# Supervision Assignments in MLBI
# Lent Term 2018
# Set 5

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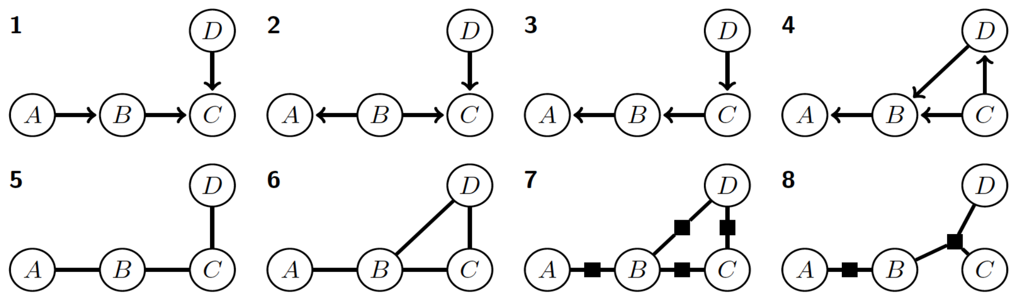
1. (*Conditional independencies and expressiveness of graphical models.*)

   Consider the following graphical models, where each square on edge indicates a factor that depends on the linked variables:




   (a) For graphs 2, 4, 6 and 8, write down all the conditional independence relationships for variable $C$ of the form $C \perp\!\!\!\perp \mathbf{X}, \mathbf{Y}$, where $\mathbf{X}$ and $\mathbf{Y}$ can be sets of other variables.

   (b) Two graphs are equivalent if they express *all* the same marginal and conditional independence relationships between their variables. A graph $G$ is subsumed by graph $H$ if all conditional independence relationships in $H$ are exhibited in $G$. Divide the above 8 graphs into the smallest number of non-overlapping sets of equivalent graphs, and state which of these sets of equivalent graphs are subsumed by one of other sets.

(a) 2. $C \perp\!\!\!\perp A|B$
   4. $C \perp\!\!\!\perp A|B$ (d-separates)
   6. $C \perp\!\!\!\perp A|B$
   8. $C \perp\!\!\!\perp A|B$

(b) 1. $B \perp\!\!\!\perp D|\emptyset; C \perp\!\!\!\perp A|B; B \perp\!\!\!\perp D|\emptyset; A \perp\!\!\!\perp D|B.$
   2. $B \perp\!\!\!\perp D|\emptyset; C \perp\!\!\!\perp A|B; B \perp\!\!\!\perp D|\emptyset; A \perp\!\!\!\perp D|B.$
   3. $B \perp\!\!\!\perp D|C; C \perp\!\!\!\perp A|B; A \perp\!\!\!\perp D|B$ (d-separates); $A \perp\!\!\!\perp D|C$ (d-separates).
   4. $C \perp\!\!\!\perp A|B; D \perp\!\!\!\perp A|B.$ both d-separate.
   5. $C \perp\!\!\!\perp A|B; D \perp\!\!\!\perp A|B; D \perp\!\!\!\perp A|C; B \perp\!\!\!\perp D|C.$
   6. $C \perp\!\!\!\perp A|B; D \perp\!\!\!\perp A|B.$
   7. $C \perp\!\!\!\perp A|B; D \perp\!\!\!\perp A|B.$

8. $C \perp\!\!\!\perp A|B; D \perp\!\!\!\perp A|B$.
Set $A : \{1, 2\}$
Set $B : \{3, 5\}$
Set $C : \{4, 6, 7, 8\}$
All conditional independence relationships in Set $C$ are exhibited in $A$ and $B$, therefor both $A$ and $B$ are subsumed by $C$. All conditional (but not marginal) independence relationships in Set $A$ are exhibited in $B$ and $C$, therefor both $B$ and $C$ are subsumed by $A$. Not all conditional independence relationships in Set $B$ are exhibited by any of the other sets, therefor no set is subsumed by $B$.

2. (*Constructing directed graphs.*)

You are the doctor on the Star Trek Enterprise and you are attempting to use Bayesian methods to help your diagnosis abilities. You would like to represent your knowledge about the following seven binary random variables describing the state of your patients on any given visit

```
M = has the disease microsoftus
L = has the disease linuxitis
A = has the disease applosis
V = is a vulcan (V=0 means "is a human")
H = has high temperature
P = likes pizza
B = has blue spots on face
```

You would like to build a directed graphical model which captures the following background knowledge:

```
Microsoftus is a rare disease.
Linuxitis and applosis are very rare diseases.
There are about four times as many humans as vulcans on the ship.
Vulcans have higher probability of getting microsoftus than humans.
Most vulcans like pizza, some humans like pizza.
Microsoftus usually causes high temperature and blue spots on the face.
Linuxitis always causes high temperature.
Applosis sometimes causes blue spots on the face.
```

(a) Draw a directed graphical model representing the relationships between the above variables. If you need to make any additional assumptions to draw your graph, state clearly what they are

(b) For each variable in your graph, define a conditional probability table for that variable given the settings of its parents. Use the above background knowledge and convert those statements into probability tables which you think reasonably represent them. You will have to make up numbers for what terms like "rare", "most", and "usually" mean.

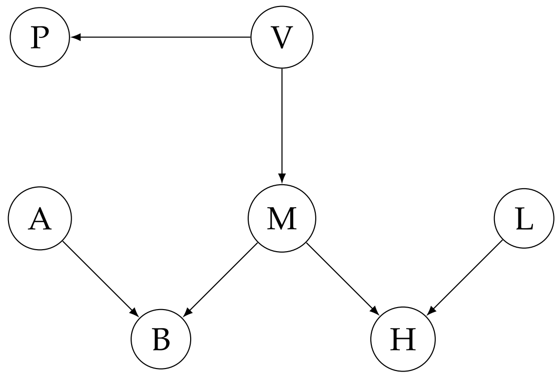
(c) BONUS: Construct a junction tree for your directed graph, drawing out the intermediate factor graph, undirected graph and chordal graph. Use the minimum deficiency search variable elimination order, and show the clique factors on the resulting junction tree.
http://www.gatsby.ucl.ac.uk/teaching/courses/ml1/lect5-handout.pdf

(d) BONUS: Using Shafer-Shenoy propagation on the junction tree, compute the probability

$$P(\texttt{patient is a vulcan} \mid \texttt{patient has blue spots and high temperature})$$

Show each message computed. You may compute the messages by hand, or using `MATLAB` as you wish. Does this probability match your intuitions?

(a) The directed graphical model representing the relationships described in part (a) is shown below. In constructing this, we assume that there is no dependence between the diseases (having one disease does not alter the probability of having another) and that there are no additional relationships between the variables than those described. We also assume that symptoms $B, H, A$ can only occur from having a disease and for not if the person is disease-free.




(b) Here follow the CPTs, one for each of the variables in the system described.

Table 1: CPT for variable V

| $V = 1$ | $V = 0$ |
|---|---|
| 0.2 | 0.8 |

Table 2: CPT for variable A

| $A = 1$ | $A = 0$ |
|---|---|
| 0.05 | 0.95 |

Table 3: CPT for variable L

| $L = 1$ | $L = 0$ |
|---|---|
| 0.05 | 0.95 |

Table 4: CPT for variable P

| | $P = 1$ | $P = 0$ |
|---|---|---|
| $V = 0$ | 0.4 | 0.6 |
| $V = 1$ | 0.8 | 0.2 |

Table 5: CPT for variable M

| | $M=1$ | $M=0$ |
|---|---|---|
| $V=0$ | $0.08\dot{3}$ | $0.91\dot{6}$ |
| $V=1$ | $0.1\dot{6}$ | $0.8\dot{3}$ |

Table 6: CPT for variable B

| | $B=1$ | $B=0$ |
|---|---|---|
| $A=0, M=0$ | 0 | 1 |
| $A=0, M=1$ | 0.7 | 0.3 |
| $A=1, M=0$ | 0.4 | 0.6 |
| $A=0, M=1$ | 0.85 | 0.15 |

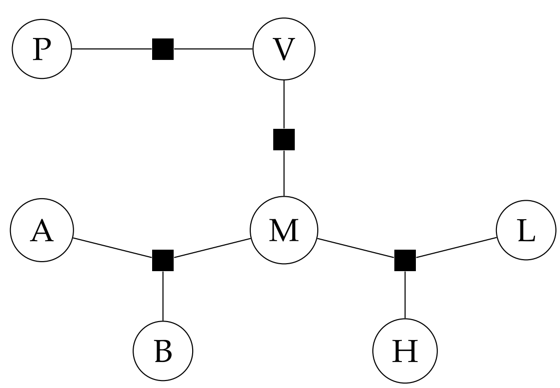
Table 7: CPT for variable H

| | $H=1$ | $H=0$ |
|---|---|---|
| $L=0, M=0$ | 0 | 1 |
| $L=0, M=1$ | 0.7 | 0.3 |
| $L=1, M=0$ | 1 | 0 |
| $L=1, M=1$ | 1 | 0 |

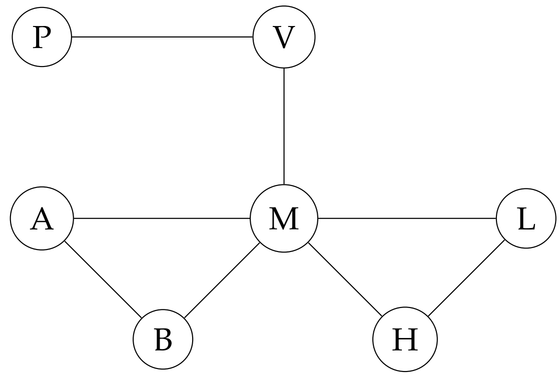
Care has been taken to ensure that even though some numbers had to be made up, the relationships between variables under various dependencies are consistent.

(c) We attempt to turn the DAG from part (a) into a junction tree, by first turning it into a factor graph and then an undirected graph (moralising) before grouping together maximal cliques and breaking off minimum weight separators to form a junction tree.

(i) Factor graph

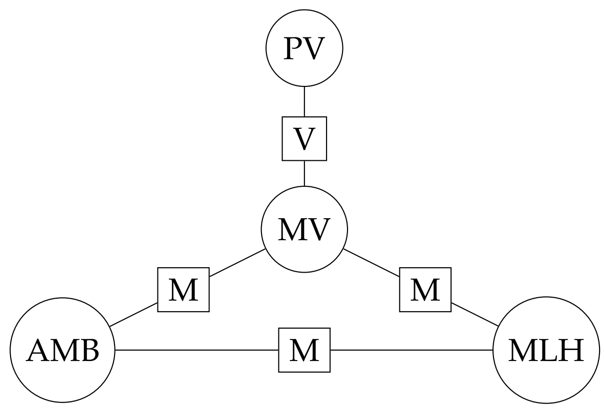



(ii) Undirected Graph (Moralising)




As you can see this is already a triangulated graph, so there is no need to perform variable elimination etc. to get a chordal graph from which to build the junction tree — this is already suitable.

(iii) Junction Tree




and by breaking one of the $M$ separators (between $MV$, $MLH$) we end up with a straight tree as below: