0.1 Industrial Organization, Week 1 Answers

$$Q_s = 400p - 100; \quad Q_d = 1100 - 200(p+t)$$
$$\rightarrow p_s = \frac{Q_o}{400} + \frac{1}{4}; \quad p_d = \frac{1100 - Q_d}{200} - t$$
$$Q_s = Q_d \leftrightarrow p^* = 2 - \frac{1}{3}t; \quad Q^* = 700 - \frac{400}{3}t$$

Surplus consumer(geometry way):

$$\begin{aligned} S_c = Base * Height/2 =& (5.5 - p^*) * Q^*/2 = \frac{1}{2}(5.5 - t - 2 + \frac{1}{3}t) * (700 - \frac{400}{3}t) \\ =& \frac{1}{2}(3.5 - \frac{2}{3}t) * (700 - \frac{400}{3}t) \\ =& \frac{1}{2}(3.5\frac{200}{200} - \frac{2*200}{3*200}t) * (700 - \frac{400}{3}t) \\ =& \frac{1}{400}(700 - \frac{200}{3}t) * (700 - \frac{400}{3}t) \\ =& \frac{1}{2}(700 - \frac{400}{3}t)^2 \end{aligned}$$

Surplus producer(geometry):

$$\begin{aligned} S_c = Base * Height/2 =& (p^* - 1/4) * Q^*/2 = \frac{1}{2}(2 - \frac{1}{3}t - 1/4) * (700 - \frac{400}{3}t) \\ =& \frac{1}{2}(\frac{7}{4} - \frac{1}{3}t) * (700 - \frac{400}{3}t) \\ =& \frac{1}{2}(\frac{700}{400} - \frac{400}{3*400}t) * (700 - \frac{400}{3}t) \\ =& \frac{1}{800}(700 - \frac{400}{3}t)^2 \end{aligned}$$

In the special cases:

$$t = 0 \rightarrow p^* = 2; \quad Q^* = 700; \quad S_c = 1225; \quad S_p = 612.5$$
$$t = 1 \rightarrow p^* = \frac{5}{3}; \quad Q^* = \frac{1700}{3} = 566.66...; \quad S_c = 802.8; \quad S_p = 401.4$$
$$DWL = 1225 + 612.5 - 802.8 - 401.4 - 566.6 = 66.7$$

In case some of you want to dig deeper, when the functions are not linear you are sometimes forced to into calculus, so this is the more general approach: Surplus consumer(calculus way):

$$
\begin{aligned}
S_c &= \int_0^{Q^*} (p_d - p^*) dq \\
&= \int_0^{Q^*} \left(\left(\frac{1100 - q}{200} - t \right) - \left(2 - \frac{1}{3}t \right) \right) dq \\
&= \int_0^{Q^*} \left(\frac{1100 - q}{200} - \frac{2}{3}t - 2 \right) dq \\
&= \int_0^{Q^*} \left(\frac{700 - q}{200} - \frac{2}{3}t \right) dq \\
&= \left| \left(\frac{700q}{200} - \frac{q^2}{400} - q\frac{2}{3}t \right) \right|_0^{Q^*} \\
&= \left| q \left(\frac{700}{200} - \frac{q}{400} - \frac{2}{3}t \right) \right|_0^{Q^*} \\
&= \left(700 - \frac{400}{3}t \right) \left(\frac{700}{200} - \frac{700 - \frac{400}{3}t}{400} - \frac{2}{3}t \right) \\
&= \left(700 - \frac{400}{3}t \right) \left(\frac{700}{400} - \frac{t}{3} \right) \\
&= \frac{1}{400} \left(700 - \frac{400}{3}t \right)^2
\end{aligned}
$$

Surplus producer(calculus way):

$$
\begin{aligned}
S_p &= \int_0^{Q^*} (p^* - p_o) dq \\
&= \int_0^{Q^*} \left(\left(2 - \frac{1}{3}t \right) - \left(\frac{q}{400} + \frac{1}{4} \right) \right) dq \\
&= \int_0^{Q^*} \left(\frac{7}{4} - \frac{1}{3}t - \frac{q}{400} \right) dq \\
&= \left| \left(\frac{7q}{4} - \frac{q}{3}t - \frac{q^2}{800} \right) \right|_0^{Q^*} \\
&= \left| q \left(\frac{7}{4} - \frac{1}{3}t - \frac{q}{800} \right) \right|_0^{Q^*} \\
&= \left(700 - \frac{400}{3}t \right) \left(\frac{7}{4} - \frac{1}{3}t - \frac{700 - \frac{400}{3}t}{800} \right) \\
&= \left(700 - \frac{400}{3}t \right) \left(\frac{700}{800} - \frac{1}{6}t \right) \\
&= \frac{1}{800} \left(700 - \frac{400}{3}t \right)^2
\end{aligned}
$$