Appropriation

***Abstract*—What is the probability of a public good problem?**

There are N objects with different values. Suppose we randomly choose one, clearly the probability of choosing the largest one is $\frac{1}{N} = P_{\geq}(samplinglargest)$. Suppose now that after we pick an object we put it back into its place and sample again, if we do this k times, what is the probabiliy that all our samples are smaller than the largest one? $P(samplinglargestwithkdraws) = P_{<}(samplinglargest)^k$

$$P_{\geq}(\lambda) = \frac{1}{N} \tag{1}$$

$P_{\geq}(\lambda)$:Probability that we sample the largest value

$P_{<}(\lambda)$:Probability we don't sample the largest value

$\Pi_{<}(\lambda)$:Prob that we fail to sample the largest value after taking the max of N samples

$$\begin{aligned}
\Pi_{<}(\lambda) =& P_{<}(\lambda)^N \\
=& exp^{Nln\left[1-P_{\geq}(\lambda)\right]} \\
\approx& exp^{-NP_{\geq}(\lambda)}
\end{aligned}$$

$$\begin{aligned}
\Pi_{<}(\lambda_{\frac{1}{2}}) =& \frac{1}{2} \\
P_{\geq}(\lambda_{\frac{1}{2}}) =& 1 - \left(\frac{1}{2}\right)^{\frac{1}{2}} \\
\approx& \frac{ln2}{N}
\end{aligned}$$

more generally the probability of samling a value that exceeds or is equal to λ_p is

$$P_{\geq}(\lambda_p) = \approx \frac{ln\frac{1}{p}}{N}$$

$$E[X^p] < \infty \leftrightarrow R_n^p = \frac{max(X_1^p, ..., X_n^p)}{\sum_{i=1}^p X_i^p} \to 0, \text{as n} \to \infty$$