What is the probability of a public good?

***Abstract*—What is the probability of a public good problem?**

A great deal of attention has recently been put foward to the problem of public goods. Coincidentally in parallel, inequality research has skyrocketed. Seldom has somebody tried to put the two together.

A public good is a good which is not excludable and not rival. A public good PROBLEM is when the good adds more value than it costs but there is no incentive to contribute. Why? Because one can always believe that the OTHERS will finance it. We will always be assuming that the public good is socially worthwhile. They can only bid their entire willingness to pay

Suppose that the willingness to pay is uniform. Say all consumers value the public good at 1. If the public good costs 10, and there are 10 consumers, then the probability of it being financed is 1. If there are 11 consumers, then consumers may wish to opt for a mixed equilibrium,

Mixed equilibrium is

$$\alpha = \frac{V-c}{V-\frac{c}{2}} \tag{1}$$

I. With n players

The condition is:

$$nV - c > 0 \tag{2}$$

$$a(n) = \frac{V-\frac{c}{n-1}}{V-\frac{c}{n}} \tag{3}$$

So the probability of a public good is:

$$V < \frac{c}{n-1} \tag{4}$$

$$\rightarrow P(n,c) = a^{11} \tag{5}$$

$$V < \frac{c}{n-2} \tag{6}$$

$$\rightarrow P(n,c) = a^n + B(n,n-1)a^{n-1}(1-a) \tag{7}$$

$$\sum_{i=0}^{\lceil \frac{c}{V} \rceil} B(n,n-i)a^n(1-a)^i \tag{8}$$

II. Probability of a public good?

If the probability of an event is P, we should expect NP realizations in N. So what is the probability that after N realizations only 1 event has occured? $P = \frac{1}{N}$

R	Finance	Not
Finance	V-c/2, V-c/2	V-c, V
Not	V, V-c	0,0

$$P_{\geq}(\lambda) = \frac{1}{N} \tag{9}$$

$$\Pi_{<}(\lambda)\text{says the probability that: } X_{max} < \lambda \tag{10}$$

$$\tag{11}$$

Let f(x) be the pdf of X

$$Z = X_1 + X_2 + ...X_n \tag{12}$$

$$X_{max} = max\{X_i; i = 1, ..., N\} \tag{13}$$

$$G_{<}(\lambda) \tag{14}$$

$$\tag{15}$$

where G is the CDF of X_{max}

$$G_{<}(\lambda) = [F_{<}(\lambda)]^N \tag{16}$$

$$= [1 - F_{\geq}(\lambda)]^N \tag{17}$$

$$= exp\{Nln\{1 - F_{\geq}(\lambda)\}\} \tag{18}$$

$$\approx exp\{-NF_{\geq}(\lambda)\} \tag{19}$$

$$P(X > c \cap max(X) > c) \tag{20}$$

What is probability

First compute the probability that there is a public good worth undertaking. That is the sum of the willingness to pay is higher than the cost of the project. Then find the probability that NO individual willingness to pay is higher than the project. Show that variance of the distribution reduces.

Note that the assumption that willingness to are independent might be the worst case scenario.