0.1 Only the chef invests

Three kinds of people are needed to make the boat trip work. The tycoon who wants to go on the boat trip, the chef who can cook and a skipper. The chef can undertake a private investment to learn how to cook before the boat trip. To go on the boat trip, they also need a boat.

If the chef invested the value is:

$$V(chef, tycoon, skipper1, skipper2) = V(all) = 240$$

The value added of everybody if the tycoon or chef owns the boat is then:

$$V(all) - V(all \setminus chef) = 240$$
$$V(all) - V(all \setminus tycoon) = 240$$
$$V(all) - V(all \setminus skipper1) = 0$$
$$V(all) - V(all \setminus skipper2) = 0$$

So the chef and the tycoon will split the cake bewteen them:

$$P(chef) = P(tycoon) = \frac{V(all)}{2} = \frac{240}{2} = 120$$
$$P(skipper1) = P(skipper2) = 0$$

On the other hand if skipper1 owns the boat:

$$V(all) - V(all \setminus chef) = 240$$
$$V(all) - V(all \setminus tycoon) = 240$$
$$V(all) - V(all \setminus skipper1) = 240$$
$$V(all) - V(all \setminus skipper2) = 0$$

This means skipper 1 can also get an equal share of the value.

$$P(chef) = P(tycoon) = P(skipper1) = 80$$
$$\frac{V(all)}{3} = \frac{240}{3} = 80$$
$$P(skipper2) = 0$$

So we can see that if the chef's investment is under 80, he will always invest. If his investment is between 80 and 120, then he will only invest if the chef or tycoon own the boat. If it is greater than 120, he will never invest.

0.2 The chef and the skipper invests

As before but now both the skipper1 and the chef can invest and each investment increases value by 240. Additionally both their investments cost 100.

If the chef invested the value is:

$$V(chef, tycoon, skipper1, skipper2) = V(all) = 480$$

The value added of everybody if the tycoon owns the boat:

$$V(all) - V(all \setminus chef) = 240$$
$$V(all) - V(all \setminus tycoon) = 480$$
$$V(all) - V(all \setminus skipper1) = 240$$
$$V(all) - V(all \setminus skipper2) = 0$$

So the chef and the tycoon will split the 240 between them and the chef and the skipper the other 240 between them:

$$P(chef) = P(skipper) = \frac{V(all)}{2} = \frac{240}{2} = 120$$
$$P(tycoon) = \frac{V(all)}{2} = \frac{240}{2} + \frac{240}{2} = 240$$
$$P(skipper2) = 0$$

On the other hand if skipper1 owns the boat:

$$V(all) - V(all \setminus chef) = 240$$
$$V(all) - V(all \setminus tycoon) = 480$$
$$V(all) - V(all \setminus skipper1) = 480$$
$$V(all) - V(all \setminus skipper2) = 0$$

This means skipper 1 can also get an equal share of the value.

$$P(skipper1) = P(tycoon) = 200$$
$$P(chef) = 80$$
$$P(skipper2) = 0$$

On the other hand if chef owns the boat:

$$V(all) - V(all \setminus chef) = 480$$
$$V(all) - V(all \setminus tycoon) = 480$$
$$V(all) - V(all \setminus skipper1) = 240$$
$$V(all) - V(all \setminus skipper2) = 0$$

This means chef can also get an equal share of the value added of skipper1.

$$P(chef) = P(tycoon) = 200$$
$$P(skipper1) = 80$$
$$P(skipper2) = 0$$

On the other hand if skipper2 owns the boat:

$$V(all) - V(all \setminus chef) = 240$$
$$V(all) - V(all \setminus tycoon) = 480$$
$$V(all) - V(all \setminus skipper1) = 240$$
$$V(all) - V(all \setminus skipper2) = 480$$

This means chef can also get an equal share of the value added of skipper1.

$$P(chef) = P(tycoon) = 160$$
$$P(skipper1) = P(skipper1) = 80$$

To summarize, if tycoon owns the boat, everything happens. If the chef owns the boat, only his, same for skipper 1. If skipper 2 owns the boat no investment occurs.