SPHERES AND NORMALS

just another primitive

BASED ON MIT 6.837

slides adapted & project started code translated to Swift by Dion Larson
adapted course materials available for free here
original course materials available for free here

Recap of planes, cameras & ray tracing

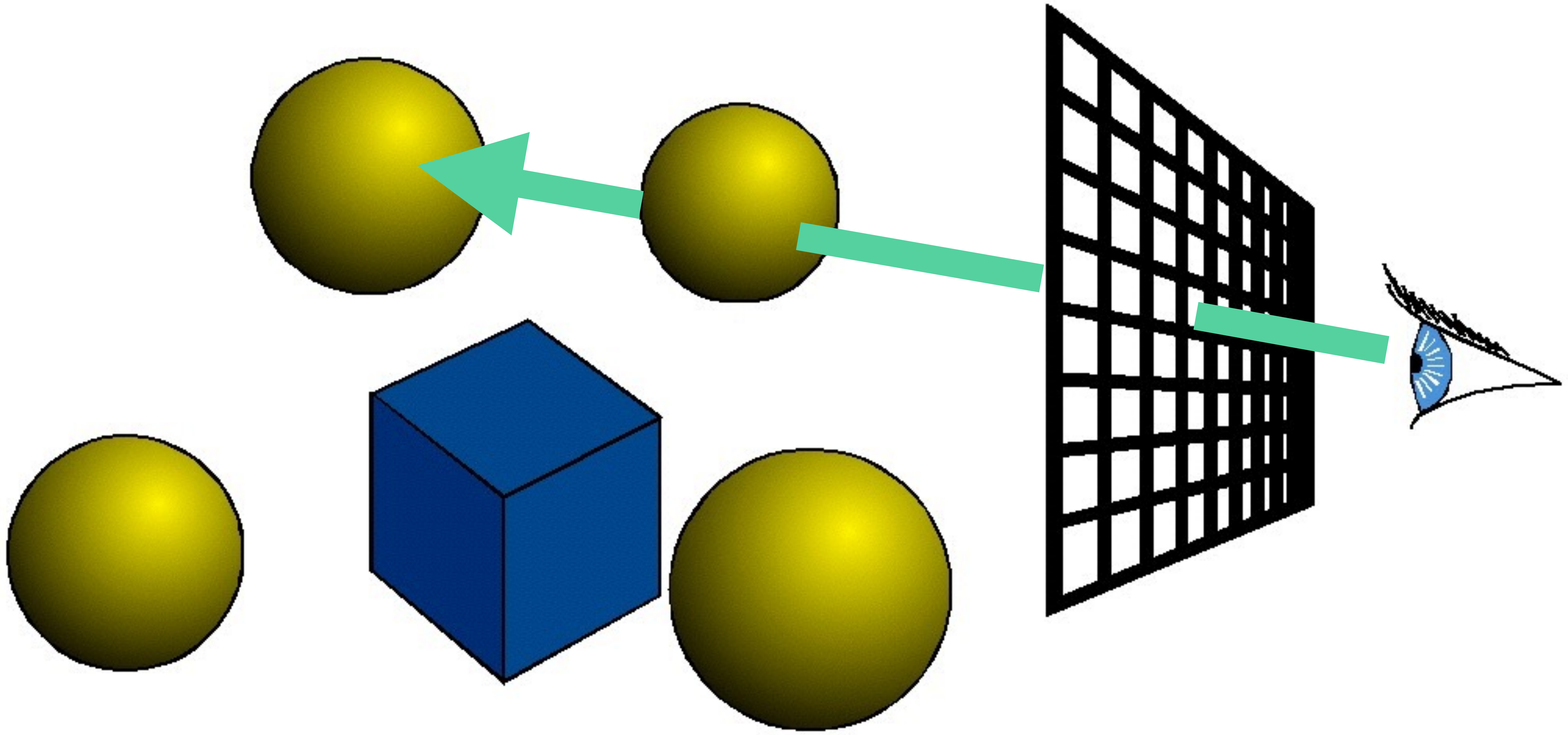
Mathematical toolbox

Ray-sphere intersection

Normals images

Next week

Mob programming (ray tracing loop & planes)



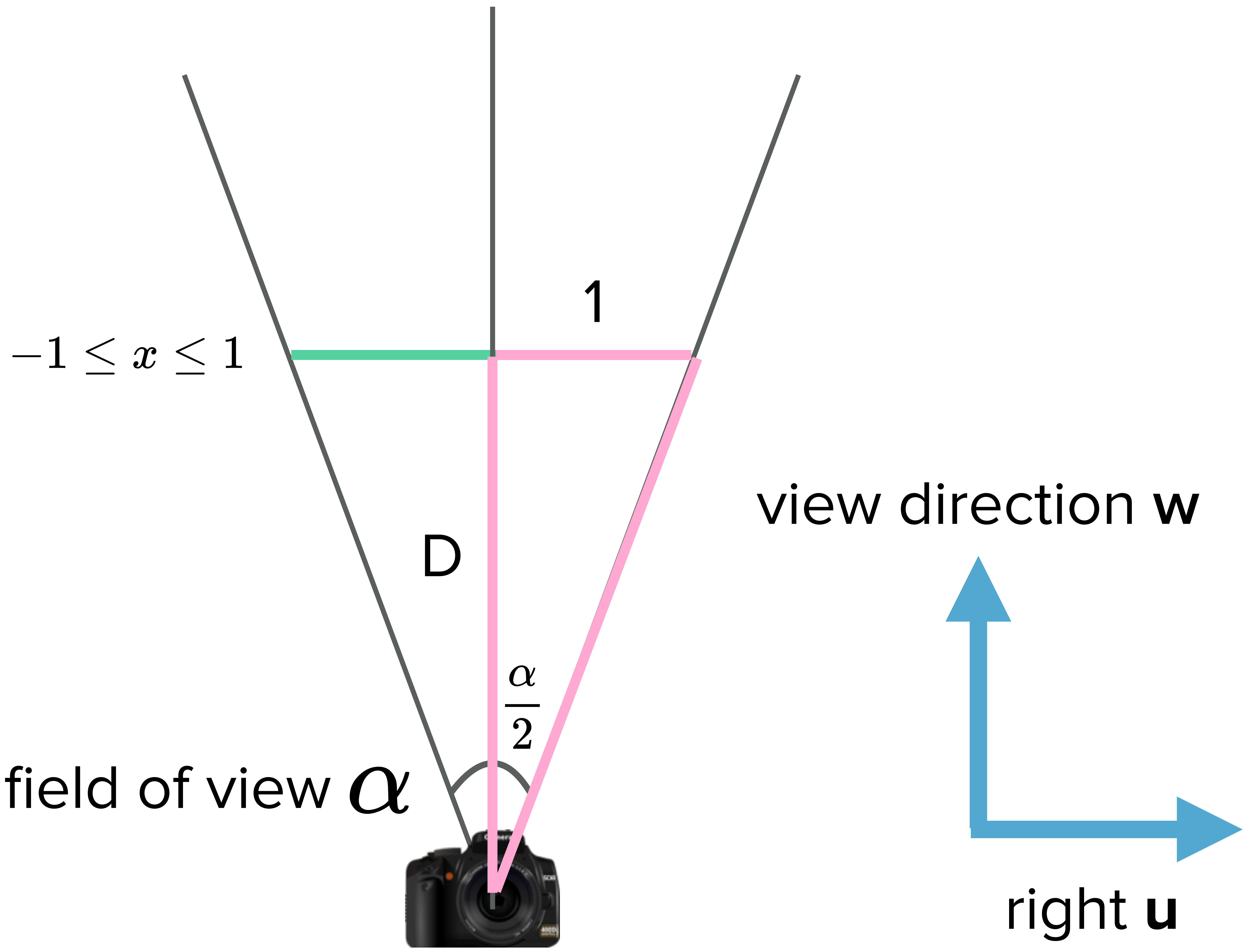
```
for every pixel
  construct ray from eye to pixel
    for every object in the scene
      find intersection t with ray
      save closest t
  shade closest using lights, normal, material
```

RAY GENERATION IN 2D

What is the distance D to the screen so that the normalized coordinates go to 1?

$$\tan \frac{\alpha}{2} = \frac{1}{D}$$

$$D = \frac{1}{\tan \frac{\alpha}{2}}$$



RAY GENERATION IN 2D

Calculate the ray

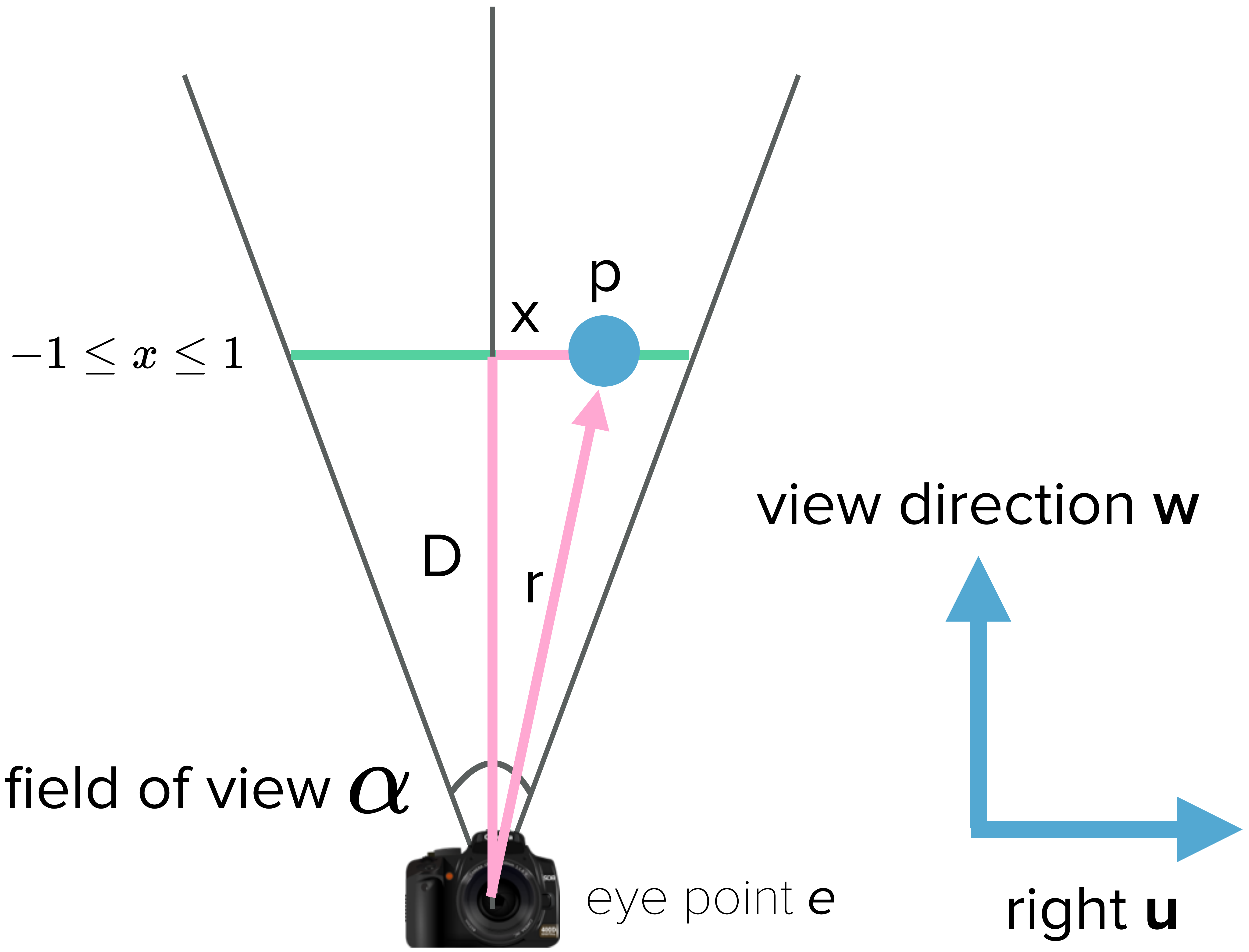
$$r = p - e = (xu, Dw)$$

Normalize it to get direction

$$d = \frac{r}{||r||}$$

Any point on ray can be expressed by

$$P(t) = e + td$$



3D WORKS JUST THE SAME

y is same as **x** but accounts for aspect ratio

$$r = x * u + aspect * y * v + D * w$$

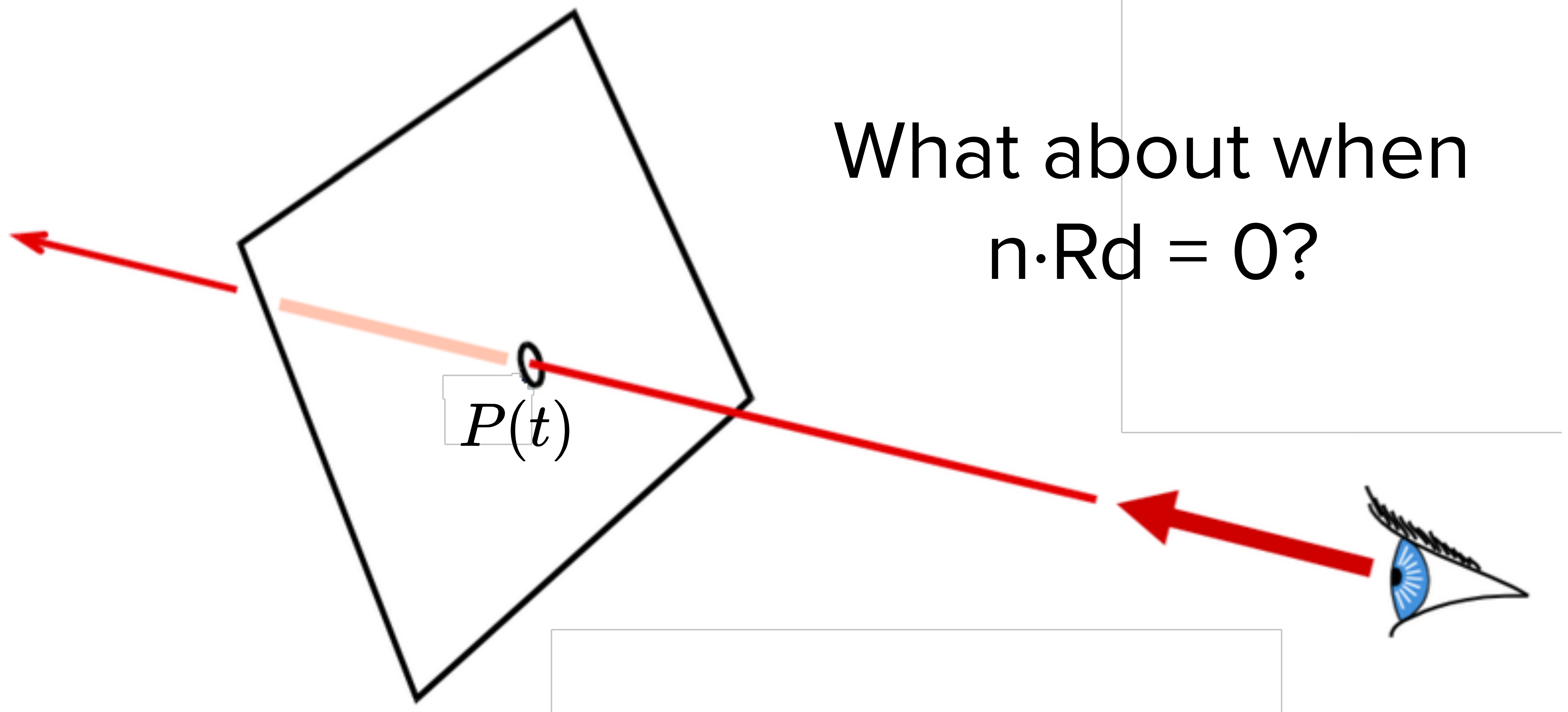
Aspect ratio is for non-square views (16:9, etc)

Allows us to use [-1, 1] for image coordinates

RAY-PLANE INTERSECTION

Intersection happens when both equations satisfied

Insert explicit ray equation into implicit plane equation and solve for *t*



What about when n·Rd = 0?

$$P(t) = R_o + tR_d$$

$$H(P) = n \cdot P + D = 0$$

$$n \cdot (R_o + tR_d) + D = 0$$

$$t = \frac{-(D + n \cdot R_o)}{n \cdot R_d}$$

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NORMALIZING VECTORS

Create a vector with same direction but length of one

$$normalize(A) = \frac{A}{||A||}$$

DOT PRODUCT

Extremely useful for light calculations in shading step

$$\mathbf{A} \cdot \mathbf{B} = \sum_{i=1}^{3} A_i B_i = A_x B_x + A_y B_y + A_z B_z$$

$$\mathbf{A} \cdot \mathbf{B} = \|\mathbf{A}\| \, \|\mathbf{B}\| \cos \theta$$

When perpendicular

$$\mathbf{A} \cdot \mathbf{B} = 0$$

When parallel

$$\mathbf{A} \cdot \mathbf{B} = \|\mathbf{A}\| \, \|\mathbf{B}\|$$

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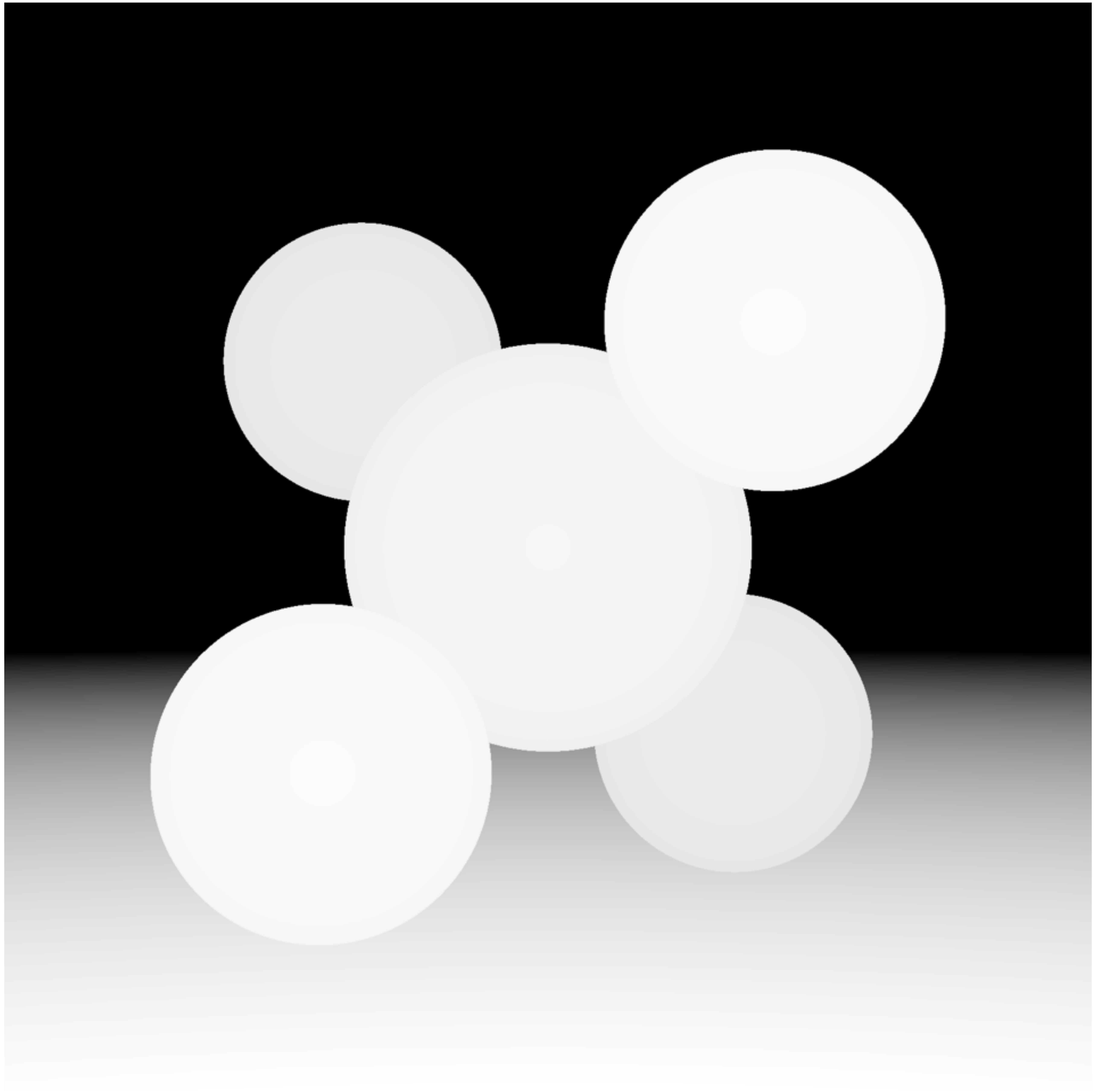
RENDER DISTANCE TO GEOMETRY

Camera representation

Plane intersection

Sphere intersection



SPHERE EQUATION

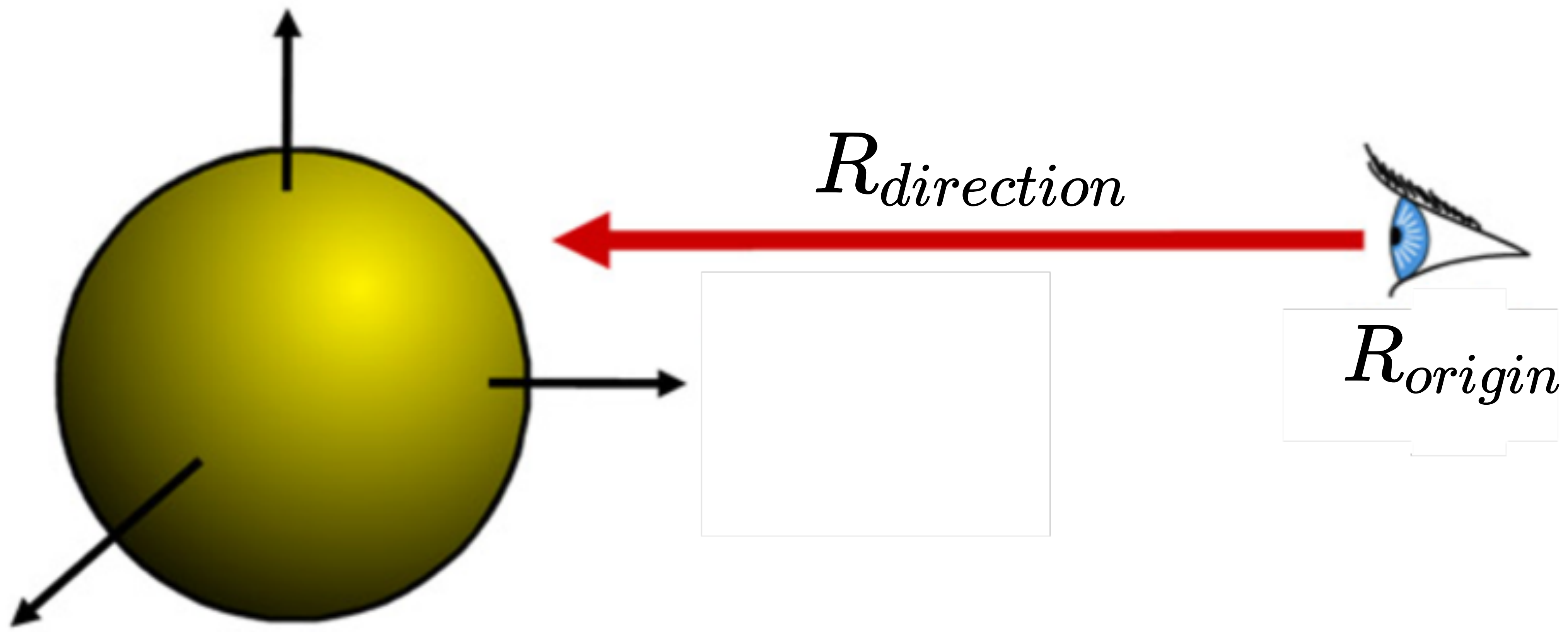
Implicit sphere equation

$$H(P) = ||P||^2 - r^2 = P \cdot P - r^2 = 0$$

Assume sphere is centered at origin

Move the ray's origin instead!

$$R_{origin} = R_{real\ origin} - H_{center}$$



EXPLICIT VS IMPLICIT

Ray equation is explicit $$P(t) = R_o + tR_d$$

Parametric, generates points

Hard to verify point is on ray

Sphere equation is implicit $$H(P) = P \cdot P - r^2 = 0$$

Solution of equation, does not generate points

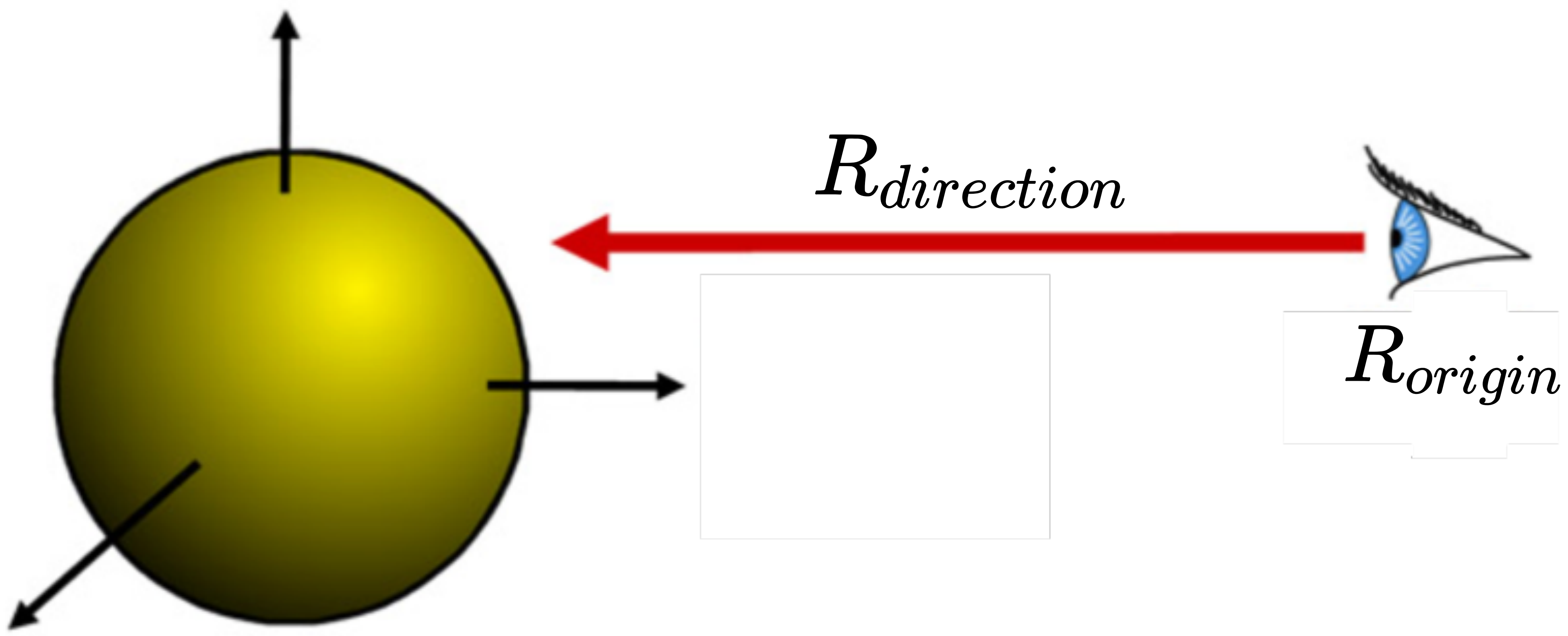
Verifies point is on the plan

RAY-SPHERE INTERSECTION

Insert explicit ray equation into implicit plane equation and solve for *t*

$$P(t) = R_o + tR_d$$

$$H(P) = P \cdot P - r^2 = 0$$



$$(R_o + tR_d) \cdot (R_o + tR_d) - r^2 = 0$$

$$R_d \cdot R_d t^2 + 2R_d \cdot R_o t + R_o \cdot R_o - r^2 = 0$$

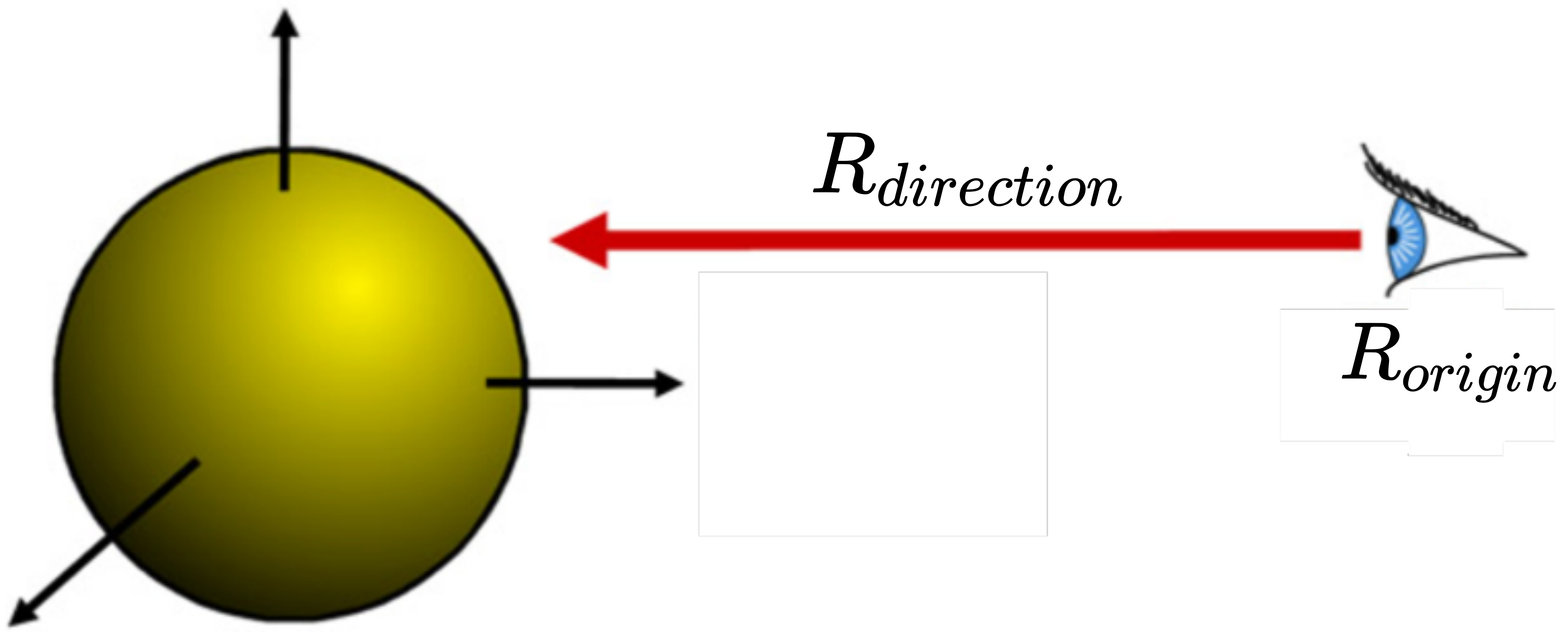
IT'S QUADRATIC!

Quadratic $$at^2 + bt + c = 0$$

$$a = ||R_d||^2$$

$$b = 2R_d \cdot R_o$$

$$c = R_o \cdot R_o - r^2$$



Discriminant $$d = \sqrt{b^2 - 4ac}$$

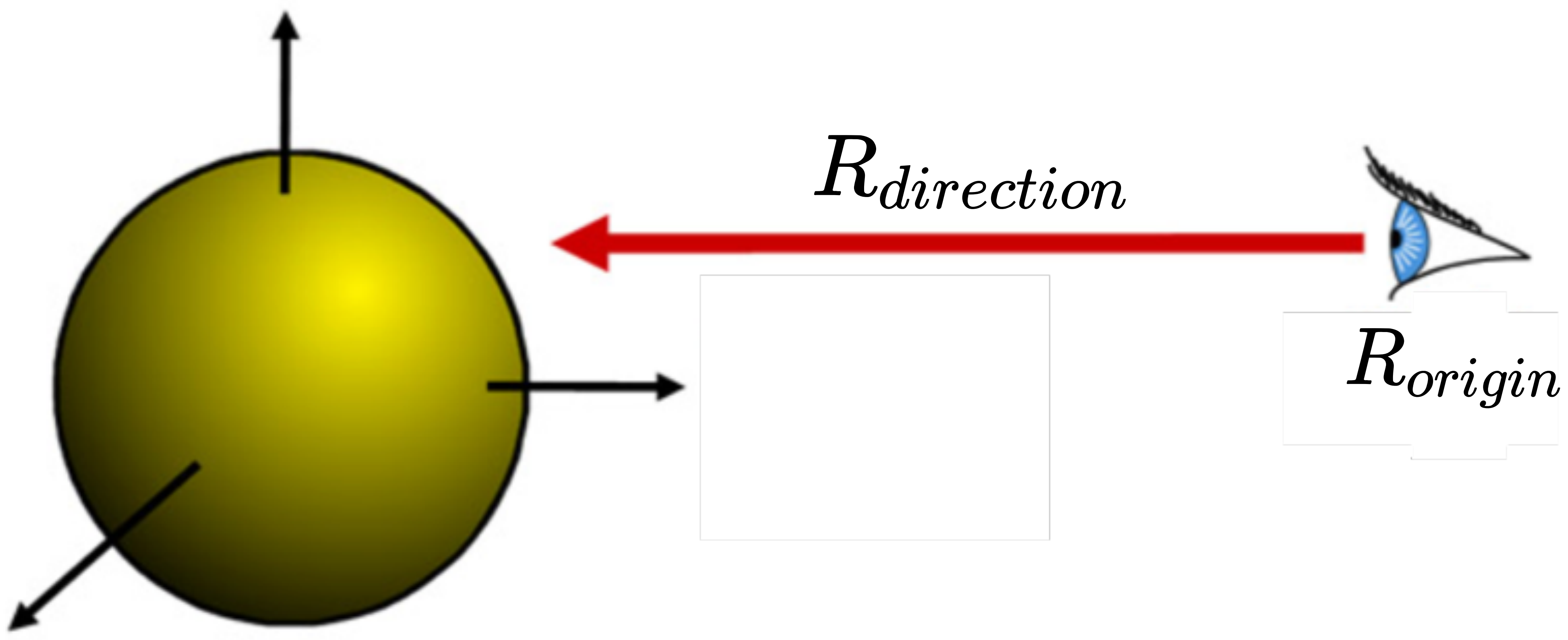
Solutions $$t_{\pm} = \frac{-b \pm d}{2a}$$

RAY-SPHERE INTERSECTION

3 cases depending on sign of $b^2 - 4ac$

What do cases correspond to?

Which *t* should you choose?



SPHERE NORMALS

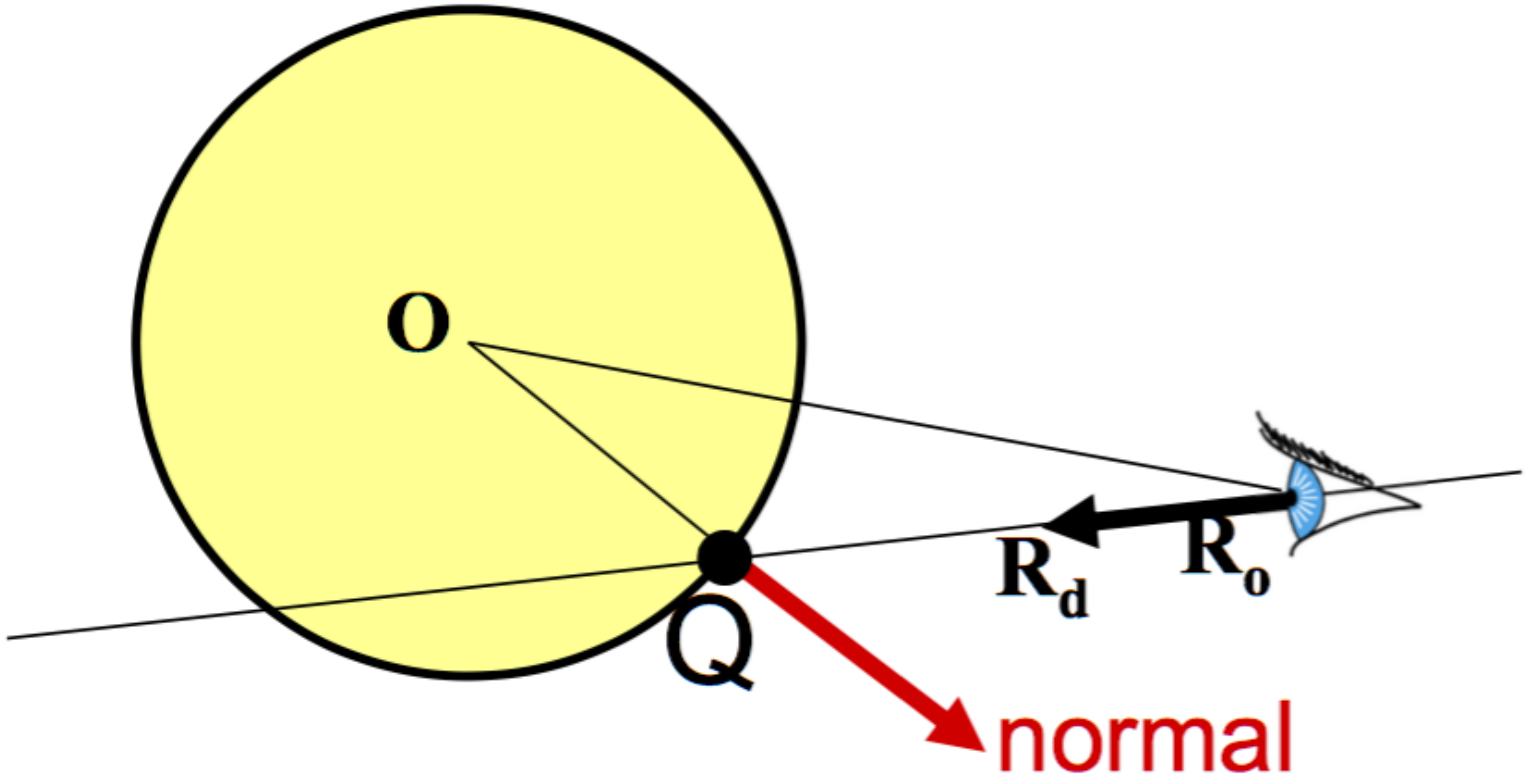
Simply

$$normalize(Q)$$

Where

$$Q = P(t)$$

or the intersection point
(for spheres centered at origin)



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NORMAL IMAGES

Color representation of normals

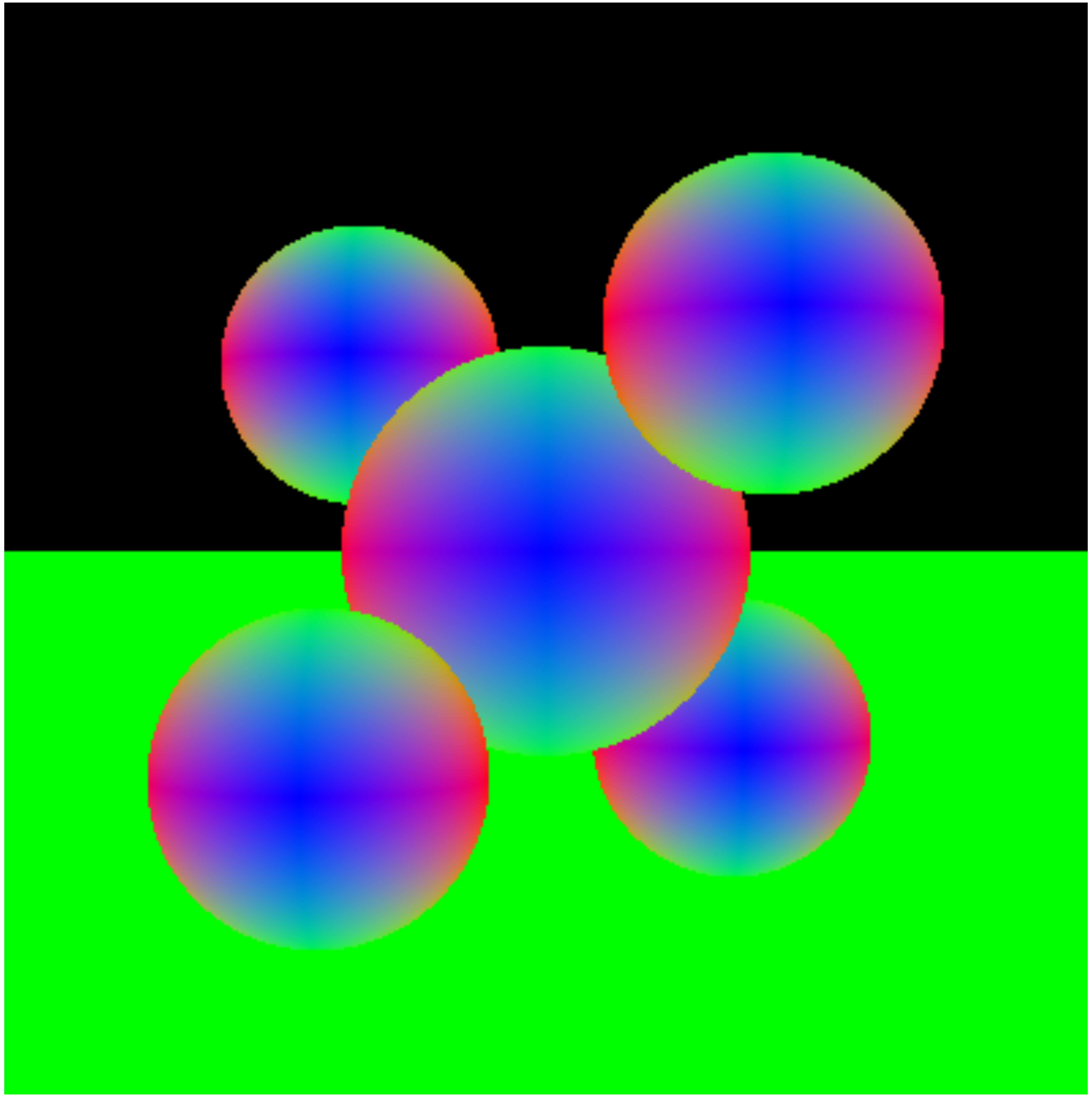
Maps

$$N_x, N_y, N_z$$

to

$$Red, Green, Blue$$

Will make debugging in shading step much easier



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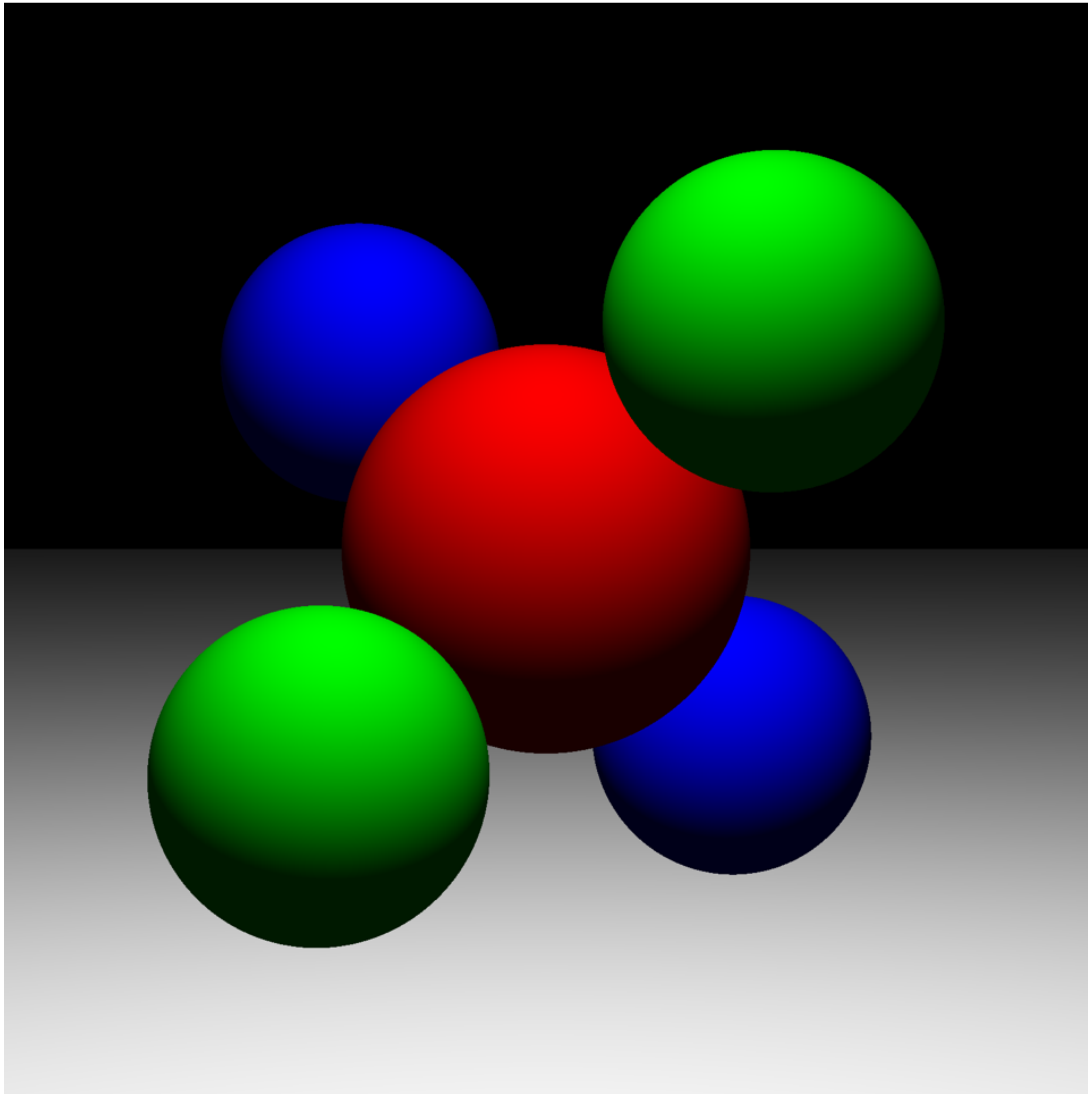
MATERIALS & SHADING

Lights

Diffuse shading



DUE NEXT SESSION

sphere intersection, normals image

reference the guide

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