

Industrial Organization, Week 5

Hotelling

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Agenda

- 1 Big picture
- 2 Hotelling
- 3 Location only
- 4 Price and location: linear costs
- 5 Price and location: quadratic costs

Dynamic

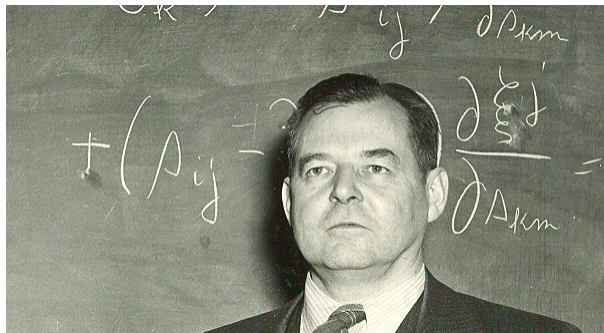
- ▶ Plan: We look at our third dynamic model today
- ▶ The idea of Hotelling is that firms can choose how much to differentiate
- ▶ We will look at the implication of this with dynamic and static competition

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Harold Hotelling

- ▶ American Mathematician, Born in 1895, Columbia/Stanford/Washington
- ▶ "Stability in Competition" in Economic Journal in 1929
- ▶ Georgist



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Utility function

- ▶ Cost function of consumer: $\tau(|x - l_i|) = t|x - l_i|$
- ▶ Pleasure to consumer: r
- ▶ Utility: $v_i(x) = r - t|x - l_i|$
- ▶ \bar{p}

Profit function

$$\pi_i(l_i, l_j) = \begin{cases} (\bar{p} - c)(l_i - l_j)/2 & \text{if } l_i < l_j \\ (\bar{p} - c)/2 & \text{if } l_i = l_j \\ (\bar{p} - c)[1 - (l_i - l_j)/2] & \text{if } l_i > l_j \end{cases}$$

General conclusion

- ▶ If firms do not choose their prices:
- ▶ They choose not to differentiate

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Choosing prices and location

- ▶ $t = 0$: Firms choose where to locate
- ▶ $t = 1$: Firms choose prices
- ▶ $t = 2$: Consumers go shopping
- ▶ Therefore to solve the problem we proceed in this way:
- ▶ Step 1: Find the indifferent consumer
- ▶ Step 2: Use the indifferent consumer to find the optimal price
- ▶ Step 3: Use the optimal price and indifferent consumer to find the location

Linear costs

The indifferent consumer is found here:

$$r - \tau(\hat{x} - l_1) - p_1 = r - \tau(l_2 - \hat{x}) - p_2 \quad (1)$$

$$\hat{x} = \frac{l_1 + l_2}{2} - \frac{p_1 - p_2}{2\tau} \quad (2)$$

$$(3)$$

We have two conditions for the indifferent consumer to be between the two other firms.

$$\begin{cases} \hat{x} \geq l_1 \leftrightarrow p_1 \leq p_2 + \tau(l_2 - l_1) \\ \hat{x} \leq l_2 \leftrightarrow p_1 \geq p_2 + \tau(l_2 - l_1) \end{cases}$$

Indifference point

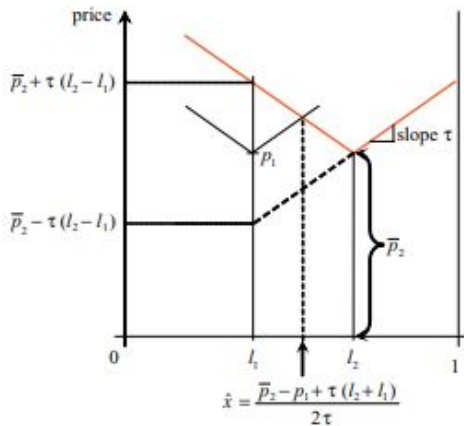


Figure 5.1 Consumer choice in the linear Hotelling model

Choosing prices and location

$$\pi_i(p_1, p_2; l_1, l_2) = \begin{cases} 0 & \text{if } p_1 > p_2 + \tau(l_2 - l_1) \\ (p_1 - c)\left(\frac{l_1 + l_2}{2} + \frac{p_2 - p_1}{2}\right) & \text{if } |p_1 - p_2| \leq \tau(l_2 - l_1) \\ (p_1 - c) & \text{if } p_1 < p_2 - \tau(l_2 - l_1) \end{cases}$$

Indifference point

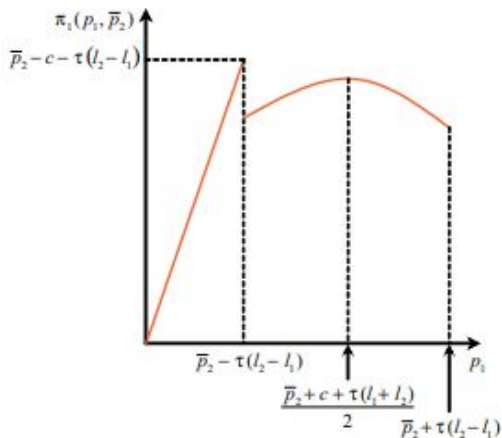


Figure 5.2 Profit function in the linear Hotelling model

Complicated conclusion

- ▶ Differentiation does not necessarily predict a single outcome
- ▶ if firms are far enough apart, there is a unique equilibrium
- ▶ But they have a tendency to prefer moving to the center

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Quadratic cost

The cost function

$$t(|x - l_i|) = \tau(x - l_i)^2$$

The indifferent consumer

$$\rightarrow \hat{x}(p_1, p_2) = \frac{l_1 + l_2}{2} - \frac{p_1 - p_2}{2\tau(l_1 - l_2)}$$

The profit

$$\pi_1 = (p_2 - c)[\hat{x}(p_1, p_2)]$$

After we take the derivative

$$p_1^* = c + \frac{\tau}{3}(l_2 - l_1)(2 + l_1 + l_2)$$

And plug it back into

$$\pi_1^* = \frac{1}{18}\tau(l_2 - l_1)(2 + l_1 + l_2)^2$$

If we optimize wrt to the location, we find that $l = 0$

Conclusion

- ▶ Effect 1: Want to be close to center to increase market size
- ▶ Effect 2: Differentiation decreases competition