

## 0.1 Industrial Organization, Challenge 1

A) Let us note some preliminary observations. 1) The owner always wants the manager to work, even if he has to pay the manager 99 cents, that still leaves him better than nothing. 2) If the owner sees a zero, he has no reason to pay the manager anything, he can even sue. 3) The owner knows that if he sees a 2, the manager worked.

Let us say that the owner's compensation vector is  $(x, y, z)$ , such that he pays  $x$  if he sees 0,  $y$  if he sees 1 and  $z$  if he sees a 2.

**Case 1, The manager sees a 0:** From the manager's point of view, she is selecting the highest utility from unemployed (U), employed but lazy (L), and employed and working (W). If  $U$  she can earn 20 cents, so  $U = 20$ . For her to prefer employment, EITHER  $L > 20$  OR  $W > 20$ . However recall that  $W$  costs her 30 cents, so the conditions are actually  $x > 20$  OR  $y - 30 > 20$ . From these two the owner wishes her to choose to be employed and work. So he needs to make sure that  $y - 30 > x$ . Thankfully this is not difficult, the owner can just set  $x = 0$  or even a negative number (he will sue the manager if he sees a 0). So to motivate the manager to work when she sees a 0, he needs to make sure that set  $y > 50$ .

**Case 2, The manager sees a 1:** In this case she is choosing between showing a manager a 1 or a 2. If she works hard she can get  $W = z - 30$  whilst if she is lazy she can get  $L = y$ . Clearly the owner wants her to work so he must ensure that  $W > L$ . Which gets us  $z > y + 30$ .

Now we need only combine the two things we have. We have that  $y > 50$  and that  $z > y + 30$ . Putting these together, we know that  $y = 50 + \epsilon$  and that  $z = 80 + 2\epsilon$ . If the owner cannot pay less than 1 cent increments then  $y = 51$  and  $z = 82$ . So the total wage scheme is  $(0, 51, 82)$ .

B) As seen above, the owner has no reason to incentivize not working. As such, in equilibrium there will be no zeros but there will be both 2's and 1's.

C) If she only sees the result of the coin toss after she is employed then the expected value of the wages has to be higher than the unemployment. As we said in the first part, it is never an equilibrium to be lazy. In this situation there are numerous equilibria. She has 50 – 50 to earn either  $y$  or  $z$ . So what must be met is:  $20 < \frac{y}{2} + \frac{z}{2} - 30$ . Whilst keeping in mind the incentive compatibility constraint, that it  $z - 30 > y$ , which is interpreted, she must always want to work whenever she ends up seeing a 1. So, let us say that:  $z + \epsilon = y + 30$ . Plug this back into  $20 < \frac{y}{2} + \frac{y+30-\epsilon}{2} - 30$  and re-arrange,  $35 + \epsilon < y$ . So  $y = 35 + 2\epsilon$  and  $z = 65 + \epsilon$ . If we assume that the lowest increment is a cent, then we have that the new wage vector is  $(0, 37, 66)$ .

So notice that if she has the information earlier, the owner is forced to pay her more than if she has it later.