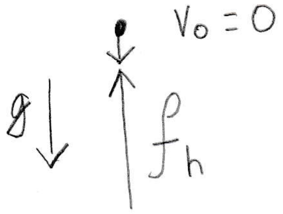


$$\Sigma F = m\frac{dV}{dt}$$
$$mg = m\frac{dV}{dt}$$
$$V(t) = gt$$
$$y(t) = \frac{1}{2}gt^2$$

$f_h \equiv$ gaya hambat udara

$= -kV$, $K \equiv$ konst. hambat

$$\Sigma F = m\frac{dV}{dt}$$

$$mg - kV = m\frac{dV}{dt} \longrightarrow \frac{dV}{dt} = g - \frac{k}{m}V \quad \times \frac{dt}{g-\frac{k}{m}V}$$

$$\int_0^V \frac{dV}{g-\frac{k}{m}V} = \int_0^t dt$$

$\rightarrow u \equiv g - \frac{k}{m}V \rightarrow dU = -\frac{k}{m}dV$

$$-\frac{m}{k}\int\frac{dU}{U} = t$$

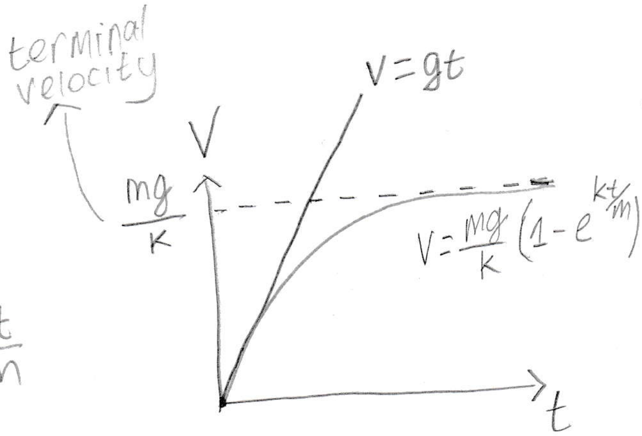
$$\ln\left|g-\frac{k}{m}V\right|\Big|_{V=0}^{V} = -\frac{kt}{m}$$

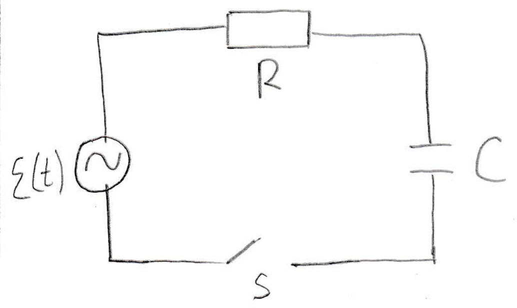
$$\ln\left(g-\frac{k}{m}V\right) - \ln g = -\frac{kt}{m}$$

$$\ln\left[1-\frac{k}{mg}V\right] = -\frac{kt}{m}$$

$$1-\frac{k}{mg}V = e^{-\frac{kt}{m}}$$

$$\boxed{V(t) = \frac{mg}{k}\left(1-e^{kt/m}\right)}$$





$$\mathcal{E}(t) = \mathcal{E}_0 \cos(\omega t)$$

$$iR + \frac{q}{C} = \mathcal{E}(t)$$

$$\frac{dq}{dt} + \frac{1}{RC} q = \frac{\mathcal{E}_0}{R} \cos(\omega t)$$

↳ Tidak bisa diseparasi

Faktor Integrasi

PDB orde-1 linear : $\boxed{a_1(x) y' + a_0(x) y = b(x)}$, $y' \equiv \frac{dy}{dx}$, $a_1 \neq 0$

$\times \frac{1}{a_1}$

solusi : $y(x)$ $\boxed{y' + P(x) y = Q(x)}$, $P(x) \equiv \frac{a_0(x)}{a_1(x)}$, $Q(x) \equiv \frac{b(x)}{a_1(x)}$

Tinjau $Q = 0$:

$$y' + Py = 0 \longrightarrow y' = -Py \quad \times \frac{dx}{y}$$

$$\int \frac{dy}{y} = -\int P\,dx$$

$$\ln|y| = -\int P\,dx + C$$

$$y = e^{-\int P\,dx + C} = e^{-\int P\,dx} \underbrace{e^{C}}_{A}$$

$$y = A e^{-I} \qquad I \equiv \int P\,dx \longrightarrow \frac{dI}{dx} = P$$

$$\frac{d}{dx}\left(y e^{I} = A\right)$$

$$= y' e^{I} + y \frac{d}{dx} e^{I}$$

$$= y' e^{I} + y \frac{dI}{dx} e^{I} = e^{I}\left(y' + y \frac{dI}{dx}\right)$$

$$= e^{I}\left(y' + Py\right)$$

$$\boxed{y' + Py = Q} \quad \times e^I$$

$$e^I(y' + Py) = e^I Q$$

$$\underbrace{e^I(y'+Py)}$$

$$(ye^I)' \equiv \frac{d}{dx}(ye^I)$$

$$\Rightarrow \frac{d}{dx}(ye^I) = e^I Q \quad \times dx$$

$$\int d(ye^I) = \int e^I Q\, dx$$

$$ye^I = \left(\int e^I Q\, dx\right) + C \quad \times e^{-I}$$

$$\boxed{y(x) = e^{-I}\left(\int e^I Q\, dx\right) + Ce^{-I}} \leftarrow \text{solusi}$$

$$I \equiv \int P\, dx$$

$\boxed{e^I} \rightarrow$ Faktor Integrasi

Cth:

$$xy' + 2y = 1 \quad \times \frac{1}{x} \qquad y' + Py = Q$$

$$\boxed{y' + \frac{2}{x}y = \frac{1}{x}}, \quad P = \frac{2}{x}, \; Q = \frac{1}{x}$$

faktor integrasi: $e^I \rightarrow e^{2\ln x} = e^{\ln x^2} = x^2$

$$I = \int P\, dx = \int \frac{2\,dx}{x} = 2\ln x$$

$$y' + \frac{2}{x}y = \frac{1}{x} \quad \times e^I = x^2$$

$$x^2 y' + 2xy = x \qquad \boxed{\frac{d}{dx}(x^2 y) = 2xy + x^2 y'}$$

$$\frac{d}{dx}(x^2 y) = x$$

$$x^2 y = \int x\, dx + C = \frac{1}{2}x^2 + C$$

$$\boxed{y(x) = \frac{1}{2} + \frac{C}{x^2}} \rightarrow y' = -\frac{2C}{x^3}$$

Proof:

$$-\cancel{\frac{2C}{x^2}} + 1 + \cancel{\frac{2C}{x^2}} = 1$$

$$1 = 1 \quad \square$$

$$x^2 y' + 3y = x^3$$

$$\boxed{y' + \frac{3}{x^2} y = x} \longrightarrow y' + \frac{3}{x^2} y = x \quad \times e^{-\frac{3}{x}}$$

$$P = \frac{3}{x^2}$$

$$Q = x$$

$$e^I = e^{3\int \frac{1}{x^2} dx} = e^{-\frac{3}{x}}$$

$$\int x^{-2} dx = -\frac{1}{x}$$

$$\underbrace{e^{-\frac{3}{x}} y' + \frac{3}{x^2} e^{-3/x} y}_{} = x e^{-3/x}$$

$$\frac{d}{dx}\left(e^{-\frac{3}{x}} y\right) = x e^{-3/x}$$

$$e^{-\frac{3}{x}} y = \int x e^{-3/x} dx + C$$

$$y' + y = e^x \rightarrow P = 1, \; Q = e^x$$

$$e^x y' + e^x y = e^{2x} \quad \times e^x \quad \Big| \quad e^I = e^x$$

$$\frac{d}{dx}(e^x y) = e^{2x}$$

$$e^x y = C + \int e^{2x} dx$$

$$= C + \frac{1}{2} e^{2x}$$

$$\boxed{y(x) = \frac{1}{2} e^x + C e^{-x}}$$

$$\boxed{\frac{dq}{dt} + \frac{1}{RC} q = \mathcal{E}_0 \cos(\omega t)} \quad P = \frac{1}{RC} \quad Q = \mathcal{E}_0 \cos \omega t$$

$$\times e^{t/RC} \qquad e^I = e^{t/RC}$$

$$e^{t/RC} \frac{dq}{dt} + \frac{1}{RC} e^{t/RC} q = \mathcal{E}_0 e^{t/RC} \cos(\omega t)$$

$$\frac{d}{dt}\left(e^{t/RC} q\right) = \mathcal{E}_0 e^{t/RC} \cos(\omega t)$$

$$e^{t/RC} q = K + \mathcal{E}_0 \int e^{t/RC} \cos(\omega t)\, dt$$

$$\partial \rightarrow \int p\, dV = pV - \int V dp, \quad p = e^{t/RC} \rightarrow dp = \frac{1}{RC} e^{t/RC} dt$$

$$dV = \cos \omega t\, dt \rightarrow V = \frac{1}{\omega} \sin \omega t$$

$$\int e^{\frac{t}{RC}} \cos \omega t \, dt = e^{t/RC} \sin \omega t - \frac{1}{\omega RC} \int e^{t/RC} \sin \omega t \, dt$$

$$2 \rightarrow \int \underbrace{e^{t/RC}}_{p} \underbrace{\sin \omega t \, dt}_{dV} = -\frac{1}{\omega} e^{t/RC} \cos \omega t + \frac{1}{\omega RC} \int e^{t/RC} \cos \omega t \, dt$$

$$dp = \frac{1}{RC} e^{t/RC} \, dt$$

$$V = -\frac{1}{\omega} \cos \omega t$$

$$\Rightarrow \int e^{t/RC} \cos \omega t \, dt$$

$$= e^{t/RC} \sin \omega t - \frac{1}{\omega RC} \left(-\frac{1}{\omega} e^{t/RC} \cos \omega t \right) - \frac{1}{\omega RC} \left(\frac{1}{\omega RC} \int e^{t/RC} \cos \omega t \, dt \right)$$

$$\left(1 + \frac{1}{\omega^2 R^2 C^2} \right) \int e^{t/RC} \cos \omega t \, dt = e^{t/RC} \sin \omega t + \frac{1}{\omega^2 RC} e^{t/RC} \cos \omega t$$

$$\int e^{t/RC} \cos \omega t \, dt = \frac{e^{t/RC}}{1 + \frac{1}{\omega^2 R^2 C^2}} \left[\sin \omega t + \frac{\cos \omega t}{\omega^2 RC} \right]$$

$$e^{t/RC} q(t) = K + \frac{\varepsilon_0 e^{t/RC}}{1 + \frac{1}{\omega^2 R^2 C^2}} \left[\sin \omega t + \frac{\cos \omega t}{\omega^2 RC} \right]$$

$$q(t) = K e^{-t/RC} + \frac{\varepsilon_0}{1 + \frac{1}{\omega^2 R^2 C^2}} \left[\sin \omega t + \frac{\cos \omega t}{\omega^2 RC} \right]$$

Persamaan Bernoulli

PDB orde-1 non-linear : $\boxed{y' + P(x) y = Q(x) y^n}$, $n \neq 1$

Definisikan : $z \equiv y^{1-n}$

$$z' = \frac{dz}{dx} = (1-n) y^{-n} y'$$

$$y' + Py = Qy^n \qquad \times (1-n) y^{-n}$$

$$\underbrace{(1-n)y^{-n}y'}_{z'} + (1-n)P\underbrace{y^{1-n}}_{z} = (1-n)Q$$

$$\boxed{z' + (1-n)P\,z = (1-n)Q}$$ PDB orde-1 Linear dlm $z(x)$

$\downarrow$ faktor integrasi

$$\boxed{\text{Solusi : } z(x)} \longrightarrow \boxed{y(x) = (z(x))^{\frac{1}{1-n}}}$$

$$y' + y = xy^{2/3}$$

$\rightarrow P(x) = 1$, $Q(x) = x$, $n = 2/3$

$$z = y^{1/3} \rightarrow z' = \frac{1}{3}y^{-2/3}y'$$

$$y' + y = xy^{2/3} \qquad \times \tfrac{1}{3}y^{-2/3}$$

$$\underbrace{\frac{1}{3}y^{-2/3}y'}\ + \frac{1}{3}y^{1/3} = \frac{1}{3}x$$

$$\boxed{z' + \frac{1}{3}z = \frac{1}{3}x}$$ $\rightarrow P = 1/3$, $Q = 1/3\,x$

$$\times e^{\int \frac{1}{3}dx} = e^{x/3}$$

$$\underbrace{e^{\frac{x}{3}}z' + \frac{1}{3}e^{\frac{x}{3}}z} = \frac{1}{3}xe^{\frac{x}{3}}$$

$$\int d\left(e^{\frac{x}{3}}z\right) = \frac{1}{3}\int xe^{\frac{x}{3}}\,dx$$

$u = x \qquad dv = e^{\frac{x}{3}}dx$

$du = dx \qquad v = 3e^{\frac{x}{3}}$

$$e^{\frac{x}{3}}z = C + \frac{1}{3}\int xe^{\frac{x}{3}}\,dx$$

$$e^{\frac{x}{3}}z = C + \frac{1}{\cancel{3}}\left[\cancel{3}e^{\frac{x}{3}}x - \cancel{3}\int e^{\frac{x}{3}}dx\right]$$

$$= C + e^{\frac{x}{3}}x - 3e^{\frac{x}{3}}$$

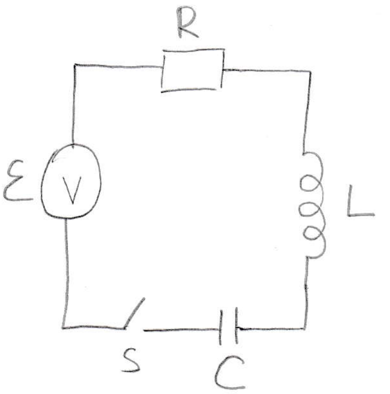
$$z(x) = x - 3 + Ce^{-x/3}$$

$$y(x) = \left(x + Ce^{-x/3} - 3\right)^3$$

Pers. Dif. Biasa orde-2 Linear

(PDB orde-2 Linear)

$$\sum \bar{F} = m\bar{a} = m\frac{d^2\bar{r}}{dt^2}$$



$$V_R + V_L + V_C = \varepsilon(t)$$

$$iR + L\frac{di}{dt} + \frac{q}{C} = \varepsilon(t)$$

$$\boxed{\ddot{q} + \frac{R}{L}\dot{q} + \frac{1}{LC}q = \frac{\varepsilon(t)}{L}}$$

$$i = \frac{dq}{dt}$$

$$\ddot{q} \equiv \frac{d^2q}{dt^2}$$

$$\nabla^2 \bar{E} = c^2 \frac{\partial^2 \bar{E}}{\partial t^2}$$

Bentuk Umum: PDB orde-2 Linear $\boxed{a_2 y'' + a_1 y' + a_0 y = b}$, $y' \equiv \frac{dy}{dx}$

$a_2 = a_2(x) \neq 0$, $b = b(x)$

$a_1 = a_1(x)$

$a_0 = a_0(x)$

PDB orde-2 Linear dg Koef. Konstan

$$\boxed{a_2 y'' + a_1 y' + a_0 y = b(x)}$$

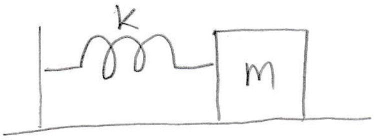
- homogen, $b(x) = 0$
- tak-homogen, $b(x) \neq 0$

• Homogen

$$\boxed{a_2 y'' + a_1 y' + a_0 y = 0} \qquad \boxed{y = 0}$$

Solusi trivial

Cth: $y'' + 3y' + 5y = 0$



$$\boxed{\ddot{x} + \omega^2 x = 0}, \quad \omega^2 \equiv \frac{k}{m}$$

$$\text{RLC}: \ddot{q} + \frac{R}{L}\dot{q} + \frac{1}{LC} q = 0$$

$y' \equiv \frac{dy}{dx}$

$y'' \equiv \frac{d^2y}{dx^2}$

Definisikan operator:

$$D \equiv \frac{d}{dx} \rightarrow y' = Dy$$

$$y'' = D^2 y = D(Dy)$$

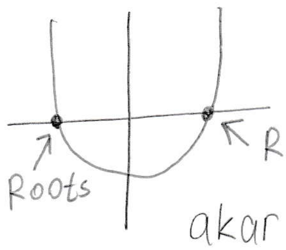
$$a_2 D^2 y + a_1 D y + a_0 y = 0$$

$$\underbrace{(a_2 D^2 + a_1 D + a_0)}_{=0} \underbrace{y}_{\neq 0} = 0$$

$$\Rightarrow \boxed{a_2 D^2 + a_1 D + a_0 = 0}$$

$ax^2 + bx + c = 0$

$\hookrightarrow x_{1,2} = -\frac{b \pm \sqrt{b^2 - 4ac}}{2a}$



akar = perpotongan fungsi dgn sumbu x

$$\underline{ax^2 + bx + c = 0} \times \frac{1}{a} \Rightarrow x^2 + \beta x + \gamma = 0$$

$$\beta \equiv b/a$$

$$\gamma = c/a$$

$\hookrightarrow (x + x_1)(x + x_2) = 0$

$$D^2 + a_{12} D + a_{02} = 0$$
$$(D - a_+)(D - a_-) = 0$$
$$a_\pm = \frac{-a_{12} \pm \sqrt{a_{12}^2 - 4a_{02}}}{2}$$

Diskriminan $\equiv \underline{a_{12}^2 - 4a_{02}}$

$$D^2 y + a_{12} D y + a_{02} y = 0$$
$$\boxed{(D + a_+) \underbrace{(D + a_-) y}_{V(x)} = 0}$$
$$\Downarrow$$
$$\boxed{(D + a_+) V(x) = 0}$$
$$\downarrow$$
$$\boxed{V' + a_+ V = 0} \checkmark$$

Definisikan :
$$\boxed{V(x) \equiv (D + a_-) y}$$
$$\downarrow$$
$$\boxed{y' + ay = V(x)} \checkmark$$

1 PDB orde-2 = 2 PDB orde-1

$$1 \text{ PDB orde-2} = \left\{ \begin{array}{l} V' + a_+ V = 0 \\ \\ y' + a_- V = 0 \end{array} \right\} 2 \text{ PDB orde-1}$$

$$V' + a_+ V = 0 \Rightarrow \frac{dV}{dx} = -a_+ V$$
$$\times \frac{dx}{V}$$
$$\int \frac{dV}{V} = -a_+ \int dx$$
$$\ln V = -a_+ x + K$$
$$\boxed{V(x) = C_1 e^{-a_+ x}}, \quad C_1 \equiv e^K$$

$$y' + a_- y = C_1 e^{-a_+ x}$$
$$\times \boxed{e^{a_- x}} \to \text{faktor Integrasi}$$
$$\frac{d}{dx}\left(e^{a_- x} y\right) = C_1 e^{(a_- - a_+) x}$$
$$e^{a_- x} y = \frac{C_1}{a_- - a_+} e^{(a_- - a_+) x} + B$$
$$\boxed{y(x) = A e^{-a_+ x} + B e^{-a_- x}}$$

$$y'' + 5y' + 6y = 0$$
$$(D^2 + 5D + 6)y = 0$$
$$(D+3)(D+2)y = 0$$
$$\boxed{y(x) = Ae^{-3x} + Be^{-2x}}$$

Proof :
$$y' = -3Ae^{-3x} - 2Be^{-2x}$$
$$y'' = 9Ae^{-3x} + 4Be^{-2x}$$

$$\cancel{9Ae^{-3x}} + \cancel{4Be^{-2x}} - \cancel{15Ae^{-3x}} - \cancel{10Be^{-2x}} + \cancel{6Ae^{-3x}} + \cancel{6Be^{-2x}} = 0$$
$\therefore y(x) = Ae^{-3x} + Be^{-2x}$, merupakan solusi. $\square$

$$y'' + y' - 2y = 0$$
$$(D+2)(D-1)y = 0$$
$$\boxed{y(x) = Ae^{-2x} + Be^{x}}$$

Klasifikasi solusi bedasarkan diskriminan :
$$a_\pm \equiv a_{12}/2 \pm \sqrt{\frac{a_{12}^2}{4} - a_{02}}$$
$$\text{Diskriminan} \equiv \frac{a_{12}^2}{4} - a_{02} \begin{cases} > 0, & a_+ \neq a_- \in \mathbb{R} \\ = 0, & a_+ = a_- \in \mathbb{R} \\ < 0, & a_+, a_- \in \mathbb{C} \end{cases}$$

$$(D + a_+)(D + a_-)y = 0$$
$$\Rightarrow (D + a_k)\underbrace{(D + a_k)y}_{\equiv} = 0$$
$$(D + a_k)\, V(x) = 0 \rightarrow V' + a_k V = 0 \Rightarrow V(x) = C_1 e^{-a_k x}$$
$$(D + a_k)y = C_1 e^{-a_k x}$$
$$\Rightarrow y' + a_k y = Ce^{-a_k x} \quad \times e^{a_k x}$$
$$\frac{d}{dx}(e^{a_k x} y) = C_1 \rightarrow e^{a_k x} y = C_1 x + C_2$$

$$\boxed{y(x) = C_1 e^{-a_K x} + C_2 x e^{-a_K x} = (A+Bx) e^{-a_K x}}$$

↳ General solution of twin roots

$$y'' + 4y' + 4y = 0$$

$$(D+2)(D+2)y = 0 \rightarrow \boxed{\cancel{y(x) = Ae^{-2x} + Be^{-2x}}} \rightarrow \text{naive solution}$$

$$\boxed{y(x) = Ae^{-2x} + Bxe^{-2x}}$$

↳ The Real solution

$\sqrt{-4} = \sqrt{-1}\sqrt{4} = 2i$, $i \equiv \sqrt{-1} \equiv$ bil. imajiner

$3+2i \rightarrow$ bil. kompleks

$$a_\pm = -\frac{a_{12}}{2} \pm \sqrt{\frac{a_{12}^2}{4} - a_{02}} \Rightarrow \alpha \equiv -a_{12}/2$$

$$\beta \equiv a_{02} - a_{12}^2/4$$

u/kasus $a_{12}^2 < a_{02}$,

$$a_\pm = \alpha \pm \sqrt{-\beta^2} = \alpha \pm i\beta$$

$$\boxed{y(x) = Ae^{(\alpha+i\beta)x} + Be^{(\alpha-i\beta)x}}$$

$$= e^{\alpha}\left(Ae^{i\beta x} + Be^{-i\beta x}\right)$$

Relasi Euler: $e^{\pm i\beta x} = \cos(\beta x) \pm i \sin(\beta x)$

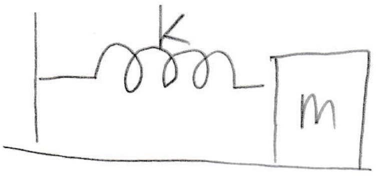
$$\Rightarrow Ae^{i\beta x} = A\cos\beta x + iA\sin\beta x$$

$$Be^{-i\beta x} = B\cos\beta x - iB\sin\beta x$$

$+$

$$= (A+B)\cos\beta x + i(A-B)\sin\beta x$$

$$y(x) = e^{\alpha x}(C_1 \sin\beta x + C_2\cos\beta x)$$



$$\ddot{x} + \omega^2 x = 0 \qquad \Big| \; i = \sqrt{-1}, \; i^2 = -1$$

$$(D^2 + \omega^2)x = 0$$

$$(D + i\omega)(D - i\omega)x = 0$$

$$x(t) = Ae^{i\omega t} + Be^{-i\omega t}$$

$$= C_1 \sin \omega t + C_2 \cos \omega t$$