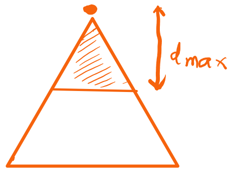
Backward Induction

Limitation: Computation heavy for large games eg: Chess, Go,...

So we adopt some speedup methods:

1. Depth-limited Search:



$$U_{agent}(s,d) = \begin{cases} utility(s) & \text{if } isEnd(s) \\ eval(s) & \text{if } d=0 \\ \max_{a\in actions(s)} U_{agent}(succ(s,a), d-1), & \text{if } player(s) = agent \\ \min_{a\in actions(s)} U_{agent}(succ(s,a), d-1), & \text{if } player(s) = opponent \end{cases}$$

eval(s) is a domain specific function denoting the possible utility to agent.

Eg: In chess, eval(s) = army + mobility + King-safety + ...

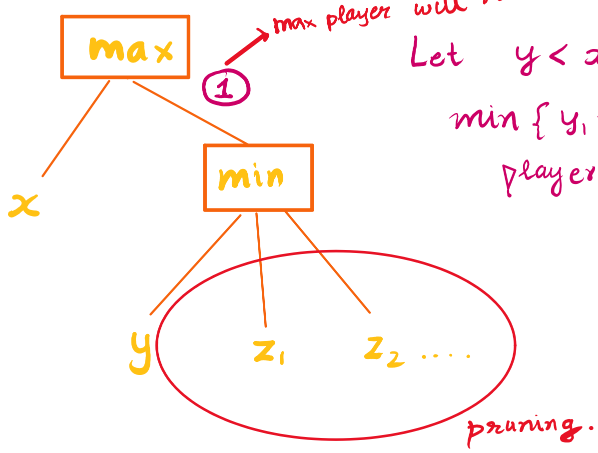
where army $= 10^{100}(K-K') + 9(Q-Q') + 5(R-R') + \ldots$ →No. of kings you have − opponent have.

mobility $= c \times \#$ of (legal moves − legal moves')

Co-efficients indicate importance of each piece. King has ∞ importance given by 10^{100}

2. Pruning (α-β pruning)

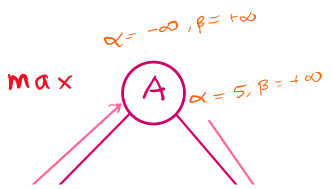
Consider a min-max scheme as below:



max player will never pick this.

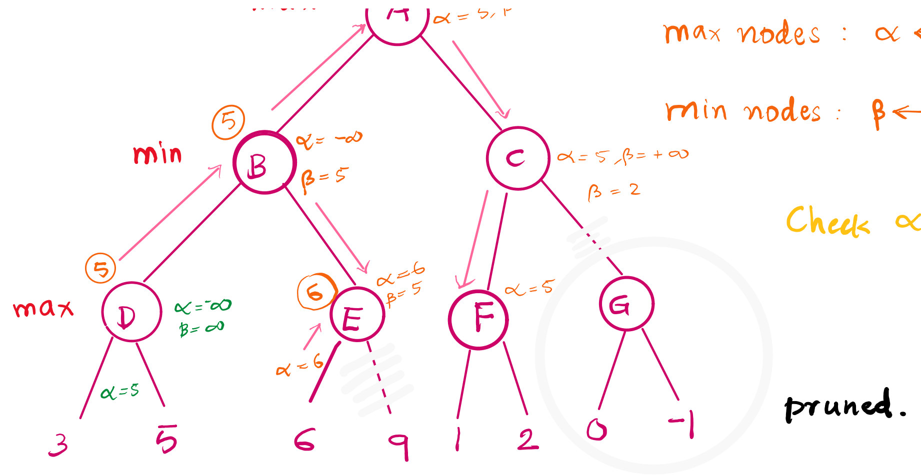
Let $y < x$. Now, min player will choose $\min\{y, z_1, z_2 \ldots\}$. So since $y < x$, max player won't choose (1)

Example:



$\alpha = -\infty \quad \beta = +\infty$

max nodes : $\alpha \leftarrow \max\{\alpha, \text{value encountered}\}$



max nodes : $\alpha \leftarrow \max\{\alpha, \text{value encountered}\}$

min nodes : $\beta \leftarrow \min\{\beta, \text{value encountered}\}$

Check $\alpha \geq \beta$?

Simultaneous move Games

Consider the game of football with Goalkeeper and a shooter:

Shooter \ Goalkeeper	L	C	R
L	1	1	1
C	1	-1	1
R	1	1	-1

Assume the shooter is an awesome left side shooter

1 : goal!
-1 : save!

These types of games are called matrix games:

Two player, zero sum, simultaneous move games are matrix games.

Shooter always chooses left (as it is his best). But if shooter chooses L, whatever goalkeeper chooses doesn't help him save.

(L, L) is a simultaneous move equilibrium.

Equilibrium: a tuple of action from which no player gains by an unilateral deviation (strictly greater than current utility)

unilateral deviation → other players remain at the state of tuple of actions and only concerned player is moving

For a normal leftside shooter:

Shooter \ Goalkeeper	L	C	R
L	-1	1	1
C	1	-1	1

Observe that there is no equilibrium in this case

Shooter	C	1	−1	1
	R	1	1	−1

Lets compare these two cases:

	L	C	R	min
L	1	1	1	1
C	1	−1	1	−1
R	1	1	−1	−1
max:	1	1	1	

max min = 1

min max = 1

For equilibrium observe that

min-max = max-min !

Saddle-points!

So our objective is to find saddle points exist / not?

	L	C	R	min
L	−1	1	1	−1
C	1	−1	1	−1
R	1	1	−1	−1
max:	1	1	1	

max-min = −1

min-max = 1

$$\max_{s_1} \min_{s_2} u(s_1, s_2) = \underline{v}$$

$$\min_{s_2} \max_{s_1} u(s_1, s_2) = \overline{v}$$

We've:

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$$u(s_1,s_2) \geq \min_{s_2} u(s_1,s_2) \quad \forall s_1, s_2.$$

Let $\min_{s_2} u(s_1^*,s_2) = \max_{s_1} \underbrace{\min_{s_2} u(s_1,s_2)}_{f(s_1)}$

So,
$$\max_{s_1} u(s_1,s_2) \geq u(s_1^*,s_2) \geq \min_{s_2} u(s_1^*,s_2)$$

$$\min_{s_2} \max_{s_1} u(s_1,s_2) \geq \max_{s_1} \min_{s_2} u(s_1,s_2)$$

$$\Rightarrow \overline{v} \geq \underline{v}$$