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Invited Review

Strategic financial risk management and operations research

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Abstract

Risk management has become a vital topic for financial institutions in the 1990s. Strategically, asset/liability management systems are important tools for controlling a firm's financial risks. They manage these risks by dynamically balancing the firm's asset and liabilities to achieve the firm's objectives. We discuss such leading international firms as Towers Perrin, Frank Russell, and Falcon Asset Management, which apply asset/liability management for efficiently managing risk over extended time periods. Three components of asset/liability management are described: 1) a multi-stage stochastic program for coordinating the asset/liability decisions; 2) a scenario generation procedure for modeling the stochastic parameters; and 3) solution algorithms for solving the resulting large-scale optimization problem.

Keywords: Stochastic optimization; Financial engineering; Nonlinear programming

1. Introduction

Financial engineering involves the design, development, and implementation of innovative financial instruments and processes, and the formulation of creative solutions to problems in finance (Finnerty, 1988). During the last two decades, we have witnessed an increased use of operations research techniques for many diverse aspects of financial engineering. Operations research tools such as decision analysis, statistical estimation, simulation, stochastic processes, optimization, decision support systems, and artificial intelligence are becoming indispensable in several domains of financial operations (Merton, 1995; Zenios, 1993). Many factors have contributed to the growth of financial engineering including technological advances, changing regulations, globalization of financial markets, increased competition, ability to solve complex financial models, price volatility (Marshall and Bansal, 1992). Besides expertise in economic and financial theory, a financial engineer requires competence in a variety of operations research skills. This broader role of a financial engineer continues to bolster the interplay between operations research and financial engineering.

Financial engineers are involved in many important areas of an organization (see Fig. 1) (Marshall and Bansal, 1992). In corporate finance, financial engineers develop new instruments or add new features to an existing instrument in order to secure the funds necessary for the operation of large-scale busi-

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nesses. In securities and derivative products trading, financial engineers develop innovative trading strategies. In investment and money management, they develop new investment vehicles such as 'high yield' mutual funds, money market funds, and the repo market. They have also developed systems for transforming high risk investment instruments to low risk instruments through such devices as repackaging and overcollaterization.

Financial engineers have been heavily involved in risk management. Risk management assesses the types of risk of different securities and constructs and maintains portfolios with the specified risk–return characteristics (Dahl et al., 1993). A financial engineer identifies and measures the risks, then creates an instrument to achieve a desired outcome. Risk management has become a central topic for the management of financial institutions in 1990s (Merton, 1995). A 1993 survey on international banking in The Economist is devoted entirely to risk management of banks and its implications for bank managers and bank regulators in the future (Freeman, 1993). With the availability of a variety of sophisticated quantitative models and optimization tools, there is now a greater opportunity to manage risk more efficiently. Risk management provides an excellent domain for the use of operations research tools, more so than any other activity of financial engineering.

Optimization models have made a significant impact on several dimensions of risk management (Zenios, 1993). The scope of risk management is such that we cannot cover all the optimization models for efficiently managing risks in a single review. Consequently, we focus this review on asset/liability management (ALM) via multi-stage stochastic optimization. Asset/liability management is an important dimension of risk management in which the exposure to various risks is minimized by holding the appropriate combination of assets and liabilities so as to meet the firm's objectives.

Multi-stage stochastic optimization brings together all major financial-related decision in a single and consistent structure (Mulvey, 1996b). It integrates investment strategies (also known as asset allocation strategies), liability decisions (e.g., borrowings) and savings strategies (or re-investment decisions) in a comprehensive fashion. A multi-stage financial system forms the basis for assessing and managing risks in large institutional organizations, including banks, savings and loans, insurance companies, pension plans, and government entities. Individual investors can also use the methodology for managing their financial affairs over time (Berger

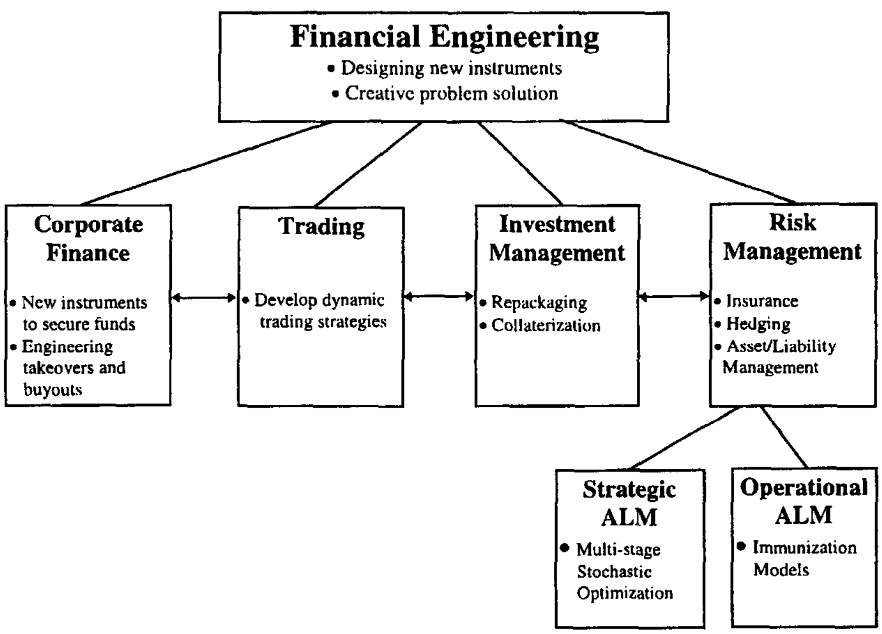


Fig. 1. Financial engineering activities.

and Mulvey, 1996). Leading international financial firms apply multi-stage stochastic programs for managing asset-liability risk over extended time periods (Mulvey and Ziemba, 1995; Ziemba and Mulvey, 1996). Prominent examples include Towers Perrin (Mulvey, 1996a), State Farm Insurance, Falcon Asset Management, Frank Russell (Cariño et al., 1994), ORTEC (Boender, 1995), Unilever, and Home Account© (Berger and Mulvey, 1996). The asset-liability management systems assist pension plan investors, banks, insurance companies and other leveraged institutions. In each case, asset investment decisions are combined with liability choices in order to maximize the investor's wealth over time.

There is a great need for integrative approaches to financial analysis and planning. The globalization of financial markets and the introduction of complex products such as exotic derivatives has increased volatility and risks. Strides in computers and information technology has eliminated any delays between the occurrence of an event and the impact on the markets – within the home country and internationally. Consideration of uncertainties is critical in financial planning. Major uncertainties include the returns of investment instruments, future borrowing rates and external deposit/withdrawal streams. Investors often seek to develop long-term strategies that hedge against uncertainties.

The rest of the review is organized as follows. We present two formulations of a multi-stage financial planning model in Section 2. The first extends the generalized network flow model of Mulvey and Vladimirou (1992), in which uncertainty occurs in the arc multipliers. The second formulation is a dynamic stochastic control model of Maranas et al. (1996), which utilizes dynamic control mechanisms to reduce the number of variables in the model. The reduction, however, comes at a cost of nonconvexity in the objective function. A critical issue in financial optimization is the modeling of uncertainty. Typically, uncertainty is represented by a moderate number of discrete realizations of the stochastic quantities, constituting distinct scenarios. Section 3 presents a discussion of scenario generation. Section 4 summarizes the various solution techniques available for solving the two models. Three prominent implementations of multi-stage stochastic programs are reviewed in Section 5. We discuss several barriers to asset/liability management via multi-stage stochastic optimization in Section 6. New product development aspects of financial engineering are examined in Section 7, with Section 8 providing several avenues for future research in asset/liability management.

2. Multi-stage stochastic optimization

Realism and computational tractability are always the main influence factors in the implementation of mathematical models. A modeler should be aware of the tradeoff between the achievable degree of realism and the computational complexity of a mathematical program. This tradeoff is even more pronounced in the case of stochastic programs, because incorporation of uncertain parameters dramatically increases the size and complexity of the resulting program. Nevertheless, for investment problems consideration of uncertainties in the financial environment is necessary; trading realism for simplicity could be too costly in these applications. A realistic model for multiperiod investment planning should ideally include the following essential features (Mulvey and Vladimirou, 1989):

1. *multiperiodicity* that captures the dynamic aspects of the problem, including fluctuating asset returns over time and balancing of trading activities with internal asset earnings, liabilities and anticipated external cash flows;
2. adequate treatment of *uncertainty* in important parameters, including any external cash flows and uncertain returns, so as to ensure that budget and liquidity requirements are met over time and that opportunity costs under various economic conditions are properly assessed;
3. ability to account for the decision maker's *risk bearing attitudes*;
4. consideration of *transaction costs* reflecting commission fees and other expenses incurred in trading activities;
5. consideration of assets and liabilities in *a single integrated model* that addresses the whole financial planning problem and complies with accounting practices;
6. *understandability*, so that the model can be convincingly explained to investment managers who will ultimately be the end users of the application;

7. ability to capture other factors involved in practical decision making, including growth and budgetary requirements, as well as legal, institutional or policy provisions pertinent to the investor's problem.

Clearly, the condition of simplicity and understandability seems to conflict with the other requirements which stress realism, detail, and complexity. We present a generalized network model for multiperiod investment planning which essentially captures most of the features listed above (Mulvey and Vladimirou, 1989). The model can be represented graphically so that it is easily understood by nontechnical users. Moreover, the use of network models offers substantial computational advantages, thus maintaining tractability of practical stochastic programs.

We divide the entire planning horizon T into two discrete time intervals T_1 and T_2 where $T_1 = 0, 1, \dots, \tau$ and $T_2 = \tau + 1, \dots, T$. The former corresponds to periods in which investment decisions are made. Period τ defines the date of the planning horizon; we focus on the investor's position at the beginning of period τ. Decisions occur at the beginning of each time stage. Much flexibility exists. An active trader might see his time interval as short as minutes, whereas a pension plan advisor will be more concerned with much longer planning periods such as the dates between the annual Board of Director's meeting. It is possible for the steps to vary over time – short intervals at the beginning of the planning period and longer intervals towards the end. T_2 handles the horizon at time τ by calculating economic and other factors beyond period τ up to period T. The investor cannot render any active decisions after the end of period τ.

Asset investment categories are defined by set $A = 1, 2, \dots, I$, with category 1 representing cash. The remaining categories can include broad investment groupings such as stocks, bonds, and real estate. The categories should track well-defined market segment. Ideally, the co-movements between pairs of asset returns would be relatively low so that diversification can be done across the asset categories.

In our approach, uncertainty is represented by a number of distinct realizations. Each complete realization of all uncertain parameters gives rise to a *scenario*; we denote by S the discrete set of all scenarios. Several scenarios may reveal identical values for the uncertain quantities up to a certain period – i.e., they share common information history up to that period (see Fig. 2). Scenarios that share common information up to a specific period must yield the same decisions up to that period. We will address the representation of the information structure through a condition known as *nonanticipativity*.

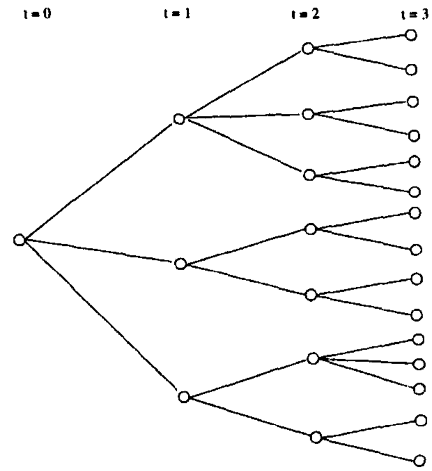


Fig. 2. Scenario tree.

We assume that the portfolio is rebalanced at the beginning of each period. Alternatively, we could simply make no transaction except reinvest any dividend and interest – a buy and hold strategy. For convenience, we also assume that the cashflows are reinvested in the generating asset category and all the borrowing is done on a single period basis.

For each $i \in A$, $t \in T_1$, and $s \in S$, we define the following parameters and decision variables.

Parameters:

$r_{i,t}^s$ = $1 + \rho_{i,t}^s$, where $\rho_{i,t}^s$ is the percent return for asset i, time period t, under scenario s (projected by the stochastic modeling subsystem).

π_s Probability that scenario s occurs, $\sum_{s=1}^{S} \pi_s = 1$.

w_0 Wealth in the beginning of time period 0.

$\sigma_{i,t}$ Transaction costs incurred in rebalancing asset i at the beginning of time period t (symmetric transaction costs are assumed, i.e., cost of selling equals cost of buying).

β_t^s Borrowing rate in period t under scenario s.

Decision variables:

$x_{i,t}^s$ Amount of money in asset category i, in time period t, under scenario s, after rebalancing.

$v^s_{i,t}$ Amount of money in asset category i, in the beginning of time period t, under scenario s, before rebalancing.

w^s_t Wealth at the beginning of time period t, under scenario s.

$p^s_{i,t}$ Amount of asset purchased for rebalancing in period t, under scenario s.

$d^s_{i,t}$ Amount of asset i sold for rebalancing in period t, under scenario s.

b^s_t Amount borrowed in period t, under scenario s.

With these definitions in place, we can present the deterministic equivalent of the stochastic asset allocation problem.

(Model SP)

$$\text{Max } Z = \sum_{s=1}^{S} \pi_s f(w^s_\tau) \tag{1}$$

s.t.

$$\sum_i x^s_{i,0} = w_0 \quad \forall s \in S, \tag{2}$$

$$\sum_i x^s_{i,\tau} = w^s_\tau \quad \forall s \in S, \tag{3}$$

$$v^s_{i,t} = r^s_{i,t-1} x^s_{i,t-1} \quad \forall s \in S, \quad t = 1, \dots, \tau, \quad i \in A, \tag{4}$$

$$x^s_{i,t} = v^s_{i,t} + p^s_{i,t}(1 - \sigma_{i,t}) - d^s_{i,t} \quad \forall s \in S, \quad i \neq 1, \quad t = 1, \dots, \tau, \tag{5}$$

$$x^s_{1,t} = v^s_{1,t} + \sum_{i \neq 1} d^s_{i,t}(1 - \sigma_{i,t})1 - \sum_{i \neq 1} p^s_{i,t} - b^s_{t-1}(1 + \beta^s_{t-1}) + b^s_t \quad \forall s \in S, \quad t = 1, \dots, \tau, \tag{6}$$

$$x^s_{i,t} = x^{s'}_{i,t} \quad \text{for all scenarios } s \text{ and } s' \text{ with identical past up to time } t. \tag{7}$$

Model (SP) is a generalized network problem with nonanticipativity and other side constraints (see Fig. 3). Much realism can be incorporated to this model by adding additional variables and constraints (Mulvey, 1996b). For example, variables for cash inflows, cash outflows and paydown of principle can be added without destroying the generalized network structure. Cash inflows can depict savings for an investor in each period, whereas cash outflow may represent investment of funds to pay for consumption expenditures. In the real-world, investors restrict their investments in asset categories for a diversity of purposes such as company policy, legal and historical rules and other considerations. We limit their structure to linear restrictions $B^s x^s = b^s$, where B is a matrix of coefficients that depend upon scenario s. For example, investors may set a lower limit – say 5% – on cash for liquidity considerations. Investors may wish to restrict their foreign exchange exposure to 10–20% of their portfolio's value. While nonanticipativity constraints can be handled efficiently within a generalized network context, other types of side constraints will render the model more difficult.

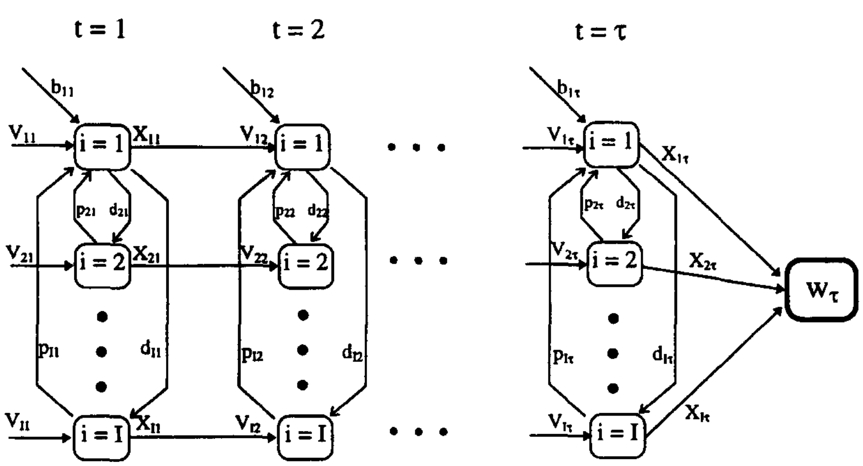


Fig. 3. Generalized network model for each scenario $s \in S$.

The nonlinear objective function (1) can take several different forms. If the classical mean–variance function is employed, then (1) becomes

$$\text{Max } Z = \eta \, \text{Mean}(w_\tau) - (1 - \eta)\text{Var}(w_\tau),$$

where Mean(w_τ) is the average total wealth and Var(w_τ) is the variance of the total wealth across the scenarios at the end of period τ. Parameter η indicates the relative importance of variance as compared with the expected value. This objective leads to an efficient frontier of wealth at period τ. An alternative to mean–variance is the von Newmann–Morgenstern expected utility of wealth at period τ. Here, the objective becomes

$$\text{Max } Z = \sum_{s=1}^{S} \pi_s \, \text{Utility}(w_\tau^s),$$

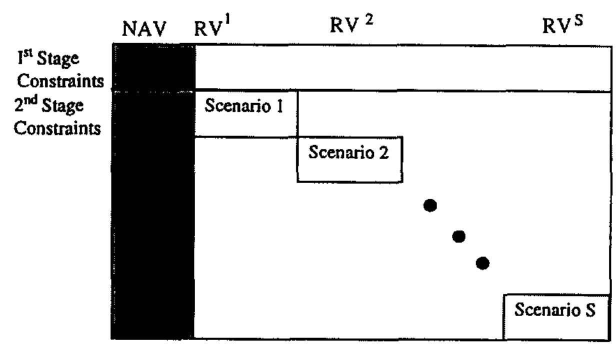
where Utility(W) is the VM utility function (Keeney and Raiffa, 1993). The two objective functions are equivalent under certain conditions on the distribution of returns and the shape of the utility function (Kroll et al., 1984). A third approach for addressing a multi-period model extends the expected utility model. Most investors are interested not only in their wealth at the end of some planning horizon, but also they prefer one set of trajectories over another – even when the results at the horizon are identical. Temporal preference can be handled via a multi-objective formulation rather than a single expected utility function. More details about the alternative objective functions are given in Mulvey (1996b).

Constraint (2) guarantees that the total initial investments equals the initial wealth. Constraint (3) represents the total wealth in the beginning of period τ. This constraint can be modified to include assets, liabilities, and investment goals. The modified result is called the surplus wealth (Mulvey, 1989). Most investors make allocation decisions without reference to liabilities or investment goals. Mulvey employs the notion of surplus wealth to the mean–variance and the expected utility models to address liabilities in the context of asset allocation strategies. Constraint (4) depicts the wealth $v_{i,t}^s$ accumulated at the beginning of period t before rebalancing in asset i. The flow balance constraint for all assets except cash for all periods is given by constraint (5). This constraint guarantees that the amount invested in period t equals the net wealth for asset. Constraint (6) represents flow balancing constraint for cash. Nonanticipativity constraints are represented by (7). These constraints ensure that the scenarios with the same past will have identical decisions up to that period. While these constraints are numerous, solution algorithms take advantage of their simple structure.

Model (SP) is a split variable formulation of the stochastic asset allocation problem. This formulation has proven successful for solving the model using techniques such as progressive hedging algorithm of Mulvey and Vladimirou (1991a) and quadratic diagonal approximation of Mulvey and Ruszczynski (1995). Split variable formulation is also found beneficial by direct solvers that use the interior point method (Lustig et al., 1991a). By substituting constraint (7) back in constraints (2)–(6), we obtain a compact formulation of the stochastic allocation problem. Constraints for this formulation exhibit a dual block diagonal structure. This formulation may be better for some direct solvers (Lustig et al., 1991a). Examples of constraint structures for both formulations are shown in Figs. 4 and 5.

2.1. Dynamic stochastic control models

An alternative formulation for the multi-stage financial planning problem is to pose the model using a dynamic stochastic control framework. This approach has a long history dating back to the early work of Samuelson (1969) and Merton (1969). Also, see Brennan et al. (1996), Chow (1993), Davis (1993), Dixit and Pindyck (1994) and their refer-



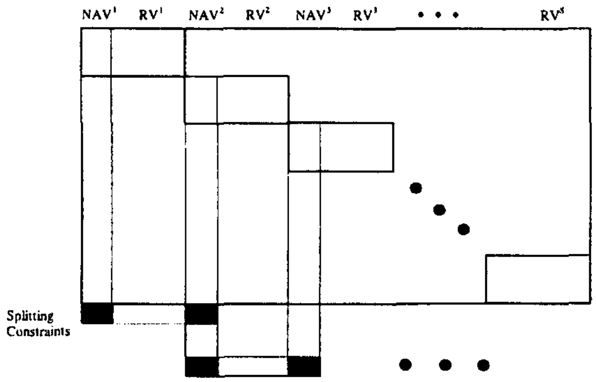


Fig. 5. Split variable formulation (NAV – nonanticipativity variables, RV – remaining variables).

ences for more recent techniques. These approaches attempt to find closed-form solutions – in continuous time or in discrete time – or they employ dynamic programming.

In this subsection, we discuss a variant of the dynamic stochastic control approach to modeling the stochastic asset allocation problem. The basic idea is to set up the optimization problem under uncertainty as a stochastic control model using a popular control policy. This model reallocates the portfolio in the end of each period such that the asset proportions meet the specified targets. The continuous sample space is represented via a discrete approximation. The discrete approximation offers a wider range of application and is easy to implement. While the fixed mix model reduces the number of decision variables considerably, it results in a nonconvex objective function (Maranas et al., 1996). Finding a global optimum to a nonconvex problem continues to be a challenge in optimization.

Let λ_i be the fraction of the wealth in asset category i. Note that $0 \leq \lambda_i \leq 1, \forall i$, and $\sum_{i=1}^{I} \lambda_i = 1$. At the beginning of each period, the investor balances his portfolio. The rules ensure that a fixed percentage on one's wealth is invested in each asset category – at least at the beginning of each period. Using the multi-period extension of Markowitz's mean–variance model, the dynamic stochastic control problem is formulated as follows.

(Model DSP)

$$\text{Max } Z = \eta \operatorname{Mean}(w_T) - (1-\eta)\operatorname{Var}(w_T) \tag{8}$$

s.t.

$$\operatorname{Mean}(w_T) = \sum_{s=1}^{S} \pi_s w_T^s,$$

$$\operatorname{Var}(w_T) = \frac{1}{S-1} \sum_{s=1}^{S} \left[w_T^s - \operatorname{Mean}(w_T) \right]^2,$$

$$w_T^s = w_0 \prod_{t=1}^{T} \left[\sum_{i=1}^{I} r_{i,t}^s \lambda_i \right] \quad \forall s \in S,$$

$$\sum_{i=1}^{I} \lambda_i = 1, \quad 0 \leq \lambda_i \leq 1,\ i = 1, \dots, I. \tag{9}$$

Model DSP is a nonconvex optimization problem involving single linear equality constraint (9) and a moderate number of variables λ_i, $i = 1, \dots, I$. The objective function is a nonconvex multivariable polynomial function in λ_i involving multiple local minima (Maranas et al., 1996).

Other control strategies are available, such as portfolio insurance (Perold and Sharpe, 1988), but these often lead to non-linear and non-convex optimization problems. At times, closed form solutions can be found for the stochastic control model (Davis and Norman, 1990; Tasker et al., 1988). In many cases, however, we must approximate the model via discretization of the sample decision space.

3. Scenario generation

Critical to any stochastic optimization is the issue of modeling the stochastic elements. Stochastic parameters for integrative financial planning problem can be placed in three groupings: 1) a small set of economic factors; 2) projected returns for asset categories as implied by the values of the economic factors in the prior group; and 3) projected liabilities based on the implied values of the same economic factors. Several goals must be kept in mind when modeling the stochastic parameters. The model structure must build on sound economic principles. Interest rates, for example, must be consistent with the returns for the underlying fixed-income asset categories. Basic trends should be preserved whenever possible – such as mean reversion in interest rates over extended horizons. And the projections should be evaluated with respect to their fit with historical data and trends.

Scenario analysis offers an effective, and easily understood tool for addressing the stochastic elements in a multi-stage financial model. We define a scenario as a single deterministic realization of all uncertainties over the planning horizon. Ideally, the process constructs scenarios that represent the universe of possible outcomes (Glynn and Iglehart, 1989; Dantzig and Infanger, 1993). This objective differs from generation of a single scenario, say for forecasting and trading strategies. We are interested in constructing a *representative* set of scenarios that are both optimistic and pessimistic within a risk analysis framework. Such an effort was undertaken by Towers Perrin (one of the largest actuarial firms in the world) using a system called CAP:Link (Mulvey, 1996a). The system entails a cascading of a set of submodels, starting with the interest rate component. Towers Perrin employs a version of the Brennan and Schwartz (1982) two factor interest rate model. The other submodels are driven by the interest rates and other economic factors. Towers Perrin has implemented the system in over 14 countries in Europe, Asia, and North America. Fig. 6 presents the scenario-generation system of Towers Perrin.

Scenario generation requires the estimation of the input parameters for the modeling of the economic factors. The ability to choose the 'correct' or 'best' set of parameters is essential if such models are to have practical value. Economic factors required for projected returns and liabilities can be estimated via techniques such as maximum likelihood (Broze et al., 1995), GMM – generalized method of moments (Hansen, 1982), SME – simulated moment estimation (Duffie and Singleton, 1993), and IPE – Integrated Parameter Estimation (Mulvey et al., 1996a). Of these approaches, SME and IPE apply to a wider range of problems since they require no explicit relationships between the parameter vector and the model variables, instead replacing them with their simulated versions. The methods of maximum likelihood and GMM employ historical data to estimate parameters and then apply the estimated values for forecasting. This practice requires historical data. In many instances, there may be inadequate relevant historical information available for estimation and forecasting. In SME and IPE, a lack of historical data can be replaced by informed opinion, so that even in the absence of a sample a model can be produced to assist in planning for the future.

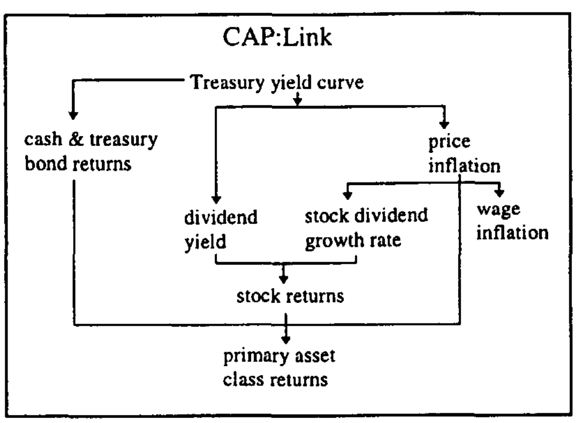


Fig. 6. The Towers Perrin Scenario Generation System.

While SME permits the target sets to include only moments, IPE permits any critical statistic or condition which can be calculated through a simulated process. IPE also allows for a general measure of utility as compared to SME's quadratic measure. The formulation includes constraints that specify tolerances between the summary statistics and its target. In the IPE calibration process, the parameters are set so that the generated scenarios match the estimated distributions and statistical relationships as closely as possible. For long term asset and liability systems, for example, it is necessary to match the tails of the distributions so that risks are accurately portrayed. IPE fits tails through quantile matching. IPE is in fact a multi-attribute optimization process that provides an intuitive approach to sensitivity analysis in calibration and comparing various statistics. Setting the relative importance of a statistic in the set is an adaptive procedure, which depends on the goals of the investor.

By its nature, parameter estimation presents a nonlinear optimization problem. However, conventional nonlinear programming algorithms are not suited for solving IPE due to non-convexities in the objective function. Instead, Mulvey et al. (1996a) employ adaptive memory programming to solve this nonconvex optimization problem (Glover et al., 1994). They compare the performance of IPE to that of maximum likelihood and generalized method of moments on a short interest rate model using UK

Treasury bill rates from 1980 to 1995. IPE provides estimates at least as good as maximum likelihood and the generalized method of moments – both in terms of efficiency and bias. Its primary value, however, lies in its ability to easily extend to a variety of complicated models such as Tower's Perrin's CAP:Link system. The flexibility of IPE in its selection of penalty and objective weights makes it a suitable choice for scenario generation.

4. Solution techniques

A number of different algorithmic approaches have been proposed for solving stochastic programs. Van Slyke and Wets (1969) showed with their L-shaped method that Benders decomposition can be applied to solving two-stage stochastic programs. Dantzig and Glynn (1990) and Infanger (1994) used importance sampling in a Benders decomposition framework. A classical approximation scheme for solving two-stage stochastic linear programs with stochastic right-hand sides is to calculate lower and upper bounds on the objective value via the inequalities of Jensen. Several variants of this approach have appeared in the literature (Infanger, 1994; Ermoliev, 1983).

In this section we review a number of algorithms available to solve the asset allocation models. We focus on solutions to multi-stage stochastic programs possessing discrete-time decisions with a modest number of scenarios – typically under 1000 to 3000 – and nonlinear objective functions addressing risk aversion. The model's size depends on the number of decision variables and the form of the nonanticipativity rules. If Model SP is selected, the model becomes a convex program who size hinges on the number of scenarios that are placed in S. If a dynamic control approach (Model DSP) is used, problem size can be much smaller since conditional decisions are greatly restricted. DSP, however, is a nonconvex optimization problem.

4.1. Direct solvers

The simplest approach when the objective is linear is to use an efficient linear programming solver. Although simpler to handle, LP does not represent risk aversion well. Lustig et al. (1991a) solve the multi-stage asset allocation problem with a linear objective function using OB1 and MINOS. OB1 is a primal–dual interior point algorithm for solving linear programs (Lustig et al., 1991a). MINOS is a nonlinear programming code that can also solve LP (Murtagh and Saunders, 1987). On a compact formulation, MINOS outperformed OB1 on several test problems. The split formulation, however, significantly reduced the time required by OB1 to yield the fastest solution times.

When the objective is nonlinear, the general purpose nonlinear programming codes MINOS or GRG (Smith and Lasdon, 1992) can be used for solution. However, the nonlinear interior point methods have advantages over these codes. For example, in mean–variance applications, the covariance matrix can be factored to convert the mean–variance function into a separable function. This is achieved by a modest increase in the number of constraints. Vanderbei and Carpenter (1993) show that nonlinear interior point methods can take advantage of the separable structure despite the increase in the number of constraints. A similar transformation is possible with the expected utility objectives as discussed in Berger et al. (1994); see also Zenios (1993).

Primal–dual interior point algorithms can be specialized to solve nonlinear stochastic optimization problems. Carpenter et al. (1993) extend a primal–dual interior point procedure for linear programs to the case of convex separable quadratic objectives. The extension is tested on the asset allocation problems of Mulvey and Vladimirou (1989) and compared to MINOS. The primal–dual interior point method compared favorably with MINOS, especially for the larger test problems. In the direct solution of nonlinear programs via interior point methods, the primary computational step is the factorization of the normal equations ADA^t, where A is the coefficient matrix and D is a diagonal matrix (Lustig et al., 1994). This factorization is typically done by means of the Cholesky (LL^t) method. A major difficulty when applying these algorithms to stochastic optimization problems has to do with the sparsity structure of A. Considerable research has been devoted to preordering schemes to reduce the amount of fill-in that occurs during the algorithm. At least five techniques have been developed to reduce the problems associated with dense columns: 1) splitting the dense variables (Lustig et al., 1991a); 2) performing a Shur complement (Lustig et al., 1994); 3) solving the dual LP (Birge et al., 1994); 4) generalizing the Sherman–Morrison–Woodbury formula (Birge and Qi, 1988); and 5) tree dissection (Berger et al., 1996).

The tree structure of multi-stage program causes difficulty for the standard pre-ordering methods employed by CPLEX (1993) and LOQO (Vanderbei, 1994). In particular the multiple min-degree procedure gives rise to considerable fill-in. Berger et al. (1996) develop an alternative ordering heuristic, called tree dissection, for the structure of multi-stage stochastic programs. They apply the tree dissection approach to a six-stage financial investment problem. The tests were conducted using a single Silicon Graphics Workstation (R8000) at SGI headquarters in Mountain View, California. The approach was considerable faster than LOQO showing that realistic size multi-stage programs can now be solved directly.

Ideas of using parallel computing for stochastic programs have been around for quite some time (Wets, 1985; Dantzig, 1988; Hillier and Eckstein, 1990; Nielson and Zenios, 1993; Yang and Zenios, 1995; Zenios and Lasken, 1988; Jessup et al., 1994). More recently, Rothberg and Gupta (1993) developed an extremely efficient method for carrying out sparse matrix factorization in a parallel environment. Rothberg's factorization coupled with tree dissection concepts provide some very encouraging results for stochastic programs. Initial evidence indicates that parallel direct solvers will be able handle stochastic programs with over 10 000 scenarios within several minutes of runtime in a parallel environment.

4.2. Decomposition algorithms

Considerable progress has been made in the design of efficient decomposition algorithms for solving multi-stage stochastic programs since the original L-shape proposal by Van Slyke and Wets (1969). These algorithms take advantage of the stochastic program's structure; see Birge et al. (1994), Birge and Holmes (1992), Dantzig and Infanger (1993), Ermoliev and Wets (1988), Gassmann (1990), Infanger (1994), Mulvey and Vladimirou (1991b), Rockafeller and Wets (1991), Dempster and Thompson (1995), Fan and Cariño (1994), and Nielson and Zenios (1993) for examples. In several cases, implementation have occurred in a parallel or distributed computing environment. Motivating the use of decomposition algorithms, we note that the size of stochastic programs quickly grows as a function of the number of scenarios. As an example, our generated six-stage financial planning problem with 156 variables and 96 constraints per scenario grows to a problem with 100 scenarios – beyond the range of most current NLP solvers.

A number of decomposition algorithms are based on the augmented Lagrangian function, such as the progressive hedging algorithm (PHA) and the diagonal quadratic approximation (DQA). PHA applies to the variable split form of the multi-stage stochastic program. The nonanticipativity constraints are placed in the objective function as penalty and multiplier terms, and are progressively enforced by an iterative procedure. Mulvey and Vladimirou (1991a) compare the performance of the progressive hedging algorithm to alternative solution strategies on a set of linear and nonlinear portfolio management problems. The general purpose optimizer MINOS (Murtagh and Saunders, 1987) solve these test problems in their compact form. This is the most efficient program formulation for MINOS because it results in the smallest constraint matrix – i.e. the size of the basis is minimized. The linear problems were also solved using the primal–dual interior code (OB1) of Lustig et al. (1991b). For nonlinear test cases, they employ an extension of the primal–dual interior point method to convex, separable optimization programs (Carpenter et al., 1993). The staircase formulation obtained by partial variable splitting, is employed in these terms. On linear problems the progressive hedging algorithm was faster than MINOS. It was also faster than OB1 when the compact form was used. Interior point outperformed PHA for staircase structures. On nonlinear problems, PHA maintains its superiority over MINOS, particularly on large test problems. The progressive hedging algorithm also fares well against interior point algorithm on nonlinear problems, outperforming it in several cases.

DQA forms an augmented Lagrangian function by dualizing nonanticipativity constraints. The DQA algorithm approximates the Lagrangian at the current iterate by a quadratic and separable term (Mulvey and Ruszczynski, 1995). The outer loop revises the dual variable by the method of multipliers, whereas the inner loop consists of separable quadratic or convex terms. DQA is a flexible scheme which can be implemented in many ways, in particular, in a parallel distributed environment. Mulvey and Ruszczynski (1995) compare the performance of DQA with highly specialized methods for linear two-stage problems. The most successful methods found so far are based on Benders decomposition, suggested for stochastic programming in Van Slyke and Wets (1969). MSLiP (Gassmann, 1990) is a recent implementation of this idea, which allows for solving linear multistage problems in a nested formulation. Mulvey and Ruszczynski (1995) show that the specialized decomposition techniques MSLiP and DQA outperform MINOS.

4.3. Computational results for the dynamically balanced problem

The dynamic stochastic control problem is a nonconvex optimization problem that is difficult to solve globally. Numerous global optimization algorithms are available. We discuss two recent algorithms that are applied to financial models.

Maranas et al. (1996) present an efficient global optimization algorithm for solving the multi-period investment problem. Due to noncovexities, multiple local solutions exist which render the location of the global one a very difficult task. Traditional local algorithms can only guarantee convergence to a local solution at best, thus failing sometimes to locate the optimal recommendation. Since the algorithm of Maranas et al. finds a global solution, the best tradeoff between risk and expected profit can be established for the multi-period investment problem. As a consequence, stochastic control provides a viable alternative to stochastic programming. This global optimization algorithm is based on a convex lower bounding of the original objective function and the successive refinement of converging lower and upper bounds by means of a standard branch and bound procedure. The procedure is applied to a realistic investment problem involving 9 assets, 20 time periods and 100 scenarios. A comparison of the efficient frontier generated with the global optimization approach with the one obtained using GRG is

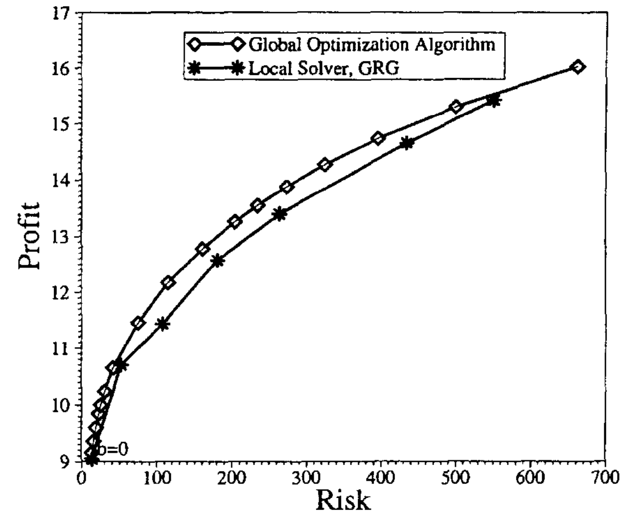


Fig. 7. Comparison with local solver.

shown in Fig. 7. Clearly, the local solver fails to correctly generate the efficient frontier instead yielding a suboptimal curve.

Real world considerations, such as taxes and transaction costs, cause grave difficulties for continuous global optimization algorithms such as the one discussed before (Maranas et al., 1996). Glover et al. (1994) develop a new approach to solve the fixed mixed problem that combines Tabu Search and a variable scaling procedure. A discretization of the solution space is used to advantage by Tabu Search. They first apply Tabu Search to the asset allocation problem of Maranas et al. without transaction costs. For this problem, their approach finds global solution without fail requiring time comparable to global optimization. When the transaction costs and taxes are added to the model, Tabu Search generated the entire efficient frontier in about 17 CPU minutes on a Silicon Graphic Workstation. Thus, Tabu Search is a viable alternative to global optimization algorithms, particularly when taxes and transactions costs are included in the fixed mix model.

5. Applications and implementations

Leading international financial firms apply multistage stochastic program for managing asset-liability risk over extended time periods. Prominent examples include: Towers Perrin, State Farm Insurance, Falcon Asset Management, Frank Russell, Unilever, and

Midas Debt Management System (Dempster, 1996). The asset liability management systems assist pension plan investors, banks, insurance companies, and other leveraged institutions. Below, we discuss a few prominent examples.

Towers Perrin, an international benefits consultancy, develops asset/liability systems (ALM) to aid in long-term financial planning. These systems begin with a set of simulations for future economic conditions – typically 500 to 1000. Each path depicts a plausible future scenario reflecting interactions of the economic variables. Ideally, the combination of all scenarios spans the range of potential outcomes. The success of alternative investment and liability strategies is observed by simulating the strategies over the scenarios. Towers Perrin measures risks in several ways – including volatility at the horizon and downside risk. Unilever, a $55 billion multi-national company uses software from Towers Perrin to analyze their pension funds in many European countries (a majority of their $11 + billion are managed via the TP system). The software includes CAP:Link, the economic and market scenario generator, FIN:Link to calculate financial results under each scenario, LIAB: Link for the actuarial cashflows, and OPT:Link to optimize the multi-period model.

A second noteworthy example of integrative ALM is the Russell–Yasuda–Kasai System for insurance companies (Cariño et al., 1994). The Frank Russell company designed a multi-period model to integrate assets and liabilities for the Yasuda–Kasai insurance company. This model consists of a stochastic linear program. The overall objective is to maximize the firm's expected profit net of penalty costs at the end of the planning horizon subject to constraints on meeting certain accounting ratios and other general linear restrictions. The result is a large-scale optimization model that is solved via a group of IBM workstations.

Individuals must plan their savings, investment, and borrowing strategies in the face of considerable uncertainties. Not only are returns on assets such as stocks and real-estate unknown, but also individuals display liabilities and goals whose cashflows depend on stochastic events. Take the case of a floating rate mortgage. The actual payments are a function of interest rates at predetermined points in time. Or suppose that the investor is saving to buy a house in 5 years. The price of the house depends upon a number of issues – including inflation and factors relating to supply and demand. These aspects are clearly stochastic. A PC-based software system integrates asset allocation decisions with savings and borrowings strategies in order to realize the individual's goals in a fashion commensurate with the investor's risk attitudes (Berger and Mulvey, 1996). As with previous ALM systems, risk is calculated as it relates to the projected uses for the funds – *investing for a purpose*. Individual financial planning can be rather elaborate: we must pay taxes on income; we possess mechanisms for deferring taxes (such as retirement account); our lifespans are highly uncertain – leading to difficulties in determining the planning horizon as well as end effects. We possess multiple objectives involving consumption and savings. The individual system fashions these disparate issues into an integrative risk management system – optimizing a time dependent and multi-criteria preference function. Its goal is to construct a virtual financial plan for the individual over the World Wide Web – with strategic advise from the ALM system.

6. Risk analysis ladder and barriers

A key idea in multi-period ALM systems is to carry out the strategic investment decisions in concert with the uses of the funds: *Invest for a purpose*. By addressing dynamic investment liability issues, the ALM systems depict a natural evolution in the progress of risk analysis depicted in Fig. 8. Model details and realism increase as we move up the ladder. At the top, full organizational risks are evaluated and managed. Currently, most financial organizations occupy the first three rungs. They fail to integrate the longer term impacts of their investment or liability management decisions on the total organization.

There are a couple of reasons for the reluctance of financial organizations to move up the risk analysis ladder. First, contingent asset/liability system (ALM) requires a vast number of computations. At each stage during the planning horizon, the ALM must design decision variables for each scenario. Most of these conditional variables will never occur;

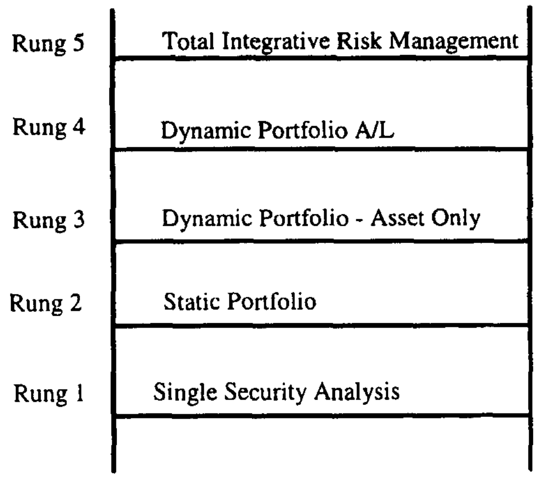


Fig. 8. The risk analysis ladder.

there is after all only one path for the future. A full set of variables is needed so that a spectrum of possibilities can be estimated – to pinpoint risks. Typically, the range of alternatives is portrayed as a scenario tree (Fig. 2). This tree structure grows larger with each time stages. Research is underway to find efficient algorithms for solving the resulting stochastic program; we are now able to address planning problems with over 15 000 scenarios, 1 000 000 equations, and 2 000 000 decision variables using a single modern workstation (Mulvey and Ruszczynski, 1995). Parallel and distributed computers will greatly reduce these run times.

A second barrier to implementing the ALM technology is the reluctance of financial regulators to require market based computations. For example, in the case of pension plans, actuarial rules have only recently dictated market discount factor for liabilities (FAS 87). As institutions are unable to keep their financial house in order (via rogue traders, misunderstanding of risks for derivatives, over concentration in single markets, and over-leverage), stockholders and the public are beginning to force financial institutions to carry out comprehensive risk management. The past several years has seen many disasters: Orange Country, Barings, and Lloyds of London. ALM scenario planning is an ideal framework for reducing the occurrences of these risk mis-management.

A third barrier involves the need to model the future course of economy, along with the accompanying returns on assets and liabilities. There is no perfect set of forecasting equations, of course. But as markets become freer and more open to economic incentives, we are able to build scenario systems that generate a reasonable range of alternatives – scenarios that are internally consistent and based on sound economic theory.

7. Financial engineering in ALM systems

Financial engineers assist in the development of new products when existing products are unsuitable or the new product is such that it will create new opportunities for the clients. New products may be the result of a concerted product development effort or they may simply result from a client's specific needs. Successful new products typically tend to be the outgrowth of an appropriately designed structure and a carefully orchestrated process (Marshall and Bansal, 1992). We can employ asset liability management systems to assist in the construction of these products. In fact, the ALM system can generate new products – for each customer's specific needs. By observing the behavior of the wealth paths over time and by scenario, and by taking into account the investor's time preference, we can often discover a set of scenarios that cause the negative returns. We can then design tailored options to reduce the impact of the sequence of events from occurring. The process entails solution to a series of interrelated optimization problems that represent goals associated with different individuals and scenarios. In addition, these interrelated problems are not just 'one shot' occurrences but must be addressed over time. Integrative Population Analysis provides a framework that enables successively encountered problems in such applications to be solved effectively and efficiently (Glover et al., 1995). This approach integrates a refinement of the learning process of target analysis with sensitivity analysis from mathematical programming.

8. Some future directions

The Towers Perrin system depends upon a set of decision rules at each stage of the planning period.

The TP actuaries and asset consultants employ variants of the dynamically balanced strategy for the pension plans that they advise. In contrast, the Russel–Yasuda–Kasai system does not fix upon any set of rules; rather it takes advantage of the branching structure in the decision tree subject to the various constraints imposed by regulatory and policy restrictions, etc. Thus, the process is directly linked to the asset/liability modeling system. Bringing the intuitive approach of TP and the theoretical validity of RYK together seems to be worthwhile area of future research.

Another worthwhile area of future research in stochastic programming is the study of estimation risk (Bawa et al., 1979). ALM systems unrealistically assume that the parameters that characterize the joint probability distributions of asset returns have known values. These parameters are estimated from historical data and the estimates are treated as if they were the true parameter values. Errors of estimation are not taken into account, thereby ignoring an important of source of uncertainty in stochastic modeling. These issues are important because a decision that is optimal in the absence of parameter uncertainty is not necessarily optimal or even close to optimal in the presence of such uncertainty (Mulvey et al., 1996b). Studies that will theoretically and empirically examine the consequences of estimation risk on asset/liability strategies are needed.

Many investment decisions are rendered in isolation of the overall organization's objectives. Take the case of pension plans. Funding and investment strategies rarely coordinate with other financial decisions, such as capital budgeting. The practicality of performing this coordination has been prohibitive. The ALM systems discussed in this paper illustrate the benefits of stepping up the risk analysis ladder. The systems provide a comprehensive measurer of organizational risks. The final step is to move to the top rung – Total Integrative Risk Management. Here, the ALM system coordinates the entire financial enterprise (Mulvey et al., 1995). For example, it might develop a funding strategy for a pension plan that is consistent with the profit/loss potential for the parent company (e.g. adding contribution under highly profitable scenarios, and reducing contribution during periods of stress). Of course, financial organizations do just this. But an integrative risk management system improves coordination and allows for more aggressive investments and increased long term growth. The total integrative risk management technology is now practical as real-time information becomes readily available for all, and as computational costs continue to decline at a dramatic rate. It is now a matter of disseminating the technology.

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