

Causal Concepts Illustrated with DAGs

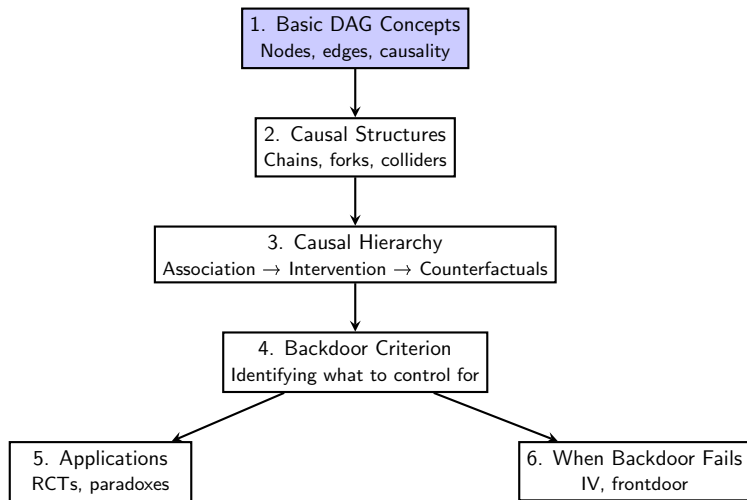
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Outline

- 1 Introduction to Causality
- 2 Basic DAG Concepts
- 3 Causal Structures
- 4 The Causal Hierarchy
- 5 Backdoor Criterion
- 6 Applications and Paradoxes
 - Experimental Design
- 7 Other ways of getting a backdoor

Roadmap: Building Causal Understanding

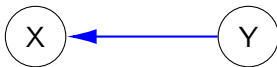
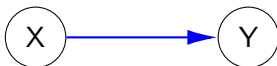


Causality and Empiricism

- Directed Acyclic Graphs (DAGs) represent causal relationships
- The general term for these relationships is **association**
- **Correlation** is a special kind of association

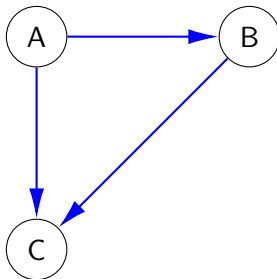
Key Insight

The following two causal structures are empirically identical - no associative measure can distinguish between them:



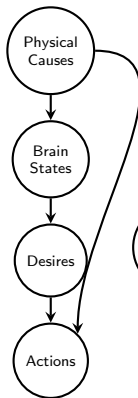
Introduction to DAGs

- **Directed Acyclic Graphs (DAGs)** represent causal relationships
- **Nodes** = variables
- **Edges** = causal relationships (direction matters)
- **No cycles** allowed (cannot be both cause and effect of itself)

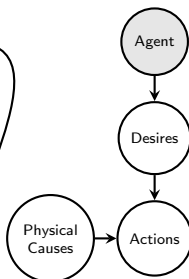


Example: Free Will vs Determinism: Four Philosophical Positions

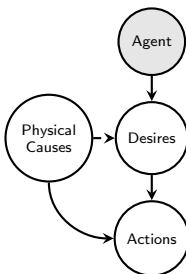
Hard Determinism



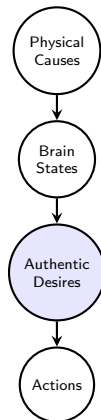
Hard Libertarian



Soft Libertarian



Compatibilism



Solid = full causation

Dashed = partial influence

Gray = uncaused

Blue = authentic

Necessary Causes

- A cause is **necessary** for an effect if the effect cannot occur without the cause
- Logical Form: $\neg X \rightarrow \neg Y$
- Entailment 1: $P(X) \geq P(Y)$
- Entailment 2: $P(Y|\neg X) = 0$



Example

Oxygen is necessary for fire - without oxygen, combustion cannot occur

Sufficient Causes

- A cause is **sufficient** for an effect if the cause guarantees the effect
- Logical Form: $X \rightarrow Y$
- Entailment 1: $P(X) \leq P(Y)$
- Entailment 2: $P(Y|X) = 1$



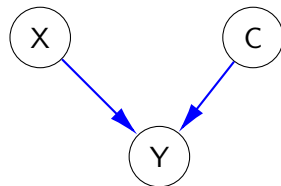
Double arrow indicates sufficiency

Example

Being a triangle is sufficient for being a polygon - every triangle is necessarily a polygon

Overdetermination

- **Overdetermination** occurs when multiple causes independently can bring about the effect
- Form: $(X \rightarrow Y) \wedge (C \rightarrow Y)$
- Entailment: $P(Y) \geq \max(P(X), P(C))$

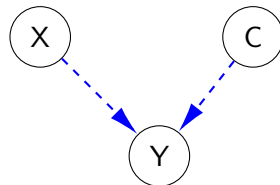


Example

Multiple gunshots - any single shot would be sufficient to cause death

Underdetermination

- **Underdetermination** occurs when multiple causes are jointly necessary but individually insufficient for the effect.
- Form: $(X \wedge C) \rightarrow Y$, but $\neg(X \rightarrow Y)$ and $\neg(C \rightarrow Y)$.
- Entailment:
 $P(Y|X \wedge C) \gg P(Y|X), P(Y|C)$,
with $P(Y|X) \approx 0$ and $P(Y|C) \approx 0$.

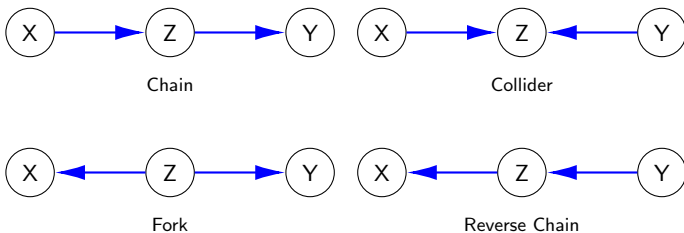


Example

Fire requires both oxygen and fuel - neither alone is sufficient to produce fire

The Four Basic Causal Structures

$$(X \not\perp Y) \parallel Z \text{ or } (X \perp Y) \parallel Z$$



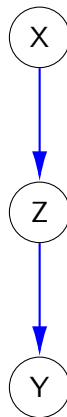
Causal Chains (Mediation)

- **Causal Chains** show mediated relationships
- X causes Z which causes Y
- X is an **indirect cause** of Y
- Z **mediates** the relationship between X and Y

Example

Education \rightarrow Skills \rightarrow Income

Education affects income through developing skills

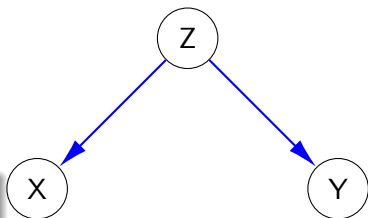


Common Cause (Confounding)

- A **Common Cause** can create a spurious correlation
- Z causes both X and Y
- X and Y appear correlated but have no direct causal relationship

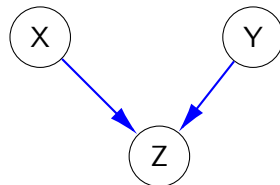
Example

Summer weather causes both ice cream sales and drowning deaths - they're correlated but neither causes the other



Colliders

- A **Collider** is where multiple causes influence a common effect
- Unlike common causes, conditioning on a collider can **create** spurious correlations
- X and Y are independent until we condition on Z



Example

Intelligence and work ethic both cause success - among successful people, these traits appear negatively correlated

The Causal Hierarchy: Three Levels of Reasoning

Pearl's Causal Hierarchy

Three distinct levels of causal reasoning, each more powerful than the last

① Association (Seeing/Observing)

- What is? How are variables related?
- $P(Y|X)$ - conditional probability
- Purely statistical, no causal claims

② Intervention (Doing/Acting)

- What if I do? What happens if we change X ?
- $P(Y|do(X))$ - interventional probability
- Requires causal knowledge beyond correlation

③ Counterfactuals (Imagining)

- What if things had been different?
- $P(Y_x|X' \neq x)$ - probability Y would be y if X had been x
- Requires most complete causal understanding

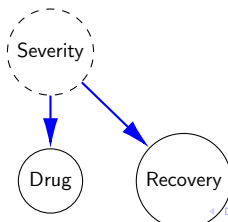
Level 1: Association and Observation

Mathematical Framework

- Joint distribution: $P(X, Y)$
- Conditional probability: $P(Y|X) = \frac{P(X, Y)}{P(X)}$
- Independence: $X \perp\!\!\!\perp Y \iff P(Y|X) = P(Y)$

What We Can Answer

- "What is the probability of disease given symptom?"
- "Are education and income correlated?"
- "What patterns exist in the data?"

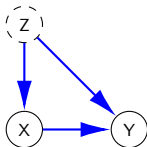


Levels 2 Intervention (do-calculus)

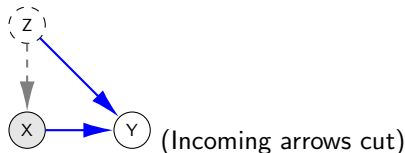
Level 2:

- $P(Y|do(X = x))$ - probability of Y when we set X to x
- Differs from conditioning: $P(Y|do(X = x)) \neq P(Y|X = x)$ in general
- Answers: "What if we force everyone to take the drug?"

$$P(Y|X = x)$$



$$P(Y|do(X = x))$$



Level 3: Counterfactuals

- $Y_x(u)$ - value Y would take for unit u if X were set to x
- Individual Treatment Effect: $ITE(u) = Y_1(u) - Y_0(u)$
- Answers: "What if this specific patient had taken the drug?"

What Requires Counterfactuals vs Intervention

Level 2 (Intervention):

- $ATE = E[Y|do(X = 1)] - E[Y|do(X = 0)]$
- ATT, CATE (population averages)

Level 3 (Counterfactual):

- ITE for specific individual
- "Would X have prevented Y ?" (necessity)
- "Would X have caused Y ?" (sufficiency)

Backdoor Criterion: Causal Effect of X on Y

Water System Analogy

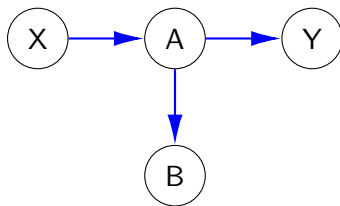
Think of it like a water system. There must not be any indirect paths from X to Y:

- Regular variables are **open gates** - if controlled, they close
- Colliders are **closed gates** - if controlled, they open

Three-Step Process

- 1 Draw your DAG
- 2 List every backdoor path from X to Y
- 3 Find a set of controls such that all backdoor paths are closed

Backdoor Example 1

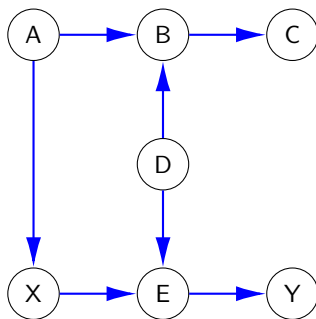


Control sets: $\{\emptyset, \{B\}\}$

Explanation

No backdoor paths exist from X to Y, so no controls needed. Controlling for B is also valid but unnecessary.

Backdoor Example 2

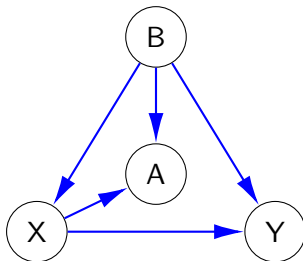


Control sets: $\{\emptyset\}$ and all subsets of $\{A, C, D\}$ except $\{B\}$, $\{B, C\}$ and E should never be controlled for.

Explanation

Backdoor path $X \leftarrow A \rightarrow B \leftarrow D \rightarrow E \rightarrow Y$ exists, but B is a collider on this path, so it's naturally blocked.

Backdoor Example 3

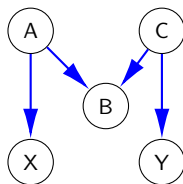


Control sets: $\{\{B\}, \{A, B\}\}$

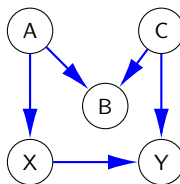
Explanation

Backdoor path $X \leftarrow B \rightarrow Y$ must be blocked by controlling for B.

Backdoor Examples 4 & 5



Example 4



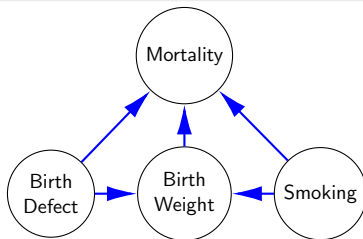
Example 5

- **Example 4 and 5 control sets:** $\{\emptyset, \{A, B\}, \{B, C\}, \{A, B, C\}\}$

Simpson's Paradox: Birth Weight Example

Paradox

Low birth-weight children born to smoking mothers have a lower infant mortality rate than low birth-weight children of non-smokers.



Explanation

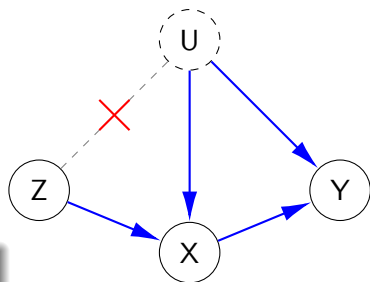
Birth weight is a collider - conditioning on it creates spurious correlation between smoking and mortality within the low birth-weight group.

Randomized Controlled Trials (RCTs)

- **Gold standard** for causal inference
- Random assignment breaks all backdoor paths
- Creates independence: $Z \perp\!\!\!\perp U$
- Eliminates confounding by design

Key Properties

- Z (assignment) affects X (treatment)
- Z is independent of all confounders U
- Effect identified by comparing groups



Randomization breaks the $U \rightarrow Z$ path

RCTs as Edge-Cutters

Key Insight

Random assignment **blocks all incoming edges** to the treatment node, except from the randomization device itself.

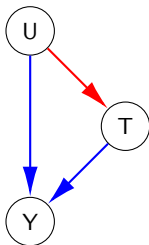
- Randomization makes treatment statistically independent of all pre-treatment variables
- Formally: $P(T|\text{Parents}(T)) \rightarrow P(T|R)$
- This independence breaks confounding paths

Result

$$T \perp\!\!\!\perp \{U, X, \dots\} | R$$

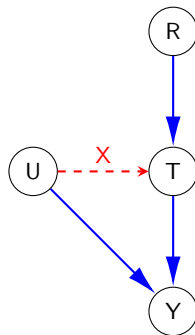
Visual Representation: Before and After

Observational Setting:



Confounding path: $U \rightarrow T \rightarrow Y$

After Randomization:



Only path to T : through R

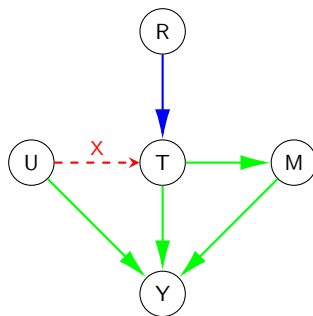
Which Edges Are Affected?

Edges that **ARE** blocked:

- All edges **into** treatment from pre-treatment variables
- Confounding paths through treatment
- Selection into treatment based on characteristics

Edges that are **NOT** blocked:

- Edges **from** treatment to outcomes
- Direct effects of confounders on outcomes
- Mediating paths: $T \rightarrow M \rightarrow Y$



Blocked edge
Active edges

Mathematical Formulation

Independence Achievement

Randomization achieves: $T \perp\!\!\!\perp \{U, X, \dots\} | R$

This means:

$$\mathbb{E}[Y | T = 1] - \mathbb{E}[Y | T = 0] = \text{ATE} \quad (1)$$

Why? No backdoor paths from T to Y :

- All paths $U \rightarrow T \rightarrow Y$ are blocked at the first arrow
- Only variation in T comes from R (random)
- Association between T and Y must be causal

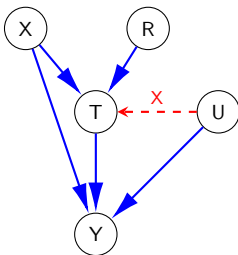
Key Assumption

No interference, perfect compliance, no attrition

Variations in Randomization

Stratified/Blocked Randomization

Randomize within levels of X : $P(T|X, U) \rightarrow P(T|X, R)$



- Edges from all variables except X and R are cut
- Must condition on X in analysis
- Improves precision if X predicts Y

What RCTs Don't Fix

Important Limitations

Randomization only addresses confounding, not other issues:

- ➊ **Attrition/Selection Bias:** Post-treatment selection can reopen paths
- ➋ **Non-compliance:** Creates gap between ITT and ATE
- ➌ **Measurement Error:** Doesn't fix measurement issues
- ➍ **External Validity:** Only identifies effects in study population

Remember

- Don't condition on post-treatment variables (reopens paths)
- Don't control for mediators (blocks part of causal effect)
- Watch for differential attrition (creates selection bias)

Key Takeaways

- 1 **RCTs as surgical edge removal:** Randomization precisely cuts confounding edges while preserving causal paths
- 2 **Independence is key:** $T \perp\!\!\!\perp$ Pre-treatment variables $| R$
- 3 **Know what's blocked:**
 - Edges INTO treatment (blocked)
 - Edges FROM treatment (preserved)
- 4 **Design implications:** Understanding edge-cutting helps design better experiments (stratification, clustering)
- 5 **Limitations remain:** RCTs solve confounding but not all causal inference problems

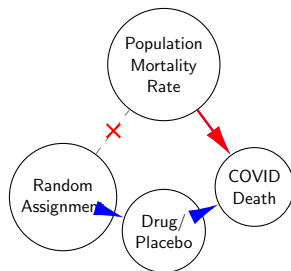
RCTs and Vaccine Drug Trials: Population Mortality Rates

- **Critical insight:** Treatment effect depends on baseline mortality rate
- Higher severity populations → larger observable effects
- Same drug shows different effect sizes across populations

Why This Matters

Drug preventing 50% of deaths:

- If 50% of population infected → 25% better than placebo.
- If 10% of population infected → 5% better than placebo.

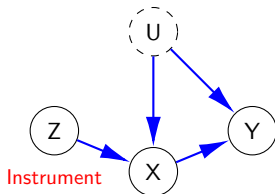


Population rate affects outcomes but not assignment

What if you can't close the backdoors?

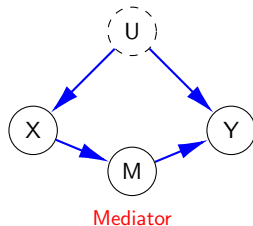
Two options to measure effect of X on Y without closing backdoor:

Instrumental Variable



Find something that causes X which doesn't have a backdoor

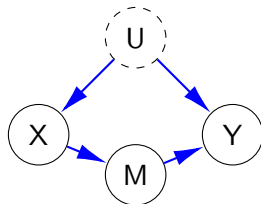
Frontdoor Criterion



Find an intermediary variable between X and Y

Frontdoor Criterion

- Used when there are **unobserved confounders**
- M satisfies frontdoor criterion if:
 - 1 X affects Y only through M (complete mediation)
 - 2 No unobserved confounders of X-M relationship
 - 3 No unobserved confounders of M-Y relationship (given X)

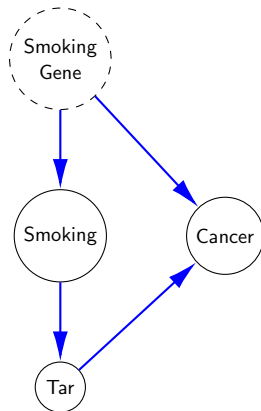


Frontdoor Example: Smoking and Lung Cancer

- Does smoking cause lung cancer?
- Problem: Genetic factors may cause both smoking behavior and cancer susceptibility
- Solution: Use frontdoor criterion with tar deposits as mediator

Logic

Smoking \rightarrow Tar deposits in lungs \rightarrow Lung cancer

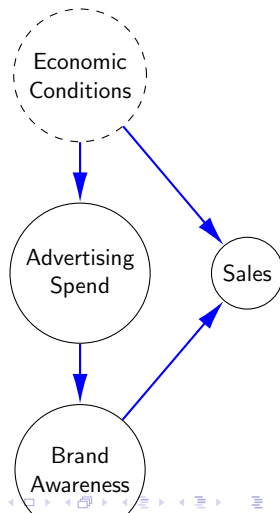


Frontdoor Example 2: Marketing and Sales

- Does advertising spending increase sales?
- Problem: Economic conditions affect both marketing budgets and consumer spending
- Solution: Use frontdoor criterion with brand awareness as mediator

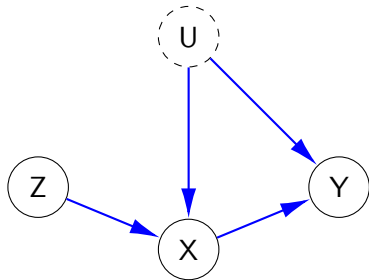
Logic

Advertising \rightarrow Brand awareness \rightarrow Product sales



Instrumental Variables

- Used when there are **unobserved confounders**
- Z is an instrument if:
 - 1 Z affects X (relevance)
 - 2 Z affects Y only through X (exclusion)
 - 3 Z is unrelated to U (exogeneity)

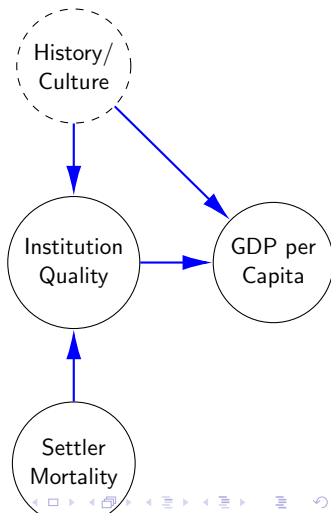


IV Example 1: Colonial Origins of Development

- **Acemoglu, Johnson, Robinson (2001)**
- How do institutions affect development?
- Problem: Good institutions and high GDP could both be caused by unobserved factors
- Solution: Instrument settler mortality

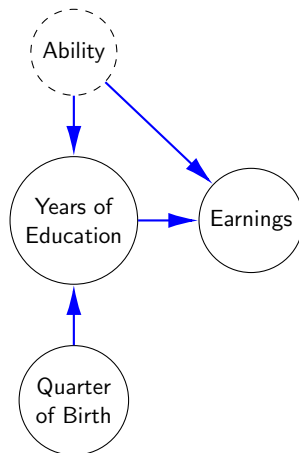
Logic

High settler mortality → Extractive institutions → Poor modern institutions → Low GDP today



IV Example 2: Returns to Education

- **Angrist & Krueger (1991)**
- What is the causal effect of education on earnings?
- Problem: Ability affects both education and earnings
- Solution: Use quarter of birth as instrument



Logic

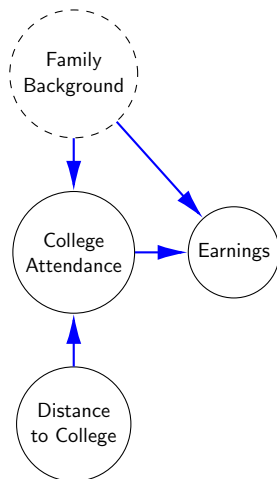
Born in Q4 → Start school older → Drop out with less education (due to compulsory schooling laws)

IV Example 3: College Proximity and Returns to Education

- **Card (1995)**
- Alternative approach to estimating returns to education
- Problem: Family background affects both education and earnings
- Solution: Use distance to nearest college as instrument

Logic

Living near a college → Lower cost of attendance → More likely to attend → Higher earnings

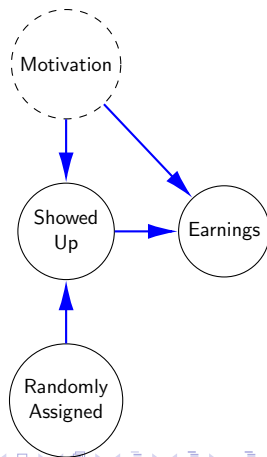


IV Example 4 Example: Job Training Partnership Act (JTPA)

- **JTPA Evaluation Study**
- How does job training affect earnings?
- Problem: Motivation is unobserved confounder
- Solution: Use frontdoor criterion with "showing up" as mediator

Logic

Random assignment → Actually showing up
→ Job training received → Higher earnings



Summary

- DAGs provide a visual language for causal reasoning
- Different structures (chains, forks, colliders) have different statistical properties
- Backdoor criterion helps identify valid control sets
- Frontdoor criterion and instrumental variables help when backdoor fails
- Understanding these concepts prevents common pitfalls in causal inference

Questions?