In Last Lecture we'd seen following Voting Rules:

1. Plurality
2. Borda's rule
3. STV
4. Condorcet consistency (If a candidate beats all other candidates in pairwise election, that candidate should win)
5. Copelant voting.

Q: What are the other desirable properties? **Unanimity!**

Defn: A voting rule f is unanimous if for every profile P such that $P_1(1) = P_2(1) = \dots P_n(1) = a$ (say) it holds that $f(P) = a$.

Observe that all voting discussed above are all unanimous.

Manipulable

If $f(P_i, P_{-i}) = b$ and ith player want a to win, it adopts some P_i' such that $f(P_i', P_{-i}) = a$

This is called "manipulation".

P_i	P_i'	P_{-i}
⋮	⋮	⋮
	b	
a	⋮	⋮
⋮	a	
b	⋮	
⋮	⋮	

Defn: A voting rule f is manipulable if $\exists i \in N$, profile $P = (P_i, P_{-i})$ such that $f(P_i', P_{-i})$ is more preferred by i under P_i then $f(P_i, P_{-i})$ for some P_i'.

A voting rule, f is non-manipulable if f does not satisfy the above

We derive our voting rule to be non-manipulable

Are the above rules manipulable?

1. Plurality:

Ex:

1	2	3	4	5	6	P_5'
a	a	b	b	c	d	b

	b	a	c
Initially tie. But if 5 chooses P_5', it can manipulate b to win.	a	b	a
	d	c	d

2. Borda's rule:

Ex:

1	2	3	4	5	6	P_5'
a	a	b	b	c	d	b
b	b	a	a	b	a	c
c	d	c	c	a	b	d
d	c	d	d	d	c	a

Borda's score:

$a \to 13$
$b \to 13$
$c \to 6$
$d \to 4$

Here again we've a tie and 5 adopt P_5' to manipulate b to win

3. STV is also manipulable HW

4. Copeland:

			P_2'
a	b	c	c
b	c	a	b
c	a	b	a

Copeland Score:

$a \to 1$
$b \to 1$
$c \to 1$

WLOG say a is copeland winner. If 2 adopts P_2', copeland winner becomes 'c'.

So all voting rules we considered are manipulable

Dictatorial: $\exists d \in N$ such that $f(P_d, P_{-d}) = P_d(1)$

This mechanism is unanimous and non-manipulable.

Gibbard - Satterthwaite:

If voters can have all possible strict preferences over the candidates and $|A| \geq 3$, then every unanimous and non-manipulable voting rule is a dictatorship

Stable Matchings

Refer slides in course webpage.

How to find a stable matching algorithmically?

Gale-Shapley Deferred acceptance

$w_1 > w_4 > w_2 > w_3$ m_1

$w_3 > w_2 > w_4 > w_1$ m_2

$w_1 > w_2 > w_3 > w_4$ m_3

$w_2 > w_1 > w_4 > w_3$ m_4

w_1 $m_2 > m_1 > m_4 > m_3$

w_2 $m_1 > m_2 > m_3 > m_4$

w_3 $m_3 > m_1 > m_2 > m_4$

w_4 $m_4 > m_2 > m_1 > m_3$

Men-proposing Version:

Round 1: Each unmatched man proposes to the most preferred woman who hasn't rejected him

In given ex:

$m_1 \longrightarrow w_1$

$m_2 \longrightarrow w_3$

$m_3 \longrightarrow w_1$

$m_4 \longrightarrow w_2$

Round 2: Each woman tentatively accepts the most preferred man from existing proposals.

Tentative match after Round 2:

$m_1 — w_1$

$m_2 — w_3$

m_3 — rejected :(

$m_4 — w_2$

This keeps on repeating. After next round:

$m_3 \rightarrow w_2$.

After this, w_2 accepts m_3 and rejects m_4

So: $m_3 - m_2$, m_4 - empty

After next round:

$m_4 \rightarrow w_1$.

w_1 keeps m_1 itself. m_4 - empty

In next round: $m_4 \longrightarrow w_4$

w_4 hasn't got any match till now so accepts.

Hence, at the end of this round, everyone gets matched.

function mpStableMatching: M: set of men W: set of women such that $|M| = |W| = n$

- Initialize all $m \in M$, $w \in W$ as free
- while $\exists m$ who is free:
  - m proposes w who is most preffered by m and has not rejected m
  - If w is free: (m,w) is a tentative match
  - If some (m′,w) already exists:
    - * If w prefers m over m′: (m,w) tentative match, m′ free
    - else: m free.
- all tentative matchings are made final.

Claim: DA algorithm converges in polynomial time.

- Every man makes $\leq n$ proposals

- Every man makes $\leq n$ proposals
- There are n men, so there could be atmost n^2 proposals

$\therefore O(n^2)$ algorithm.

Claim: DA algorithms always return a perfect matching (ie, all vertices are matched)

- No women is matched to more than one man
- Every women is either tentatively matched (OR) gets multiple proposals and keeps one.
- Once a woman is tentatively matched, never unmatched again
- DA runs till all the men are matched

Claim: DA gives a pairwise stable matching:

Let $\mu_p : M \to W$ be matching f^n such that

$\mu_p(m) =$ women to which m is matched to
$\mu_p(w) =$ man to which w is matched to

A matching is pairwise unstable if
$\exists P, m, w, m', w'$ such that:

$P_{w'}: m > m'$
$P_m : w' > w$

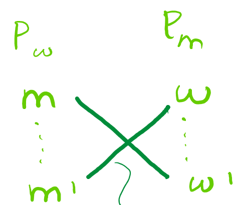
$$\left.\begin{array}{l} \mu_p(m) = w \\ \mu_p(m') = w' \end{array}\right\} \text{ and } \begin{array}{l} w' P_m w \\ m \, P_{w'} \, m' \end{array}$$

w' is more prefered by m

We call (m, w') a blocking pair.

Proof: Suppose DA gives a pairwise unstable matching.
$\therefore$ $\exists P$, blocking pair (m, w)

This happens because m would've been rejected by w', which happens only when w' was tentatively matched with some one above m. This leads to contradiction



matching given by DA

as w ends up getting m' by DA