

0.1 Industrial Organization, Week 2 Answers

0.1.1 Monopoly solution using price

So the tricky thing in this question is that I gave you the demand q function of price but the cost as a function of quantity. As such there are two equivalent ways of writing the problem. First we write it as a function of price:

$$\begin{aligned}\pi(p) &= pq(p) - c(q) \\ &= p(60 - p) - \frac{1}{2}q^2 \\ &= 60p - p^2 - \frac{1}{2}(60 - p)^2 \\ &= p(60 - p) - \frac{1}{2}(60^2 + p^2 - 120p)\end{aligned}$$

Once we have eliminated q from the equation we can take the partial derivative.

$$\begin{aligned}\frac{\delta\pi(p)}{\delta p} &= 60 - 2p - p + 60 \\ &= 120 - 3p = 0 \\ &\rightarrow p = 40; q = 20\end{aligned}$$

0.1.2 Monopoly solution using quantity

For completion we can do the same thing with but this time eliminate the price: First re-write the price, $p = 60 - q$ and then write the profit as function of quantity:

$$\begin{aligned}\pi(q) &= p(q)q - c(q) \\ &= (60 - q)q - \frac{1}{2}q^2\end{aligned}$$

Once we have eliminated p we can take the partial derivative and set it equal to zero.

$$\begin{aligned}\frac{\delta\pi(q)}{\delta q} &= 60 - 2q - q = 0 \\ &= 60 - 3q = 0 \\ &\rightarrow q = 20; p = 40\end{aligned}$$

0.1.3 Perfect competition

Perfect competition outcome:

In perfect competition, the price is equal to the marginal cost, the marginal cost is simply the derivative of the cost function q

$$\begin{aligned}q &= 60 - q \\ &\rightarrow q = 30; p = 30\end{aligned}$$

So the price is lower and the quantity is greater.

0.1.4 Deadweight loss

This time, the cost function intersects the $(0, 0)$ point, so we can simply use the price to get the base of the triangle. Perfect competition outcome surplus is: $60 * 30/2 = 900$

So monopoly outcome surplus is more complicated: There is the upper triangle which is the consumer surplus, $(60 - 40) * 20\frac{1}{2} = 200$ The producer surplus will be the rectangle and the lower triangle. The lower triangle is: $20 * 20\frac{1}{2} = 200$ For the rectangle, we must first compute the lower right point. This is where the equilibrium quantity, intersects the marginal cost curve $20 = q$. So the rectangle has $(40 - 20) * 20 = 400$ Total is: 800

Dead weight loss is 100

0.1.5 Graph

[Click here](#) for the graph:

