Industrial Organization, Week 6 Individual Answers

Answers to questions

Indifferent consumer
$$r - t(x - l_1)^2 - p_1 = r - t(x - l_2)^2 - p_2$$
 Solving for x should give us
$$\overline{x} = \frac{p_2 - p_1 + t(l_2^2 - l_1^2)}{2t(l_2 - l_1)}$$
 We now set the profit function of 1
$$\pi_1 = p \left(\frac{p_2 - p_1 + t(l_2^2 - l_1^2)}{2t(l_2 - l_1)} \right)$$
 The price is then
$$\frac{\delta \pi_1}{\delta p_1} = 0 \to p_1 = \frac{t}{3} \left(l_2(2 + l_2) - l_1(2 + l_1) \right)$$
 By symmetry
$$\frac{\delta \pi_2}{\delta p_2} = 0 \to p_2 = \frac{t}{3} \left(l_1(2 + l_1) - l_2(2 + l_2) \right)$$
 Plug this into profit
$$\pi_1 = \frac{1}{18} t(l_2 + l_1 + 2) \left(l_2^2 + 2l_2 - l_1(l_1 + 2) \right)$$
 Take derivative
$$\frac{\delta \pi_1}{\delta l_1} = \frac{t}{18} \left(l_2^2 - 2l_2 l_1 - 3l_1^2 - 8l_1 - 4 \right)$$

Which is always negative when location is contained within 0-1, which means that profit is maximized when firm 1 places itself all the way to the left! So the equilibrium is that firms maximally differentiate!