the conditions on the information of decision makers. Sozou (1998); Dasgupta and Maskin (2005) suggested that a decision maker is learning over time, which allows for PR. This approach implicitly assumes constructivist rationality similar to that of Smith (2003). In the most basic sense, the methodological approach is to posit the cognitive situation of the agent and to deduce his discounting rule.

The reasoning behind the model of Sozou (1998), for example, is that agents do not know the hazard rate of an event and learn about the hazard rate over time. The logic behind this is that the agents use Bayesian updating to gradually learn over time, the longer an event does not occur, the more likely it is that it will not occur (decreasing hazard rate). Using this approach, it is shown that an exponential distribution yields hyperbolic discounting.

On the other hand, Dasgupta and Maskin (2005) assume that the agent knows that an event will occur for certain but it is unclear when. The mechanics behind the model are that since an event will occur at some future time, the closer we are to that future time the more certain one of the events will occur very soon. This is because it is initially assumed that the probability of early realizations is the same for both gambles. This, in turn, means that the chance for early realization is more valuable for larger payouts than smaller payouts. They later extend their result to say that the density is not the same but that the probability of one event increases relatively more over time which allows for a wider class of hazard functions. This provides a model for how hyperbolic discounting can describe PR under uncertainty.

This paper takes a different approach. Our model consists of a decision maker, who chooses between two known and different payoffs to be received at known and different times by comparing the growth rates of total wealth associated with each option. The model is further specified by assumptions about the wealth dynamics of the decision maker and the time frame of the decision. In some specifications, the model produces forms of discounting – including hyperbolic – which predict PR. In another specification, standard exponential discounting under multiplicative growth is recovered, which does not predict PR. Thus, we propose that a model that assumes a growth rate maximizing decision maker under various assumptions is consistent with a wide range of experimental evidence.

The main contribution of this paper is to shed new light on the possible explanations for PR and hyperbolic discounting, while demonstrating this can be achieved without being inconsistent with the standard exponential discounting. The importance of these findings lies in the absence of rationality criteria – the same model and the same criteria can produce different types of discounting. We stress that the importance lies in specifying the dynamics of the problem in question.

The paper also contributes to the growing branch of ergodicity economics (Peters and Gell-Mann, 2016; Berman, Peters and Adamou, 2017; Peters and Adamou, 2018), which proposes an alternative to mainstream decision theories, such as expected utility theory and prospect theory, namely that agents maximize the growth of their resources averaged over time. This joins recent evidence on the effect changes in the dynamics of wealth have on decision makers under uncertainty (Meder et al., 2019).

The paper is organized as follows. Section 2 lays out our model and the basic setup of the problem we are addressing. In Section 3 we present different specifications for the problem in question. We describe how a decision maker will discount payoffs in each specification under our model, giving rise to preference reversal. We conclude in Section 4.

2 Model

Our model consists of a decision maker at time t_0 choosing between two future cash payments, one earlier than the other, whose amounts and payment times are known with certainty. The two options are:

a. an earlier payment of $\Delta x_a > \$0$ at time $t_a > t_0$; and

b. a later payment of $\Delta x_b > \Delta x_a$ at time $t_b > t_a$.

We have confined our attention to $\Delta x_b > \Delta x_a$ because we assume a larger and earlier payment is trivially always preferred. We also assume that the decision maker knows his net wealth at time t_0, which we denote by $x(t_0)$. In general, $x(t)$ denotes the net wealth of the decision maker at time t. This setup is illustrated in Fig. 1.

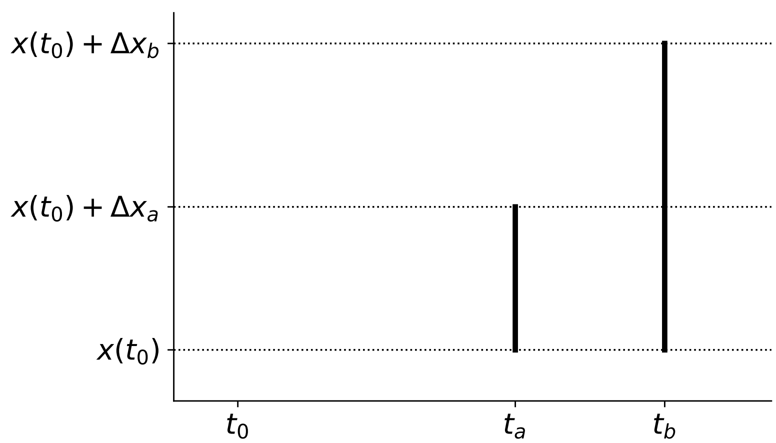

Figure 1: The basic setup of the model. A decision maker faces a choice at time t_0 between option a, which guarantees a payment of Δx_a at time t_a, and option b, which guarantees a payment of $\Delta x_b > \Delta x_a$ at time $t_b > t_a$.

We note that in this setup there is no uncertainty in the payoffs or in the times in which they are realized. Thus, there is no risk.

This setup corresponds to a standard question that arises in the context of temporal discounting, *e.g.* "would you prefer to receive \$100 tomorrow or \$200 in a month's time?" Despite its apparent simplicity, answering this question requires additional assumptions. Or, put another way, the problem is underspecified. One extra assumption needed concerns the dynamics under which the decision-maker's wealth grows. Often it is assumed that wealth grows exponentially, compounding continuously at a constant riskless rate like funds in a savings account. Another assumption concerns the time frame of the decision, specifically whether a decision-maker accepting the earlier payment at t_a is free immediately to make his next decision, or whether he must wait until the later time t_b (or, indeed, some other time) before the decision can be repeated. Such assumptions are needed to compute decision-maker's maximand – in our model, the growth rate of his wealth – so that the options can be compared quantitatively.

We will describe four different specifications of this basic setup. In each we will calculate the growth rates, g_a and g_b, of wealth associated with options a and b. The decision maker prefers the option whose growth rate is larger.

We will also infer the discount factor (DF) from this analysis. This is the multiplicative factor, δ, by which the later payout, Δx_b, must be multiplied to equal the earlier payout, Δx_a, when the payout amounts and times are such that the decision maker is indifferent between the two options. In symbols,

$$\delta \equiv \left. \frac{\Delta x_a}{\Delta x_b} \right|_{g_a = g_b}, \tag{2.1}$$

i.e. the ratio of payments under the constraint that the growth rates of wealth are equal.

As we show below, this setup predicts decisions equivalent to hyperbolic and exponential discounting under different specifications. Some specifications of the model predict preference reversal. Our model differs from many standard models in the literature by assuming that decision makers maximize the growth rate of their wealth, rather than the expected change in their utility.

3 Results

3.1 Specification

We begin by describing four different specifications for our basic setup. Each specifies two aspects necessary to quantify the growth rate of wealth: the time frame of the decision; and the dynamics under which wealth evolves.

3.1.1 Time frame

The time frame is a key aspect, often left unspecified in similar setups in the literature. Consider the following scenarios:

1. Dana, the real estate developer, loves to work and always wants to keep busy with her building projects, she always gets paid at their completion. Dana has a choice between a project that lasts three months and a project that lasts six months.

2. Every year, Nate the Naval officer must go for either a three month long mission or a six month long mission. He is given the choice at the beginning of every year(both missions finish before the end of the year). He is paid right after his mission is completed.

In the first scenario, the time frame depends on the choice made. We call this the *elastic* time frame because Dana is more flexible to pursue other opportunities if she chooses the shorter project. On the other hand, if she chooses the longer project, it locks her in a for a longer time period, which means it also changes when she will have another choice.

In the second scenario, the important element to note is that no matter which choice is made, it will not affect the timing of future choices. Said otherwise, the time frame is independent of the choice, so we say it is *fixed*.

In our model, we must choose the time period over which the growth rates of wealth in each option are computed. We can choose it to be the time period associated with each payment, *i.e.* $t_a - t_0$ for option a and $t_b - t_0$ for option b. This specification corresponds to Dana's situation, the elastic time frame specification. Or we can choose it always to be the longer time period, $t_b - t_0$, resembling Nate's dilemma, the fixed time specification.

3.1.2 Wealth dynamics

The wealth dynamics can also take different forms. A standard assumption would be that wealth grows exponentially in time, at a riskless rate r. We label this dynamic as multiplicative. This dynamic corresponds to investing wealth in income-generating assets, in which the income is proportional to the amount invested. This is the dynamic traditionally assumed in temporal discounting, and when present values are calculated of future expected payouts. In this case it is also assumed that the payout itself is re-invested at the risk-free rate.

Another possible form is additive dynamics. Under this dynamic wealth grows linearly in time, at a rate k, and it is not invested in income-generating assets. It is equivalent to assuming a flow of wealth at some rate, *e.g.* labor income. In this case, there is essentially no re-investment of the payout – the income generated by this dynamic is not proportional to wealth as in the multiplicative dynamics.

The definition of the wealth growth rate differs for the different dynamics. The growth rate between time $t + \Delta t$ and t under additive dynamics is $\frac{x(t+\Delta t)-x(t)}{\Delta t}$ and under multiplicative dynamics it is $\frac{\log x(t+\Delta t)-\log x(t)}{\Delta t}$, see also (Peters and Gell-Mann, 2016; Peters and Adamou, 2018).

We will discuss the four specifications, as illustrated in Fig. 2. In each case we will: compute the growth rates g_a and g_b associated with each option; compare them to determine the conditions under which each option is preferred; elicit the form of temporal discounting equivalent to our decision model; and, finally, determine whether PR is predicted.

3.2 Case A – Elastic time frame with additive dynamics

Specification: the period for computing the growth rate is that between the decision and the chosen payout; and the wealth dynamics are additive, with growth rate k.

We begin by writing down the final wealth under the two options, evaluated at t_a

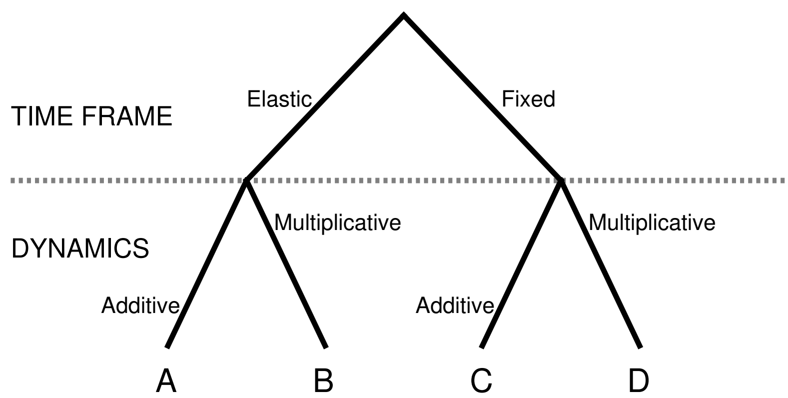

Figure 2: The four model specifications, determined by specifying a time frame and wealth dynamics. The labels A, B, C, and D, are used for the different cases.

and t_b respectively:

$$x_a(t_a) = x(t_0) + \Delta x_a + k(t_a - t_0)\,; \tag{3.1}$$

$$x_b(t_b) = x(t_0) + \Delta x_b + k(t_b - t_0)\,. \tag{3.2}$$

The growth rates are:

$$g_a = \frac{x_a(t_a) - x(t_0)}{t_a - t_0} = \frac{\Delta x_a}{t_a - t_0} + k\,; \tag{3.3}$$

$$g_b = \frac{x_b(t_b) - x(t_0)}{t_b - t_0} = \frac{\Delta x_b}{t_b - t_0} + k\,. \tag{3.4}$$

It follows that the criterion $g_a > g_b$ is

$$\frac{\Delta x_a}{t_a - t_0} > \frac{\Delta x_b}{t_b - t_0}\,. \tag{3.5}$$

This criterion suggests that, under this specification, the only thing that matters to the decision maker is the linear payment rate of each option.

If we treat the payment amounts, Δx_a and Δx_b, and payment times, t_a and t_b, as fixed parameters of the problem, then we can elicit the dependence of the decision on the decision time, t_0. When the payments are far ahead in the future, *i.e.* as $t_0 \to -\infty$, the denominators in the growth rates approach each other and $g_a < g_b$ since we have assumed $\Delta x_a < \Delta x_b$. When the earlier payment is imminent, *i.e.*

as $t_0 \to t_a$, g_a grows without bound while g_b remains finite and so $g_a > g_b$. In other words, as time passes, our decision model under this specification predicts preference reversal from the later, larger payment to the earlier, smaller payment. This is illustrated in Fig. 3.

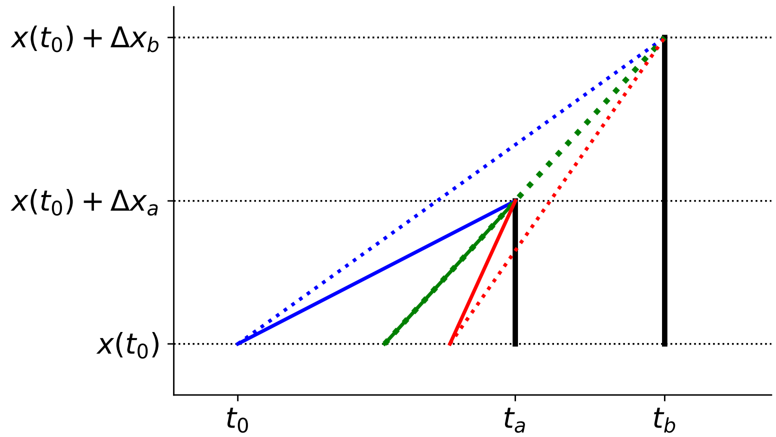

Figure 3: An illustration of preference reversal in case A. Initially, option b is preferable, as reflected in the slopes of the blue lines. The solid blue line shows the payout rate of option a and the dashed blue line shows the payout rate of option b. At a later time t_0^{PR}, both options imply equal growth (green lines), and preference reversal occurs. At later times (red lines) option a is preferable.

We can compute the decision time, t_0^{PR}, at which preference reversal occurs by setting $g_a = g_b$ to give

$$t_0^{\mathrm{PR}} = \frac{\Delta x_b t_a - \Delta x_a t_b}{\Delta x_b - \Delta x_a}. \tag{3.6}$$

We can also find the effective discount factor under this specification. When $g_a = g_b$, we have

$$\delta = \frac{\Delta x_a}{\Delta x_b} = \frac{t_a - t_0}{t_b - t_0} = \frac{1}{1 + \frac{t_b - t_a}{t_a - t_0}}, \tag{3.7}$$

where we have made the final manipulation to express δ in hyperbolic form. We see that the discount factor depends on two time periods: that between decision and the earlier payment, $t_a - t_0$, which we will call the *horizon*; and that between the two payments, $t_b - t_a$, which we which we will call the *delay*.[1] If we define $H \equiv t_a - t_0$

[1] Indeed, the problem is fully specified by these two time periods and the two payment amounts. The actual times, t_0, t_a, t_b, are not needed to specify the problem because, when computing growth rates, only elapsed times matter. The time origin is arbitrary.

and $D \equiv t_b - t_a$, we can write the discount factor as

$$\delta = \frac{1}{1 + D/H}, \tag{3.8}$$

which is expressed in the conventional way as a hyperbolic function of the delay, D. The psychological degree of discounting parameter used in mainstream models is replaced here by $1/H$, the reciprocal of the horizon. As the horizon gets shorter, $1/H$ becomes larger, δ gets smaller, and the later payment becomes less favourable. No knowledge of the decision-maker's psychology is required in this setup, other than that he prefers his wealth to grow faster.

Finally, we note that the background growth rate, k, of the decision-maker's wealth does not appear in the decision criterion. This is because wealth growth under additive dynamics is not affected by exogenous cash flows: the gain $k\Delta t$ over period Δt occurs regardless of other payments received. This contrasts with multiplicative dynamics, where payments can be subjected to the growth process through re-investment.

3.3 Case B – Elastic time frame with multiplicative dynamics

Specification: the time frame for computing the growth rate is time to the chosen payment; and the wealth dynamics are multiplicative, with growth rate r.

We follow the same steps as in case A. Wealth evolves to:

$$x_a\left(t_a\right) = x\left(t_0\right) e^{r(t_a - t_0)} + \Delta x_a\,; \tag{3.9}$$

$$x_b\left(t_b\right) = x\left(t_0\right) e^{r(t_b - t_0)} + \Delta x_b\,. \tag{3.10}$$

The corresponding growth rates are:

$$g_a = \tfrac{1}{t_a - t_0} \log\left(\tfrac{x_a(t_a)}{x(t_0)}\right) = \tfrac{1}{t_a - t_0} \log\left(1 + \tfrac{\Delta x_a}{x(t_0) e^{r(t_a - t_0)}}\right) + r \tag{3.11}$$

$$g_b = \tfrac{1}{t_b - t_0} \log\left(\tfrac{x_b(t_b)}{x(t_0)}\right) = \tfrac{1}{t_b - t_0} \log\left(1 + \tfrac{\Delta x_b}{x(t_0) e^{r(t_b - t_0)}}\right) + r\,. \tag{3.12}$$

This setting displays preference reversal: $g_a < g_b$ for t_0 sufficiently far away from t_a (long horizon); and $g_a > g_b$ for t_0 sufficiently close to t_a (short horizon). No closed-form expression for the reversal time, t_0^{PR}, is available.

Similarly, the discount factor δ cannot be derived explicitly. However, if we assume small payments relative to wealth, *i.e.* $\Delta x_a \ll x(t_0)\, e^{r(t_a-t_0)}$ and $\Delta x_b \ll x(t_0)\, e^{r(t_b-t_0)}$, then, setting $g_a = g_b$ and using the first-order approximation $\log(1+\epsilon) \approx \epsilon$ for $\epsilon \ll 1$, we get

$$\delta = \frac{\Delta x_a}{\Delta x_b} \approx \frac{(t_a - t_0)e^{r(t_a-t_0)}}{(t_b - t_0)e^{r(t_b-t_0)}} = \frac{e^{r(t_a-t_b)}}{1 + \frac{t_b-t_a}{t_a-t_0}}\,. \tag{3.13}$$

Using the previous definitions of H and D, we can write this as

$$\delta \approx \frac{e^{-rD}}{1 + D/H}\,, \tag{3.14}$$

which is a hybrid of hyperbolic and exponential discounting. We note again that only the elapsed times, H and D, appear in the discount factor. However, that the background wealth growth rate, r, no longer cancels out when dynamics are multiplicative, as does k when they are additive.

3.4 Case C – Fixed time frame with additive dynamics

Now we assume additive dynamics as in case A, but with a fixed time frame so that the outcomes of both choices are compared at t_b. The wealths evolve to:

$$x_a(t_b) = x(t_0) + \Delta x_a + k(t_b - t_0)\,; \tag{3.15}$$

$$x_b(t_b) = x(t_0) + \Delta x_b + k(t_b - t_0)\,. \tag{3.16}$$

The growth rates are:

$$g_a = \frac{x_a(t_b) - x(t_0)}{t_b - t_0} = \frac{\Delta x_a}{t_b - t_0} + k\,; \tag{3.17}$$

$$g_b = \frac{x_b(t_b) - x(t_0)}{t_b - t_0} = \frac{\Delta x_b}{t_b - t_0} + k\,. \tag{3.18}$$

Note that the wealth and its growth rate under option b are the same as in case A, since they were already evaluated at t_b there.

Since we have assumed $\Delta x_b > \Delta x_a$, option b is always preferred to option a. This is a trivial case – if we assume additive wealth dynamics and comparing the growth rates at the same time (or assuming repetition over fixed periods), then the only thing that matters to the decision-maker is payout size. In this case, the discount factor δ cannot be defined, since the later, larger payout is always preferred and the indifference condition is never satisfied.

3.5 Case D – Fixed time frame with multiplicative dynamics

Finally, we assume multiplicative dynamics and a fixed time frame. This is the specification that corresponds to the standard assumptions usually considered in temporal discounting – that wealth is continuously compounding at the risk-free rate and that payouts are re-invested at this rate.

The chief difference from case B is that the earlier payment, Δx_a, if chosen, is treated as growing exponentially from t_a to t_b. The wealths evolve from t_0 to t_b as follows:

$$x_a(t_b) = x(t_0)\, e^{r(t_b-t_0)} + \Delta x_a e^{r(t_b-t_a)}\,; \tag{3.19}$$

$$x_b(t_b) = x(t_0)\, e^{r(t_b-t_0)} + \Delta x_b\,. \tag{3.20}$$

The corresponding growth rates are:

$$g_a = \frac{1}{t_a-t_0}\log\left(\frac{x_a(t_a)}{x(t_0)}\right) = \frac{1}{t_b-t_0}\log\left(1+\frac{\Delta x_a e^{r(t_b-t_a)}}{x(t_0)e^{r(t_b-t_0)}}\right)+r \tag{3.21}$$

$$g_b = \frac{1}{t_b-t_0}\log\left(\frac{x_b(t_b)}{x(t_0)}\right) = \frac{1}{t_b-t_0}\log\left(1+\frac{\Delta x_b}{x(t_0)e^{r(t_b-t_0)}}\right)+r\,. \tag{3.22}$$

Note that the evolution of wealth under option b is the same as in case B.

The criterion $g_a > g_b$ is actually very simple, since only the second term in the logarithm is different and so only this must be compared. Thus, $g_a > g_b$ if

$$\Delta x_a e^{r(t_b-t_a)} > \Delta x_b\,, \tag{3.23}$$

or, in terms of the delay, if

$$\Delta x_a e^{rD} > \Delta x_b\,. \tag{3.24}$$

The discount factor is similarly easily expressed by setting the growth rates to be equal. Then we get $\Delta x_a e^{rD} = \Delta x_b$ and

$$\delta = \frac{\Delta x_a}{\Delta x_b} = e^{-rD}\,, \tag{3.25}$$

which is the standard exponential discounting result. The interpretation is straightforward: if it is possible to re-invest the earlier payment such that, by the time of the later payment, it will exceed the later payment amount, then option a is preferable to option b (and *vice versa*). Note that, with this specification, the horizon is irrelevant. All that matters is the payment amount after possible re-investment.

4 Discussion

This paper describes a model in which a decision maker chooses between two payoffs realized at different points in time by comparing the growth rate of wealth associated with each option.

The main finding is that discounting can be interpreted as growth rate optimization. We find that depending on the wealth dynamics assumed by the decision maker, growth rate optimization can be equivalent to hyperbolic discounting, in which case it predicts preference reversal. It can also be equivalent to a mixed case of hyperbolic and exponential discounting, which also implies preference reversal. Under multiplicative dynamics, we find that growth-rate optimization reproduces standard exponential discounting. This reveals the standard form of discounting as just one of many possible forms of discounting, each of which is optimal under a different type of wealth growth.

A fundamental question about the model is why would someone maximize growth instead of expectation value of utility? This, of course, differs from the traditional framework for analyzing decisions in economics, usually taking it as axiomatic that people maximize utility. Yet, this is only one way of studying optimal choice under different conditions. At the same time it is important to question where does the utility optimizing mechanism come from?

The framework we used here has an answer – evolutionary mechanisms. That is, as time passes some utility functions dominate over others. The utility functions that do so over the long run are growth-optimal. Technically, they are ergodicity mappings: maximizing the expected value of such a utility function is guaranteed to maximize wealth over time.

This paper discusses discounting from a theoretical perspective. An important complementary step of this research would be comparing the theoretical predictions of the results to empirical and experimental results. In particular, the predicted discount factors and discount rates can be compared to results from controlled experiments. This is planned for future work.

An additional extension is the inclusion of risk. The standard explanations to hyperbolic discounting consist of a behavioral response to risk (Sozou, 1998; Dasgupta and Maskin, 2005), while here we showed that hyperbolic discounting can be observed even in the absence of uncertainties in the payouts or in their timing. Adding uncertainty to our model might create additional forms of temporal discounting, which might be more realistic and closer to empirical evidence.

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