

0.1 Only the chef invests

Three kinds of people are needed to make the boat trip work. The tycoon who wants to go on the boat trip, the chef who can cook and a skipper. The chef can undertake a private investment to learn how to cook before the boat trip. To go on the boat trip, they also need a boat.

If the chef invested the value is:

$$V(chef, tycoon, skipper1, skipper2) = V(all) = 240$$

The value added of everybody if the tycoon or chef owns the boat is then:

$$\begin{aligned} V(all) - V(all \setminus chef) &= 0 \\ V(all) - V(all \setminus tycoon) &= 0 \\ V(all) - V(all \setminus skipper1) &= 240 \\ V(all) - V(all \setminus skipper2) &= 240 \end{aligned}$$

So the chef and the tycoon will split the cake between them:

$$\begin{aligned} P(chef) &= P(tycoon) = \frac{V(all)}{2} = \frac{240}{2} = 120 \\ P(skipper1) &= P(skipper2) = 0 \end{aligned}$$

On the other hand if skipper1 owns the boat:

$$\begin{aligned} V(all) - V(all \setminus chef) &= 0 \\ V(all) - V(all \setminus tycoon) &= 0 \\ V(all) - V(all \setminus skipper1) &= 0 \\ V(all) - V(all \setminus skipper2) &= 240 \end{aligned}$$

This means skipper 1 can also get an equal share of the value.

$$\begin{aligned}P(chef) &= P(tycoon) = P(skipper1) = 80 \\ \frac{V(all)}{3} &= \frac{240}{3} = 80 \\ P(skipper2) &= 0\end{aligned}$$

So we can see that if the chef's investment is under 80, he will always invest. If his investment is between 80 and 120, then he will only invest if the chef or tycoon own the boat. If it is greater than 120, he will never invest.

0.2 The chef and the skipper invests