

So we start with a random variable, recall that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$

$$\phi(x) = \frac{1}{\sqrt{2\Gamma(\frac{1}{2})}} e^{-\frac{x^2}{2}}$$

So the same variable squared is:

$$\begin{aligned} f(x) &= \frac{\delta}{\delta x} Pr(X^2 \leq x) = \frac{\delta}{\delta x} Pr(-\sqrt{x} \leq X \leq \sqrt{x}) \\ &= \frac{\delta}{\delta x} \frac{1}{\sqrt{2\Gamma(\frac{1}{2})}} \int_{-\sqrt{x}}^{\sqrt{x}} e^{-\frac{u^2}{2}} = \frac{\delta}{\delta x} \frac{2}{\sqrt{2\Gamma(\frac{1}{2})}} \int_0^{\sqrt{x}} e^{-\frac{u^2}{2}} \\ &= \frac{2}{\sqrt{2\pi}} e^{-\frac{\sqrt{x^2}}{2}} \frac{\delta}{\delta x} \sqrt{x} = \frac{2}{\sqrt{2\pi}} e^{-\frac{x}{2}} \frac{1}{2\sqrt{x}} \\ &= \frac{e^{-\frac{x}{2}}}{\sqrt{2\pi x}} \end{aligned}$$

or using change of random variable formula

$$f_Y(y) = \sum_i f_X(g_i^{-1}(y)) \text{abs} \frac{\delta g_i^{-1}(y)}{\delta y}$$