

Problems, paradoxes and probability

The main theme of the course is to present basic concepts in probability theory and relate them to their history and arrive at an understanding how these are used to make decisions. Problem or a paradox and then resolve it through some concept, either from philosophy or probability theory. The goals will be to give students the tools to think critically about how to make decisions. Additionally, some of the areas I will be presenting will represent cutting edge research avenues which are open to be understood from mathematical, economic, sociological, philosophical, and even theological frameworks. This content is relatively difficult but it is a fast track for understanding cutting edge material in the academic world. The goal will be to give an expansive view of the problems involved when trying to construct confidence about events in the worlds.

The first few sessions will aim at a comprehension of basic tools of probability theory. Each class will involve examples of the concepts, and will have brief assignments on the material. Most of the material will simply be presented on a blackboard/whiteboard but depending on the class some PowerPoint or handouts may complement it. Problems can be assigned every week or biweekly, the textbooks can be used or the site <http://www.cut-the-knot.org/> for more advanced students.

1) Elementary probability: Fair and biased, Chapter 3-4 IAN Peterson Chapter 2

Explain the classical definition of probability in order to communicate the concept of **bias** as the property that not every outcome is equally likely.

Independence, how much information does one thing give us about another? Give an example of an airplane with 2 engines vs one with 3 engines. Ask the students to reveal what is wrong with the Gambler who will bet on red because he has seen 12 consecutive blacks on the roulette wheel.

When do we add probabilities and when do we multiply them? We add them when the events are mutually exclusive, and multiply them if they compound.

Homework: selected problems from the textbooks

2) Conditional probability: Bayes, Chapter 4-7 Ian Peterson, Chapter 4, Chapter 6 and 10

Present the [Monty Hall problem](#); and [Boy or Girl paradox](#) work, explain how it works from a Bayesian point of view.

Homework: Selected problems OR Assignment

Assignment: Watch this [video](#) and read the [Wikipedia](#) article, explain the main issues with the Sleeping beauty problem.

3) [St Petersburg paradox](#) and it's solution([Kelly](#) vs ergodic vs utility), Ian, Chapter 8-9, Peterson, Chapter 4

Explain what expected value is by giving intuitive examples. Read the [wager passage](#) of Pascal and explain the argument. Give some objections and some answers to the objections. After

explaining the St Petersburg paradox I will discuss how it can be solved by a change in the optimand.

[Reference for optimal gambling, Chapter 2 of Ergodic economics](#)

4) Ambiguity: [Ellsberg](#) and [Allais](#). Chapter 10 from IAN. Chapter 4 from Peterson

What is the difference between Risk and Uncertainty? What is a [Dutch book](#) explanation? Explain how the two paradoxes relate to betting and subjective probability theory.

Assignment: An essay question about the differences between Classical vs Frequentist VS Bayesian.

5) Independence and Association, Pearl 2009 Chapter 2&3 Peterson Chapter 9

What is independence? Correlation vs causation, examples

Explain what association and correlation is, introduce the concept of a collider and talk a little bit about causality. Explain why Fischer was not convinced about smoking and cancer and explain how the backdoor criterion is insufficient to prove smoking causes cancer whilst the front door criterion works.

6) Risk seeking and research: (optional session, this session is more technical so only if the rest have proceeded as planned). Dubins and Savage

Give an [example](#) in detail.

Explain how it links to [Savage's](#) treatment of decision theory under constraints.

Give some more examples of Bayesian decision theory, including some cutting edge work in economics.

Main textbooks:

Probability: Hacking, Ian, and Hacking Ian. *An introduction to probability and inductive logic*. Cambridge university press, 2001.

Decision theory: Peterson, Martin. *An introduction to decision theory*. Cambridge University Press, 2017.

Pearl, Judea, and Dana Mackenzie. [Book of why](#): *the new science of cause and effect*. Basic Books, 2018.

Secondary books:

Resnik, Michael D. *Choices: An introduction to decision theory*. U of Minnesota Press, 1987.

Pearl, Judea. "[Causal inference in statistics](#): An overview." *Statistics surveys* 3 (2009): 96-146.

Dubins, Lester E., et al. *How to gamble if you must: Inequalities for stochastic processes*. Courier Corporation, 2014.