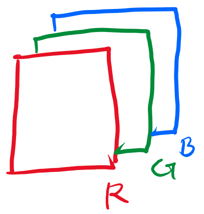
Lecture 14: Convolutional Neural Network

09 March 2024 11:01

Consider a coloured image. This image has three different channels (R, G, B)

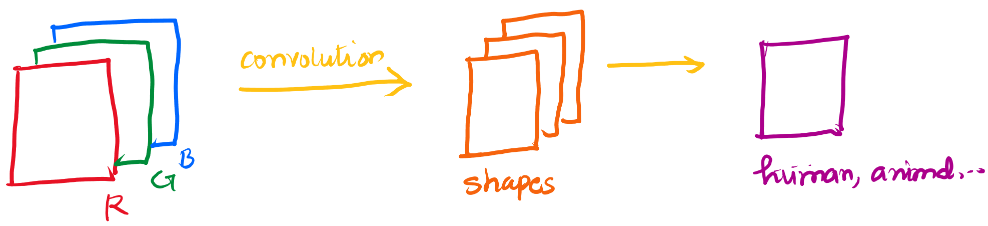


This is given by $1024 \times 1024 \times 3$ matrix.

For a NN using this image as input, the input layer will have $\sim 3\times10^6$ input nodes.
If hidden layer has $\sim 10^3$ nodes, $w \sim 3\times10^9$!

Convolution is used to reduce the image size and retain only useful information, without compromising the information content.

For an image we pass each channel through a convolution layer:



Convolution:

For any two functions $f(x)$, $g(x)$, the convolution is given by:

$$(f * g)(x) = \int_{-\infty}^{\infty} f(t)\, g(x-t)\, dt$$

Refer Wiki page of convolution:
https://en.wikipedia.org/wiki/Convolution

Consider a 5x5 grayscal image (we've nos b/w 0 to 255)
(Black) to (white)

image

1	20	20	20	1
20	1	1	1	20
20	1	1	1	20
1	20	1	20	1
1	1	20	1	1

$n \times n$

$*$

filter

1	0	-1
1	0	-1
1	0	-1

$f \times f$

$=$

filtered image

21	0	-21
38	0	-38
0	0	0

$(n-f+1)(n-f+1)$

This filter acts as a vertical edge detector.

If the filter is: $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$, it acts as a horizontal edge detector.

Filters are used to find shapes or useful information via convolution.

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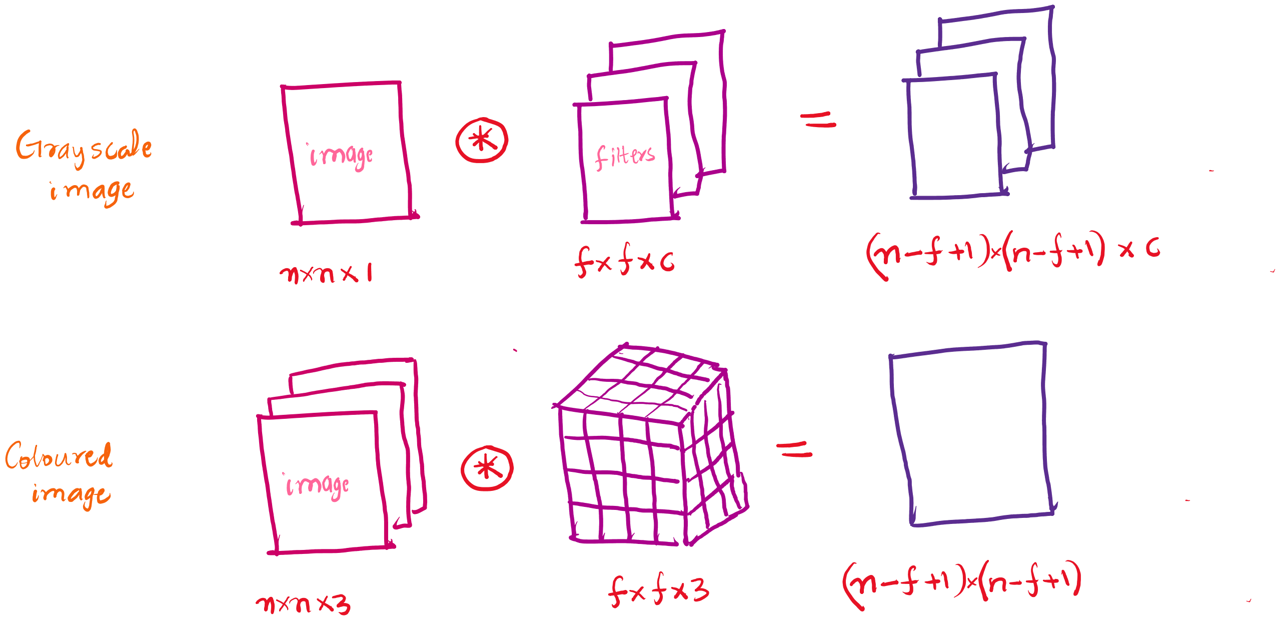
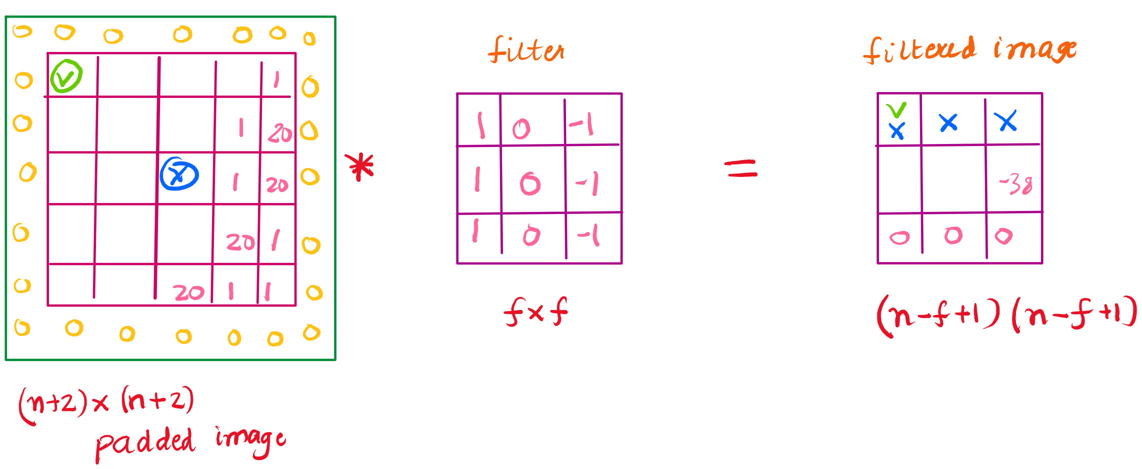


How will we find these filters? We need to "train" the weights of filters

Padding

Consider our example of convolution of 5×5 grayscale image with 3×3 filter.



"VALID Convolution" – without padding.

"SAME Convolution" – Padding such that final (filtered image size is same as original, given the filter

eg: [6×6] with padding and 3×3 filter gives [6×6] output

ie, $n' = n + 2p$, $(n' - f + 1) = n$

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$\Rightarrow p = \frac{f-1}{2}$, f is odd

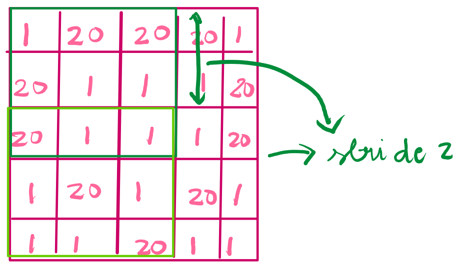
We use padding so that we don't loose information around the borders of the image.

Stride:

If we use a filter with a stride, gives a smaller image. For all that we discussed till now, stride is 1.

$$(5 \times 5) \circledast (3 \times 3) = (2 \times 2)$$

stride 2.

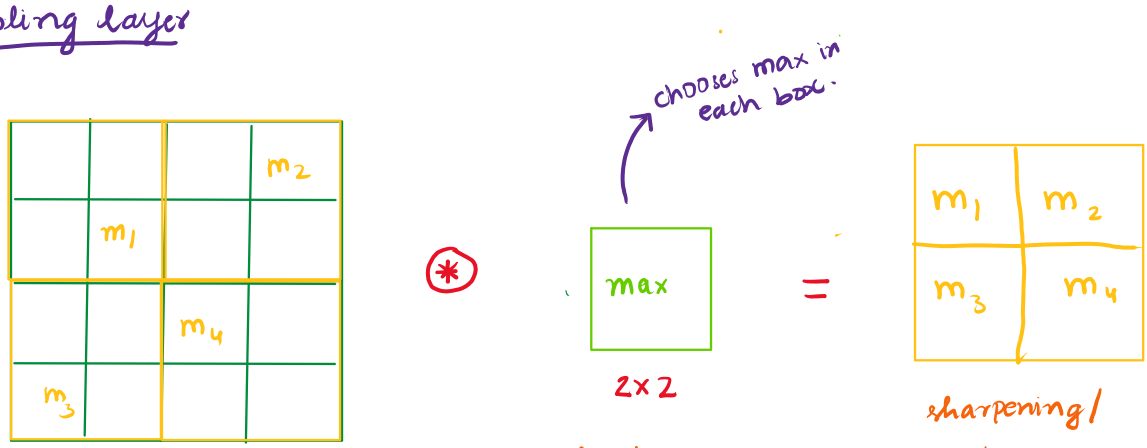


Stride ignores the part of the image where convolution can't be entirely within image.

$$\underset{n \times n}{\text{Image}} \circledast \underset{f \times f}{\text{filter}} = \text{filtered image } \left\lfloor \frac{n-f}{s} + 1 \right\rfloor \times \left\lfloor \frac{n-f}{s} + 1 \right\rfloor$$

stride = s.

Max pooling layer



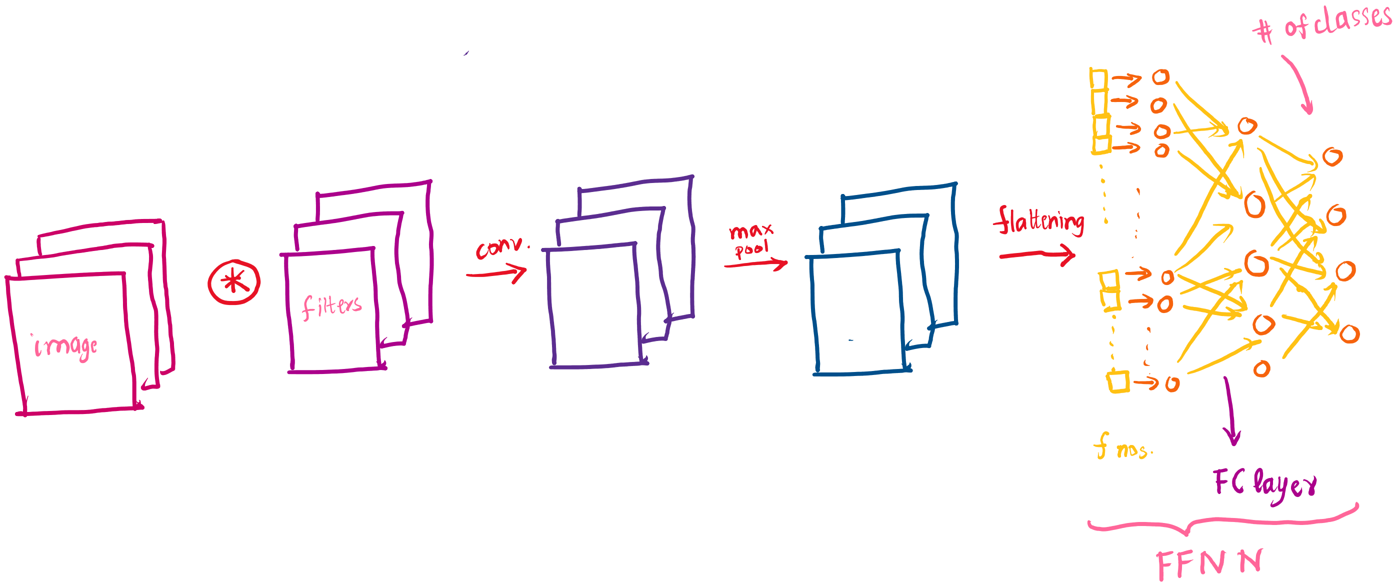
4×4 $\circledast$ 2×2 =

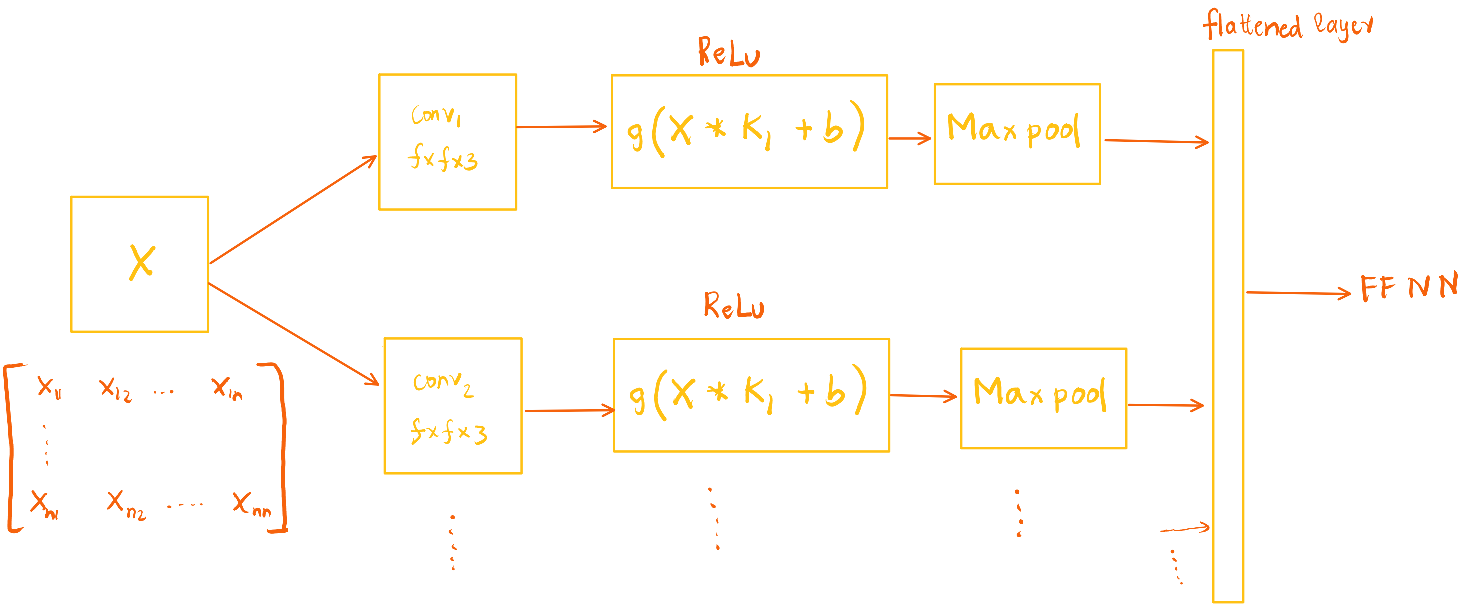
stride = 2 → typically strides = size of the filter.

sharpening / contrast increase

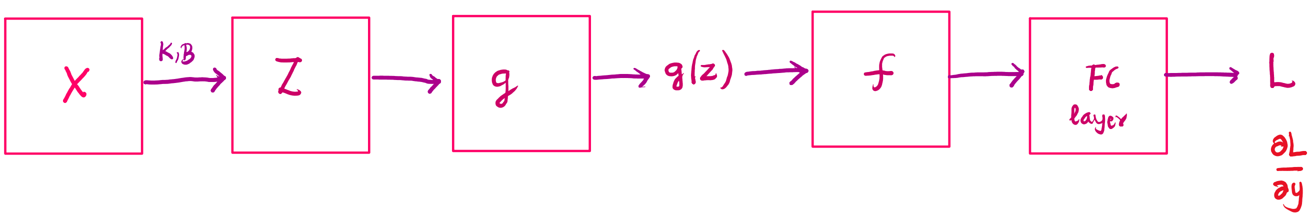
Other alternatives: Average pooling.

> Convolutional layers extract features from images.
> How will we use these features to classify?
> By using a fully connected layer (FC layer) : FFNN.





Backpropagation in CNN:



$$K^{(t+1)} \leftarrow K^{(t)} - \eta \left.\frac{\partial L}{\partial K}\right|_t$$

$$B^{(t+1)} \leftarrow B^{(t)} - \eta \left.\frac{\partial L}{\partial B}\right|_t$$

Let $X = \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix}$

$K = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}$ $\quad\therefore\quad X \circledast K + B = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$

$$K = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \quad \therefore \quad X \circledast K + B = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

such that $Z_{11} = X_{11}K_{11} + X_{12}K_{12} + X_{21}K_{21} + X_{22}K_{22} + B$

$\vdots$

already known via back propagation

Consider $$\frac{\partial L}{\partial K_{mn}} = \sum_{i,j} \frac{\partial L}{\partial Z_{ij}} \cdot \frac{\partial Z_{ij}}{\partial K_{mn}}$$

Eg: $$\frac{\partial L}{\partial K_{11}} = \frac{\partial L}{\partial Z_{11}} \cdot \underbrace{\frac{\partial Z_{11}}{\partial K_{11}}}_{x_{11}} + \frac{\partial L}{\partial Z_{12}} \cdot \underbrace{\frac{\partial Z_{12}}{\partial K_{11}}}_{x_{12}} + \frac{\partial L}{\partial Z_{21}} \underbrace{\frac{\partial Z_{21}}{\partial K_{11}}}_{x_{21}} + \frac{\partial L}{\partial Z_{22}} \underbrace{\frac{\partial Z_{22}}{\partial K_{11}}}_{x_{22}}$$ (See the expression of Z_{11} above)

Verify that $\frac{\partial L}{\partial K} = X \circledast \frac{\partial L}{\partial Z}$.

Now, $$\frac{\partial L}{\partial B} = \sum_{i,j} \frac{\partial L}{\partial Z_{i,j}}$$

$$\frac{\partial L}{\partial X_{mn}} = \sum_{i,j} \frac{\partial L}{\partial Z_{i,j}} \cdot \frac{\partial Z_{ij}}{\partial X_{mn}}$$

Verify that $\frac{\partial L}{\partial X} = \left(\text{zero padded } \frac{\partial L}{\partial Z}\right) \circledast \tilde{K}$

where, $\tilde{K} = K$ rotated by 180°
$$= \begin{bmatrix} K_{22} & K_{21} \\ K_{12} & K_{11} \end{bmatrix}$$