0.1 Industrial Organization, Week 1 Answers

$$Q_s = 400p - 100; \quad Q_d = 1100 - 200(p+t)$$
$$\rightarrow p_s = \frac{Q_o}{400} + \frac{1}{4}; \quad p_d = \frac{1100 - Q_d}{200} - t$$
$$Q_s = Q_d \leftrightarrow p^* = 2 - \frac{1}{3}t; \quad Q^* = 700 - \frac{400}{3}t$$

Surplus consumer(geometry way):

$$\begin{aligned} S_c = Base * Height/2 =& (5.5 - p^*) * Q^*/2 = \frac{1}{2}(5.5 - t - 2 + \frac{1}{3}t) * (700 - \frac{400}{3}t) \\ =& \frac{1}{2}(3.5 - \frac{2}{3}t) * (700 - \frac{400}{3}t) \\ =& \frac{1}{2}(3.5\frac{200}{200} - \frac{2*200}{3*200}t) * (700 - \frac{400}{3}t) \\ =& \frac{1}{400}(700 - \frac{200}{3}t) * (700 - \frac{400}{3}t) \\ =& \frac{1}{2}(700 - \frac{400}{3}t)^2 \end{aligned}$$

Surplus producer(geometry):

$$\begin{aligned} S_c = Base * Height/2 =& (p^* - 1/4) * Q^*/2 = \frac{1}{2}(2 - \frac{1}{3}t - 1/4) * (700 - \frac{400}{3}t) \\ =& \frac{1}{2}(\frac{7}{4} - \frac{1}{3}t) * (700 - \frac{400}{3}t) \\ =& \frac{1}{2}(\frac{700}{400} - \frac{400}{3*400}t) * (700 - \frac{400}{3}t) \\ =& \frac{1}{800}(700 - \frac{400}{3}t)^2 \end{aligned}$$

In the special cases:

$$t = 0 \rightarrow p^* = 2; \quad Q^* = 700; \quad S_c = 1225; \quad S_p = 612.5$$
$$t = 1 \rightarrow p^* = \frac{5}{3}; \quad Q^* = \frac{1700}{3} = 566.66...; \quad S_c = 802.8; \quad S_p = 401.4$$
$$DWL = 1225 + 612.5 - 802.8 - 401.4 - 566.6 = 66.7$$

Click here for the graph. In case some of you want to dig deeper, when the functions are not linear you are sometimes forced to into calculus, so this is the more general approach: Surplus consumer(calculus way):

$$
\begin{aligned}
S_c &= \int_0^{Q^*} (p_d - p^*) dq \\
&= \int_0^{Q^*} \left(\left(\frac{1100 - q}{200} - t \right) - \left(2 - \frac{1}{3}t \right) \right) dq \\
&= \int_0^{Q^*} \left(\frac{1100 - q}{200} - \frac{2}{3}t - 2 \right) dq \\
&= \int_0^{Q^*} \left(\frac{700 - q}{200} - \frac{2}{3}t \right) dq \\
&= \left| \left(\frac{700q}{200} - \frac{q^2}{400} - q\frac{2}{3}t \right) \right|_0^{Q^*} \\
&= \left| q \left(\frac{700}{200} - \frac{q}{400} - \frac{2}{3}t \right) \right|_0^{Q^*} \\
&= \left(700 - \frac{400}{3}t \right) \left(\frac{700}{200} - \frac{700 - \frac{400}{3}t}{400} - \frac{2}{3}t \right) \\
&= \left(700 - \frac{400}{3}t \right) \left(\frac{700}{400} - \frac{t}{3} \right) \\
&= \frac{1}{400} \left(700 - \frac{400}{3}t \right)^2
\end{aligned}
$$

Surplus producer(calculus way):

$$
\begin{aligned}
S_p &= \int_0^{Q^*} (p^* - p_o) dq \\
&= \int_0^{Q^*} \left(\left(2 - \frac{1}{3}t \right) - \left(\frac{q}{400} + \frac{1}{4} \right) \right) dq \\
&= \int_0^{Q^*} \left(\frac{7}{4} - \frac{1}{3}t - \frac{q}{400} \right) dq \\
&= \left| \left(\frac{7q}{4} - \frac{q}{3}t - \frac{q^2}{800} \right) \right|_0^{Q^*} \\
&= \left| q \left(\frac{7}{4} - \frac{1}{3}t - \frac{q}{800} \right) \right|_0^{Q^*} \\
&= \left(700 - \frac{400}{3}t \right) \left(\frac{7}{4} - \frac{1}{3}t - \frac{700 - \frac{400}{3}t}{800} \right) \\
&= \left(700 - \frac{400}{3}t \right) \left(\frac{700}{800} - \frac{1}{6}t \right) \\
&= \frac{1}{800} \left(700 - \frac{400}{3}t \right)^2
\end{aligned}
$$