

# Causal Concepts Illustrated with DAGs

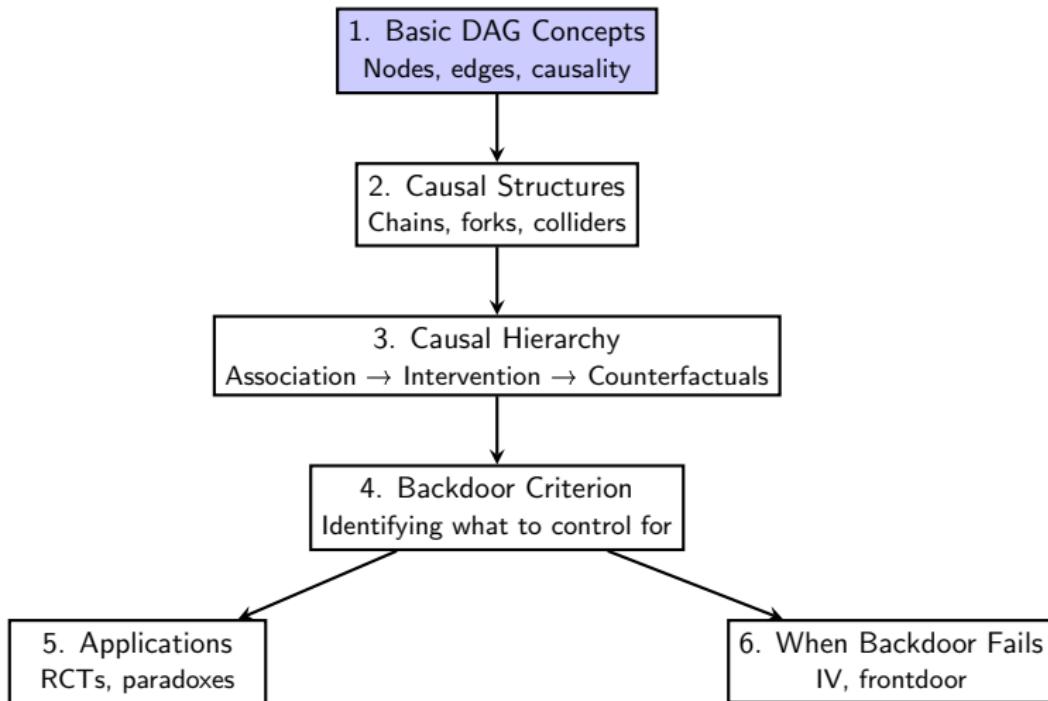
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# Outline

- 1 Introduction to Causality
- 2 Basic DAG Concepts
- 3 Causal Structures
- 4 The Causal Hierarchy
- 5 Backdoor Criterion
- 6 Applications and Paradoxes
  - Experimental Design
- 7 Other ways of getting a backdoor

# Roadmap: Building Causal Understanding



# Causality and Empiricism

- Directed Acyclic Graphs (DAGs) represent causal relationships
- The general term for these relationships is **association**
- **Correlation** is a special kind of association

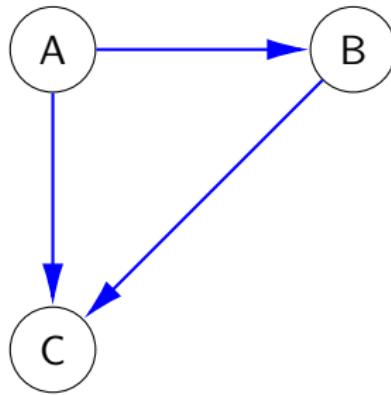
## Key Insight

The following two causal structures are empirically identical - no associative measure can distinguish between them:



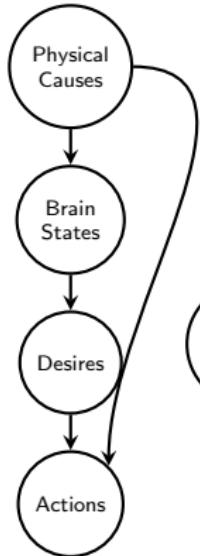
# Introduction to DAGs

- **Directed Acyclic Graphs (DAGs)** represent causal relationships
- **Nodes** = variables
- **Edges** = causal relationships (direction matters)
- **No cycles** allowed (cannot be both cause and effect of itself)

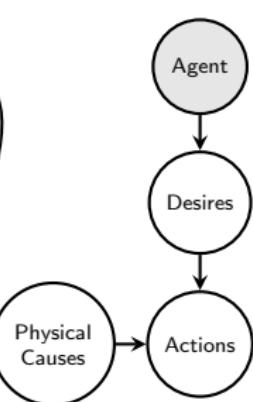


# Example: Free Will vs Determinism: Four Philosophical Positions

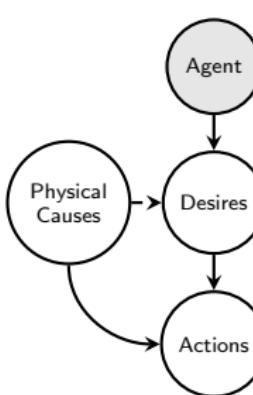
Hard Determinism



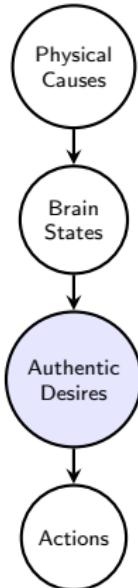
Hard Libertarian



Soft Libertarian



Compatibilism



Solid = full causation

Dashed = partial influence

Gray = uncaused

Blue = authentic



# Necessary Causes

- A cause is **necessary** for an effect if the effect cannot occur without the cause
- Logical Form:  $\neg X \rightarrow \neg Y$
- Entailment 1:  $P(X) \geq P(Y)$
- Entailment 2:  $P(Y|\neg X) = 0$



## Example

Oxygen is necessary for fire - without oxygen, combustion cannot occur

# Sufficient Causes

- A cause is **sufficient** for an effect if the cause guarantees the effect
- Logical Form:  $X \rightarrow Y$
- Entailment 1:  $P(X) \leq P(Y)$
- Entailment 2:  $P(Y|X) = 1$



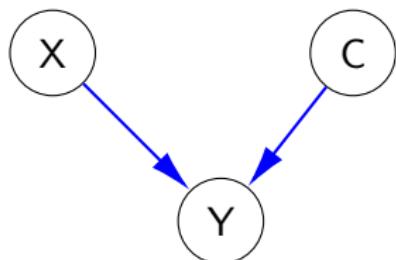
## Example

Being a triangle is sufficient for being a polygon - every triangle is necessarily a polygon

Double arrow indicates sufficiency

# Overdetermination

- **Overdetermination** occurs when multiple causes independently can bring about the effect
- Form:  $(X \rightarrow Y) \wedge (C \rightarrow Y)$
- Entailment:  $P(Y) \geq \max(P(X), P(C))$

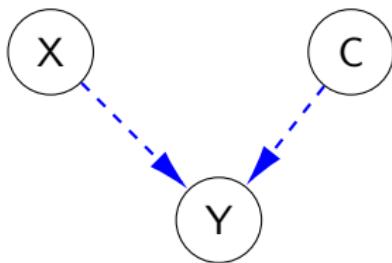


## Example

Multiple gunshots - any single shot would be sufficient to cause death

# Underdetermination

- **Underdetermination** occurs when multiple causes are jointly necessary but individually insufficient for the effect.
- Form:  $(X \wedge C) \rightarrow Y$ , but  $\neg(X \rightarrow Y)$  and  $\neg(C \rightarrow Y)$ .
- Entailment:  
 $P(Y|X \wedge C) \gg P(Y|X), P(Y|C)$ ,  
with  $P(Y|X) \approx 0$  and  $P(Y|C) \approx 0$ .

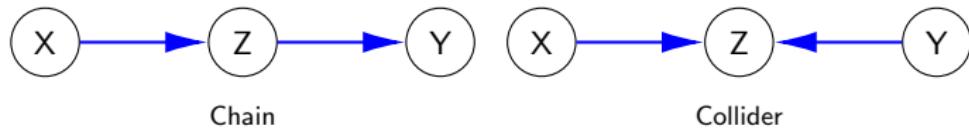


## Example

Fire requires both oxygen and fuel - neither alone is sufficient to produce fire

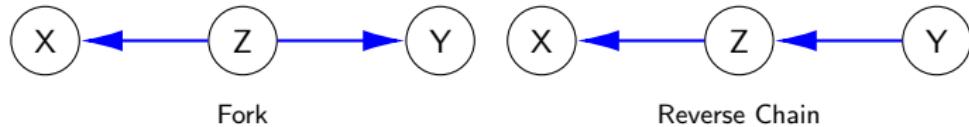
# The Four Basic Causal Structures

$$(X \not\perp\!\!\!\perp Y) \parallel Z \text{ or } (X \perp\!\!\!\perp Y) \parallel Z$$



Chain

Collider

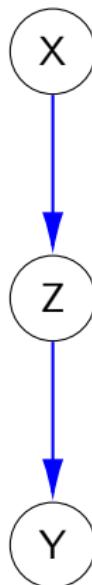


Fork

Reverse Chain

# Causal Chains (Mediation)

- **Causal Chains** show mediated relationships
- X causes Z which causes Y
- X is an **indirect cause** of Y
- Z **mediates** the relationship between X and Y



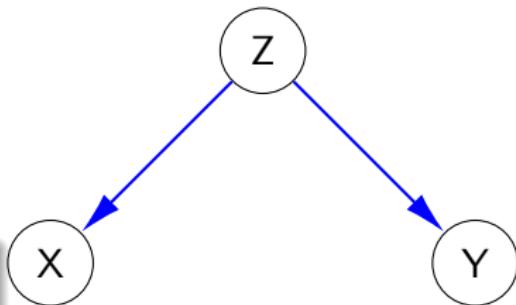
## Example

Education → Skills → Income

Education affects income through developing skills

# Common Cause (Confounding)

- A **Common Cause** can create a spurious correlation
- Z causes both X and Y
- X and Y appear correlated but have no direct causal relationship

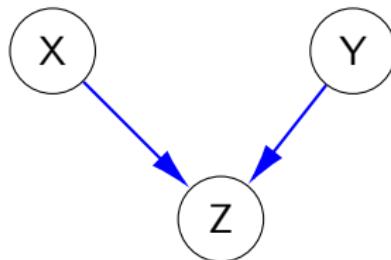


## Example

Summer weather causes both ice cream sales and drowning deaths - they're correlated but neither causes the other

# Colliders

- A **Collider** is where multiple causes influence a common effect
- Unlike common causes, conditioning on a collider can **create** spurious correlations
- X and Y are independent until we condition on Z



## Example

Intelligence and work ethic both cause success - among successful people, these traits appear negatively correlated

# The Causal Hierarchy: Three Levels of Reasoning

## Pearl's Causal Hierarchy

Three distinct levels of causal reasoning, each more powerful than the last

### ① Association (Seeing/Observing)

- What is? How are variables related?
- $P(Y|X)$  - conditional probability
- Purely statistical, no causal claims

### ② Intervention (Doing/Acting)

- What if I do? What happens if we change X?
- $P(Y|do(X))$  - interventional probability
- Requires causal knowledge beyond correlation

### ③ Counterfactuals (Imagining)

- What if things had been different?
- $P(Y_x|X' \neq x)$  - probability Y would be y if X had been x
- Requires most complete causal understanding

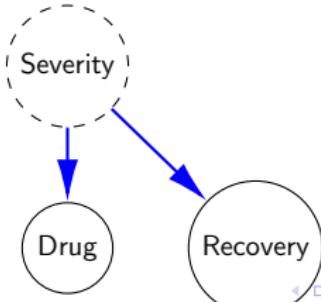
# Level 1: Association and Observation

## Mathematical Framework

- Joint distribution:  $P(X, Y)$
- Conditional probability:  $P(Y|X) = \frac{P(X,Y)}{P(X)}$
- Independence:  $X \perp\!\!\!\perp Y \iff P(Y|X) = P(Y)$

## What We Can Answer

- "What is the probability of disease given symptom?"
- "Are education and income correlated?"
- "What patterns exist in the data?"

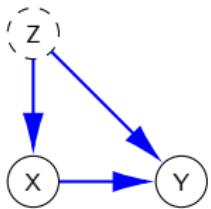


# Levels 2 Intervention (do-calculus)

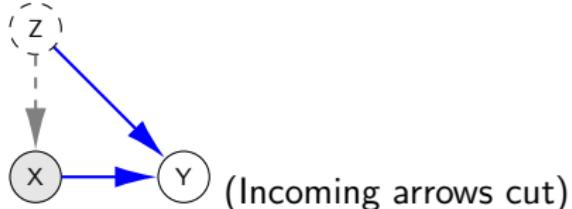
## Level 2:

- $P(Y|do(X = x))$  - probability of Y when we set X to x
- Differs from conditioning:  $P(Y|do(X = x)) \neq P(Y|X = x)$  in general
- Answers: "What if we force everyone to take the drug?"

$$P(Y|X = x)$$



$$P(Y|do(X = x))$$



## Level 3: Counterfactuals

- $Y_x(u)$  - value Y would take for unit u if X were set to x
- Individual Treatment Effect:  $\text{ITE}(u) = Y_1(u) - Y_0(u)$
- Answers: "What if this specific patient had taken the drug?"

### What Requires Counterfactuals vs Intervention

#### Level 2 (Intervention):

- $\text{ATE} = E[Y|do(X = 1)] - E[Y|do(X = 0)]$
- ATT, CATE (population averages)

#### Level 3 (Counterfactual):

- ITE for specific individual
- "Would X have prevented Y?" (necessity)
- "Would X have caused Y?" (sufficiency)

# Backdoor Criterion: Causal Effect of X on Y

## Water System Analogy

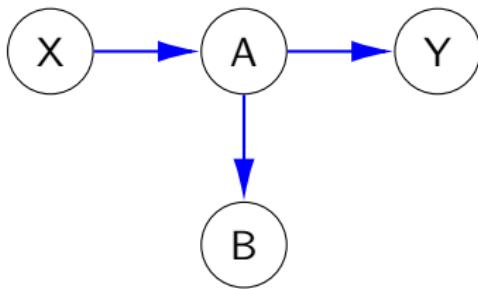
Think of it like a water system. There must not be any indirect paths from X to Y:

- Regular variables are **open gates** - if controlled, they close
- Colliders are **closed gates** - if controlled, they open

## Three-Step Process

- ① Draw your DAG
- ② List every backdoor path from X to Y
- ③ Find a set of controls such that all backdoor paths are closed

## Backdoor Example 1

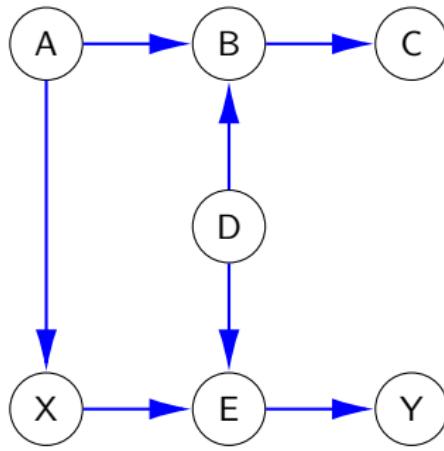


**Control sets:**  $\{\emptyset, \{B\}\}$

### Explanation

No backdoor paths exist from X to Y, so no controls needed. Controlling for B is also valid but unnecessary.

## Backdoor Example 2

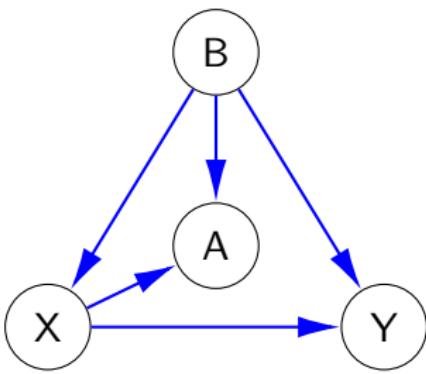


**Control sets:**  $\{\emptyset\}$  and all subsets of  $\{A, C, D\}$  except  $\{B\}$ ,  $\{B, C\}$  and  $E$  should never be controlled for.

### Explanation

Backdoor path  $X \leftarrow A \rightarrow B \leftarrow D \rightarrow E \rightarrow Y$  exists, but  $B$  is a collider on this path, so it's naturally blocked.

## Backdoor Example 3

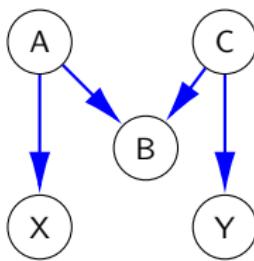


**Control sets:**  $\{\{B\}, \{A, B\}\}$

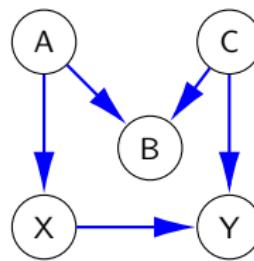
### Explanation

Backdoor path  $X \leftarrow B \rightarrow Y$  must be blocked by controlling for B.

## Backdoor Examples 4 & 5



Example 4



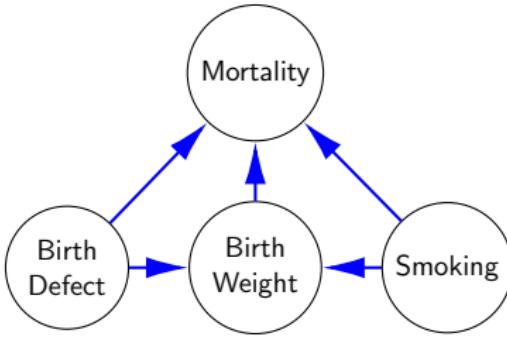
Example 5

- **Example 4 and 5 control sets:**  $\{\emptyset, \{A, B\}, \{B, C\}, \{A, B, C\}\}$

## Simpson's Paradox: Birth Weight Example

### Paradox

Low birth-weight children born to smoking mothers have a lower infant mortality rate than low birth-weight children of non-smokers.



### Explanation

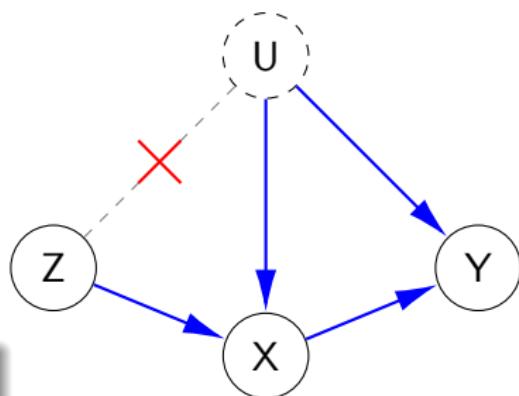
Birth weight is a collider - conditioning on it creates spurious correlation between smoking and mortality within the low birth-weight group.

# Randomized Controlled Trials (RCTs)

- **Gold standard** for causal inference
- Random assignment breaks all backdoor paths
- Creates independence:  $Z \perp\!\!\!\perp U$
- Eliminates confounding by design

## Key Properties

- Z (assignment) affects X (treatment)
- Z is independent of all confounders U
- Effect identified by comparing groups



Randomization breaks the  $U \rightarrow Z$  path

# RCTs as Edge-Cutters

## Key Insight

Random assignment **blocks all incoming edges** to the treatment node, except from the randomization device itself.

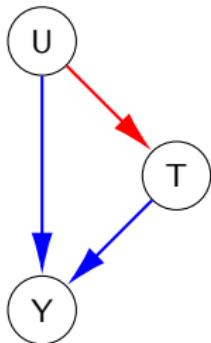
- Randomization makes treatment statistically independent of all pre-treatment variables
- Formally:  $P(T|\text{Parents}(T)) \rightarrow P(T|R)$
- This independence breaks confounding paths

## Result

$$T \perp\!\!\!\perp \{U, X, \dots\} | R$$

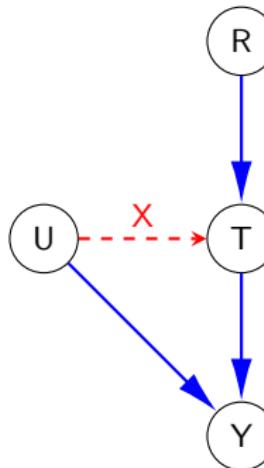
# Visual Representation: Before and After

## Observational Setting:



Confounding path:  $U \rightarrow T \rightarrow Y$

## After Randomization:



Only path to  $T$ : through  $R$

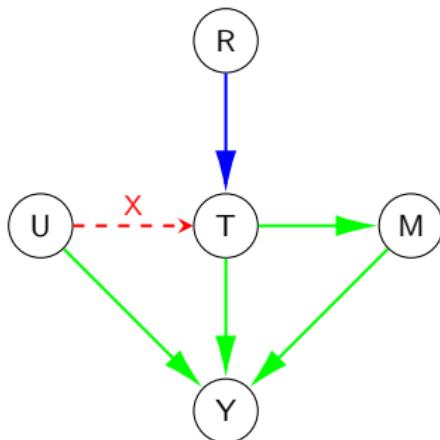
# Which Edges Are Affected?

## Edges that ARE blocked:

- All edges **into** treatment from pre-treatment variables
- Confounding paths through treatment
- Selection into treatment based on characteristics

## Edges that are NOT blocked:

- Edges **from** treatment to outcomes
- Direct effects of confounders on outcomes
- Mediating paths:  $T \rightarrow M \rightarrow Y$



Blocked edge  
Active edges

# Mathematical Formulation

## Independence Achievement

Randomization achieves:  $T \perp\!\!\!\perp \{U, X, \dots\} | R$

This means:

$$\mathbb{E}[Y|T=1] - \mathbb{E}[Y|T=0] = \text{ATE} \quad (1)$$

**Why?** No backdoor paths from  $T$  to  $Y$ :

- All paths  $U \rightarrow T \rightarrow Y$  are blocked at the first arrow
- Only variation in  $T$  comes from  $R$  (random)
- Association between  $T$  and  $Y$  must be causal

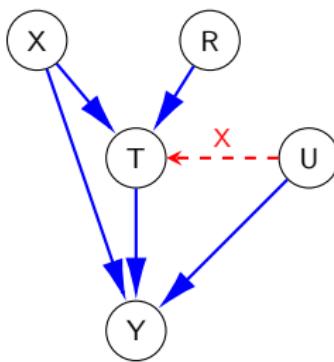
## Key Assumption

No interference, perfect compliance, no attrition

# Variations in Randomization

## Stratified/Blocked Randomization

Randomize within levels of  $X$ :  $P(T|X, U) \rightarrow P(T|X, R)$



- Edges from all variables except  $X$  and  $R$  are cut
- Must condition on  $X$  in analysis
- Improves precision if  $X$  predicts  $Y$

## Experimental Design

# What RCTs Don't Fix

## Important Limitations

Randomization only addresses confounding, not other issues:

- ① **Attrition/Selection Bias:** Post-treatment selection can reopen paths
- ② **Non-compliance:** Creates gap between ITT and ATE
- ③ **Measurement Error:** Doesn't fix measurement issues
- ④ **External Validity:** Only identifies effects in study population

## Remember

- Don't condition on post-treatment variables (reopens paths)
- Don't control for mediators (blocks part of causal effect)
- Watch for differential attrition (creates selection bias)

## Key Takeaways

- ① **RCTs as surgical edge removal:** Randomization precisely cuts confounding edges while preserving causal paths
  - ② **Independence is key:**  $T \perp\!\!\!\perp$  Pre-treatment variables |  $R$
  - ③ **Know what's blocked:**
    - Edges INTO treatment (blocked)
    - Edges FROM treatment (preserved)
  - ④ **Design implications:** Understanding edge-cutting helps design better experiments (stratification, clustering)
  - ⑤ **Limitations remain:** RCTs solve confounding but not all causal inference problems

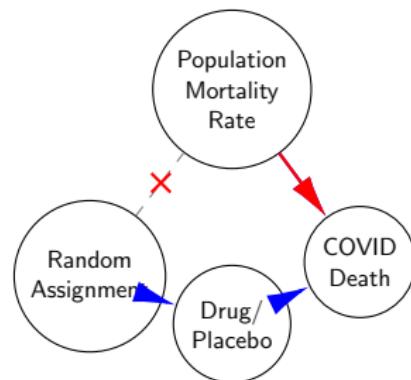
# RCTs and Vaccine Drug Trials: Population Mortality Rates

- **Critical insight:** Treatment effect depends on baseline mortality rate
- Higher severity populations → larger observable effects
- Same drug shows different effect sizes across populations

## Why This Matters

Drug preventing 50% of deaths:

- If 50% of population infected  
→ 25% better than placebo.
- If 10% of population infected  
→ 5% better than placebo.

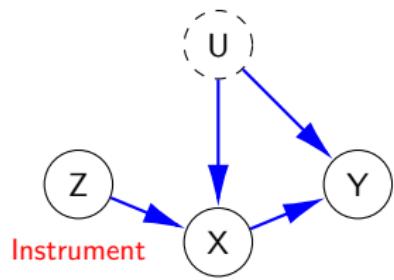


Population rate affects outcomes but not assignment

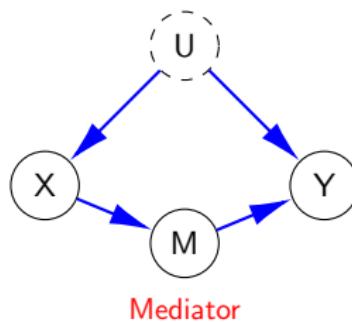
# What if you can't close the backdoors?

Two options to measure effect of X on Y without closing backdoor:

## Instrumental Variable



## Frontdoor Criterion

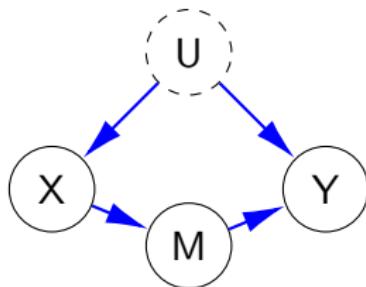


Find something that causes X which  
doesn't have a backdoor

Find an intermediary variable  
between X and Y

# Frontdoor Criterion

- Used when there are **unobserved confounders**
- M satisfies frontdoor criterion if:
  - X affects Y only through M (complete mediation)
  - No unobserved confounders of X-M relationship
  - No unobserved confounders of M-Y relationship (given X)

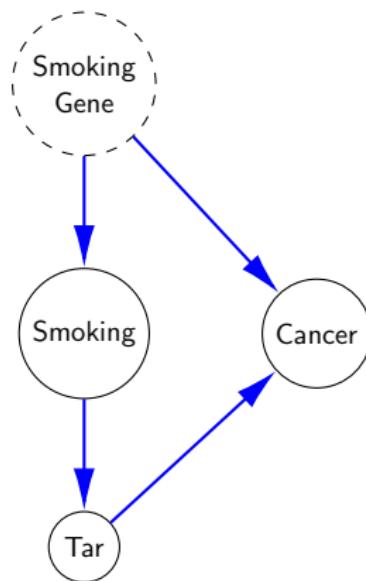


# Frontdoor Example: Smoking and Lung Cancer

- Does smoking cause lung cancer?
- Problem: Genetic factors may cause both smoking behavior and cancer susceptibility
- Solution: Use frontdoor criterion with tar deposits as mediator

## Logic

$\text{Smoking} \rightarrow \text{Tar deposits in lungs} \rightarrow \text{Lung cancer}$

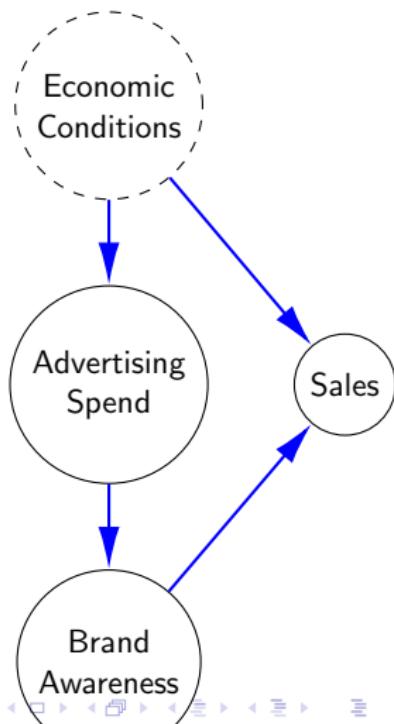


## Frontdoor Example 2: Marketing and Sales

- Does advertising spending increase sales?
- Problem: Economic conditions affect both marketing budgets and consumer spending
- Solution: Use frontdoor criterion with brand awareness as mediator

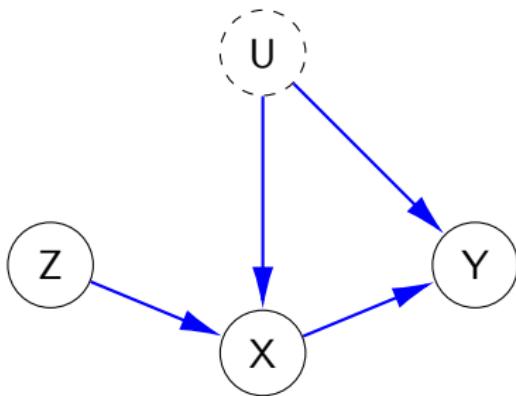
### Logic

Advertising → Brand awareness → Product sales



# Instrumental Variables

- Used when there are **unobserved confounders**
- Z is an instrument if:
  - Z affects X (relevance)
  - Z affects Y only through X (exclusion)
  - Z is unrelated to U (exogeneity)

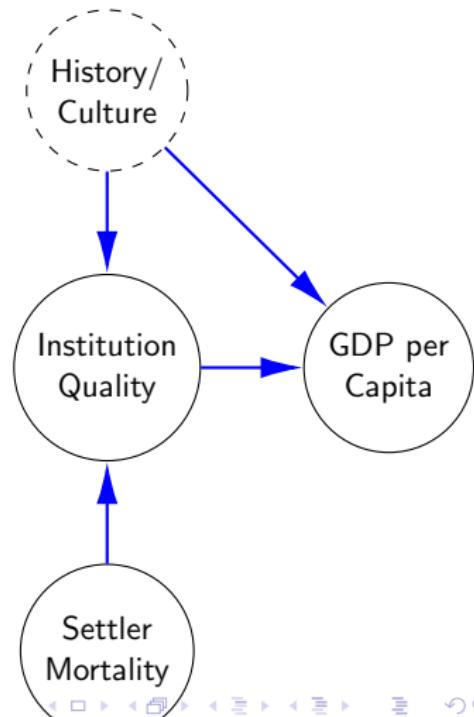


## IV Example 1: Colonial Origins of Development

- **Acemoglu, Johnson, Robinson (2001)**
- How do institutions affect development?
- Problem: Good institutions and high GDP could both be caused by unobserved factors
- Solution: Instrument settler mortality

### Logic

High settler mortality → Extractive institutions → Poor modern institutions → Low GDP today

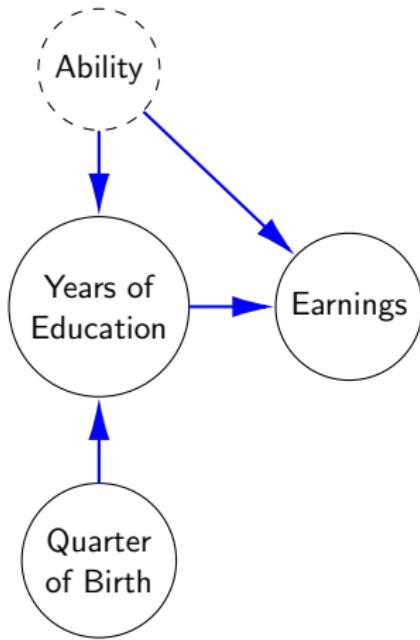


## IV Example 2: Returns to Education

- **Angrist & Krueger (1991)**
- What is the causal effect of education on earnings?
- Problem: Ability affects both education and earnings
- Solution: Use quarter of birth as instrument

### Logic

Born in Q4 → Start school older → Drop out with less education (due to compulsory schooling laws)

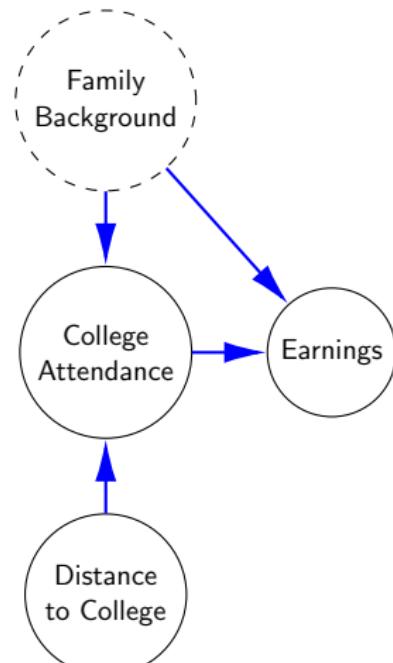


## IV Example 3: College Proximity and Returns to Education

- Card (1995)
- Alternative approach to estimating returns to education
- Problem: Family background affects both education and earnings
- Solution: Use distance to nearest college as instrument

### Logic

Living near a college → Lower cost of attendance → More likely to attend → Higher earnings

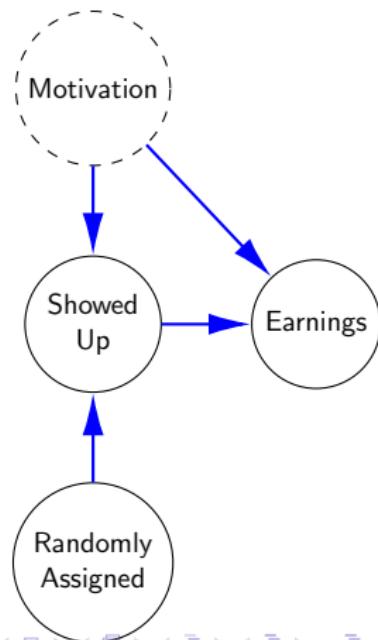


## IV Example 4 Example: Job Training Partnership Act (JTPA)

- **JTPA Evaluation Study**
- How does job training affect earnings?
- Problem: Motivation is unobserved confounder
- Solution: Use frontdoor criterion with "showing up" as mediator

### Logic

Random assignment → Actually showing up  
→ Job training received → Higher earnings



# Summary

- DAGs provide a visual language for causal reasoning
- Different structures (chains, forks, colliders) have different statistical properties
- Backdoor criterion helps identify valid control sets
- Frontdoor criterion and instrumental variables help when backdoor fails
- Understanding these concepts prevents common pitfalls in causal inference

## Questions?