Microeconomie 2 / Examen Final

Mai 2016

Exercise : Exchange and Production Economy

It is advised to do the first part before the second part.

Part I (4.5 points, difficulty **)

Assume an economy with two consumers $i = A, B$, and two goods $l = 1, 2$. The individual endowments of A and B are $\omega^A = \omega^B = (\frac{1}{2}, \frac{1}{2})$. Good 2 is the numeraire good (i.e. $p_2 = 1$). We note $p_1 = p$. The preferences of the consumer are represented by the utility functions :

$$u^A(x_1^A, x_2^A) = ln(x_1^A) + ln(x_2^A) \qquad u^B(x_1^B, x_2^B) = (x_1^B)^{\frac{1}{4}}(x_2^B)^{\frac{3}{4}}$$

1. Determine the Walrasian equilibrium (find $p = \frac{3}{5}$ and allocations $((\frac{2}{3}, \frac{2}{5}); (\frac{1}{3}, \frac{3}{5}))$. (2.5 points)
2. Check if the Walrasian equilibrium is Pareto-optimal. Which computations should be made to check that the equilibrium is in the core ? (2 points)

Part II (7 points, difficulty ** and ***)

We carry on working in the same framework with the same consumers (same preferences and endowments). A firm is created by the consumer B to produce good 2 using good 1 as input. The production function is $y_2 = \sqrt{y_1}$. We note π the firm's profit. In the following questions, the firm maximises its profit independently of the consumer B's preferences. The profit is then added to the consumer B's budget.

1. Determine the demand for good 1 of the firm and the consumers. Prove the price p is equal to $p = \frac{3+\sqrt{59}}{3}$. (2 points)
2. The production function becomes $y_2 = y_1$. Determine the demand for good 1 of the firm and the consumers by distinguish 3 cases with respect to the value of p. (2.5 points)
3. The production function becomes $y_2 = \frac{y_1}{c}$ (with $c > 0$). Determine the values of c such that the the firm is active at equilibrium (i.e. $y_1 > 0$) and the values of c such that the firm is not active (Hint : Show that, for some values of c, there is an excess demand of good 1 when the firm is active). Compare the equilibrium of question I.1 with the equilibrium with the non active firm. (2.5 points)