# Problems, paradoxes and probability

The main theme of the course is to present basic concepts in probability theory and relate them to their history and arrive at an understanding how these are used to make decisions. Problem or a paradox and then resolve it through some concept, either from philosophy or probability theory. The goals will be to give students the tools to think critically about how to make decisions. Additionally, some of the areas I will be presenting will represent cutting edge research avenues which are open to be understood from mathematical, economic, sociological, philosophical, and even theological frameworks. This content is relatively difficult but it is a fast track for understanding cutting edge material in the academic world. The goal will be to give an expansive view of the problems involved when trying to construct confidence about events in the worlds.

The first few sessions will aim at a comprehension of basic tools of probability theory. Each class will involve examples of the concepts, and will have brief assignments on the material. Most of the material will simply be presented on a blackboard/whiteboard but depending on the class some PowerPoint or handouts may complement it. Problems can be assigned every week or biweekly, the textbooks can be used or the site http://www.cut-the-knot.org/ for more advanced students.

Linear Regression concept only, concept of best fitting line

Confidence interval: estimate, type of answer

Hypothesis testing

Overbooking, what does the business want to do

If you want to have a \*95% confidence interval, how many people should you ask

1. Shopping lecture/First Lecture: What is uncertainty?
   1. Why do we care about certainty vs uncertainty?

One can simply asset that we do and be done with it. That is, we care about having true beliefs independent of their usefulness.

The stoic answer would be that we should separate the world about things we can control and things we cannot control and simply focus on caring about those that we do.

Suppose that you must come into Milestone in the afternoon and you must cross a bridge to get here. It may rain or it may not rain, which will result in you being wet or not wet upon arrival. Should you care about whether it will rain or not?

If you are purely a consequentialist, the intuitive answer is that you should only care if you have an umbrella. What is the purpose of forming beliefs about the rain if you cannot affect whether you will be wet or not? Or indeed if it is completely costless to you to carry an umbrella around (maybe the umbrella magically just follows you around like the carpet in Aladdin), then once again, there is no reason to form a belief about the outcome.

In other words, for a consequentialist to care about uncertainty it must be that 1) he has an action set which can affect outcomes he cares about. 2) The actions are not costless or at least in part mutually exclusive.

* 1. How prevalent is uncertainty?

Okay fine, there may be cases where knowing about uncertainty may be useful. But how many of those are there?

* + 1. First kind of unpredictability: Conceptual innovation

**Poppers argument:** Uncertainty is all around us, there are things we cannot predict and hence cannot put a probability on. Conceptual innovations are the easy case of non-predictability. That is, if you can predict a conceptual innovation, you have in fact, invented it. If you predict the wheel or some theological doctrine such justification by faith alone or tragic drama as an art form or Einstein’s general relativity, you have in fact also invented these. To specify what a concept IS, IS to invent it.

**Macintyre**: Suppose somebody claims they can predict which theorems in mathematics will be proved in the next 5 years will be proved. This entails that this person has a decision procedure which will allow them to say in advance that some subset of theorems will have proofs and others will not have proofs but we know by Gòdel’s incompleteness theorem that for any calculus stronger than first order predicate calculus, there is no such decision procedure, which means mathematics is fundamentally unpredictable.

If the above is true and mathematics cannot be predicted, then the future of natural sciences cannot be predicted, and if the future of natural sciences cannot be predicted, then the future of technology cannot be predicted either. Example: It was not possible to predict that Turing would have proved the effective computability theorem, which means we would not be able to predict computers and hence not be able to predict most modern technology and weapons.

* + 1. Second kind of unpredictability: Open decision

**An open decision:** You are choosing between A or B but you don't know which one you will choose. This means the outcome is unpredictable by you.

**Objection 1**: Note that it is possible that you can predict what choice you will make even though you don’t know which choice that will be. For example if you are playing chess, you may be able to predict that you will make whatever move your twin brother of identical up brining made yesterday even though you have no idea what that move actually is.

This means you cannot predict which move you will make merely by describing the choice set, but you can predict it by describing something else.

**Answer**: This means this source of unpredictability assumes no other descriptions is available other than the description of the two alternatives. So we merely refine the set of unpredictable choices to only the cases where no alternative description is available.

**Objection 2**: Can a man ever know fully know the strength of his own desires? If he does, then that is equivalent to knowing which choice they will make.

**Answer**: We refine the set of unpredictability a little further. We can say that the only way for a person to evaluate which desire is stronger is by looking backwards on what choice was actually made, similar to the weak axiom of revealed preferences.

Suppose that John is deciding whether to do A or B and this is an open decision. Smith is watching John and thinks he can predict John’s decision based on characteristics. However, Smith cannot predict his own behavior, and if he cannot predict his own behavior, then he cannot know that he will not interfere with John’s decision and hence cannot predict John’s decision. In other words, for somebody to be able to predict, they must be powerless(cannot affect what outcome occurs). It follows that if there exists anyone with power, whose actions cannot be predicted, then anything that agent can affect also cannot be predicted.

* + 1. Third kind of unpredictability: Game theoretic

**A game theoretic unpredictability:** If what somebody wants to do depends on what the others want to do, but what the other's want to do, also depend on what that somebody wants to do, there is no uniquely rational belief.

1. Lecture 1: Uncertainty vs risk
   1. -Pandora’s box
   2. -Primitives of gambling

Explain the classical definition of probability in order to communicate the concept of **bias** as the property that not every outcome is equally likely.

Independence, how much information does one thing give us about another? Give an example of an airplane with 2 engines vs one with 3 engines. Ask the students to reveal what is wrong with the Gambler who will bet on red because he has seen 12 consecutive blacks on the roulette wheel.

When do we add probabilities and when do we multiply them? We add them when the events are mutually exclusive, and multiply them if they compound.

**Homework**: selected problems from the textbooks

1. Lecture 2: Risk and probability
   1. Outcomes and probabilities, playing against nature
   2. Fair and Biased

**Recommended** reading: Chapter 3-4 IAN Peterson Chapter 2

1. Lecture 3: Conditional Probability
   1. [Monty Hall problem;](https://en.wikipedia.org/wiki/Monty_Hall_problem) and [Boy or Girl paradox](https://en.wikipedia.org/wiki/Boy_or_Girl_paradox)

**Recommended** reading: Chapter 4-7 Ian Peterson, Chapter 4, Chapter 6 and 10

Assignment: Watch this [video](https://www.youtube.com/watch?v=5Cqbf86jTro) and read the [Wikipedia](https://en.wikipedia.org/wiki/Sleeping_Beauty_problem) article, explain the main issues with the Sleeping beauty problem.

1. Lecture 3: Expected value and gambling
   1. [St Petersburg paradox](https://en.wikipedia.org/wiki/St._Petersburg_paradox) and it’s solution([Kelly](https://en.wikipedia.org/wiki/Proebsting%27s_paradox) vs ergodic vs utility)
   2. Read the [wager passage](https://en.wikipedia.org/wiki/Pascal%27s_wager#:~:text=for%20practical%20purposes.-,Explanation,has%20no%20affinity%20to%20us.&text=A%20game%20is%20being%20played%20at%20the%20extremity%20of%20this,or%20tails%20will%20turn%20up.) of Pascal
   3. Verkes on gambling with information theory

Explain what expected value is by giving intuitive examples. Read the [wager passage](https://en.wikipedia.org/wiki/Pascal%27s_wager#:~:text=for%20practical%20purposes.-,Explanation,has%20no%20affinity%20to%20us.&text=A%20game%20is%20being%20played%20at%20the%20extremity%20of%20this,or%20tails%20will%20turn%20up.) of Pascal and explain the argument. Give some objections and some answers to the objections. After explaining the St Petersburg paradox I will discuss how it can be solved by a change in the optimand.

**Recommended:** Ian, Chapter 8-9, Peterson, Chapter 4

1. Ambiguity
   1. [Ellsberg](https://en.wikipedia.org/wiki/Ellsberg_paradox#:~:text=The%20Ellsberg%20paradox%20is%20a,be%20evidence%20for%20ambiguity%20aversion.) and [Allais](https://en.wikipedia.org/wiki/Allais_paradox#:~:text=The%20Allais%20paradox%20is%20a,predictions%20of%20expected%20utility%20theory.) and explain the link to subjective probability theory
   2. Explain the concept of a [Dutch book](https://en.wikipedia.org/wiki/Dutch_book#:~:text=In%20gambling%2C%20a%20Dutch%20book,coherent%2C%20namely%20are%20being%20skewed.)

**Recommended reading**: Chapter 10 from IAN. Chapter 4 from Peterson

1. Independence and Association, Pearl 2009 Chapter 2&3 Peterson Chapter 9

What is independence? Correlation vs causation, examples

Explain what association and correlation is, introduce the concept of a collider and talk a little bit about causality. Explain why Fischer was not convinced about smoking and cancer and explain how the backdoor criterion is insufficient to prove smoking causes cancer whilst the front door criterion works.

1. Risk seeking and research: (optional session, this session is more technical so only if the rest have proceeded as planned). Dubins and Savage

Give an [example](https://www.cut-the-knot.org/Curriculum/Probability/ChessTournament.shtml#solution) in detail.

Explain how it links to [Savage’s](https://www.goodreads.com/en/book/show/19484230) treatment of decision theory under constraints.

Give some more examples of Bayesian decision theory, including some cutting edge work in economics.

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| **Main textbooks:** |
| Probability: Hacking, Ian, and Hacking Ian. *An introduction to probability and inductive logic*. Cambridge university press, 2001. |

Decision theory: Peterson, Martin. *An introduction to decision theory*. Cambridge University Press, 2017.

Pearl, Judea, and Dana Mackenzie. [Book of why](file:///C:\Users\DavidEttinger02\Documents\Mendeley%20Desktop\Pearl,%20Mackenzie\Pearl,%20Mackenzie_Unknown.pdf)*: the new science of cause and effect*. Basic Books, 2018.

**Secondary books:**

Resnik, Michael D. *Choices: An introduction to decision theory*. U of Minnesota Press, 1987.

Pearl, Judea. "[Causal inference in statistics](file:///C:\Users\DavidEttinger02\Documents\Mendeley%20Desktop\Pearl\Pearl_Unknown.pdf): An overview." *Statistics surveys* 3 (2009): 96-146.

Dubins, Lester E., et al. *How to gamble if you must: Inequalities for stochastic processes*. Courier Corporation, 2014.