Microeconomics 1 / M1: Fall 2016 / Quizz 2

December 2, 2016

Time : 1 hour 20 minutes

Exercise 1: Equivalent Variation and Compensated Variation

The two parts are independent.

Part I (4 points, difficulty *)

Miss Moto is keen on ringing the bells of a town church for 10 hours per day. She spends c on the other goods she consumes and spends x hours ringing the bells. Her utility function is :

$$U(c, x) = c + 3x$$

for $x <= 10$.

If $x > 10$, she gets extremly painful blisters, and her utility function becomes :

$$U(c, x) = c$$

Her income is 100 euros and she is allowed by the preacher to ring the bells for 10 hours. In the following questions, we always compare this initial situation with a new one.

1. Because of complains in the town, the preacher decides to allow Miss Moto to ring the bells *only* for 5 hours. That is bad news for Miss Moto and is equivalent to a loss in her income. How much is this loss ? (1 point)

2. The preacher makes a new offer : Miss Moto can ring the bells as much as she wants but she has to pay 2 euros per hour. How long will she ring the bells? Which loss in her income would cause to her the same loss in utility as this tax ? (1 point)

3. The citizens carry on complaining. The preacher increases the price to 4 euros per hour. How long Moto will ring the bells ? Which loss in her income would cause to her the same loss in utility as this tax ? (1 point)

4. In the previous questions, did you use Equivalent Variation, Compensated Variation or none of them ? Why ? (1 point)

Part II (5 points, difficulty **)

Assume a consumer whose the wealth is $m = 200$ euros and who can buy two goods in quantities x_1 and x_2 respectively. The price of the first good is $p_1 = 1$; that of the second one is $p_2 = 2$. The preferences of the consumer are represented by the utility function :

$$U(x_1, x_2) = min(2x_1, x_2)$$

The consumer faces an increase in p_1. We note the new price $p_1' = 4$.

1. Show that the compensated variation is equal to 120 euros. (2 point)

2. Show that the equivalent variation is equal to 75 euros. (2 point)

3. Represent graphically the equivalent variation and the compensated variation. (1 point)

Exercise 2: Exchange and Production Economy

It is advised to do the first part before the second part.

Part I (4.5 points, difficulty **)

Assume an economy with two consumers $i = A, B$, and two goods $l = 1, 2$. The individual endowments of A and B are $\omega^A = \omega^B = (\frac{1}{2}, \frac{1}{2})$. Good 2 is the numeraire good (i.e. $p_2 = 1$). We note $p_1 = p$. The preferences of the consumer are represented by the utility functions :

$$u^A(x_1^A, x_2^A) = ln(x_1^A) + ln(x_2^A) \qquad u^B(x_1^B, x_2^B) = (x_1^B)^{\frac{1}{4}}(x_2^B)^{\frac{3}{4}}$$

1. Determine the Walrasian equilibrium (find $p = \frac{3}{5}$ and allocations $((\frac{2}{3}, \frac{2}{5}); (\frac{1}{3}, \frac{3}{5}))$. (2.5 points)
2. Check if the Walrasian equilibrium is Pareto-optimal. Which computations should be made to check that the equilibrium is in the core ? (2 points)

Part II (7 points, difficulty ** and ***)

We carry on working in the same framework with the same consumers (same preferences and endowments). A firm is created by the consumer B to produce good 2 using good 1 as input. The production function is $y_2 = \sqrt{y_1}$. We note π the firm's profit. In the following questions, the firm maximises its profit independently of the consumer B's preferences. The profit is then added to the consumer B's budget.

1. Determine the demand for good 1 of the firm and the consumers. Prove the price p is equal to $p = \frac{3+\sqrt{59}}{3}$. (2 points)
   Hint : Look at the Help.
2. The production function becomes $y_2 = y_1$. Determine the demand for good 1 of the firm and the consumers by distinguish 3 cases with respect to the value of p. (2.5 points)
3. The production function becomes $y_2 = \frac{y_1}{c}$ (with $c > 0$). Determine the values of c such that the the firm is active at equilibrium (i.e. $y_1 > 0$) and the values of c such that the firm is not active (Hint : Show that, for some values of c, there is an excess demand of good 1 when the firm is active). Compare the equilibrium of question I.1 with the equilibrium with the non active firm. (2.5 points)

Help

To solve a quadratic equation $ax^2 + bx + c = 0$, you need to determine the discriminant $\Delta = b^2 - 4ac$ and to solve $x = \dfrac{-b \pm \sqrt{\Delta}}{2a}$.