1 Introduction

Royalty stacking is the phenomenon that when a firm enters a market it must pay numerous royalties because its product builds upon numerous previous innovations. This occurs because there is no legal obligation that the total royalty fees must remain below some threshold (such as a fixed monetary amount or a proportion of the cost of the product). Royalty stacking is similar to the the term "Patent stacking" except that the latter implies a single owner whilst the former is indifferent to the ownership structure of the stack.

The question this phenomenon raises is under what conditions would a firm prefer to use the latest technology and pay a higher royalty cost rather than use a less cutting edge technology and diminish its royalty cost? To answer this question one must look at both the firm seeking permission and the firm giving permission. If giving permission has no cost associated with it[1] then the entrant will always attach himself to the highest firm it can. So necessarily for any other structure but a chain of increasingly better innovations to occur either a cost must be present for the permission giving firm or some behavioral assumption must be assumed.

The empirical evidence, though limited is that the cost of royalty stacking can be quite significant. In smartphones, the cost of royalties has been estimated to be higher than the cost of components. Armstrong et al. (2014) Royalties come to represent such a significant portion of the cost because innovation is sequential. When each innovation builds on the previous innovations it leads to firms forming a chain of innovation. If strong intellectual property rights are available then this is equivalent to saying that an entrant must find at least one predecessor who has property rights to consent to the entry.

The usual analysis of the fragmented ownership is through the hold up problem. In the usual analysis, the more fragmented is the ownership structure of the chain, the more difficult it will be to create a new product. However the hold up problem is only a sort of worst case scenario. In practice, even if it is assumed that royalties are fixed and there is no hold up problem, there are

[1] the cost does not have to be direct, it could be a cost on the value of its license, this is explored in Katz & Shapiro (1986)

structural issues that can arise due to royalty stacking. This paper aims to highlight the conditions under which a new innovation is preferred with sequential innovation with endogenous royalties.

Why does patent stacking occur? Though formally courts are meant to recognize that royalty fees must be proportional to the value added of the innovation and not using the entire value of the product there are practical difficulties that prevents this from occurring. In practice the value added of a given royalty is not an observable quantity. This means that the value must be inferred and the metric usually employed is the difference in price between non-patented and patented innovation. Non-patented products have stronger competition: their costs of components are generally cheaper, which implies that the difference between the two prices is not only the value added of the patented technology but also the difference between industry efficiency. For a full discussion about why royalties end up in practice being a larger share than their value added, see Elhauge et al. (2008)

Royalty stacking implies that costs associated to royalties gets more important as innovations increase. There is a nuance here between royalty related *total costs* and royalties *paid*. We find that the later a firm enters, the higher the *total royalty costs*, however we also find that later entry implies a lower royalty *paid*. This is because the two costs are not independent: the royalties paid will be related to future royalty revenue and hence if a firm enters later, the willingness to accept to pay a royalty because of future royalties is decreased. Thenceforth the royalty paid depends on how much money the patent infringement permission is ultimately worth.

The reason for our result is that each patent owner can extract the surplus of all future patent holders by charging the appropriate price. This means that patentees can only earn as a function of other market opportunities they had available at the time. Intuitively, if there is only one firm in the market, an entrant can either earn the infringement profits or the non-infringement profits, however as the number of firms increases, the infringement options increase and it can make previous firms that have entered compete to sell their respective licenses. However the ability to license one's technology is a foreseeable, so the price paid to develop it will include the future royalties it will bring.

If we assume the firms are operating on the same market the effects of competition change the willingness to license. A firm may get a license from another firm that is currently producing or from one that has stopped producing. Ceteris Paribus, a firm that is not producing should charge less for a patent than one that is currently producing; this is because the presence of a competitor imposes a direct cost on the profit of a producing firm, while it imposes only the indirect cost of reducing patent value if the said firm is not producing.

The framework presented here allows for a structural interpretation of patent length. That is the disagreement payoff of an entrant is increasing as the patent length decreases. However our model shows that unless there are market complementarities, the patent length does not matter. That is, firms that are not on the cutting edge cannot charge for their license because they have less to offer than cutting edge firms. This means that whether the patent length is long or short does not matter since the disagreement payoff of the entrant is not affected.

The structure of the paper will be as follows. In the second section we will present the model and clarify our three main assumptions of market symmetry, foresightedness and monotonicity in technology. The third section will describe the equilibrium concept and the main results of the model. The fourth section will discuss what other assumptions are usually made in economic theory and how those affect the model. In the fifth section we fully resolve the three firm case. Finally in the sixth section we discuss some extensions and relaxations of the model.

2 The model

We model a sequential game where firms enter sequentially and decide if they wish to attach themselves to any of the existing firms. Consider a set of firms $\Omega = (\omega_1, \omega_2, \ldots)$ that each decides sequentially to enter the market of a given good. Consider that $N = (1, 2, \ldots, n)$ is the set of firms which enter the market, ordered by the time at which they made their decision. The game immediately ends for these in $\Omega \setminus N$; they all get a null payoff. Those in N pay a fixed cost $F \in \mathbb{R}_+$ upon entry, irrespective of their production decisions. Once in the industry and before producing, every firm has the possibility to access and improve upon the pre-existing production technologies owned by the firms that entered at an earlier date. We denote i's technology level by k_i ; k_i measures the efficiency of i at producing the good, and it is an integer between 1 and i, for any $i \in N$. This process of accessing the technologies in the pre-existing industry and innovating upon them is made possible via the formation of directed links, from a predecessor firm to the entrant, and captures the technology transfer from the former to the later.

To capture the concept of sequentially improving innovation we will represent firms with the most advanced technology to have the most predecessors. However, to build upon a technology, a firm must first bargain with the owner of the said technology to be allowed to have access to it. An entrant may infringe on any existing firm's technology; also, if it wishes to infringe on two firms' different technologies, it can do so yet it will use the most advanced technology one for producing. We model the technology decisions of the firms in N as a game of endogenous network formation. The transmission of technology from a firm h to a successor, i, occurs if and only if i has a directed link $h \rightarrow i$ to its said predecessor, h. The link means that $k_i > k_h$, i.e. a firm always successfully improves upon a technology it has access to. A strategy of link formation is denoted s_i for any $i \in N$, and s_i is the set of all of i's predecessors with which i forms links. All n firms' decisions in link formation map to a technological network g. When $i \in N$ has infringed upon many technologies, thus $|s_i|> 1$, we assume that i produces using its most efficient one: i.e. we set $k_i = \max_{\forall j \in s_i} k_j + 1$. The level of technology k_i of firm i is equal to the length of the **longest directed path** that starts at i in g plus 1[2]. If no path from i exist in g, then $k_i = 1$. Note that firm 1's technology level is $k_1 = 1$ always.

Improving upon someone's technology comes at a cost. We assume that a firm i which wants to achieve some technological level $\tilde{k}$ will bargain simultaneously with all firms h which currently own the level of technology $k_h = \tilde{k} - 1$. The outcome of the bargaining between i and h is the royalty r_h^i that i pays to h. One can see r_h^i as the cost on i for the link $h \rightarrow i$. Note that every firm can access the technology level of 1 for free. In this paper, we will consider pure strategies only. Therefore, if $\mathcal{S}_i$ is the set of all pure strategies of i then $|\mathcal{S}_i|= 2^{i-1}$; also, we call $\mathcal{S} = \mathcal{S}_1 \times \ldots \mathcal{S}_n$ the space of all firms' pure strategies.

Definition 1. *A technological network g is a directed acyclic graph (DAG). Abusing notation, we*

[2]Note that the longest path from i always has a length between zero and $i - 1$; if it is equal to zero, then $k_i = 1$.

denote $g = (s_1, \ldots, s_n)$ the network that is formed by some vector of strategies $(s_1, \ldots, s_n) \in \mathcal{S}$ played by the firms in N.

Proof. The result that g is a DAG follows from the fact that no firm can form a link to any of its successors. $\square$

The firms all have two kinds of payoffs, a royalty payoff stemming from the directed edges and a competitive market payoff. The total payoff ultimately depends on the technological level of a firm and the network structure. The payoff of firm i is denoted by $\pi_i(k_i, \mathrm{g})$, and π_i has the two following components: (i) $p_i(k_i, \mathrm{g})$ is i's *market payoff*, to be interpreted as how much profit the firm can achieve by freely competing. We assume that the larger k_i, the more efficient firm i at producing the good; thus the larger its market profit $p_i(k_i, .)$ (see assumption 1 for the full statement). And (ii) $r_i(\mathrm{g}) = r_i^+(\mathrm{g}) - r_i^-(\mathrm{g})$ is i's *royalty net revenue.* This is all royalty payments that i receives from its successors that have a link to i, minus i's royalty expenditures the later pays to all of its predecessors in s_i. The expressions are $r_i^+(\mathrm{g}) = \sum_{h:\ i \in s_h} r_i^h$ and $r_i^-(\mathrm{g}) = \sum_{h \in s_i} r_h^i$ if we follow the notations introduced in the former paragraph. The total payoff of firm i, given the technological network g and i's technology k_i is:

$$\pi_i(k_i, \mathrm{g}) = p_i(k_i, \mathrm{g}) + r_i(\mathrm{g}) - F. \tag{1}$$

The next assumption imposes a direct relation between a firm's relative location in the network and its market payoff.

Assumption 1. *Technology and payoffs*

Firms with higher technology have a higher market payoff:

$$k_i \geq k_j \quad \Leftrightarrow \quad p_j(k_j, .) \geq p_i(k_i, .),$$

where the equality holds if and only if $k_i = k_j$.

Note that this assumption does not mean that two firms that have the same technology have the same total payoff since their royalty revenues may differ. The rest of the assumptions are commonly used in any market environment.

Assumption 2. *Foresightedness*

Firms are foresighted: they anticipate the arrival of successor firms on the market, and take decisions accordingly.

Since there is risk or uncertainty in the model, this assumption implies that all firms must have positive payoffs.

Assumption 3. *Market power*
Consider any technological network g; and take any subgraph $g' \subseteq g$ *of the former network. Then:* $p_i(k_i, g') \geq p_i(k_i, g)$ *for all* $i \in g'$*. For a same technology level, a firm has a higher market payoff the less competition it faces on the market.*

In other words, firms may face both upstream and downstream competition.

Finally, we make explicit the timing of the game:

1) ENTRY AND NETWORK FORMATION. *Each firm in* Ω *sequentially decides whether it enters the industry or not; if it enters, a firm immediately chooses simultaneously its strategy of link formation which determines its technology level. Royalty payments are cleared and settled. By the end of this stage, the technological network* $g = (s_1, \ldots, s_n)$ *is formed.*

2) PRODUCTION. *The firms in* N *decide on how much they supply on the market.*

3) Payoffs are realized.

3 Subgame Perfect Nash networks and equilibrium royalty payments

Since we did not specify the market payoffs of the firms in N, we solve for the Nash equilibria of the network formation stage as well as for the equilibrium royalty payments. A firm's decisions regarding entry and its choice of technology happen simultaneously. The choice of technology determines the set of links a firm forms, as well as the royalty the later pays for each of its connections. Later on in the game, if a successor wants to form a link to a predecessor, the predecessor will then make a take it or leave it royalty offer for its successor. In other words, upon entering a firm will pay the royalties immediately to the firm it will connect to, and will only receive royalty payments when its successors enter.

We first present the results about the royalty payments between the different firms in the industry. These results will help to derive some of the properties of the subgame perfect Nash networks featured in section 5.

4 General results on equilibrium royalty payments

In this section we present partial results on the equilibrium royalty payments made by firms which infringe on the technologies of their predecessors. These results hold regardless of the network structure. The specific relations between royalty payments and network structure are explored in the next section. We start by setting clear some terminologies.

Definition 2.

- *Consider some network $g \in \mathcal{S}$. Firm $i \in N$ is a net receiver if and only if:*

$$r_i^+(g) > r_i^-(g),$$

and is a net payer otherwise.

- *Firm i is said to be a rent seeker if and only if:*

$$\pi_i(k_i, g) = r_i(g) - F.$$

  In equilibrium, all rent seekers are net receivers.

- *A firm that is not a rent seeker is called a producer. In equilibrium, all net payers are producers.*

- *In equilibrium, there is at least one producer, which is firm n.*

Proof. The second and third bullet points are implied by our assumption that firms are farsighted. For the last statement, note that n does not have any successor: thus $r_n(\mathrm{g}) \leq 0$ for any $\mathrm{g} \in \mathcal{S}$. Thence n must produce. □

The first proposition shows that when two firms acquire the same technology, then any of their common successor can improve upon their technology for free.

Proposition 1. *In equilibrium, two firms j and h that have the same technology level $\tilde{k} = k_j = k_h$ transmit $\tilde{k}$ to their common successors for free. Meaning, if $i > j, h$ is a common successor of both j and h then $r_h^i = r_j^i = 0$.*

Proof. Consider firm $i > h, j$ a common successor of both firms h and j. If i wants to acquire technology $k_i = \tilde{k} + 1$, then i rationally pays for only one link to either h or j if the royalty is strictly positive. Firms h and j are in competition for i's attachment to them; here, h and j play a prisonner's dilemma game which Nash equilibrium is $r_h^i = r_j^i = 0$. □

Two firms that decide to compete on the market with the same technology sacrifice any prospect of royalty revenue. In the next proposition, we show that a firm cannot subsidize the technology of these successors which build upon theirs.

Proposition 2. *In equilibrium, a firm that infringes on another one's technology always pays a positive royalty cost for doing so. In other words, there is no case where a firm subsidizes the technology of its successors in equilibrium.*

Proof. Consider the following scenario: there are $i-1$ firms which entered the market, and the highest technology level used so far is denoted $\bar{k}$. Consider firm i. Assume that a predecessor of i, that we shall call firm f, offers to subsidize a link from i to herself. Note that firm f has a technology k_f that is strictly worse than $\bar{k}$. This offer is rational if f intends here to deter i from further innovating beyond the technology level k_f+1. Assume that i accepts the offer: it forms a link to f and gets paid for it. If a firm among all these in $1,\ldots,i-1$ and which has a strictly higher technology than k_f+1 negotiates with i, and if the negotiation is successful, then i gets attached to this firm as well. But then i uses the technology it paid for since it is better than k_f+1. Thus f subsidized i for nothing. Therefore offering a subsidy was irrational for f. Now, if i's negotiations with all other firms were unsuccessful, then f would have been better off by offering a positive price for the link from i to her. □

The following corollary sheds light on the implications of the two propositions.

Corollary 1. *In equilibrium, (i) there are at most 2 firms that have the same technology, and (ii) both of them are producers. Also, (iii) all of their common successors have strictly superior technologies to theirs.*

Proof. (i) Assume not; i, j and m are three firms in N with $i<j<m$, and $k_i=k_j=k_m$ in the network g. Consider firm m; the later could have gotten the technology k_m+1 for free and earned more market profit. In g, m has no royalty revenue by proposition 1. If m has formed a link with either i or j instead, m could have earned a positive royalty revenue. (ii) All common successors of some firm j which acquires the technology $\tilde{k}=k_j=k_i$ with $i<j$ do not trigger any royalty revenue to either i or j by proposition 1; therefore j must produce in equilibrium by definition 2. Note that if i was not producing, equivalently if $p_i(\tilde{k},\mathrm{g})=p_j(\tilde{k},\mathrm{g})<F$, then it would have been

irrational of j to enter the market and get the technology $\tilde{k}$. Hence both i and j must produce. Finally, (iii) follows from (i). □

A firm that does not improve upon the highest technology available in the industry at the date it enters never gets any royalty revenue. We conclude this section by setting clear the relation between the set Ω of all firms which face the decision of entering the industry and the set N of these which decide to enter.

Proposition 3. *In equilibrium, if firm $\omega_i \in \Omega$ decides to enter the industry, then so did all firms $\omega_1, \ldots, \omega_{i-1}$. Put differently, $\omega_i = i$ in equilibrium for all $i \in N$.*

Proof. Assume that $\omega_1, \ldots, \omega_{i-1}$ entered the market, ω_i decided not to, and ω_{i+1} does. Hence $\omega_{i+1} = i$ with $i \in N$. Let $\pi_i(k_i, \mathrm{g})$ the payoff of ω_{i+1}; since the later firm is farsighted and rational, then $\pi_i(k_i, \mathrm{g}) \geq 0$. But then i could have earned the same payoff by entering the industry, payoff that is weakly larger than i's of 0. □

5 Network topology and its relation with royalty payments

5.1 Subgame Perfect Nash networks

We solve for the optimal decisions of the firms in terms of their choice of links. We first set clear the definition of a subgame perfect Nash network.

Definition 3. *The vector $(s_1, \ldots, s)$ of all firms' strategies is a SPNE of the game if and only if there is no alternate strategy t_i for each $i \in N$ that gives firm i a strictly larger payoff, for any $t_i \neq s_i$ with $t_i \in \mathcal{S}_i$. A network $g = (s_1, \ldots, s_n)$ is said to be a subgame perfect Nash network if and only if $(s_1, \ldots, s_n)$ is a SPNE.*

We first reveal the common features of the subgame perfect Nash networks. The first result states that a firm builds upon at most 1 technology.

Proposition 4. *Maintaining at most 1 link is a weakly dominant strategy for every firm in N.*

Proof. Let $s_i \in \mathcal{S}_i$ be any strategy for firm i that consists in maintaining at least two links in the technological network $\mathrm{g} = (s_i, s_{-i})$. Let $h \in s_i$ be the firm that has the largest technological level among all firms in s_i. Consider the alternate strategy $s'_i \subset s_i$ for i that consists of forming one single link to h; and let us call $\mathrm{g}' = (s'_i, s_{-i})$ the resulting alternate network. We show that s'_i always weakly dominates s_i. By assumption only the longest path that starts at i determines i's production technology. The longest path that starts at i has the same length whether i plays s_i or s'_i. Therefore s'_i gives the same market payoff to i than s_i does. Now, the royalty revenue of firm i. If i maintains a single link to h, then i receives revenues from its successors which infringe upon its technological level $k_i = k_h + 1$. If i maintains two links with the firms e and h in s_i, then given that $k_e < k_h$, there is at least one other firm than i that has access to the technology level $k_e + 1$. We know from proposition 1 that $i \to e$ triggers no royalty revenue to i.
Since the only difference between s_i and s'_i is the number of links maintained, and since a link is never subsidized by proposition 2, then $r_i^-(\mathrm{g}) = r_i^-(\mathrm{g}')$ if and only if the extra links are costless to i, and $r_i^-(\mathrm{g}) > r_i^-(\mathrm{g}')$ if not. So we have the result. □

Already quite a lot can be said about the topology of the subgame perfect Nash networks of the game. What should be recalled so far is that the first firms to enter the market maintain one link to their direct predecessor so as to form a chain $1 \to 2 \ldots j-1 \to j$, for some threshold firm $1 \leq j \leq n$. These firms (except for j) earn a positive royalty revenue paid by its direct successor, guaranteeing to themselves some income in case their technology is not good enough to earn a positive market profit once the full network realized. For the rest of the firms from $j+1$ to n, each may improve upon the highest technology they find when they enter the industry (thus extending the chain). Or they get a cheaper technology used already by one of its predecessors (by corollary 1 only two firms produce with the same technology in equilibrium) that dooms them to produce in equilibrium.

We now give a result that ties the connectedness properties of the technological network to the equilibrium production decisions of the firms.

Corollary 2.

(i) If g is a subgame perfect Nash network and g is not weakly connected, meaning that another firm than 1 operates with the technology level 1*, then all firms in the industry produce.*

(ii) If g is a subgame perfect Nash network and g is weakly connected, then all firms in the industry produce if firm 1 produces.

Proof. (i) Consider firm i in g such that $k_i = 1$. By proposition 1, $r_i(\mathrm{g}) = 0$; and by assumption i is farsighted. Thus i must produce in equilibrium, and $p_i(1, \mathrm{g}) \geq F$. Thus $p_j(k_j, \mathrm{g}) \geq F$ for all firms j with technology $k_j \geq 1$, i.e. for all in N. The same reasoning applies to (ii). □

In the next section, we show the implications of both the results about equilibrium royalties and best-responses in link formation.

5.2 Payoffs and royalty payments in Subgame Perfect Nash networks

In this section we further clarify the relations between the royalty payments in the industry, the payoffs of the firms and the network that is formed in equilibrium. The first result deals with these firms that all have a same production technology. It has been showed earlier on that at most two firms produce with the same technology in a subgame perfect Nash network. We now highlight the relation between their profits and royalty payments.

Remark 1. *Consider some technological network g. In equilibrium, if two firms i and j have technology $\tilde{k}$ in g, and i is j's predecessor ($i < j$), then $r_i^-(g) \geq r_j^-(g)$. Further $\pi_i(g) \leq \pi_j(g)$ if all firms that are successors of i and predecessors of j have each a technology that is strictly worse than $\tilde{k}$.*

Proof. We show that $r_i^-(\mathrm{g}) \geq r_j^-(\mathrm{g})$. Note that if there are two firms, both predecessors of i, that have the technology $\tilde{k} - 1$, then neither i nor j have paid anything for theirs. Let us continue with the case where only 1 firm, say firm f, owns $\tilde{k} - 1$ when i enters the industry. At the time i negotiates, firm f vies with the other $i - 2$ predecessors of i for i's attachment; while when j negotiates, f faces the additional competition of these successor firms of i and predecessor firms of j. More competition can only drive the cost of a link to f down, i.e. $r_i^f \geq r_j^f$. Since by proposition 4 a firm pays a strictly positive royalty to at most one of its predecessors, it follows that $r_i^-(\mathrm{g}) \geq r_j^-(\mathrm{g})$. The conclusion of the last claim in the remark follows first from $r_i^+(\mathrm{g}) = r_j^+(\mathrm{g}) = 0$ by proposition 1; second, both firms produce by corollary 1 point (ii) and $p_i(\tilde{k}, \mathrm{g}) = p_j(\tilde{k}, \mathrm{g})$ by assumption 1. □

The remark above says that there is a malus associated with being the first firm to get some technology level when another firm will acquire the same later on. The idea is that the firm which is the second to acquire the technology pays less for it, as there is more competition for winning this second firm's attachment than for the first firm. Also, if between i and j entry dates no firm has attached to i, then i has no longer any prospect of earning a royalty revenue - since it will necessarily vie against j for every single attachment. Consequently, both of these two firms have zero royalty revenue; and they earn the same market payoff as their production technologies are identical.

Discussion: let us focus on the specifics of the bargaining procedure through which an entrant acquires the technology of one of its predecessors. Consider a firm which enters the market and that does not build upon the current highest technology level - understand here that the entrant does not extend the "chain" of the network he finds upon entering. For the sake of clarity, assume that by the time entrant i enters, the highest technology $\tilde{k} = k_h$ is owned by firm $h < i$. We see two possible explanations for why i does not infringe on k_h:

(i) firm i could not afford the cost of a connection to firm h. Meaning that given that h charges a royalty that is at least equal to the *negative externality* of having i producing with the technology $\tilde{k}+1$ instead of some other less efficient one; and that the maximum h can charge to i is the differential in i's payoff when the later produces with technology $\tilde{k}+1$ instead of with some other less effective production technology; the second term is less than the first one. (Firm i can never afford to fully compensate the negative externality it imposes on firm h),

(ii) firm h strategically prevents i from being more cost effective at producing than it itself is. Here,

i could afford to compensate the negative externality imposed on h from having i producing with $\tilde{k}+1$; however h demands an extravagantly high royalty for the sake of deterring i from acquiring the superior technology $\tilde{k}+1$.

In the next proposition, we show that a predecessor strategically prevents an entrant from accessing its technology only if the predecessor manages to secure a monopoly on its technology until the end of the network formation process.

Proposition 5. *Consider any three firms h, i and j in N such that $h < i < j$. If $k_h > k_i$ and $k_j > k_h$ in equilibrium, then i could not afford technology $k_h + 1$. (h did not prevent i strategically from acquiring $k_h + 1$.)*

Proof. Assume not. Firm h predecessor of i strategically prevents i from forming the link $h \to i$; and j, successor of i, produces with technology $k_j = k_h+1$ in equilibrium. Let g be the technological network. If h strategically prevents i from getting the technology $k_h + 1$, then it cannot be that $j > i$ acquires this technology in the end. To see why: first, because if i gets a lower technology than k_h+1, h faces an increased competition for j's attachment, which consequently reduces h's expected royalty revenue (if ever the link $h \to j$ exists in g; otherwise h's royalty revenue is null). Second, because i having technology $k_h + 1$ would have deterred more subsequent firms from entering than when i acquires some inferior technology, which leaves the door open for the successors of firm i to enter and access some technology either for free or for a lower price than what they would have paid if i had connected to h. (In other words, there would have been no subgraph of the network that would have been realized if i had formed a link to h that is not a subgraph of g). Thus h would have faced less competition on the market; then h would have earned a larger market profit by assumption 3. Finally, the negative externality on the predecessors of h imposed by the strategy of the later is larger than if h had let i get the technology $k_h + 1$. First because in the end j manages to reach $k_j = k_h + 1$; second, because of the consequent increased competition just mentioned. □

A firm that chooses not to improve upon the current best technology lowers by its action the cost paid by its successors for their own technology compared to what they would have paid for the same technology otherwise. This is because a firm that chooses the aforementioned link strategy leaves the possibility for its successors to access some technology for free, which therefore increases the competition between the predecessors for winning an entrant's attachment. It follows that for any given link $i \to j$, the link is the most expensive if the network prior to j's entry is a chain $1 \to 2 \to \ldots \to i-1 \to i$ and $j = i+1$; and it is the cheapest (the link is free) if the technology i uses is also used by another predecessor of j. Other things being equal, the royalty paid by an entrant depends positively on the technology acquired by its direct predecessor.

The next proposition compares the payoffs of the first firms that enter the industry and that form a chain. We first clarify the set of firms on which the result applies then feature our statement.

Definition 4. *Consider some technological network $g = (s_1, \ldots, s_n)$ and assume that there is a subgraph g' on $m \leq n$ firms that is a chain such that: $s_i = i-1$ for all $1 \leq i \leq m$, with $2 \leq m \leq n$. The subgraph $\mathcal{C}_1^m$ will be referred to as the chain of the m incumbents of g.*

If g has a chain $\mathcal{C}_1^m$ of m incumbent firms, then the first m firms that have entered the market have played a linking strategy that consists in forming a link to their direct predecessor in the industry. The next proposition features a result on the relative royalty expenditures paid by the firms along the chain $\mathcal{C}_1^m$ in any network g with $\mathcal{C}_1^m \subset$ g.

Proposition 6. *Consider some technological subgame perfect Nash network g that has a chain $\mathcal{C}_1^m$ of m incumbents, with $2 \leq m \leq n$. Along $\mathcal{C}_1^m$, the royalty paid by a firm in equilibrium is decreasing in its index: $r_i^-(g)$ is decreasing in i, for all $i \in \{1, 2, \ldots, m\}$.*

Proof. Consider $\mathcal{C}_1^m$ the chain of m incumbents in g. All firms in $\{1, 2, \ldots, m\}$ could afford to pay for the *negative externality* they are imposing on their direct predecessor (we mean by *negative externality* what we have been defining as so in point (i) in the discussion paragraph). Because the network g is subgame perfect Nash and royalties paid are in equilibrium, both individual rationality and incentive compatibility constraints are satisfied for every firm along g$'$. Given firm i's linking strategy s_i such that $i-1 \in s_i$, the royalty paid by i to $i-1$, $r_i^-(\mathrm{g}) = r_i^{i-1}(\mathrm{g})$, to reach the technology level $k_i = i$ is in equilibrium if:

$$(\mathrm{IR}_i) \quad \pi_i(i,\mathrm{g}) \geq 0 \quad \Leftrightarrow \quad p_i(i,\mathrm{g}) - r_i^{i-1}(\mathrm{g}) + r_i^{i+1}(\mathrm{g}) - F \geq 0$$

$$(\mathrm{IC}_i) \quad \pi_i(i,\mathrm{g}) \geq \max\{\pi_i(\tilde{k}_i,\tilde{\mathrm{g}}),0\} \quad \Leftrightarrow \quad p_i(i,\mathrm{g}) - r_i^{i-1}(\mathrm{g}) + r_i^{i+1}(\mathrm{g}) - F \geq \max\{p_i(\tilde{k}_i,\tilde{\mathrm{g}}) - r_i^h(\tilde{\mathrm{g}}) - F, 0\}$$

where: $\pi_i(\tilde{k}_i,\tilde{\mathrm{g}})$ is the maximum payoff i would get by attaching to any firm $h < i-1$ instead of $i-1$, and $\tilde{\mathrm{g}}$ the network that would result if i was playing the linking strategy $\tilde{s}_i$ with $i-1 \notin \tilde{s}_i$ and $h \in \tilde{s}_i$. Note that any other link that i may have with other firm(s) than $i-1$ in g is free by proposition 4. And if $\tilde{k}_i = k_h + 1 = h+1$ in the alternate network $\tilde{\mathrm{g}}$, then $r_i^h(\tilde{\mathrm{g}}) \geq 0$ and the rest of i's link in $\tilde{\mathrm{g}}$ are free, by the same proposition; also $r_i^+(\tilde{\mathrm{g}}) = 0$ by proposition 1 since i shares the same technology as its predecessor, firm $h+1$.

Claim. *If g is subgame perfect Nash and the royalty payments are in equilibrium, then the IC constraints of all firms* $1,2,\ldots,m$ *that belong to* $\mathcal{C}_1^m$ *are binding.*

Proof. Otherwise, i's predecessor, firm $i-1$, could increase the royalty it claims against i's attachment by ϵ sufficiently small so that $i-1$ still prefers to maintain its link to $i-1$ over deleting it and forming a link to h instead, for any $2 \leq i \leq m$. $\square$

Let us rewrite the *IC* constraint of any firm $1 \leq i \leq m-1$ in equilibrium as follows:

$$(\mathrm{IC}_i) \quad r_i^{i-1}(\mathrm{g}) = \max\{\Delta p_i + r_{i+1}^i(\mathrm{g}) - r_i^-(\tilde{\mathrm{g}}) \ , \ p_i(i,\mathrm{g}) + r_i^{i+1}(\mathrm{g}) - F\},$$

with $\Delta p_i = p_i(i,\mathrm{g}) - p_i(\tilde{k}_i,\tilde{\mathrm{g}}) > 0$. Replacing what firm $i+1$ pays to i in g by $i+1$'s IC constraint, and applying this recursively gives:

$$r_i^{i-1}(\mathrm{g}) = \sum_{l=i}^{m} \max\{\Delta p_l - r_l(\tilde{g}) \ , \ p_l(l,\mathrm{g}) - F\},$$

where every term of the sum is positive (otherwise l's linking strategy s_l would be strictly dominated, for any $i \leq l \leq m$). It follows that $r_i^{i-1}(\mathrm{g})$ is an increasing function of the index i of a firm. Therefore the result. $\square$

The result introduced in the last proposition has strong implications onto the relative royalty revenues of the firms in a chain network.

Corollary 3. *Assume that the chain network on all n firms is subgame perfect Nash. In equilibrium, all firms but firm 1 are net payers and they are producers.*

Proof. Assume that $\mathrm{g} = (s_1, \ldots, s_n)$ is a chain on all n firms and g is subgame perfect Nash. Note that $s_i = i - 1$, for any $2 \leq i \leq n$; also, $\mathrm{g} = \mathcal{C}_1^n$. Applying our result from proposition 6, we have $r_i(\mathrm{g}) = r_{i+1}^i(\mathrm{g}) - r_i^{i-1}(\mathrm{g}) \leq 0$ for any $i \neq 1$. It follows from definition 2 that all $i \geq 2$ are net payers - thus producers. And firm 1 is a net receiver. $\square$

The last proposition essentially says that in a network that has a chain of m incumbents, every firm along it gets *expropriated*, through the royalty it pays, from the surplus it realizes by linking to its direct predecessor instead of by forming a link to any other firm. This expropriation process goes as follows: at the time firm 2 enters and forms a link to firm 1, the later makes the former pay for the total surplus of all subsequent firms $2, 3, \ldots, m$ in the chain. Firm 2, which ends up paying its connection to firm 1 for more than what it individually gains from it, gets its compensation by charging a royalty to firm 3 that is the sum of all of the surpluses other than its own (that it yet had to pay to firm 1); that is, the surpluses that $3, 4, \ldots, m$ will make. This process continues recursively, so that the cheapest connection in the chain is that maintained by the last firm m.

6 Case study

This section will hopefully help the reader to develop an overall understanding of the intricacies of our game. The first subsection shows how the type of competition considered may strongly imply the architecture of the subgame Nash network of the game. For the case of the competition a la Bertrand, at most two firms enter the market and only the last one produces. Next, we solve for the subgame perfect Nash equilibria of the game in the general case, but with only 3 firms in the set Ω.

6.1 The case of Bertrand competition

Consider our game and assume that the firms in N, once the network formation and royalty payment stage finished, compete a la Bertrand. It is immediate that there are either 1 or 2 producers, and these firms are these with the best technology in the industry (recall from corollary 1 that at most 2 firms can have the same technology). We directly move on to the first stage of the game. We find that there are only two architectures possible for the subgame perfect Nash networks. This result is presented in the remark and proposition below. The result about the royalty payments is featured in the second proposition.

Remark 2. *If $g = (s_1, \ldots, s_n)$ is a subgame perfect Nash network, then $k_i = i$ (i.e. $i-1 \in s_i$) for all firms $i \in N$.*

Proof. Assume that all firms $j \in \{1, \ldots i-1\}$ have $j = k_j$ however $k_i < i$. Let $\bar{k}$ the highest technology level in the industry. Since $k_i < i$, i shares the same technology as that of one of its predecessors; hence by corollary 1 i is a net payer and i must produce in equilibrium. Thus it must be that $i = n$; and $k_n = k_{n-1} = \bar{k}$ since (i) firms compete a la Bertrand thus only the most cost efficient firms produce, (ii) that the last firm to enter, firm n, always produces hence $k_n = \bar{k}$ and (ii) both firms n and $n-1$ are farsighted. Consider firm $\omega_{n+1} \in \Omega$. If the later enters and forms a link to either n or $n-1$, it gets the technology $\bar{k}+1$ and $p_{\omega_{n+1}} - F > \pi_n(\bar{k}, \mathrm{g})$. Therefore a contradiction that n is the last firm to enter the industry. □

We show in the proposition below that a subgame perfect Nash network can have only 2 architectures.

Proposition 7. *Given that the firms in N compete a la Bertrand, a nonempty subgame perfact Nash network is:*

1. *either there is only 1 firm in the industry (i.e. $N = 1$) and we denote this network g_1,*

2. *or $g = \mathcal{C}_1^2$; meaning, there are 2 firms in the industry, only firm 2 produces and firm 1 is a rent seeker.*

Proof. By remark 2, there is a subgraph of g that is a chain $\mathcal{C}_1^n$ of n incumbents; hence if the conclusion of the proposition is false, then $n > 2$. Along $\mathcal{C}_1^n$, $r_i(\mathrm{g}) \leq 0$ for any firm $2 \leq i \leq n$ by proposition 6. Since only n produces due to the type of competition considered, it must be that $r_i(\mathrm{g}) = 0$ for any $2 \leq i \leq n-1$. It follows that $\pi_i(i, \mathrm{g}) = -F < 0$ for these firms. A contradiction that i is farsighted. Thus $n = 2$. If $\mathrm{g} = \mathcal{C}_1^2$ (since firm 2 cannot have any other link than the one to firm 1), then 1 is a rent seeker since firm 2's technological is superior, and 2 is a producer and net payer since it is the last one to enter the market. □

At last we determine conditions on the value of the entry cost F for which either one of these two architectures is a subgame perfect Nash network.

Proposition 8. *Consider the 2 networks g_1 and g_2 featured in the last proposition. Then:*

(i) g_1 is the subgame perfect Nash network of the game if and only if $p_1(1, g_1) - F > \max\{p_2(2, g_2 - 2F\ ,\ 0)\}$,

(ii) g_2 is the subgame perfect Nash network of the game if and only if $p_2(2, g_2) - 2F \geq \max\{p_1(1, g_1) - F\ ,\ 0\}$,

(iii) the empty network is the subgame perfect Nash network of the game if and only if $\max\{p_1(1, g_1) - F\ ,\ p_2(2, g_2) - 2F\} < 0$.

Proof. Consider first the network g_2. If firm 1 decides to let firm 2 enter the market and form a link, then the incentive compatibility constraint of firm 2 is saturated by proposition 6. If firm 2 does not enter, its payoff would be zero; and if it enters without forming a link to firm 1, then 2 would get a negative payoff. Therefore: $r_2^1(\mathrm{g}_2) = p_2(2, \mathrm{g}_2) - F$. Since firm 1 is a rent seeker in the network g_2, it follows that $\pi_1(1, \mathrm{g}_2) = p_2(2, \mathrm{g}_2) - 2F$. If firm 1 does not let firm 2 enter, then the network that is realized is g_1 in which firm 1's payoff is $\pi_1(1, g_1) - F = p_1(1, g_1) - F$. The inequalities in (i), (ii) and (iii) follow immediately. $\square$

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