Microeconomics

September 22, 2017

Exercise 1.1: Rational preferences

Let $\succsim$ be a rational preference on X(rational and transitive). Show that

$a \succsim b$, $b \succsim c \rightarrow a \succsim c$

1) It is irreflexive and transitive

a) $a \succ b$ and $b \succ a$ so $b \succsim a$ but $a \succ b$, or $\neg b \succsim a$: contradiction!

b)If $a > b$ and $b > c \rightarrow a \succsim$ b and $b \succsim c$ then we have that $a \succsim c$, which indicates that a $\succ$ c or a $\sim$ c $\rightarrow \neg b \succsim a$ and $\neg c \succsim b$; because if $c \succsim a$, by transitivity of $\succ$, we have that $c \succsim b$: contradiction

2) $\succ$ is reflexive transitive and symmetric

a) Immediate

b) If $\begin{cases} x \succ y \\ y \succ z \end{cases}$ $and \begin{cases} y \succ z \\ z \succ y \end{cases} \rightarrow \begin{cases} x \succ z \\ z \succ x \end{cases}$

c) If $x \sim y \rightarrow \begin{cases} x \succ y \\ z \succ y \end{cases} \rightarrow y \sim x$

3) if $x \succ y \succsim z$ then $x \succ z$

$x > y \succ$ z $\rightarrow$ x $\succ$ y $\succ$ z $\rightarrow$ x $\succ$ z by transitivity.

By contradiction, if z $\succ$ x, by transitivity, z $\succ$ y. We have by the hypothesis that z $\sim$ y, and by transitivity of $\sim$ we have that $y \sim x$: which is a contradiction.

Exercise 1.2: Representation of preferences

Let $u : X \rightarrow R$ be utility function which represents the preferences on $\succsim$ on X, such that. $u(x) \geq u(y) \iff x \succsim y, \forall x, y \in X$

Show that for all functions $f : R \rightarrow R$ which are strictly increasing $f \circ u$, also represents $\succsim$. What happens if f is increasing but not strictly?

a) $f(u(x)) \succsim f(u(y)) \iff u(x) \succsim u(y) \iff x \succsim y$

b) In this case, the first equivalence is false: $u(x) \geq u(y) \rightarrow f(u(x)) \geq f(u(y))$ holds always but the inverse relation does not. Example: f =constant $, u(z) = z \forall z \in R, x = 0, y = 1$ $f(u(x)) = f(u(y))$ and $u(x) < u(y)$

Exercise 1.3: preferences on a finite set

Let X be a finite set and $\succsim$. Show that their exists a utility function $u : X \rightarrow R$ which represents the preferences.

By induction. Let $M_1(x \in X, x \leq y \forall y \in X) \neq \emptyset$ We let $u(z) = 1 \forall z \in M_1$ If $M_1 = X$ we are done, Otherwise $M_1 \nsubseteq X$ and $X_1 = X \setminus M_1$ we let $u(z) = 2 \forall z \in M_2 = (x \in X_1, x \leq y), \forall y \in X_1$ and repeat.

This algorithm takes at most $|X|$ stages and constructs a representative utility function of $\geq$ for the values of N

Remarque: While X is countable we can represent $\geq$ by a utility function u: $x \rightarrow (0,1)$

Exercise 1.4: Weak axiom of revealed preferences

Let $X = \{x, y, z\}$ be an ensemble of alternatives, $G = \{\{x, y\}, \{x, y, z\}\}$ a sub set of X and let C be a function of choice defined on G so that $C(\{x, y\}) = \{x\}$. Show that if C satisfies the weak axiom of revealed preferences, then $C(\{x, y, z\})$ is equal to $\{x\}\{z\}, \{x, z\}$

Reminder that C verifies the weak axiom of revealed preferences if, when x is revealed to be equally preferred to y, y cannot be revealed to be strictly better than x. Said otherwise, there does not exist an $A, B \in G$ so that $x, y \in A \cap B, x \in C(A), y \in C(B)$ and $x \notin C(B)$

Suppose that y $\in C(\{x, y, z\})$ and let $A = \{x, y\}$ et $B = \{x, y\}$

Therefore we have that $y, x \in B \cap A$ $y \in C(B)$ $x \in C(A)$

According to the (WA) this implies that $y \in C(A)$, a contradiction. We need only verify that C(B) = $\{x\}$ or $\{x\}$ or $\{x, z\}$ does not contradict the (WA). But this is trivial because $A \cap B = \{x, y\}$ said otherwise, $z \notin A \cap B$ which means that it can't serve as a counterexample to the axiom.