The Optimal Scope of the Royalty Base in Patent Licensing

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Abstract

Legal scholars debate the merits of using the total value of the product, as opposed to the value of the component to which the technology contributes, as the base for a royalty in licensing contracts. In this paper we make use of the fact that these two royalty bases are equivalent to using ad valorem and per-unit royalties, respectively. We abstract from implementation and practicability considerations to analyze the welfare implications of the two rules. Ad valorem royalties tend to lead to lower prices, particularly in the context of successive monopolies. They benefit upstream innovators and do not necessarily hurt downstream producers. This benefit increases when there are multiple innovators contributing complementary technologies, as is typical of standard-setting organizations. Ad valorem royalties are even more desirable when enticing upstream investment is optimal. Our findings explain why most licensing contracts include royalties based on the value of the product.

1. Introduction

The licensing of a patented technology is one of the most important sources of revenue for many innovators, particularly when they do not participate in production in the final market. A licensing contract typically includes a royalty payment that comprises two components: a royalty rate and a royalty base. Most attention in the economic literature has been devoted to the optimal determination of the royalty rate. Much less work has been done in studying the scope of the royalty base. This scope can be determined in two principal ways. One option is the value of the sales of the entire product that incorporates the patented technol-

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ogy. Alternatively, the scope of the royalty base can be associated with the value of the component of the infringing product.[1]

The royalty rates associated with these royalty bases are denoted the ad valorem and per-unit rate, respectively. The former consists of a payment comprising a percentage of the value of the sales of the product. The latter corresponds to a constant payment based on the number of units sold. An example can help clarify the relationship between these two royalty bases and rates. Consider a producer that sells a good at a price $p = \$100$. This product comprises a component that embeds the patented technology of an upstream innovator. An ad valorem rate is a percentage s of that price for each unit sold. Therefore, the total income from this royalty corresponds to a portion s of the revenue from the sale of this product, $s \times p \times Q$, where Q is the quantity sold. If instead the royalty applies to the value of the component, say \$10, the royalty rate could be either a percentage of this value, say 20 percent, or a fixed amount per unit sold, in this case $r = \$2$. A percentage of the value of the component and a per-unit royalty are equivalent in this last case, as they do not depend on the price of the final product.[2] Nevertheless, establishing a per-unit royalty rate is easier, since it avoids specifying the value of the component in the contract to set the rate. As a result, the total revenue of the licensor would be in this case the per-unit royalty times the number of units sold, $r \times Q$.

It has been often understood among practitioners and legal scholars that in a world without legal frictions, ad valorem royalties would yield the same market outcomes as per-unit royalties if they were adjusted appropriately.[3] In the previous example, this would imply that an ad valorem royalty of $s = 2$ percent would be equivalent to a per-unit royalty of $r = \$2$. In this paper we show that such an equivalence is false. We find that in most circumstances, ad valorem royalties yield market outcomes that are welfare superior to those resulting from the use of per-unit royalties in terms of both lower prices and stronger incentives for firms to innovate.

We analyze the decision of an upstream innovator that licenses its technology to a pure downstream producer. This innovator holds all the bargaining power when choosing the royalty rate. Abstracting from the incentives to innovate—that is, assuming that all research and development (R&D) has been successfully carried out—we show that ad valorem royalties favor the innovator, whereas per-unit royalties tend to benefit the producer. Importantly, the resulting price in the final market is never higher under ad valorem royalties. The reason is that ad va-

[1] In the legal literature, these royalty bases are called the entire-market-value rule and the apportionment rule, respectively.

[2] In other words, if the value of the component is v, a percentage rate t of this value leads to a royalty payment equal to $t \times v \times Q$, and a per-unit rate r would result in a royalty payment of $r \times Q$. If $r = t \times v$, then the royalty payments are identical. Note that in both cases the royalty payment is independent of p, the price of the final product.

[3] As stated by the US Court of Appeals for the Federal Circuit, "[T]here is nothing inherently wrong with using the market value for the entire product for the infringing component or feature, so long as the multiplier accounts for the proportion of the base represented by the infringing component or feature" (*Lucent Tech., Inc. v. Gateway, Inc.,* 580 F.3d 1301, 1338–39 [Fed. Cir. 2009]).

lorem royalties impose a royalty tax on the downstream markup, which reduces the profitability of price increases—in the example, 2 percent of the increase in price is transferred to the innovator. As a result, they typically make the double-marginalization problem less severe by generating lower distortions in the final market. Only under an isoelastic demand function are prices identical under these licensing schemes. Even in that case, however, ad valorem royalties lead to lower prices when we allow for other realistic features, such as a more balanced allocation of bargaining power between the innovator and the producer.

This insight is intimately related to a classical result in the indirect-taxation literature. Suits and Musgrave (1953), citing even earlier insights in Wicksell (1896), study the least distortionary way to raise revenue from consumption taxes. They show that when there is market power, ad valorem taxes (such as the value-added tax) are preferred to per-unit taxes, as they allow for the same revenue to be raised through lower final prices. The main difference with our case is that in our setup the innovator maximizes profits (rather than minimizes market distortions) but is still interested in lower prices since they generate more sales and, with them, higher revenues.

Such a connection has not been made before in the licensing literature. This is striking since the proper definition of the royalty base has been a controversial topic in recent years, which places it at the core of many legal cases. It is of particular interest to standard-setting organizations (SSOs), forums in which firms like technology developers and implementers who produce the final goods discuss the specifications of standards for complex products such as mobile phones and Wi-Fi-enabled devices. Hundreds of patents of many innovators typically cover the technologies necessary to sell products that comply with the standard.

Traditionally, these patents have been licensed under ad valorem royalties. However, courts have recently ruled against this practice and in favor of per-unit royalties, particularly in the context of SSOs. Their claim has been that ad valorem royalties are subject to more frictions (Sherry and Teece 1999). Since the royalty bases are argued to be otherwise equivalent, these frictions make ad valorem royalties unappealing in most situations, compared with per-unit royalties. This position is summarized by the US Court of Appeals to the Federal Circuit in *Ericsson Inc. v. D-Link Systems Inc.* (773 F.3d 1201, 1227 [Fed. Cir. 2014]): "The principle, applicable specifically to the choice of a royalty base, is that, where a multi-component product is at issue and the patented feature is not the item which imbues the combination of the other features with value, care must be taken to avoid misleading the jury by placing undue emphasis on the value of the entire product," and the "reliance on the entire market value [using ad valorem royalties] might mislead the jury, who may be less equipped to understand the extent to which the royalty rate would need to do the work in such instances." As a result, unless "the patented feature drives the demand for the entire multi-component product," ad valorem royalties should be avoided. This recommendation was recently incorporated in the updated patent policy of the Institute of Electrical and Electronics Engineers (IEEE), an association that sponsors many of the most relevant standards. On February 2, 2015, the Department of Justice issued a business review letter praising the IEEE for this change (Hesse 2015) and generating controversy among academics and practitioners (see Teece 2015; Sidak 2015).

Our paper shows that this move from ad valorem to per-unit royalties proposed by courts and sponsored by SSOs and antitrust authorities is likely to lead to higher final prices, which ought to be traded off with the lower legal frictions that they might entail. Furthermore, we show that these negative welfare consequences will be more significant precisely in those contexts in which many firms contribute technologies that are combined in a single product. In particular, we show that once we introduce several upstream innovators that provide complementary technologies, an additional force appears. As is well known, the interaction of several licensors creates a classical problem of Cournot complements, known in this context as royalty stacking: by requiring a large royalty, innovators reduce the quantity that the final producer of a good sells, which creates a negative externality on the rest of the innovators. As a result, prices are higher than those that would emerge from the profit-maximizing behavior of an upstream monopolist that holds all technologies.[4] The model shows that the royalty-stacking problem is more severe under per-unit than under ad valorem royalties.

Finally, our model allows us to study the incentives for firms to innovate as a result of the royalty base used under the current patent system. This is an important dimension that has been mostly ignored in the literature and in previous legal debates that implicitly assume that all technologies are already present. We model a first stage in which both the innovator and the producer simultaneously must make an investment. The innovator invests in the R&D necessary to create the technology, while the producer invests in the implementation of the technology in a final product that can be marketed. Since the choice of the royalty base affects the allocation of profits among the firms operating in different production stages, it will also affect the incentives to invest and, consequently, social welfare.

In the context of an upstream and a downstream monopoly, ad valorem royalties are usually superior from an ex ante perspective. By allocating profits to the innovator, they spur upstream investment at the expense of the producer's incentives. However, because they mitigate the double-marginalization problem, total surplus is higher, which might partially compensate for the decrease in investment incentives of the producer. It is only when the upstream innovator has a cost of R&D that is substantially lower than the implementation cost of the downstream producer that per-unit royalties may engender more incentives to innovate and increase social welfare.

When we consider multiple innovators with complementary technologies, numerical results indicate that ad valorem royalties have the potential to foster the investment not only of those innovators but also of the producer that implements these technologies in the creation of final goods. The reason is that by increasing

[4] Spulber (2014) shows that this distortion does not arise when innovators follow quantity-setting strategies.

upstream profits, ad valorem royalties increase the productivity of the investment of all parties, which generates a positive-feedback loop. As a result, the positive effect on social welfare of the lower prices that these royalties engender is reinforced by the greater investment incentives.

By comparing the welfare implications of ad valorem and per-unit royalties, we contribute to the well-established literature on licensing contracts. This literature typically focuses on the trade-offs between fixed fees—understood as payments independent of the quantity produced—and per-unit royalties. In vertical relationships, Kamien and Tauman (1986) among others show that fixed fees are superior to per-unit royalties even when there are several downstream producers to which the technology can be licensed.[5] However, this literature pays very little attention to ad valorem royalties. This is striking since the empirical evidence shows that these royalties are prevalent. For example, in a sample of 278 contracts, Bousquet et al. (1998) show that 225 included royalties but in only nine of them were these royalties paid per unit sold.

Our results suggest that ad valorem royalties tend to spur more innovation and lead to lower final prices, which explains their popularity. Few contributions in the literature study the trade-offs between royalty types. Bousquet et al. (1998) compare ad valorem and per-unit royalties in combination with fixed fees in the case of vertical relationships like the ones we consider here. They show that when there is uncertainty regarding the demand, typical of product innovations, ad valorem royalties in combination with fixed fees are more effective for risk sharing. In contrast, in the case of cost uncertainty, typical of process innovations, the ranking between the two royalty schemes is far less clear.

Other studies analyze different trade-offs involving the two royalty bases and, in particular, their implications for raising rivals' costs (Salop and Scheffman 1983) when a vertically integrated firm licenses its technology to downstream competitors. San Martín and Saracho (2010) show that under Cournot competition ad valorem royalties constitute a more effective commitment to soften downstream competition, which raises the final price. However, Colombo and Filippini (2015) show that the opposite is true when downstream firms compete in prices. Our model abstracts from the previous effects by assuming that innovators do not have downstream operations. Although this is a limitation of our work, it is also a relevant situation inasmuch as the court cases mentioned earlier and the proposed patent reforms concern vertically disaggregated firms such as large innovators or nonpracticing entities. Furthermore, the conflicting results of San Martín and Saracho (2010) and Colombo and Filippini (2015) indicate that it would be difficult to obtain robust additional insights by introducing vertical integration into the model.

Our work is also related to the literature that studies the optimal reward for complementary technologies in patent pools or SSOs. As in our paper, Gilbert and Katz (2011) examine the incentives for innovators to carry out R&D to un-

[5] Of course, the previous results do not hold when we consider market frictions. Hernández-Murillo and Llobet (2006), for example, show that royalties can be optimal when asymmetric-information considerations are taken into account.

cover the complementary technologies that are embedded in complex products. Firms choose which technologies to pursue. Gilbert and Katz show that the optimal payoff from innovation must counterbalance two forces. On the one hand, firms cannot appropriate all the return from the innovation, which leads to underinvestment. On the other hand, for each innovation firms engage in a patent race, which leads to overinvestment. An important conclusion is that, even in the case of perfectly complementary innovations, an equal division of surplus among innovators is unlikely to be optimal, since it encourages firms to obtain either only one or all innovations. Instead, here we focus on the interaction between upstream innovators and downstream producers. Since we assume that innovators are identical and do not choose which technologies to pursue, equal division among them is optimal in our context. Furthermore, the lack of a patent-race component always leads to underinvestment, which results from the lack of appropriability of all the returns from the innovation.

The paper proceeds as follows. In Section 2 we introduce the benchmark model that includes an upstream innovator and a downstream producer and allow for different allocations of bargaining power. In Section 3 we examine the case of multiple upstream innovators. We briefly discuss some extensions of the model in Section 4. In Section 5 we conclude by discussing some policy implications. All proofs are relegated to the Appendix.

2. The Benchmark Model

Consider the market for a new product. Its development requires the participation of two firms. An upstream innovator, denoted U, uncovers the basic technology that is required for the product. Development also requires a downstream producer that adapts the technology and creates the final product that can be marketed. This firm is denoted D and faces a demand function $D(p)$, given a price p.

We treat the investment decisions of these firms symmetrically. Each firm exerts effort e_s, for $s = \mathrm{U}, \mathrm{D}$. Efforts are complementary in the development of the final product. In particular, we assume that the upstream technology is successful with probability e_{U}, and the downstream producer can adapt it successfully with probability e_{D}, so the final product can be marketed with probability $e_{\mathrm{U}}e_{\mathrm{D}}$. Firms face an increasing and convex cost of effort $C(e_s) = \frac{1}{2}e_s^2$, for $s = \mathrm{U}, \mathrm{D}$.

Research effort may engender technologies that have alternative uses beyond the product considered. These uses lead to profits $\pi_0^{\mathrm{U}} > 0$ for the innovator if its research effort succeeds and $\pi_0^{\mathrm{D}} > 0$ if the final producer succeeds. These profits can originate, for example, from different applications of the technology developed upstream or from its use in other products that the downstream producer may already sell.[6]

The innovator offers a licensing contract to the final producer of a good to allow the sale of the good. The amount to be paid is determined as the result of

[6] As we discuss later, differences in these outside profits have effects that are isomorphic to differences in the cost of effort. In particular, $\pi_0^{\mathrm{D}} > \pi_0^{\mathrm{U}}$ will lead to implications equivalent to a lower marginal cost of effort for the downstream producer.

Nash bargaining. We denote the bargaining power of the innovator and of the producer γ and $1 - \gamma$, respectively, for $\gamma \in [0, 1]$. Since the contributions of both parties are essential for the product to be marketable, the outside option in the negotiation of each of the parties is set to 0, and in case of disagreement they obtain only the profits from other uses π_0^s, for $s =$ U, D. Once the payment has been agreed on, the producer chooses the price for the final good and incurs a marginal cost of production $c \geq 0$.

As discussed in the introduction, these licensing payments involve a royalty that the producer will have to pay according to the royalty base specified in the contract. We compare the two relevant ones. The first base consists of the units sold, $q = D(p)$, and the producer pays a per-unit royalty depending on them. The second base consists of the total gross revenue $pD(p)$, and the producer pays a proportion of it in the form of an ad valorem royalty.

To summarize, the timing of the model is as follows. In the first stage, both firms simultaneously choose their level of effort. If effort leads to a successful product, in the second stage firms negotiate the licensing agreement. In the last stage, the final price is set by the downstream producer.[7] Notice that the structure of the model implies that contracts are incomplete. Effort is not ex ante contractible. Furthermore, although firms may set ex ante the type of contract to be used (the royalty base), the specific royalty rate is chosen only after the value of the innovation has been uncovered.[8]

In Sections 2.1 and 2.2 we characterize the subgame-perfect equilibrium of the game. We start by comparing the equilibrium prices under the two royalty bases. We then proceed to examine how the incentives to innovate are affected by the royalty base.

2.1. Equilibrium Royalties and Prices

The price that maximizes profits for the producer depends on the royalty base used. Under per-unit royalties the producer incurs a marginal cost of $c + r$, where r is the per-unit royalty payment that the innovator receives for each unit sold. Profit maximization implies that the producer will charge an optimal price $p^*(r)$ increasing in r. A higher per-unit royalty is passed through a higher final price for the product. The innovator obtains total profits equal to the per-unit royalty rate times the quantity sold, which depends on the final price, $rD[p^*(r)]$.

We denote the ad valorem royalty s. This means that the producer retains a proportion $1 - s$ from the revenue of each unit sold. With some abuse of notation, we denote the optimal price $p^*(s)$. As in the case of per-unit royalties, this

[7] Notice that we do not allow the negotiation of the royalty to be related to the price that is set in the final market. This assumption is consistent with standard licensing contracts. Furthermore, in a context in which a producer engages in bilateral negotiations with several innovators that own patents that need to be licensed, writing contracts that determine the final price would not be compatible with innovators having different interests.

[8] This kind of incompleteness is common in the literature on profit sharing and is used in studies such as Romano (1994) in the context of retail price maintenance contracts. In standard-setting environments, fair, reasonable, and nondiscriminatory (FRAND) commitments are well described by this timing.

price is increasing in the royalty rate but for a different reason. When s increases from each unit sold, the producer incurs the same cost of production c but obtains a lower revenue. This lower profitability entices the firm to produce less and raise the price. The innovator receives a share s of the total revenue of the producer, $sp^*(s)D[p^*(s)]$.

To isolate the different effects at work in the model, we start with the case in which the innovator has all the bargaining power, $\gamma = 1$. This case corresponds to the situation in which the innovator chooses the royalty rate. The next result shows that under very weak regularity assumptions over the demand, ad valorem royalties always lead to prices lower than (or equal to) per-unit royalties. In other words, the double-marginalization problem typical of vertical relations like the one assumed in this model is less severe under ad valorem royalties.

Proposition 1. Set $\gamma = 1$ and assume that $D(p)$ is a twice-continuously differentiable demand function with a price elasticity $\eta(p)$ increasing in p. Then, under successive monopolies,

1) if an ad valorem and a per-unit royalty lead to the same final price, the ad valorem royalty results in higher profits for the innovator, and

2) the ad valorem royalty that maximizes the innovator's profits leads to a lower final price than the per-unit royalty that maximizes the innovator's profits.

First, notice that the result holds for a large family of demand functions, the isoelastic one being a limiting case. It includes typical functions like the linear demand function and, more generally, log-concave demand functions, a standard class of demand specifications that guarantee that the problem of the producer is well behaved.

Proposition 1.1 shows that, for a given final price, an ad valorem royalty allows the upstream innovator to extract a larger share of the surplus from the relationship with the downstream producer. Remarkably, although this result has never been stated in the context of licensing contracts, it is the counterpart of a classical result in public finance. Early literature on indirect taxation compares taxes based on the units sold (per unit) or the value of sales (ad valorem). In the context of a market monopolist, Suits and Musgrave (1953) show that, contingent on raising the same revenue, ad valorem taxes turn out to be less distorting and are, therefore, superior from a social welfare standpoint.

To interpret this result, it is useful to review how the trade-off of the producer when choosing the optimal price is resolved as a function of the royalty rate. As mentioned above, an increase both in the per-unit and the ad valorem royalty leads to an increase in the optimal price but through a different mechanism. A higher per-unit royalty increases the marginal cost by the same amount independently of the quantity produced, $D(p)$. A higher ad valorem royalty, however, has an impact on each unit sold that depends on the current price. Since the revenue of the firm is concave in the price, any increase beyond the monopoly increase implies a larger-than-linear drop in revenue.

Since the cost of raising the price under ad valorem royalties is increasing in the price, the downstream producer chooses not to pass through as much of an increase in the royalty rate as in the case of per-unit royalties, for which the drop in income is proportional to the number of units sold. This implies that the innovator can choose an ad valorem royalty that extracts a higher share of the total surplus while implementing the same price in the final market that could be achieved under per-unit royalties.

Proposition 1.2 characterizes the profit-maximizing ad valorem royalty rate and shows that it induces a lower final price for the product.[9] When the innovator increases the royalty rate, a trade-off arises between the capacity to extract more surplus and the reduction of that surplus as a result of a higher price. Under ad valorem royalties, both the innovator and the producer care about maximizing total revenues $p(s)D[p(s)]$, from which each firm obtains a portion. This implies that the incentives are better aligned, and the innovator does not want to set a royalty too high because that would cause a significant price increase and depress the share of the surplus that it can extract, $sp(s)D[p(s)]$. Under per-unit royalties, the innovator does not internalize such an effect and induces a higher price in order to extract a higher revenue from each unit sold.

The previous result also indicates that, under ad valorem royalties, the innovator is better off. Notice that because the equilibrium price is lower, consumer surplus also increases. The effect on the final producer, however, arises from the combination of two opposing forces. On the one hand, ad valorem royalties raise total profits because they lead to a lower equilibrium price that, irrespective of the royalty base, is above the monopoly price because of double marginalization. On the other hand, the innovator retains a higher proportion of these profits.

As an illustration of the previous effects, in Figure 1 we show the case in which demand is linear, $D(p) = 1 - p$. In the top three graphs we reproduce the equilibrium prices and profits that arise from the previous expressions. The case in which $c = 0$ is particularly illuminating of the different effects at work. Under per-unit royalties, a higher income for the innovator comes at the expense of a higher perceived cost by the downstream producer, which leads to a price above the monopoly price. Under ad valorem royalties, however, there is no conflict between the innovator and the producer, since both firms are interested in maximizing total revenue. Hence, no matter how large s is, the downstream producer always chooses the monopoly price.[10] In other words, there is no pass-through of a higher ad valorem royalty into a higher price. Hence, it is optimal for the innovator to choose $s = 1$. This case constitutes an extreme illustration of the different forces that operate under the two royalty bases in proposition 1. As the marginal cost increases, the double-marginalization effect becomes more relevant under ad valorem royalties, narrowing the gap between the resulting price and the one that

[9] Gaudin and White (2014) derive a counterpart to this result in the context of indirect taxation and show that the result is robust to other assumptions about downstream competition.

[10] This result holds, of course, for any demand function since the downstream producer maximizes $\max_p (1 - s)pD(p)$, which leads to an optimal price p^* equal to the monopoly price and, thus, independent of the royalty paid.

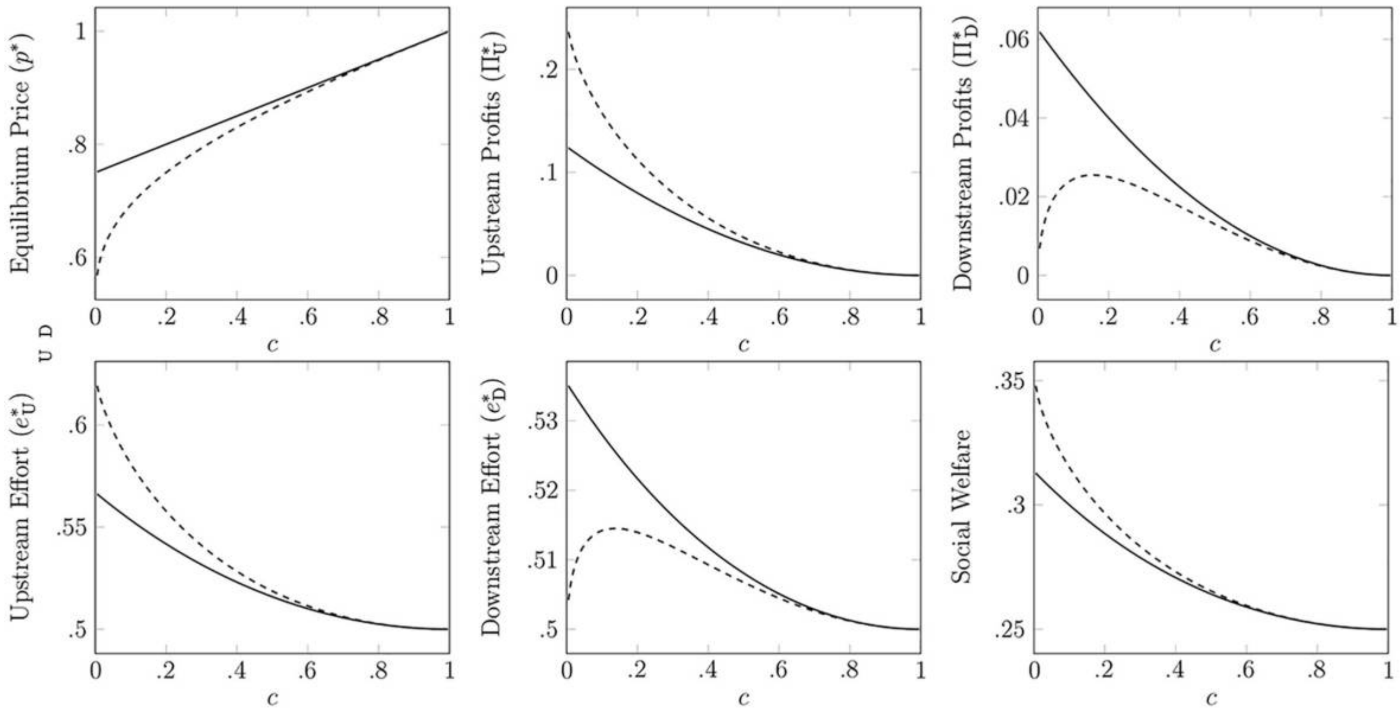

Figure 1. Equilibrium prices, ex post profits, effort levels, and social welfare with per-unit (*solid line*) and ad valorem (*dashed line*) royalties for changes in the marginal cost c, with demand function $D(p) = 1 - p$ and benchmark parameters $\pi_0^U = \pi_0^D = 1$.

emerges under per-unit royalties. An implication of this result is that in many technological products for which fixed costs are the main cost component, as opposed to variable production costs, ad valorem royalties are especially beneficial for consumers.

Figure 1 also shows that upstream profits are higher under ad valorem royalties, while the opposite is true for the downstream producer. An interesting result, however, is that ad valorem royalties generate profits for the producer that are not monotonic in the cost. To use the intuition about the price discussed above, when the marginal cost is low, the innovator can charge a high royalty without distorting the price much. As the marginal cost increases, however, this royalty must be decreased to keep the price low so that revenues are not excessively depressed, which increases downstream profits.

It is especially interesting to consider the case of the isoelastic demand function $D(p) = p^{-\eta}$ with $\eta > 1$. As proposition 1 states, this is a corner case, and, indeed, in the next lemma we show that for $\gamma = 1$, prices are identical regardless of whether the innovator chooses per-unit or ad valorem royalties. Profits, however, are different in each case.

Lemma 1. Consider an isoelastic demand function $D(p) = p^{-\eta}$ with $\eta > 1$ and set $\gamma = 1$. Then

1) the equilibrium price is the same under per-unit and ad valorem royalties, and

2) the innovator's profits are higher under ad valorem royalties, and, consequently, the producer's profits are higher under per-unit royalties.

Furthermore, the profits of the innovator under ad valorem royalties coincide with the profits of the producer under per-unit royalties.

The fact that equilibrium prices are identical under per-unit and ad valorem royalties implies that the two effects that delivered proposition 1.2 cancel out here. Remember that the optimal royalty rate arises from balancing out the capacity to extract surplus with the reduction of this surplus when prices increase. In the general case, under ad valorem royalties the innovator chooses to lower the royalty rate—compared with the rate that would lead to the same price under per-unit royalties—as a way to lower the final price and increase total revenue. This increase partially benefits from the lower elasticity associated with a lower price and, thus, the larger boost in demand. When demand elasticity is constant, this last effect does not exist, and the incentives to lower the price are reduced to the point that they cancel out with the loss in profits resulting from the lower royalty.

Obviously, if the final prices are the same in equilibrium, aggregate profits must also be the same. However, the way they are split is different in each case. The innovator benefits more from ad valorem royalties. This result is consistent with proposition 1.1, and it would explain why in the court cases we have seen in recent years patent holders defend ad valorem royalties whereas implementers argue in favor of per-unit royalties. Interestingly, the profits of one firm in the

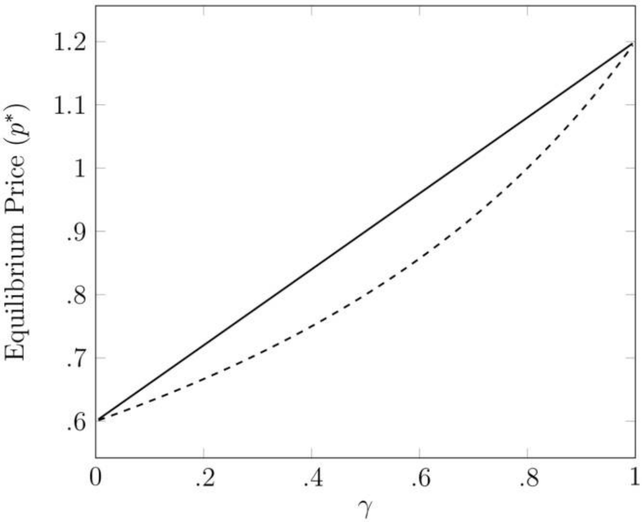

Figure 2. Final price under per-unit (*solid line*) and ad valorem (*dashed line*) royalties given bargaining power γ, with parameter values $c = .3$ and $\eta = 2$.

case of per-unit royalties coincide with the profits of the other under ad valorem royalties. This property will become useful in Section 2.2 when we discuss the investment decisions of both firms.

Our setup also allows us to understand whether different allocations of the bargaining power have different effects on per-unit and ad valorem royalties. Remember that we assume that the two parties split the surplus according to Nash bargaining. This means that the royalty maximizes $\Pi_U^{\gamma}\Pi_D^{1-\gamma}$, where Π_U and Π_D are the profits of the upstream innovator and downstream producer, respectively.

To be able to compare ad valorem and per-unit royalties in the context of the model, we need to isolate the effect of different allocations of bargaining power from the generic effects discussed in proposition 1. This is easy if we focus on the case of an isoelastic demand where, as we saw in lemma 1, the price is the same in both cases if $\gamma = 1$. Proposition 2 shows that this equivalence is not, however, a generic result for $\gamma \neq 1$.

Proposition 2. Under an isoelastic demand function, for any bargaining power $\gamma \in (0, 1)$, per-unit royalties lead to strictly higher prices than do ad valorem royalties. For $\gamma = 0$ and $\gamma = 1$, the price is independent of the royalty base.

The equilibrium price is the same under both royalty bases only when $\gamma = 1$, as stated in lemma 1, or $\gamma = 0$. This last case arises from the fact that when all the bargaining power is allocated downstream, the producer wants to set the minimal possible royalty, $r^* = s^* = 0$, which results in the monopoly price in both cases. Interestingly, in the intermediate situation, when $\gamma \in (0, 1)$, the price is systematically higher under per-unit royalties, which exacerbates the double-marginalization problem. This difference is shown in Figure 2. The reason is that both parties internalize a part of the double-marginalization problem that the vertical relationship generates, and ad valorem royalties are more effective in aligning the incentives of innovator and producer, as both take into account the effect on the price in the negotiation to a larger extent.

Having characterized the impact on equilibrium downstream prices of alternative royalty base regimes, we now move to the first stage, in which firms choose their investment efforts simultaneously. Throughout the rest of the paper, we make the following assumptions.

Assumption 1. Demand is isoelastic, $D(p) = p^{-\eta}$, with $\eta > 1$.

Assumption 2. The innovator holds all bargaining power, $\gamma = 1$.

These assumptions introduce analytic tractability and allow us to disentangle the effects of ad valorem and per-unit royalties analyzed in proposition 1 from those that emerge once we consider innovation incentives or other market structures. These assumptions also mean that the positive effects that ad valorem royalties generate through lower prices will be undervalued.

2.2. First-Stage Effort

In the first stage, firms simultaneously choose their investment. As in Section 2.1, we denote the generic expression for profits of the upstream innovator and downstream producer resulting from the royalty and posterior pricing stages Π_{U} and Π_{D}, respectively.

The producer chooses effort e_{D} to maximize profits from the new product and profits from the alternative use of the innovation, net of the cost of this effort. That is, the optimal effort arises from

$$\max_{e_{\mathrm{D}}} e_{\mathrm{U}} e_{\mathrm{D}} \Pi_{\mathrm{D}} + e_{\mathrm{D}} \pi_0^{\mathrm{D}} - \frac{1}{2} e_{\mathrm{D}}^2. \tag{1}$$

In a symmetric way, the level of effort e_{U} that maximizes profits for the innovator can be obtained from

$$\max_{e_{\mathrm{U}}} e_{\mathrm{U}} e_{\mathrm{D}} \Pi_{\mathrm{U}} + e_{\mathrm{U}} \pi_0^{\mathrm{U}} - \frac{1}{2} e_{\mathrm{U}}^2. \tag{2}$$

Expressions (1) and (2) imply that investment (or effort) is more beneficial when profits are higher both from the sale of the product and from other uses that it might have. The effort of both firms is complementary in the success of the product considered but independent with regard to the alternative uses of their investments. The next result is, under assumptions 1 and 2, a direct consequence of the symmetry in profits of both firms under per-unit and ad valorem royalties.

Proposition 3. Under an isoelastic demand function and $\gamma = 1$, ex ante social welfare levels are higher with ad valorem royalties if and only if $\pi_0^{\mathrm{D}} \geq \pi_0^{\mathrm{U}}$.

To understand this result, it is useful to start with the case in which both firms obtain the same profits from the alternative uses of the innovation. Remember that the producer benefits more from the production of the good under per-unit royalties and the innovator benefits more under ad valorem royalties. Thus, when $\pi_0^{\mathrm{D}} = \pi_0^{\mathrm{U}}$, under per-unit royalties a higher proportion of the investment is made by the producer, whereas under ad valorem royalties a higher proportion is made the upstream innovator. However, from lemma 1, since profits of the innovator under one royalty scheme are identical to the profits of the producer under the other, the total probability of success $e_{\mathrm{U}}e_{\mathrm{D}}$ is the same in both cases, which leads to identical social welfare in both situations, given that the final price is also the same.

Raising π_0^{D} above π_0^{U} leads to an increase in the investment of the producer. In equilibrium, because of the complementarity of efforts, the investment of the innovator also increases, albeit by a smaller amount. Facing a convex cost of effort, the producer who is already making a large investment is not affected as much as the innovator by the higher proportion of profits that per-unit royalties allocate downstream. Thus, a switch to ad valorem royalties increases the probability of success, since it boosts the incentives of the innovator, who faces a low marginal cost of effort, while having a smaller negative effect on the producer.[11]

The same results could be obtained if we assumed that outside profits were the same for both firms but the marginal costs of effort were different. In particular, if the cost of effort was lower for the producer so that it would naturally tend to invest more, ad valorem royalties would be optimal, as they would spur the innovator's effort without taxing too much of the producer's effort. For example, if the marginal costs of effort were $C_{\mathrm{U}}'(e_{\mathrm{U}}) = a_{\mathrm{U}} + e_{\mathrm{U}}$ and $C_{\mathrm{D}}'(e_{\mathrm{D}}) = a_{\mathrm{D}} + e_{\mathrm{D}}$, ad valorem royalties would lead to higher welfare levels if and only if $a_{\mathrm{D}} \leq a_{\mathrm{U}}$.

The case studied here, satisfying assumptions 1 and 2, simplifies the exposition, but it is easy to see that dropping these assumptions would make ad valorem royalties superior under more general conditions. To see the effects of relaxing assumption 1, consider the case of linear demand. Figure 1 includes the simulation of effort decisions and ex ante social welfare assuming that outside profits are identical, $\pi_0^{\mathrm{D}} = \pi_0^{\mathrm{U}}$. Although per-unit royalties spur the investment of the downstream producer and ad valorem royalties spur the investment of the innovator, the two effects are not of the same magnitude when the demand is not isoelastic. The probability of success is higher under ad valorem royalties since the double-marginalization effect is smaller when the elasticity of demand $\eta(p)$ is increasing in p, and, thus, total profits are higher in this case. The combination of the higher overall incentives to innovate and a lower price makes the level of social welfare higher under ad valorem royalties.

Consider now the case in which not all bargaining power is in the hands of the upstream innovator and $\pi_0^{\mathrm{D}} = \pi_0^{\mathrm{U}}$. We know that, at the other extreme, when $\gamma = 0$, ad valorem and per-unit royalties coincide, and they allocate all profits to the producer, which makes both royalty bases equivalent. When the bar-

[11] The results would be unchanged if we used consumer or producer surplus as our welfare measure since the price is the same under both royalty bases and so is surplus.

gaining power of the innovator increases, two forces emerge. On the one hand, the lower price and higher total profits that ad valorem royalties entail feed back into greater overall incentives to invest. On the other hand, ad valorem royalties tend to allocate more profits to the innovator, which distorts the incentives of the producer to exert effort. The first force dominates when γ is low, since Figure 2 shows that price differences increase as more bargaining power is allocated to the innovator. A lower price increases the marginal return to effort by expanding market size. When γ is low, the innovator is likely to choose a low level of investment because of the low profits, and so the marginal cost of effort is low. When γ is high, however, price differences shrink as γ increases, and the second effect dominates, which reduces the advantage of ad valorem royalties because of the decrease in the producer's effort. As proposition 3 indicates, in the limit, when $\gamma = 1$, the probability of success is the same in both cases.

The previous discussion is also helpful in understanding the consequences of enlarging the family of contracts we consider. Suppose, in particular, that in our benchmark model we allow for fixed fees, understood as payments that the producer makes independently of the quantity sold (alone or in combination with royalties). In that case, both parties would agree on a contract that includes a royalty of 0 and a fixed fee that splits total surplus according to each firm's bargaining power. As a result, the first force dominates, since no double marginalization will take place and total surplus is maximized.

The second force, however, might also become more significant in this case. In particular, suppose that $\gamma = 1$, so the innovator has all the bargaining power. It is still the case that total surplus is higher under fixed fees than under royalties (either per unit or ad valorem). Nevertheless, in that case, the equilibrium fixed fee would extract all profits from the producer, who, in anticipation, would choose to exert minimum effort. Thus, in instances in which the effort of the party that holds little bargaining power is important, fixed fees will lead to lower profits and lower incentives to innovate.[12]

3. Multiple Upstream Innovators

In recent years, the debate about the royalty base has gained relevance because of disputes over royalty rates in the case of patents related to SSOs. Court decisions like those mentioned in the introduction have generally found in favor of moving from ad valorem to per-unit royalties. Remarkably, in those decisions the number of innovators that contribute the technologies that are embedded in a product has not been considered relevant. In this section we show that it is precisely in the context of SSOs that ad valorem royalties are most preferable and the move toward per-unit royalties is most harmful.

Here we extend the previous framework by assuming that there are $N_U > 1$

[12] Even in less extreme cases, the bad risk-sharing properties of fixed fees makes them unattractive as a way to split surplus in licensing agreements. In practice, they are often observed in combination with royalties but for different reasons. For example, they arise in settlements as a way for licensees to compensate patent holders for previous infringements.

components necessary for the final product, each one researched by a different innovator. Component i is researched by upstream firm i, for $i = 1, \ldots, N_{\mathrm{U}}$. We assume that these components are symmetric, so the cost for innovator i of exerting effort e_{U}^{i} is $C(e_{\mathrm{U}}^{i}) = \frac{1}{2}(e_{\mathrm{U}}^{i})^2$. Similarly, the probability of success is symmetric among innovators and equal to

$$E(e_{\mathrm{D}}, e_{\mathrm{U}}^{1}, \ldots, e_{\mathrm{U}}^{N_{\mathrm{U}}}) = \left[\sum_{i=1}^{N_{\mathrm{U}}} (e_{\mathrm{U}}^{i})^{\alpha}\right]^{1/\alpha} e_{\mathrm{D}},$$

where $\alpha \in [0, 1]$ measures the degree of independence of the different components. When the value of α is high, the effect of innovator i's effort on the productivity of innovator j's effort is small. As α decreases, innovations become more complementary. A low value of α is consistent with the existence of SSOs that aim to coordinate the firms that provide technologies for a product.[13] As before, we assume that the alternative uses of each component lead to profits $\pi_0^{\mathrm{U}} > 0$ for each innovator. To abstract from the effects discussed in Sections 2.1 and 2.2 favoring ad valorem royalties, we conduct our analysis under assumptions 1 and 2.

In the first stage, firms simultaneously choose their research efforts. The unique downstream producer chooses e_{D} to maximize

$$\max_{e_{\mathrm{D}}} E(e_{\mathrm{D}}, e_{\mathrm{U}}^{1}, \ldots, e_{\mathrm{U}}^{N_{\mathrm{U}}}) \times \Pi_{\mathrm{D}} + e_{\mathrm{D}}\pi_0^{\mathrm{D}} - \frac{1}{2}e_{\mathrm{D}}^2.$$

Innovator i chooses e_{U}^{i} to maximize

$$\max_{e_{\mathrm{U}}^{i}} E(e_{\mathrm{D}}, e_{\mathrm{U}}^{1}, \ldots, e_{\mathrm{U}}^{N_{\mathrm{U}}}) \times \Pi_{\mathrm{U}} + e_{\mathrm{U}}^{i}\pi_0^{\mathrm{U}} - \frac{1}{2}(e_{\mathrm{U}}^{i})^2.$$

The same kind of complementarity discussed in the case with one innovator operates here between different innovators. If we keep profits from marketing the product constant, increases in N_{U} raise the probability of success of the innovation and thus the incentives of all parties to invest. Of course, profits in the final stages of the game will change as N_{U} increases, which affects this result. We now analyze the direction of these changes and, in particular, how the profits of the innovators and the producer are shaped under the different bases for the royalty rate as N_{U} increases. As in the case of one innovator, we start by comparing equilibrium prices and profits under per-unit and ad valorem royalties.

3.1. Equilibrium Royalties and Prices

With per-unit royalties, the producer has a marginal cost $c + R$, where $R \equiv \sum_{i}^{N_{\mathrm{U}}} r_i$. That is, the producer cares only about the total royalty charged by all innovators. That marginal cost determines the producer's profit-maximizing price. This is a standard problem that has been extensively discussed in the literature, starting with Shapiro (2001), and its implications are well known. Innova-

[13] Nevertheless, we assume that all components are required for the final product, as in the case of standard essential patents. In Section 4 we discuss the effects of relaxing this assumption and allowing for innovations to be substitutes.

tors choose their royalty rate to maximize their own profits without internalizing the fact that the higher their royalty is, the lower will be not only their sales but also the sales of the other innovators. This negative externality implies that innovators will choose a royalty that will be too high, even higher than the one that an upstream monopolist would choose if it owned all innovations. This effect is termed royalty stacking, and it is typical of all markets in which firms sell complementary products.

Under ad valorem royalties, the total royalty rate is again $S \equiv \sum_i^{N_U} s_i$. As in the previous case, the producer cares only about the total royalty paid and the proportion of the revenue that it can keep, $(1 - S)pD(p)$. This means that the royalty-stacking effect discussed in the case of per-unit royalties will also arise here. Proposition 4 shows, however, that the strength of this effect differs.

Proposition 4. Consider the case in assumptions 1 and 2. Suppose that innovation efforts have been successful. Under ad valorem royalties, the equilibrium price in the final market is always lower. Furthermore, the innovator's profits are higher under ad valorem royalties, whereas the producer's profits are higher under per-unit royalties.

Some of the results obtained in the one-innovator case are preserved here. In particular, it is still true that ad valorem royalties tend to favor innovators rather than the producer, whereas the opposite is true for per-unit royalties. An important difference, however, is that ad valorem royalties lead to lower downstream prices even under an isoelastic demand function. Thus, ex post consumer surplus is always higher under ad valorem royalties.

For the reasons discussed in proposition 1, if under per-unit and ad valorem royalties the price in the final market were the same, innovators would obtain higher profits under the latter. As in the case with one innovator, the optimal royalty trades off the effect on the final price and total surplus with the part of the surplus that the firm can appropriate. We argued that ad valorem royalties allocate a larger proportion of the surplus to the innovator, and the marginal gains from raising the royalty are low, which reduces the equilibrium distortions.

The only exception is the case of an isoelastic demand, where the innovator has more incentives to increase the royalty because this generates less of an effect on total revenues. The result in proposition 4 shows that when $N_U > 1$, even with an isoelastic demand we obtain a lower equilibrium price under ad valorem royalties. The reason is that, compared to the case with one innovator, increasing the royalty rate is less profitable since it generates a distortion that reduces total surplus, but the proportion of the total revenue it can appropriate is smaller since total revenue is divided among all innovators.

3.2. First-Stage Effort

To understand ex ante incentives, we need to rely on numerical analysis. In Figures 3 and 4 we provide examples of the effects of the different parameters of

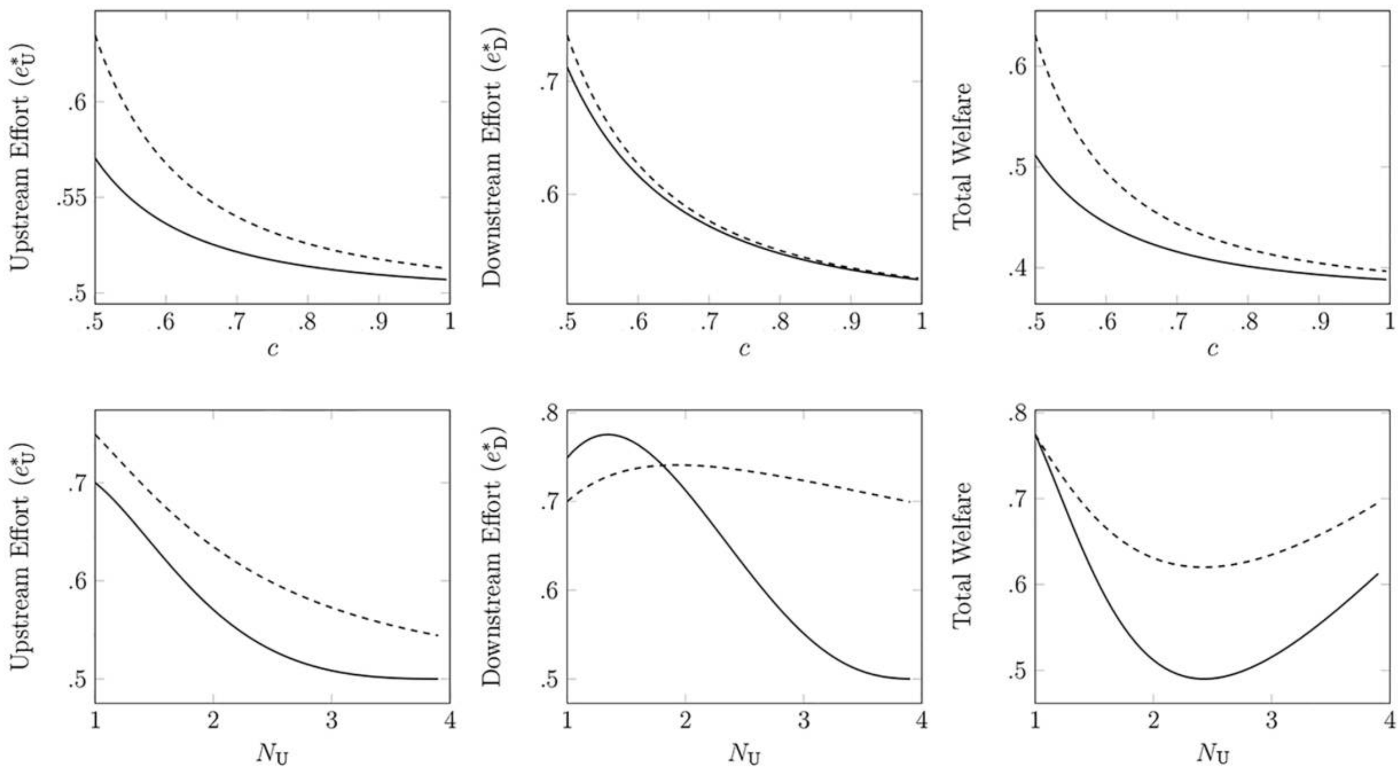

Figure 3. Effort by upstream innovators and the downstream producer and total welfare under the use of per-unit (*solid line*) and ad valorem (*dashed line*) royalties for changes in the marginal cost c and the number of upstream developers N_U, with baseline parameter values $\pi_0^D = \pi_0^U = .5$, $\alpha = .7, N_U = 2, \eta = 4$, and $c = .5$.

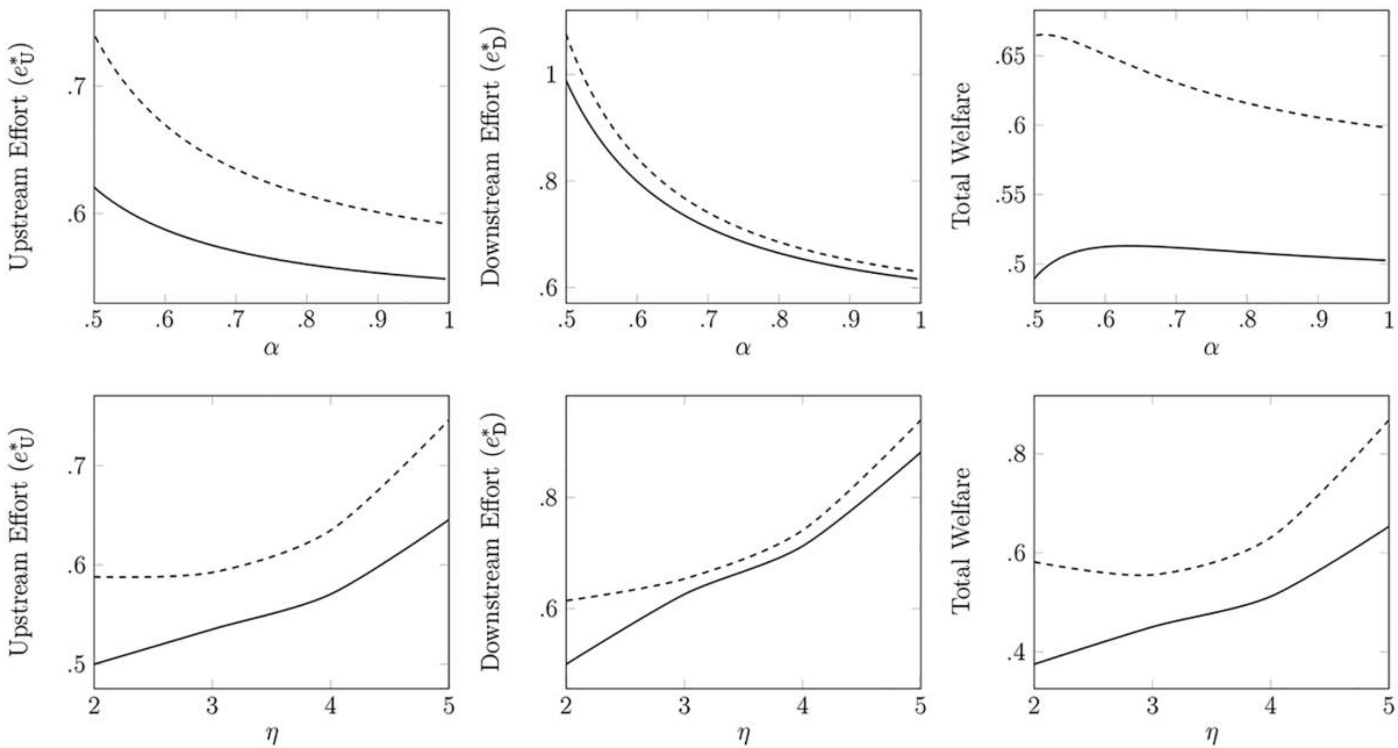

Figure 4. Effort by upstream innovators and the downstream producer and total welfare under the use of per-unit (*solid line*) and ad valorem (*dashed line*) royalties for changes in the substitubility between innovations α and the elasticity of the demand η, with baseline parameter values $\pi_0^D = \pi_0^U = .5$, $\alpha = .7$, $N_U = 2$, $\eta = 4$, and $c = .5$.

the model on the equilibrium decisions in the first stage of the model, upstream and downstream effort, and on social welfare.

The results indicate that ad valorem royalties typically translate into greater investment by upstream innovators. This effect is particularly important for low values of c and α. In the case of the former, as in the one-innovator case, the result stems from the fact that the double-marginalization effect under ad valorem royalties is small when c is low, which implies a large optimal royalty s^* and large upstream profits that spur investment. For the latter, notice that low values of α imply that the investments of different innovators are more complementary. As a result, the higher profits that ad valorem royalties imply are reinforced by the increased investments of other innovators.

In the case of the downstream producer, the equilibrium effect on investment is the combination of two forces. On the one hand, ad valorem royalties lead to lower downstream profits contingent on success, as illustrated in proposition 4, which feed back into lower incentives to invest. On the other hand, as mentioned just above, innovators invest more and, because of the complementarity-of-investments assumption, the marginal productivity of downstream investment rises. The numerical results indicate that the second force typically dominates, and downstream investment also rises under ad valorem royalties. The only exception corresponds to the case in which $N_{\mathrm{U}} < 2$. As shown in Section 2.2, when there is only one innovator, under ad valorem royalties the producer's profits are lower, and so are the incentives to innovate.

Total welfare levels are generally higher under ad valorem royalties for two reasons. First, ad valorem royalties lead to lower final prices, which favor consumers. Second, they typically generate an increase in total investment and in the resulting probability of the product's success. The difference in total surplus is particularly large when we consider more complementary components and the effect of ad valorem royalties on the incentives of innovators to invest is highest.

4. Extensions

The results in this paper are robust to numerous changes in the model. Here we discuss two that are developed in more detail in Llobet and Padilla (2014).

4.1. Weakly Complementary Components

The implicit assumption in Section 3 is that the success of all components is necessary for a product to be marketable. In the context of some SSOs, however, this might not always be the case. The standard often includes components that are optional, and downstream producers might decide whether to license them. This reduces the market power of innovators, as their royalty demands might exclude their technology from the standard.

In Llobet and Padilla (2014) we address this case by assuming that each innovation allows the producer to reduce the marginal cost of production but that no innovation is essential for the final product. Innovations have a varying degree of complementarity (or even substitutability) in terms of cost reductions, and we show that the main result of that paper, namely, the lower price that ad valorem royalties entail, remains unchanged.

We show, however, that an additional effect emerges. When complementarity of innovations is weak, the total royalty stack is small and so is the price. As the complementarity increases, both the total royalty and the equilibrium price increase for the usual reasons. However, when complementarity is sufficiently strong, the price decreases because of the significant reductions in the cost that more technologies generate, which induces a decrease in the price that compensates for the increased royalty rate. While ad valorem royalties always lead to lower prices, the difference with per-unit royalties is particularly important for intermediate values of complementarity, precisely when the royalty stack is largest. This result also suggests that in the case of products that embed numerous technologies and for which the complementarities are very important, such as those typically coordinated by SSOs, the endogenous decisions to which innovators conform the standard might mitigate the royalty-stacking problem.

4.2. Downstream Competition

The main driver of our model is the different pass-through that the ad valorem and per-unit royalty rates induce in the final market. When there is perfect competition in the final market, per-unit and ad valorem royalties are identical since they generate the same pass-through, at least when there is a unique innovator. If we consider that N_D identical firms compete in quantities, we can transition between those extreme cases in a monotonic way.

Once we endogenize the effort decisions in the first stage, the results do not change qualitatively from those described in our benchmark model. In the case of several innovators with complementary components, we show that ad valorem royalties, by allocating higher profits upstream, generate more effort because of the feedback loop between the complementary decisions of these innovators. When there are multiple downstream producers, however, allocating profits downstream through per-unit royalties does not generate such a positive feedback. The reason is that producers sell substitute products. As a result, an increase in downstream profits generates an increase in the returns from being a successful monopolist in the market but also a lower probability that the firm becomes a monopolist and can benefit from that increase in profits. This second force limits the investment incentives of the producers.

5. Concluding Remarks

We have shown that, under many circumstances, ad valorem royalties, which are based on the value of sales, yield superior outcomes from the standpoints of both consumer welfare and total welfare than do per-unit royalty rates, which are based on the value of the components of the infringing product that are covered by the patented technology. In our analysis, ad valorem royalties are better from the perspectives of consumer welfare and total welfare for two reasons. First, they mitigate the double-marginalization problem that naturally arises in technology markets characterized by market power at the licensing and manufacturing levels of the vertical chain. This is because innovators internalize to a greater extent the impact on the product price of an increase in their royalty rate when this royalty is based on firms' revenues. Second, investments by both innovators and producers are typically greater under ad valorem royalties because overall industry profits, and hence the incentives to innovate, are greater when the double-marginalization problem is less severe. This effect is stronger, which makes ad valorem royalties even more attractive, when bargaining power is distributed between the parties or when there are multiple innovators licensing complementary technologies. Because ad valorem royalties shift profits upstream, they partially compensate for the tendency to underinvest by owners of complementary technologies.

Our results can be used to develop normative implications. Courts have emphasized the advantages of using per-unit royalties, for example, as a way to prevent royalty payments that are considered too high when the royalty rate applies to the price of the product. We show that these frictions need to be weighed against the effects that mandating per-unit royalties would have, particularly in the context of SSOs and their standard essential patents. Although the frictions that courts are trying to prevent might become more apparent in those environments, it is there that ad valorem royalties might help deliver a lower royalty stack, more incentives to innovate, and, as a result, greater consumer and social welfare.

Appendix

Proofs

A1. Lemma A1 and Proof

Lemma A1. A twice-continuously differentiable profit function $\Pi(p) = (p - c)D(p)$ is quasi-concave if the elasticity of the demand $\eta(p) = -D'(p)p/D(p)$ is increasing in p.

Proof. A single-variable function $\Pi(p)$ is quasi-concave if it is either always increasing or always decreasing in p or if there exists a p^* such that $\Pi(p)$ is increasing for $p < p^*$ and decreasing for $p > p^*$. Let us assume, toward a contradiction, that neither of the two previous conditions is true. In that case, at least one of the solutions to the first-order condition must characterize a minimum. This first-order condition can be written as the standard Lerner index $(p^* - c)/p^* = 1/\eta(p^*)$, which shows that when $\eta(p)$ is increasing in p, there can be at most one solution since the left-hand side is always increasing while the right-hand side is always decreasing.

We now show that this unique critical value p^* always defines a maximum. We have

$$\frac{\partial \eta}{\partial p} = -\frac{D'(p)D(p) + pD''(p)D(p) - D'(p)^2}{D^2(p)}.$$

This expression is positive if and only if

$$D''(p) \leq \frac{D'(p)^2}{D(p)} - \frac{D'(p)}{p},$$

We can now compute

$$\Pi''(p^\star) = D''(p^\star)(p^\star - c) + 2D'(p^\star) < D'(p^\star)\left[1 - \frac{1}{\eta(p^\star)}\right] = D'(p^\star)\frac{p^\star - c}{p^\star} < 0,$$

where the first inequality comes from the upper bound on $D''(p)$ originating from the previous expression. Thus, the first-order condition determines a maximum, and the profit function is quasi-concave. Q.E.D.

A2. Lemma A2 and Proof

Lemma A2. Assume that $D(p)$ is a twice-continuously differentiable demand function with a price elasticity $\eta(p)$ increasing in p. Then, under per-unit royalties, the optimal price has an upper bound

$$p^\star_{\text{pu}} \leq \frac{(c + r^\star)^2}{c}.$$

Proof. Using Lerner's rule, we have that under per-unit royalties the optimal price is determined to be

$$\left[1 - \frac{1}{\eta(p^\star_{\text{pu}})}\right](p^\star_{\text{pu}}) = c + r, \tag{A1}$$

where we have made explicit the dependency of the demand elasticity η on p. Using the implicit-function theorem, we have

$$\frac{\partial p^\star_{\text{pu}}}{\partial r} = 1 \Big/ \left\{\left[1 - \frac{1}{\eta(p)}\right] + \frac{p}{\eta(p)^2}\eta'(p)\right\} > 0. \tag{A2}$$

In the first stage, the innovator chooses the royalty to maximize $rD[p^\star_{\text{pu}}(r)]$, which results in a first-order condition

$$D[p^\star_{\text{pu}}(r^\star)] + r^\star D'[p^\star_{\text{pu}}(r^\star)]\frac{\partial p^\star_{\text{pu}}}{\partial r} = 0.$$

Solving for $r^\star$ and substituting equation (A2), we have

$$r^\star = -\frac{D[p^\star_{\text{pu}}(r^\star)]}{D'[p^\star_{\text{pu}}(r^\star)](\partial p^\star_{\text{pu}}/\partial r)} = \left\{p^\star_{\text{pu}}\left[1 - \frac{1}{\eta(p^\star_{\text{pu}})}\right] + \frac{(p^\star_{\text{pu}})^2}{\eta(p^\star_{\text{pu}})^2}\eta'(p^\star_{\text{pu}})\right\} \Big/ \eta(p^\star_{\text{pu}}).$$

Substituting $r^\star$ in the first-order condition for the downstream producer (equation [A1]) and rearranging terms, we obtain

$$\left[1-\frac{1}{\eta(p^\star_{\text{pu}})}\right]^2 p^\star_{\text{pu}} = c + \frac{(p^\star_{\text{pu}})^2}{\eta(p^\star_{\text{pu}})^3}\eta'(p^\star_{\text{pu}}).$$

We can now substitute the left-hand-side expression with $(c+r)^2/cp^\star_{\text{pu}}$ and rearrange terms to obtain

$$p^\star_{\text{pu}} = (c+r^\star)^2 \Big/ \left[c + \frac{(p^\star_{\text{pu}})^2}{\eta(p^\star_{\text{pu}})^3}\eta'(p^\star_{\text{pu}})\right] \leq \frac{(c+r^\star)^2}{c},$$

where the last inequality arises from $\eta'(p) \geq 0$. Q.E.D.

A3. Proof of Proposition 1

For proposition 1.1, take a per-unit royalty r. The profit-maximizing price for the downstream producer satisfies

$$\left(1-\frac{1}{\eta}\right)D(p^\star_{\text{pu}}) = c + r, \tag{A3}$$

whereas with ad valorem royalties the same price could be reached with a royalty s such that

$$(1-s)\left(1-\frac{1}{\eta}\right)D(p^\star_{\text{av}}) = c. \tag{A4}$$

Notice that to simplify notation we have dropped the argument in the demand elasticity η. From lemma A1, these first-order conditions are necessary and sufficient. Hence, the two royalty schemes lead to the same price, $p^\star_{\text{pu}} = p^\star_{\text{av}} = p^\star,$ if and only if

$$r = \frac{s}{1-s}c.$$

Upstream profits under per-unit royalties are

$$\Pi_{\text{U,pu}}(r) = rD(p^\star) = s\frac{c}{1-s}D(p^\star) < sp^\star D(p^\star) = \Pi_{\text{U,av}}(s),$$

where the last inequality comes from the fact that $(1-s)p^\star > c$.

For proposition 1.2, take the optimal per-unit royalty $r^\star$ and the equilibrium price $p^\star_{\text{pu}} = p^\star$. As shown before, this same price could be induced under an ad valorem royalty $\hat{s} = r/(c+r)$. Given the concavity of the profits of the innovator, it is enough to show that the derivative of the profits of the upstream producer evaluated at $\hat{s}$ is positive. In particular,

$$\frac{\partial \Pi_{\text{U,av}}}{\partial s}(\hat{s}) = p^\star\, D(p^\star) + \hat{s}[pD'(p^\star) + D(p^\star)]\frac{\partial p^\star}{\partial s}.$$

Using the implicit-function theorem for equations (A3) and (A4) and using the fact that under $\hat{s}$ the equilibrium price is the same, we have

$$\frac{\partial p_{\text{av}}^{\star}}{\partial s}=\frac{[1-(1/\eta)]D(p^{\star})}{1-\hat{s}}\frac{\partial p^{\star}}{\partial r}=\frac{c+r^{\star}}{1-\hat{s}}\frac{\partial p^{\star}}{\partial r}.$$

Substituting the previous first-order condition and the expression for $\hat{s}$ and using the fact that $p^{\star}D'(p^{\star}) + D(p^{\star}) = (c + r)D'(p^{\star})$ from the optimality of $p^{\star}$, we have

$$\frac{\partial \Pi_{\text{U,av}}}{\partial s}(\hat{s})=p_{\text{av}}^{\star}D(p_{\text{av}}^{\star})+\frac{r^{\star}(c+r^{\star})^2}{c}D'(p^{\star})\frac{\partial p^{\star}}{\partial r}.$$

Finally, notice that the first-order condition that pins down $r^{\star}$ implies that $D'(p^{\star})(\partial p^{\star}/\partial r) = D(p^{\star})/r^{\star}$, and substituting we have

$$\frac{\partial \Pi_{\text{U,av}}}{\partial s}(\hat{s})=\left[p_{\text{av}}^{\star}-\frac{(c+r^{\star})^2}{c}\right]D(p_{\text{av}}^{\star})<0,$$

where the last inequality comes from lemma A2. Q.E.D.

A4. Proof of Lemma 1

Consider an isoelastic demand function $D(p) = p^{-\eta}$ with $\eta > 1$. Given a per-unit royalty r, the monopoly price corresponds to $p^{\star}(r) = (c + r)\eta/(\eta - 1)$. Substituting the profit function of the innovator, we have that the optimal per-unit royalty corresponds to $r^{\star} = c/(\eta - 1)$. The final price can be computed as $p_{\text{pu}}^{\star}=c[\eta/(\eta-1)]^2$. Profits become

$$\Pi_{\text{U,pu}}^{\star}=\frac{(\eta-1)^{2\eta-1}}{\eta^{2\eta}}c^{1-\eta}$$

and

$$\Pi_{\text{D,pu}}^{\star}=\frac{(\eta-1)^{2(\eta-1)}}{\eta^{2\eta-1}}c^{1-\eta},$$

which implies that per-unit royalties allocate a higher share of the total surplus to the downstream producer, $\Pi_{\text{D,pu}}^{\star}>\Pi_{\text{U,pu}}^{\star}$.

Under ad valorem royalties, the monopoly price that the producer sets is equal to $p^{\star}(s) = \eta c/[(1 - s)(\eta - 1)]$. Substituting the profit function of the innovator, we have that the equilibrium royalty rate corresponds to $s^{\star} = 1/\eta$. The equilibrium price is equal to $p_{\text{av}}^{\star}=c[\eta/(\eta-1)]^2$. Thus, $p_{\text{pu}}^{\star}=p_{\text{av}}^{\star}$. However, profits are different. In particular,

$$\Pi_{\text{U,av}}^{\star}=\frac{(\eta-1)^{2(\eta-1)}}{\eta^{2\eta-1}}c^{1-\eta}$$

and

$$\Pi^{\star}_{\mathrm{D,av}} = \frac{(\eta-1)^{2\eta-1}}{\eta^{2\eta}} c^{1-\eta}.$$

That is, $\Pi^{\star}_{\mathrm{U,av}} = \Pi^{\star}_{\mathrm{D,pu}}$ (and, thus, $\Pi^{*}_{\mathrm{D,av}} = \Pi^{*}_{\mathrm{U,pu}}$). Q.E.D.

A5. Proof of Proposition 2

Contingent on successful development and given a royalty r, calculations included in the proof of lemma 1 imply that the final price is $p^{\star}(r) = (c + r)[\eta/(\eta - 1)]$. Profits correspond to $\Pi_{\mathrm{U}}(r) = r[p^{\star}(r)]^{-\eta}$ and $\Pi_{\mathrm{D}}(r) = [p^{\star}(r) - (c + r)]p^{\star}(r)^{-\eta}$. Hence, the equilibrium royalty results from

$$r^{\star} = \arg\max_{r} \Pi_{\mathrm{U}}(r)^{\gamma}\, \Pi_{\mathrm{D}}(r)^{1-\gamma},$$

or $r^{\star} = \gamma c/(\eta - 1)$. The equilibrium price becomes $p^{\star}_{\mathrm{pu}} = c\eta(\eta+\gamma-1)/(\eta-1)^2$, increasing in γ.

Under ad valorem royalties, given a royalty s, the final price is $p^{\star}(s) = c\eta/[(1 - s)(\eta - 1)]$. Profits can be written as $\Pi_{\mathrm{U}}(s) = s[p^{*}(s)]^{1-\eta}$ and $\Pi_{\mathrm{D}}(r) = [(1 - s)p^{\star}(s) - c]p^{\star}(s)^{-\eta}$. The equilibrium royalty results from

$$s^{\star} = \arg\max_{s} \Pi_{\mathrm{U}}(s)^{\gamma}\, \Pi_{\mathrm{D}}(s)^{1-\gamma},$$

or $s^{\star} = \gamma/\eta$. The final price is then $p^{\star}_{\mathrm{av}}(s) = c\eta^2/(\eta-1)(\eta-\gamma)$, again increasing in γ. The comparison of the two prices delivers the result. Q.E.D.

A6. Proof of Proposition 3

Given that prices in both cases are identical, $p^{\mathrm{M}}_{\mathrm{av}} = p^{\mathrm{M}}_{\mathrm{pu}} = p^{\mathrm{M}}$, consumer surplus contingent on success, CS, is also the same. Consider a given level of profits for the innovator and producer Π_{U} and Π_{D}, respectively. The social welfare function can be written as

$$W(\Pi_{\mathrm{D}}, \Pi_{\mathrm{U}}, \pi_0^{\mathrm{U}}, \pi_0^{\mathrm{D}}) = e^{\star}_{\mathrm{U}} e^{\star}_{\mathrm{D}}(\mathrm{CS} + \Pi_{\mathrm{U}} + \Pi_{\mathrm{D}}) + e^{\star}_{\mathrm{U}}\pi_0^{\mathrm{U}} + e^{\star}_{\mathrm{D}}\pi_0^{\mathrm{D}} - \frac{(e^{\star}_{\mathrm{U}})^2}{2} - \frac{(e^{\star}_{\mathrm{D}})^2}{2}, \quad \text{(A5)}$$

where $e^{\star}_{\mathrm{U}}$ and $e^{\star}_{\mathrm{D}}$ are the equilibrium levels of effort arising from expressions (1) and (2),

$$e^{\star}_{\mathrm{U}} = \frac{\pi_0^{\mathrm{D}}\Pi_{\mathrm{U}} + \pi_0^{\mathrm{U}}}{1 - \Pi_{\mathrm{U}}\Pi_{\mathrm{D}}} \quad \text{(A6)}$$

and

$$e^{\star}_{\mathrm{D}} = \frac{\pi_0^{\mathrm{U}}\Pi_{\mathrm{D}} + \pi_0^{\mathrm{D}}}{1 - \Pi_{\mathrm{U}}\Pi_{\mathrm{D}}} \quad \text{(A7)}$$

if $\Pi_{\mathrm{U}}\Pi_{\mathrm{D}} < 1$ and $e^{\star}_{\mathrm{U}} = e^{\star}_{\mathrm{D}} = \infty$ otherwise. This latter case can be safely ignored since changes in the royalty base would have no effect on effort.

Notice that $CS + \Pi_U + \Pi_D$ is constant in both cases since $\Pi^*_{U,pu} + \Pi^*_{D,pu} = \Pi^*_{U,av} + \Pi^*_{D,av}$. Furthermore, equilibrium effort levels are the same if $\pi_0^U = \pi_0^D$, which implies that the expected total surplus is the same in both cases.

Let $\pi_0^U \geq \pi_0^D$ and consider the following problem:

$$\max_{\lambda} \tilde{W}(\lambda) = \max_{\lambda} \lambda W_{pu}(\pi_0^U, \pi_0^D) + (1-\lambda) W_{av}(\pi_0^U, \pi_0^D),$$

where $W_{pu}(\pi_0^U, \pi_0^D) \equiv (\Pi_{D,pu}, \Pi_{U,pu}, \pi_0^U, \pi_0^D)$ and $W_{av}(\pi_0^U, \pi_0^D) \equiv (\Pi_{D,av}, \Pi_{U,av}, \pi_0^U, \pi_0^D)$ correspond to welfare under per-unit and ad valorem royalties. It is enough to show that $\tilde{W}$ is supermodular in λ and π_0^U, and this implies that

$$\frac{\partial W}{\partial \pi_0^U}(\Pi_D, \Pi_U, \pi_0^U, \pi_0^D) - \frac{\partial W}{\partial \pi_0^U}(\Pi_U, \Pi_D, \pi_0^U, \pi_0^D) > 0,$$

with $\Pi_D > \Pi_U$ and $\Pi_U + \Pi_D$ and $\Pi_U \Pi_D$ constant. The first term (and by symmetry the second) can be computed as

$$\begin{aligned}\frac{\partial W}{\partial \pi_0^U}(\Pi_D, \Pi_U, \pi_0^U, \pi_0^D) = {} & \frac{\pi_0^D(\Pi_U\Pi_D + 1) + 2\pi_0^U \Pi_D}{(1-\Pi_U\Pi_D)^2} CS \\ & + \frac{\pi_0^D(\Pi_U^2\Pi_D + \Pi_D) + \pi_0^U(\Pi_D\Pi_U + \Pi_D)}{(1-\Pi_U\Pi_D)^2}.\end{aligned}$$

The difference between the two corresponds to

$$(\Pi_D - \Pi_U)\left[\frac{2\pi_0^U}{(1-\Pi_U\Pi_D)^2} CS + \frac{\pi_0^D(1-\Pi_U\Pi_D) + \pi_0^U(\Pi_U + \Pi_D)}{(1-\Pi_U\Pi_D)^2}\right] > 0.$$

Q.E.D.

A7. Proof of Proposition 4

Under per-unit royalties, the optimal downstream price is equal to $p^*(R) = (c + R)\eta/(\eta - 1)$. Innovator $i = 1, \ldots, N_U$ chooses the royalty rate r_i to maximize

$$\max_{r_i} r_i [p^*(R)]^{-\eta}.$$

Focusing on a symmetric equilibrium in which $r_i = r^*$ for all i, we have $r^* = c/(\eta - N_U)$ and $p^*_{pu} = c\eta^2/(\eta-1)(\eta - N_U)$, which are defined only if $\eta > N_U$. This royalty is increasing in N_U. Profits can be computed as

$$\Pi^*_{U,pu} = \frac{(\eta - N_U)^{\eta-1}(\eta-1)^{\eta}}{\eta^{2\eta}} c^{1-\eta}$$

and

$$\Pi^*_{D,pu} = \frac{(\eta - N_U)^{\eta-1}(\eta-1)^{\eta-1}}{\eta^{2\eta-1}} c^{1-\eta},$$

which are decreasing in N_{U} because of the previous effect.

Under ad valorem royalties, the optimal price corresponds to $p^{\mathrm{M}}(S) = c\eta/(1 - S)(\eta - 1)$. Innovator i chooses s_i in the first stage to maximize

$$\max_{s_i} s_i [p^\star (S)]^{1-\eta}.$$

Focusing on a symmetric equilibrium in which $s_i = s^\star$ for all i, we have $s^\star = 1/(\eta + N_{\mathrm{U}} - 1)$ and $p^\star_{\mathrm{av}} = c\eta(\eta + N_{\mathrm{U}} - 1) / (\eta - 1)^2$, whereas before the price was increasing in N_{U}. Profits can be computed as

$$\Pi^\star_{\mathrm{U,av}} = \frac{(\eta - 1)^{2\eta - 2}}{\eta^{\eta - 1}(\eta + N_{\mathrm{U}} - 1)^\eta} c^{1-\eta}$$

and

$$\Pi^\star_{\mathrm{D,av}} = \frac{(\eta - 1)^{2\eta - 1}}{\eta^{\eta}(\eta + N_{\mathrm{U}} - 1)^\eta} c^{1-\eta}.$$

The ordering of the prices arises from the comparison between $p^\star_{\mathrm{pu}}$ and $p^\star_{\mathrm{av}}$, which is strict when $N_{\mathrm{U}} > 1$. Regarding profits, notice that $\Pi^\star_{\mathrm{D,pu}} / \Pi^\star_{\mathrm{U,pu}} = \eta/(\eta - 1) = \Pi^\star_{\mathrm{U,av}} / \Pi^\star_{\mathrm{D,av}}$, which proves the second part of the result, since $\eta/(\eta - 1) > 1$. Q.E.D.

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