Supervision Assignments in MLBI
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Set 2

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1. *(SVMs)*

   Assume that the set $S = \{(\mathbf{x}_i, y_i)\}_{i=1}^m \subset \mathbb{R}^2 \times \{-1, 1\}$ of binary examples is strictly linearly separable by a line going trough the origin, that is, there exists $\mathbf{w} \in \mathbb{R}^2$ such that the linear function $f(x) = \mathbf{w}^T\mathbf{x}, \mathbf{x} \in \mathbb{R}^2$ has the property that $y_i f(\mathbf{x}_i) > 0$ for every $i = 1, ..., m$. In this case, a linear separable SVM computes the parameters w by solving the optimisation problem:

$$P1: \quad \min_{\mathbf{w}\in\mathbb{R}^2}\{\frac{1}{2}\mathbf{w}^T\mathbf{w} : y_i\mathbf{w}^T\mathbf{x}_i \geq 1, i = 1, ..., m\}. \tag{1}$$

   (a) Show that the vector $\mathbf{w}$ solving problem P1 has the form $\mathbf{w} = \sum_{i=1}^m c_i y_i \mathbf{x}_i$ where $c_1, ..., c_m$ are some nonegative coefficients.

   (b) Show that the coefficients $c_1, ..., c_m$ in the above formula solve the optimization problem

$$P2: \quad \max\{-\frac{1}{2}\sum_{i,j=1}^m c_i c_j y_i y_j \mathbf{x}_i^T \mathbf{x}_j + \sum_{i=1}^m c_i : c_j \geq 0, j = 1, ..., m\}. \tag{2}$$

   (c) Argue that, if $(\hat{c}_1, ..., \hat{c}_m)$ solves problem P2 and $\hat{\mathbf{w}}$ solves problem P1, then $\hat{\mathbf{w}}^T\hat{\mathbf{w}} = \sum_{i=1}^m \hat{c}_i$.