THE ROYAL SOCIETY

On hyperbolic discounting and uncertain hazard rates

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The value of a future reward should be discounted where there is a risk that the reward will not be realized. If the risk manifests itself at a known, constant hazard rate, a risk-neutral recipient should discount the reward according to an exponential time-preference function. Experimental subjects, however, exhibit short-term time preferences that differ from the exponential in a manner consistent with a hazard rate that falls with increasing delay. It is shown here that this phenomenon can be explained by uncertainty in the underlying hazard. The time-preference function predicted by this analysis can be calculated by means of either (i) a direct superposition method, or (ii) Bayesian updating of the expected hazard rate. The observed hyperbolic time-preference function is consistent with an exponential prior distribution for the underlying hazard rate. Sensitivity of the predicted time-preference function to variation in the probability distribution of the underlying hazard rate is explored.

Keywords: hyperbolic discounting; time preference; preference reversal; hazard rate; survival analysis; Bayesian updating

1. TIME PREFERENCES

The systematic preference for an immediate reward over a future reward of the same magnitude has been observed in humans and other animals (Logan 1965; Rachlin & Green 1972; Tobin & Logue 1994). An immediate reward of smaller magnitude may be preferred to a delayed reward of greater magnitude. One plausible reason for this behaviour is the risk that a future reward will not be realized. A forager on a patch, for example, may be interrupted (Houston *et al.* 1982). If the reward is a future reproductive opportunity, there is a risk of the animal dying before the reproductive act (Iwasa *et al.* 1984; Houston & McNamara 1986; Candolin 1998). In human culture, a promise of a future reward may be broken (Rotter 1954).

The present value which an animal places on some given reward when the reward is due after a delay of τ can be expressed as a time-preference function $v(\tau)$. If the animal is risk-neutral, this will satisfy

$$v(\tau) = v_0\, s(\tau), \tag{1}$$

where v_0 is the value of an immediate reward, assumed here to be free of risk, and $s(\tau)$ is a survival function specifying the probability that the reward can be realized after a delay of τ.

It is convenient to define a hazard rate $h(\tau)$. This is the risk per unit time of the hazard occurring, given that it has not occurred before τ. The risk that a reward which is still available after a delay of τ is lost between a delay of τ and of $\tau+\mathrm{d}\tau$ is therefore $h(\tau)\ \mathrm{d}\tau$.

The absolute risk per unit time of the hazard occurring at a delay of τ is given by $-\mathrm{d}s/\mathrm{d}\tau$. $h(\tau)$ is obtained by dividing this absolute risk by the probability that the hazard has not already occurred before τ, i.e. dividing by the survival function $s(\tau)$. This yields

$$h(\tau) = -\frac{1}{s}\frac{\mathrm{d}s}{\mathrm{d}\tau}. \tag{2}$$

(a) *Time preferences under a constant hazard*

If the hazard rate h has a constant value λ for all τ, equation (2) gives

$$s(\tau) = \exp(-\lambda\tau). \tag{3}$$

And hence, from equation (1),

$$v(\tau) = v_0 \exp(-\lambda\tau). \tag{4}$$

Exponential time preferences of the form in equation (4) have been widely used in economic models of consumer behaviour (see, for example, Hirshleifer (1970), Auerbach & Kotlikoff (1987), or Varian (1996)).

(b) *Observed time preferences*

In experiments, animals exhibit time preferences which are not exponential, but instead fall off with delay at a decreasing proportional rate. A review of early results may be found in Ainslie (1975). Mazur (1987) employed an adjusting-delay titration procedure on pigeons, and found that a good empirical fit to the data is given by a hyperbolic function

$$v(\tau) = \frac{v_0}{1+k\tau}, \tag{5}$$

where k is a constant, with a larger value of k denoting more rapid discounting. Richards *et al.* (1997) obtained similar results using an adjusting-amount titration

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procedure on rats. Rachlin *et al.* (1991) also obtained results consistent with hyperbolic discounting for human subjects choosing between hypothetical rewards. Another human study of this type (Green *et al.* 1994) found age-dependent deviation from this hyperbolic form, but the overall finding from experiments is that behaviour is better explained by a hyperbolic, rather than an exponential, time-preference function.

Therefore, there is a need for a good biological theory of why non-exponential discounting occurs. Kacelnik (1997) describes a hypothesis which does not invoke risk, but is based instead on opportunity cost. It proposes that (i) animals try to maximize their reward rate per unit time, and (ii) a reward is treated as part of a repeated sequence. The time spent waiting for a delayed reward is therefore time wasted. If the reward has a value V and handling time T, and delay is denoted (as before) by τ, the rate of reward gain per unit time is $V/(T+\tau)$, which has a hyperbolic form.

This rate maximization theory (i) does not apply to one-off rewards, and (ii) requires that the animal is not engaged in other productive activities while waiting for a delayed reward. These considerations limit its wider applicability to discounting phenomena at large. Rate maximization could still be plausible as a specific explanation for results of delay-reward animal experiments. In an experimental test, however, the predicted maximization of expected reward rate was not observed (Bateson & Kacelnik 1996).

Can a theory based on risk explain non-exponential discounting? The survival function corresponding to the hyperbolic time-preference function, in equation (5), is given by

$$s(\tau) = \frac{1}{1 + k\tau}. \tag{6}$$

A survival function of this form, which falls off more slowly than the exponential, implies a hazard rate which falls with increasing delay (Kagel *et al.* 1986; Green & Myerson 1996). An explanation for why the hazard rate should fall in this way is required.

2. SURVIVAL FUNCTIONS UNDER AN UNCERTAIN HAZARD

We consider a potential reward which is subject to a hazard at a constant but unknown rate λ, drawn from a known probability distribution $f(\lambda)$. λ will now be referred to as the underlying hazard rate. The survival function may be obtained from the prior distribution for λ, either by direct superposition or by Bayesian updating. It will be shown that hyperbolic time preferences are consistent with an exponential prior distribution for the hazard rate.

(a) *Direct superposition method*

Begin with the case where the underlying hazard rate can take one of a discrete set of n values. Let the probability of an underlying hazard rate of λ_1 be p_1, that of λ_2 be p_2, and so on up to a probability of p_n for an underlying hazard rate of λ_n . The survival function is then given by

$$s(\tau) = p_1 \exp(-\lambda_1\tau) + p_2 \exp(-\lambda_2\tau) \ldots + p_n \exp(-\lambda_n\tau). \tag{7}$$

Suppose now that the underlying hazard rate is determined by some continuous probability distribution $f(\lambda)$. Generalizing equation (7) to the continuous case, the survival function is given by

$$s(\tau) = \int_0^\infty f(\lambda) \exp(-\lambda\tau) \mathrm{d}\lambda. \tag{8}$$

The integral operation described by expression (8) is known as a Laplace transform. Thus, the survival function is given by the Laplace transform of the prior distribution of the underlying hazard rate. This is a convenient result as Laplace transforms are widely employed in mathematical problems involving differential equations (Rainville 1963), and their properties are well understood.

(b) *Bayesian updating method*

The basis for this approach is as follows: information that the hazard has not materialized after a certain delay can be used to update an estimate of λ. If the hazard does not materialize as the delay increases, it will seem progressively more likely that λ is relatively small.

The prior probability density function for λ is given by $f(\lambda)$. The event that the hazard has not materialized (and hence that the reward is still available) after a delay τ is denoted by X. Applying Bayes' theorem for a continuous distribution (Lindley 1965), the distribution of λ conditional on event X is given by

$$f(\lambda|x) = \frac{f(\lambda)\, p(X|\lambda)}{\displaystyle\int_0^\infty f(\lambda)\, p(X|\lambda) \mathrm{d}\lambda}. \tag{9}$$

In this expression, $p(X|\lambda)$ denotes the probability of event X for a given value of λ. This is given by the exponential survival function $s(\tau)$ in equation (3), the substitution of which yields

$$f(\lambda|X) = \frac{f(\lambda) \exp(-\lambda\tau)}{\displaystyle\int_0^\infty f(\lambda) \exp(-\lambda\tau) \mathrm{d}\lambda}. \tag{10}$$

The recipient of the potential reward does not know the exact value of the underlying hazard rate. From the recipient's point of view, the hazard risk per unit time will reflect this uncertainty in λ. The recipient's hazard rate $h(\tau)$ is given by a weighted-mean estimate of λ, where the weighting function is given by the conditional distribution for λ specified in equation (10). Hence $h(\tau)$ is equal to the posterior expectation of λ, conditional on the hazard not occurring up to a delay of τ

$$h(\tau) = \frac{\displaystyle\int_0^\infty \lambda f(\lambda|X) \mathrm{d}\lambda}{\displaystyle\int_0^\infty f(\lambda|X) \mathrm{d}\lambda} = \frac{\displaystyle\int_0^\infty \lambda f(\lambda) \exp(-\lambda\tau) \mathrm{d}\lambda}{\displaystyle\int_0^\infty f(\lambda) \exp(-\lambda\tau) \mathrm{d}\lambda}. \tag{11}$$

Expression (11) may also be obtained via the direct superposition method, by substituting equation (8) into equation (2). Thus the direct superposition and Bayesian updating methods are mathematically equivalent.

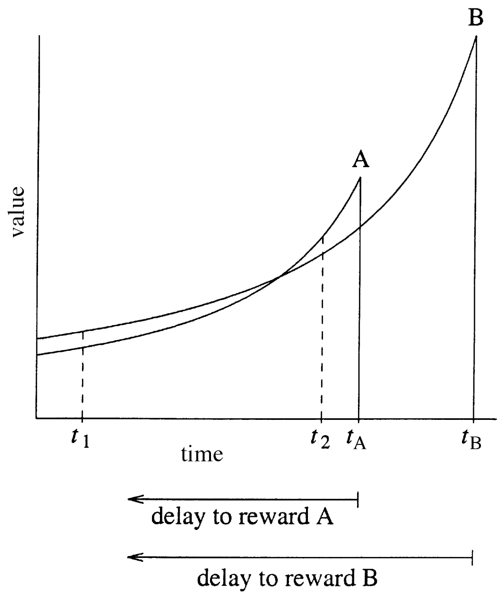

Figure 1. The time-preference reversal effect. The curves show how the value to a recipient of a reward A due at a time t_A and a larger reward B due at a later time t_B change over time. At a very early time t_1, when there is a long delay to both rewards, B is preferred over A. At a later time t_2, when reward A is imminent, it is preferred over B.

(c) *Explaining the hyperbolic time-preference function*

It remains to be shown that there is a prior probability density function for λ which can account for the observed hyperbolic time-preference function. Substituting equation (6) into equation (8), the prior distribution $f(\lambda)$ must satisfy

$$\int_0^\infty f(\lambda)\exp(-\lambda\tau)\mathrm{d}\lambda = \frac{1}{1+k\tau}. \qquad (12)$$

This is solved by

$$f(\lambda) = \frac{1}{k}\exp(-\lambda/k). \qquad (13)$$

Hence, the hyperbolic time-preference function is explained by an uncertain underlying hazard rate, with an exponential prior distribution for λ. The constant k of equation (5) arises from the parameter characterizing this exponential distribution.

3. TIME-PREFERENCE REVERSALS

Non-exponential time-preference curves can cross (Strotz 1956) and consequently the preference for one future reward over another may change with time (Green *et al.* 1981). This is illustrated in figure 1.

Some authors have interpreted this time-preference reversal effect as indicating non-rational time preferences (Ainslie 1975; Strotz 1956). Thus I may appear to be temporally inconsistent if, for example, I prefer the promise of a bottle of wine in three months over the promise of a cake in two months, but I prefer a cake immediately over a promise of a bottle of wine in one month.

There is, however, no inconsistency if I perceive a promised future reward not as a sure thing, but instead as having a probability attached to it. This can be illustrated numerically. Let the intrinsic value of a cake be two units, and that of a bottle of wine three units. Suppose also that cake and wine have an exponential prior probability distribution for the hazard rate λ as given in equation (13) and hence a hyperbolic survival function as given in equation (6), with k=1 month^{-1}. The probability of a reward being available after delays of zero to three months are shown in table 1. The corresponding expected values for a reward of cake or wine are also shown. It can be seen from the table that a cake immediately is worth more than a promise of wine after a month, while a promise of wine after three months is worth more than a promise of cake after two months. So my preferences are indeed consistent with maximizing my expected reward.

Table 1. *Numerical illustration of time-preference reversal*

(The table shows the expected values of rewards of cake or wine which are due after delays ranging from zero to three months. The probabilities are given by the hyperbolic survival function, expression (6) in the main text, with k=1 month^{-1}. In this example, an immediate cake should be preferred over a promised reward of wine after a delay of a month, but a promised reward of wine after a delay of three months should be preferred over a promised reward of cake after a delay of two months.)

delay τ	probability s	expected value of cake $s \times v_{\mathrm{cake}}$	expected value of wine $s \times v_{\mathrm{wine}}$
zero	1	2	3
one month	$\frac{1}{2}$	1	$\frac{3}{2}$
two months	$\frac{1}{3}$	$\frac{2}{3}$	1
three months	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$

intrinsic value of cake: $v_{\mathrm{cake}} = 2$
intrinsic value of wine: $v_{\mathrm{wine}} = 3$

4. SENSITIVITY ANALYSIS

Figure 2 shows various possible prior probability distributions for the underlying hazard rate. The case where there is no uncertainty in λ is represented schematically by a dashed line. For comparability, the horizontal scaling is such that all the distributions have the same mean value of λ.

Figure 3 shows the corresponding survival functions, with time-scales chosen so that each curve has a median survival time of one. It can be seen that curves C and E are intermediate between the exponential and hyperbolic curves. Curve D falls off even more rapidly than the hyperbolic for small delays, and even more slowly for large delays.

For figure 4, the hyperbolic curve B is as for figure 3. For each of the other curves, the time-scaling parameter has been chosen so as to minimize the vertical least-squares error between the curve and the hyperbolic over the range zero to five. The sum-of-squares difference between curve C and the hyperbolic is 20% of that between curve A (the exponential) and the hyperbolic. The sum-of-squares difference between curve D and the hyperbolic is 44% of that between curve A and the

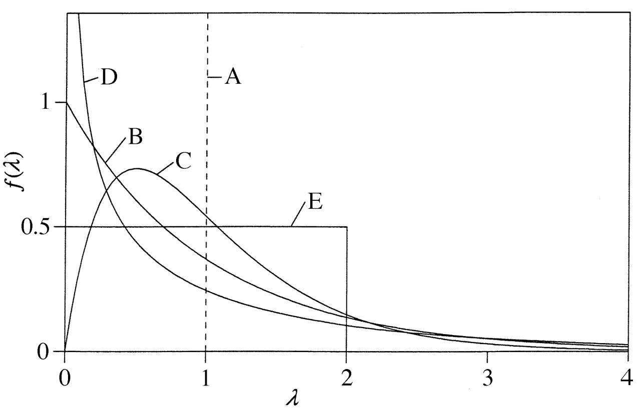

Figure 2. Alternative prior distributions for the underlying hazard rate λ. The horizontal scalings are such that each distribution has the same mean value of one. A: schematic representation (dotted line) of a Dirac delta function, corresponding to prior certainty in λ. B: exponential prior. C: gamma/Erlang prior with $c=2$. D: gamma prior with $c=0.5$. E: uniform prior.

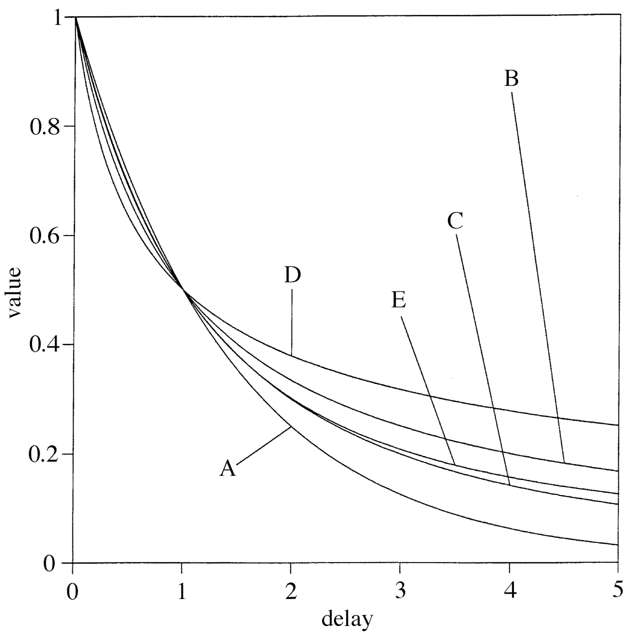

Figure 3. Time-preference curves: common median survival time. A: exponential curve, corresponding to prior with no uncertainty. B: hyperbolic curve, corresponding to exponential prior. C: curve corresponding to gamma/Erlang prior with $c=2$. D: curve corresponding to gamma prior with $c=0.5$. E: curve corresponding to uniform prior.

hyperbolic. The sum-of-squares difference between curve E and the hyperbolic is 14% of that between curve A and the hyperbolic.

These results indicate that moderate change to the prior distribution for the hazard rate gives a survival function which remains reasonably close to hyperbolic. Thus the time-preference function predicted by this theory, which is proportional to the survival function, has modest sensitivity to changes in the prior distribution of the hazard rate.

The analysis above has shown that the observed hyperbolic time-preference function is consistent with an exponential prior distribution for the hazard rate $f(\lambda)$. How sensitive is the predicted time-preference function to changes in $f(\lambda)$? The exponential distribution is a special case of the gamma distribution: it is therefore instructive to examine the predicted time-preference function under more general gamma distribution priors for the hazard rate. A uniform prior distribution for the hazard rate will also be considered.

(a) *Gamma prior distribution for the hazard rate*

The gamma distribution is a general class of distribution which may be written as

$$f(\lambda) = \frac{(\lambda/b)^{c-1}\exp(-\lambda/b)}{b\Gamma(c)}, \tag{14}$$

where $\Gamma(c)$ is the gamma function. The parameter c, known as the shape parameter, can take any positive value. If c is an integer, the distribution may be referred to as an Erlang distribution; this describes the sum of c independent exponential variates (Evans *et al.* 1993).

At $c=1$, the distribution is exponential: the probability density is highest at $\lambda=0$, and falls off with increasing λ. When c is increased above one, the distribution becomes mound shaped. As c is increased further, the distribution tends toward a normal distribution, with an increasingly small ratio of standard deviation over mean. In the limit of c approaching infinity, the distribution tends to a Dirac delta function, representing a hazard rate of c/b with no uncertainty. At the other extreme, when c is less than one, the distribution is even more skewed than the exponential distribution.

Substituting equation (14) into equation (8), the survival function is given by the Laplace transform of the gamma distribution. This is a standard result (Evans *et al.* 1993),

$$s(\tau) = \frac{1}{(b\tau+1)^c}. \tag{15}$$

An exponential prior distribution ($c=1$) yields a hyperbolic survival function, as previously derived. Letting c tend to infinity, keeping c/b constant, the method of limits yields an exponential survival function. For values of c greater than one but less than infinity, the shape of the survival function is intermediate between hyperbolic and exponential. When c is less than one, the survival function falls off even more slowly than the hyperbolic for large delays.

(b) *Uniform prior distribution for the hazard rate*

Let λ be uniformly distributed in the range 0 to k:

$$\begin{aligned} f(\lambda) &= \frac{1}{b} \quad (0 \leqslant \lambda \leqslant b) \\ f(\lambda) &= 0 \quad (\text{otherwise}). \end{aligned} \tag{16}$$

Substituting into equation (8) yields

$$s(\tau) = \frac{1}{b\tau}[1-\exp(-b\tau)]. \tag{17}$$

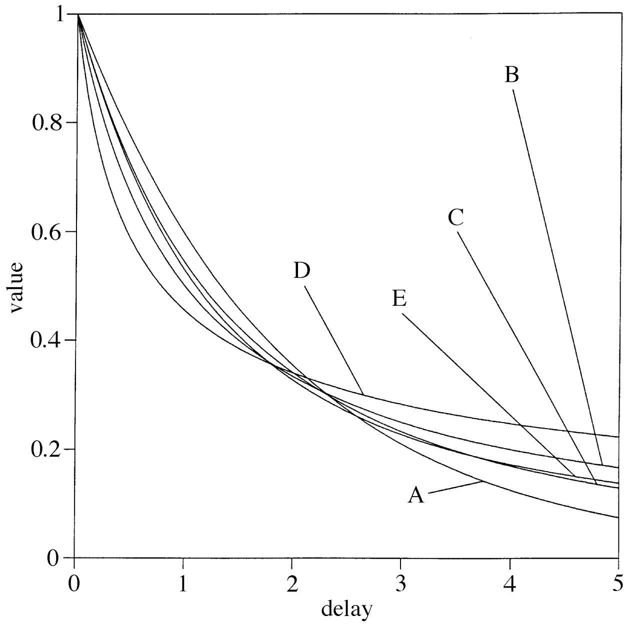

Figure 4. Time-preference curves: least-squares fit to the hyperbolic. A: exponential curve, corresponding to prior with no uncertainty. B: hyperbolic curve, corresponding to exponential prior. C: curve corresponding to gamma/Erlang prior with $c=2$. D: curve corresponding to gamma prior with $c=0.5$. E: curve corresponding to uniform prior.

5. DISCUSSION

This paper has considered discounting as a function of the risk that a delayed reward will not be received. It has shown that hyperbolic discounting is explained by an uncertain hazard rate with an exponential prior distribution. The survival function for the reward, and hence the time-preference function, may be calculated either by direct superposition or by Bayesian updating. The direct superposition method is more straightforward, yielding a survival function which is given by the Laplace transform of the distribution for the hazard rate. The Bayesian updating formulation is, however, conceptually useful; it illustrates how, if a hazard does not materialize after a certain delay, this information may be used to update an estimate of the hazard rate.

Bayesian methodology is based on updating probabilities in the light of events. Can this be applied to experimental scenarios in which a choice between alternative rewards and delays must be made in advance? In these circumstances, a 'true' Bayesian would need to carry out a thought experiment to determine what the estimated survival probability would be after a given delay were experienced. It is not suggested here that animals have such cognitive abilities. The evolution of near-optimal behaviour in a given environment does not require the capacity for conscious optimization on the part of the animal concerned (McNamara & Houston 1980). It may be said that animals act as if they are carrying out either Laplace transforms or Bayesian updating thought experiments, but decision rules are acquired through natural selection.

Are there any ecological processes which could give rise to an exponential distribution for the underlying hazard rate λ? One such process would be where λ itself is proportional to the lifetime of a survival process, with a constant and fixed probability of termination per unit time. Suppose for example that (i) the hazard event in question is a predation event, with a hazard rate proportional to the number of predators in a given locality, and (ii) predators are produced earlier in the season at a constant rate, until some random event with a fixed probability per unit time occurs. Then the prior probability distribution for the number of predators, and hence for the underlying hazard rate for the predation event, will be exponential.

The sensitivity analysis has shown that the predicted time-preference function is not unduly sensitive to the exact form of the prior distribution for the underlying hazard rate. For animals which experience heterogeneous ecological environments, it is not surprising that there should be some uncertainty in underlying hazard rates. The qualitative experimental finding that time preferences fall off more slowly than the exponential is consistent with any arbitrary distribution with non-zero spread for λ. Assessment of the theory will therefore require detailed quantitative analysis of ecological and experimental data.

The rapidity with which an animal discounts a delayed reward, specified by the parameter k in this model, is not presumed to be universally constant; it will depend, through natural selection, on the ecological hazards faced by the animal's ancestors. For example, Tobin & Logue (1994) found that animals with a higher specific metabolic rate appear to be more impulsive, which implies a larger value of k. The theory proposed in this paper can be tested by comparing such behavioural differences with differences in animals' ecological environments.

Future work will compare the predictions of the theory with time-preference data. Specifically, the following merit further study:

(i) Detailed analysis of time-preference data (including new and recent data) to establish more precisely what form of hazard rate distribution is consistent with the data.
(ii) Comparative work on time preferences across species and across rewards within a given species, comparing these data to ecological parameters which are likely to reflect hazard rates.

Finally, it is noted that human discounting behaviour may be affected by learning about the economic environment. For monetary rewards in particular, a choice between a smaller, sooner pay-off and a larger, later pay-off is likely to depend not only on innate time preferences, but also on market interest rates on loans and savings. This could account for the results of Green *et al.* (1994), indicating that as age group increases—from children to young adults to older adults—the discounting of hypothetical monetary rewards becomes progressively closer to the exponential. Studies based on rewards which cannot readily be traded in the economy may reveal people's innate time preferences more closely.

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