0.1 Industrial Organization, Week 4 Answers

0.2 Part A

0.2.1 Part 1

Start with the profit function of the second player	$\pi_2 = (1000 - q_2 - q_1)q_2 - c_2 q_2$
Deriving, setting to zero, and re-arranging we get	$q_2 = \dfrac{1000 - c_2 - q_1}{2}$
We now set the profit function of the first player	$\pi_1 = (1000 - q_2 - q_1)q_1 - c_1 q_1$
We plug in the the quantity of the second firm	$\pi_1 = (1000 - \dfrac{1000 - c_2 - q_1}{2} - q_1)q_1 - c_1 q_1$
Derive with respect to quantity and solve	$q_1 = \dfrac{1000 + c_2 - 2c_1}{2}$
Plug this quantity into the second firms quantity	$q_2 = 250 - \dfrac{3c_2}{4} - \dfrac{c_1}{2}$
Plug in cost values to get price and quantities	$q_2 = 246.25; q_1 = 482.5; p = 271.25$
Do the same thing with the profits:	$\pi_2 = 116403; \pi_1 = 60639.1$
Now let us compute the consumer surplus	$CS = (1000 - 271.25)(246.25 + 482.5)\dfrac{1}{2} = 265538$
So welfare is: the same thing with the profits:	$W = 265538 + 116403 + 60639.1 = 442580$

0.2.2 Part 2

We now move on to the Cournot	$\pi_2 = (1000 - q_2 - q_1)q_2 - c_2 q_2$
The reaction function of firm 2 is the same	$q_2 = \dfrac{1000 - c_2 - q_1}{2}$
By symmetry	$q_1 = \dfrac{1000 - c_1 - q_2}{2}$
Plug these into each other:	$q_1 = \dfrac{1000 + c_2 - 2c_1}{3}; q_2 = \dfrac{1000 + c_1 - 2c_2}{3};$
Plug in the numerical values again	$q_1 = 321.667; q_2 = 326.667; p = 351.667$

So the equilibrium price is higher with Cournot than with Stackelbrg, Stackelberg has a higher amount of goods sold than Cournot.

0.2.3 Part 3

Here the question is simply, given that the $c_1 = 30$, is there a c_2, such that $q_2 > q_1$ in Stackleberg?

$$
\begin{aligned}
&\text{Start with the inequality} && q_2 > q_1 \\
&\text{Plug in the equations we computed in part 1} && 250 - \frac{3c_2}{4} - \frac{c_1}{2} > \frac{1000 + c_2 - 2c_1}{2} \\
&\text{Plug in the cost of the firs firm} && 250 - \frac{3c_2}{4} - \frac{30}{2} > \frac{1000 + c_2 - 60}{2} \\
&\text{Move things around abit} && 235 - \frac{3c_2}{4} > 470 + \frac{c_2}{2} \\
&\text{Until we have} && -164 > c_2
\end{aligned}
$$

Which is either impossible or it some subsidy scheme where somebody pays firm 2 164 for every unit it produces. So in the situation here, the first mover advantage is more important than the cost.

The graph I showed in class is here: Click here for the graph: