Industrial Organization, Week 5
Hotelling

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03 March 2021

Agenda

Dynamic

- Plan: We look at our third dynamic model today
- The idea of Hotelling is that firms can choose how much to differentiate
- We will look at the imlication of this with dynamic and static competition

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Harold Hotelling

- American Mathematician, Born in 1895, Columbia/Stanford/Washington
- "Stability in Competition" in Economic Journal in 1929
- Georgist

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Utility function

- Cost function of consumer: $\tau(|x - l_i|) = t|x - l_i|$
- Pleasure to consumer: r
- Utility: $v_i(x) = r - t|x - l_i|$
- $\overline{p}$

Profit function

$$\pi_i(l_i, l_j) = \begin{cases} (\overline{p} - c)(l_i - l_j)/2 & \text{if } l_i < l_j \\ (\overline{p} - c)/2 & \text{if } l_i = l_j \\ (\overline{p} - c)[1 - (l_i - l_j)/2] & \text{if } l_i > l_j 0 \end{cases}$$

General conclusion

- If firms do not choose their prices:
- They choose not to differentiate

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Choosing prices and location

- $t = 0$: Firms choose where to locate
- $t = 1$: Firms choose prices
- $t = 2$: Consumers go shopping
- Therefore to solve the problem we proceed in this way:
- Step 1: Find the indifferent consumer
- Step 2: Use the indifferent consumer to find the optimal price
- Step 3: Use the optimal price and indifferent consumer to find the location

Linear costs

$$r - \tau(\hat{x} - l_1) - p_1 = r - \tau(l_2 - \hat{x}) - p_2 \tag{1}$$

$$\hat{x} = \frac{l_1 + l_2}{2} - \frac{p_1 - p_2}{2\tau} \tag{2}$$

$$\tag{3}$$

$$\begin{cases} \hat{x} \geq l_1 \leftrightarrow p_1 \leq p_2 + \tau(l_2 - l_1) \\ \hat{x} \leq l_2 \leftrightarrow p_1 \geq p_2 + \tau(l_2 - l_1) \end{cases}$$

Indifference point

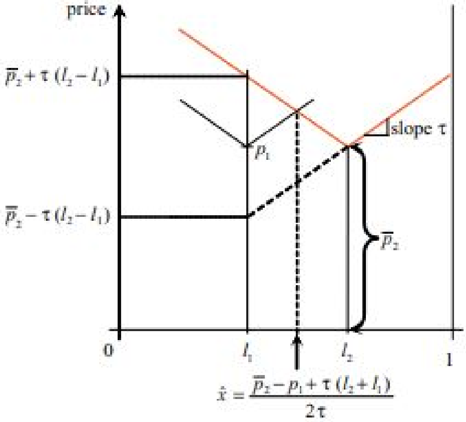

Figure 5.1 Consumer choice in the linear Hotelling model

Choosing prices and location

$$\pi_i(p_1, p_2; l_1, l_2) = \begin{cases} 0 & \text{if } p_1 > p_2 + \tau(l_2 - l_1) \\ (p_1 - c)(\frac{l_1+l_2}{2} + \frac{p_2-p_1}{2}) & \text{if } |p_1 - p_2| \leq \tau(l_2 - l_1) \\ (p_1 - c) & \text{if } p_1 < p_2 - \tau(l_2 - l_1) \end{cases}$$

Indifference point

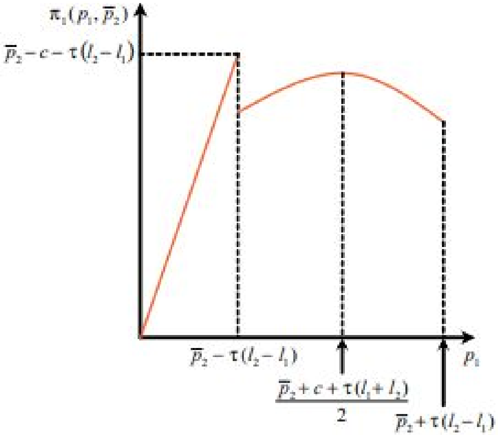

Figure 5.2 Profit function in the linear Hotelling model

Complicated conclusion

- Differentiation does not neccesarily predict a single outcome
- if firms are far enough apart, there is a unique equilibrium
- But they have a tendency to prefer moving to the center

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Quadratic cost

The cost function	$t(\|x - l_i\|) = \tau(x - l_i)^2$
The indifferent consumer	$\rightarrow \hat{x}(p_1, p_2) = \frac{l_1 + l_2}{2} - \frac{p_1 - p_2}{2\tau(l_1 - l_2)}$
The profit	$\pi_1 = (p_2 - c)[\hat{x}(p_1, p_2)]$
After we take the derivative	$p_1^* = c + \frac{\tau}{3}(l_2 - l_1)(2 + l_1 + l_2)$
And plug it back into	$\pi_1^* = \frac{1}{18}\tau(l_2 - l_1)(2 + l_1 + l_2)^2$

If we optimize wrt to the lo

Conclusion

- Effect 1: Want to be close to center to increase market size
- Effect 2: Differentiation decreases competition