󰀀 WEEK 08 FULL PLAYBOOK – GRAPH FUNDAMENTALS: REPRESENTATIONS, BFS, DFS & TOPOLOGICAL SORT

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󰀀 QUICK START & STRUCTURE

This playbook is organized into **5 learning layers**, one for each day of the week. Each layer builds on the previous, moving from **foundational concepts** to **practical algorithms** to **real-world systems**.

󰀀 Learning Arc

DAY 1: GRAPH MODELS & REPRESENTATIONS
↓
DAY 2: BREADTH-FIRST SEARCH (BFS)
↓
DAY 3: DEPTH-FIRST SEARCH (DFS) & TOPOLOGICAL SORT
↓
DAY 4: CONNECTIVITY & BIPARTITE GRAPHS
↓
DAY 5: STRONGLY CONNECTED COMPONENTS (SCC) [Optional Advanced]

󰀀 How to Use This Playbook

For Quick Review (30 minutes):

1. Read Section 2 structure overview
2. Skim the visual diagrams
3. Review the complexity tables
4. Check the mastery checklist

For Focused Learning (3-4 hours):

1. Follow Days 1-4 sequentially
2. Work through the traces and diagrams
3. Attempt the practice problems
4. Check understanding on the quiz

For Deep Mastery (15-20 hours):

1. Follow the full 5-day arc
2. Code every algorithm from scratch
3. Complete all practice problems
4. Study real-world system examples
5. Understand Week 8 to Week 9 connections

For Interview Prep (5-10 hours):

1. Focus on Days 1-4 core patterns
2. Know BFS and DFS backwards and forwards
3. Understand topological sort use cases
4. Practice the provided LeetCode problems

LEARNING OUTCOMES

By End of Week 08, You Should Master:

Conceptual:

- [ ] Distinguish between directed/undirected, weighted/unweighted graphs
- [ ] Understand graph representations and their trade-offs
- [ ] Explain implicit graphs (grids, state spaces)
- [ ] Trace BFS and DFS on paper without errors
- [ ] Understand why BFS finds shortest paths in unweighted graphs
- [ ] Know DFS tree edge types (tree, back, forward, cross edges)
- [ ] Explain cycle detection using DFS
- [ ] Describe topological sort and its use cases
- [ ] Understand connected components and bipartite testing
- [ ] Know Strongly Connected Components (optional advanced)

Implementation:

- [ ] Code BFS with queue
- [ ] Code DFS with recursion and iterative stack
- [ ] Implement cycle detection
- [ ] Implement topological sort (DFS post-order and Kahn's algorithm)
- [ ] Test for bipartite graphs using 2-coloring
- [ ] Find connected components
- [ ] Handle edge cases (disconnected vertices, self-loops, multiple edges)

Problem-Solving:

- [ ] Recognize when to use BFS vs DFS
- [ ] Model problems as graphs (especially implicit graphs)
- [ ] Solve shortest path problems on unweighted graphs
- [ ] Resolve task dependencies using topological sort
- [ ] Group nodes using connectivity algorithms
- [ ] Model graph problems using standard patterns

▯ SECTION 1: CONTEXT & MOTIVATION

The Engineering Problem

Imagine you're building a **social network** like LinkedIn. You have millions of users (nodes) and connections between them (edges). Now you need to solve real problems:

1. **Shortest connection path:** Given person A and person B, what's the shortest chain of mutual connections?
2. **Friend groups:** Are there isolated groups of friends? How many groups?
3. **Recommendation:** Who should I connect with? (Find users similar to my network)
4. **Dependency management:** A software project has 100 tasks. Some tasks depend on others. In what order should we execute them?

These problems require understanding **graph structure and navigation**. A naive approach would be to check every possible path—exponential time, impossible at scale. You need **BFS** (for shortest paths) and **DFS** (for structure exploration).

This week, you'll learn the foundational tools that power:

- Social networks (Facebook, LinkedIn)
- Recommendation engines (Netflix, Spotify)
- Route planning (Google Maps, Uber)
- Compiler design (dependency resolution)
- Database query optimization
- Network analysis (internet routing, web crawling)

The Constraint: At LinkedIn's scale, even O(V + E) algorithms need to be implemented perfectly. A bug in graph traversal might crash the entire recommendation system.

The Opportunity: Master BFS/DFS this week, and you unlock the entire graph algorithm universe. Next week: shortest paths (Dijkstra), minimum spanning trees, maximum flow. In Week 11: graph DP (on DAGs). These are ALL built on the foundations you learn now.

Why Graphs Matter: The Hidden Dependency Graph

Consider a simple JavaScript project:
main.js depends on utils.js and auth.js
utils.js depends on config.js
auth.js depends on config.js and utils.js

Visualized as a directed graph:
main.js ──→ utils.js ──→ config.js
↓ ↑
auth.js ────┘

Question: In what order should we load these files?

Naive approach: Try all 5! = 120 permutations. At scale with 1000 modules, this is impossible.

Smart approach: Use **topological sort**—a linear-time algorithm that respects dependencies. We load config.js, then utils.js and auth.js, then main.js.

This is the power of understanding graph algorithms. They transform impossible problems into elegant linear-time solutions.

󰀀 SECTION 2: COURSE STRUCTURE OVERVIEW

Week 08 at a Glance

Day	Topic	Core Concept	Complexity	Real-World Application
1	Graph Models & Representations	Adjacency List/Matrix, Implicit Graphs	O(V+E) traversal	Social networks, Maps
2	Breadth-First Search (BFS)	Queue-based level-order traversal	O(V+E) time, O(V) space	Shortest paths, Level-order
3	Depth-First Search (DFS) & Topological Sort	Stack-based exploration, Post-order traversal	O(V+E) time, O(V) space	Cycle detection, Task scheduling
4	Connectivity & Bipartite Graphs	Component finding, 2-coloring	O(V+E) time	Grouping, Constraint satisfaction
5	Strongly Connected Components (SCC) [Optional]	Kosaraju/Tarjan algorithm	O(V+E) time	Web graph analysis, Compiler optimization

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□ SECTION 3: BUILDING THE MENTAL MODELS

Day 1: Graph Types & Representations

□ The Hook: Why Representation Matters

A graph is fundamentally simple: nodes and edges. But **how you represent** it determines performance:

- **Adjacency matrix:** Fast edge lookups O(1), but $O(V^2)$ space. Good for dense graphs.
- **Adjacency list:** Efficient traversal O(V+E), space-efficient. Good for sparse graphs.
- **Edge list:** Most flexible, slowest lookups.

Real-world impact: At Facebook scale (3 billion users), storing a dense adjacency matrix is impossible—you'd need 9 billion × 9 billion = 81 quintillion entries. You **must** use adjacency lists.

□ Mental Model: Graph as Relationship Map

Think of a graph like a **social network**:

- **Nodes (vertices)** = People
- **Edges** = Relationships
- **Directed edge** = "Follows" (Twitter: A follows B, but B might not follow A)
- **Undirected edge** = "Friends with" (Facebook: mutual relationship)
- **Weighted edge** = "Distance" (Maps: road length between cities)

□ Graph Type Classifications

By Direction:

- **Undirected:** Relationships are mutual (Twitter friendship)
- **Directed:** Relationships have direction (Twitter follow)

By Weights:

- **Unweighted:** All edges equal (social distance = 1 connection)
- **Weighted:** Edges have values (distance in km, cost in $, time in seconds)

By Structure:

- **Sparse:** Few edges relative to nodes (social network: ~100 friends vs billions of people)
- **Dense:** Many edges (complete graph: every node connects to every other)
- **Cyclic:** Contains cycles (most real graphs)
- **Acyclic (DAG):** No cycles (dependency graphs, task scheduling)

Representations in Depth

Representation 1: Adjacency List

Structure:
Array of lists, where each index represents a node,
and the list at that index contains its neighbors.

Node 0: [1, 2]
Node 1: [0, 2, 3]
Node 2: [0, 1]
Node 3: [1]

Memory: O(V + E)
Edge lookup: O(degree of node) = O(V) worst case
Traversal: O(V + E)
Best for: Sparse graphs, most interview problems

C# Implementation:
```
// Adjacency list representation
Dictionary<int, List<int>> adjacencyList = new()
{
{ 0, new List<int> { 1, 2 } },
{ 1, new List<int> { 0, 2, 3 } },
{ 2, new List<int> { 0, 1 } },
{ 3, new List<int> { 1 } }
};

// Accessing neighbors of node 0: adjacencyList[0] → [1, 2]
```

Representation 2: Adjacency Matrix

Structure:
2D array where matrix[i][j] = 1 if edge exists from i to j,
else 0 (or the weight if weighted).

	0	1	2	3
0	0	1	1	0
1	1	0	1	1
2	1	1	0	0
3	0	1	0	0

Memory: $O(V^2)$
Edge lookup: O(1)
Traversal: $O(V^2)$ must check all pairs
Best for: Dense graphs, edge queries

C# Implementation:
```
// Adjacency matrix representation
int[][] adjacencyMatrix = new[]
{
new[] { 0, 1, 1, 0 },
new[] { 1, 0, 1, 1 },
new[] { 1, 1, 0, 0 },
```

new[] { 0, 1, 0, 0 }
};

// Check if edge exists from 0 to 1: adjacencyMatrix[0][1] = 1 (yes)

Representation 3: Edge List

Structure:
List of (source, destination) or (source, destination, weight) tuples.

[(0, 1), (0, 2), (1, 0), (1, 2), (1, 3), (2, 0), (2, 1), (3, 1)]

Memory: O(E)
Edge lookup: O(E)
Traversal: O(E), then need to build adjacency for neighbors
Best for: Flexible graph operations, minimal storage

Implicit Graphs: The Hidden Graph Structure

Not all graphs are explicit. Many problems have **implicit graphs** where nodes and edges are defined by rules rather than stored.

Example 1: Grid as Graph

Maze represented as a grid:

0 1 2 3
0 S . . .
1 . # . #
2 . . . G
3 # . . .

S = Start (node), G = Goal (node)
. = Walkable cell (node)

= Wall (not a node)

Edges: Each '.' connects to adjacent '.' cells (up, down, left, right)

Example 2: State Space as Graph

8-puzzle problem:
Node = state of puzzle (e.g., [1,2,3,4,5,6,7,8,0])
Edge = valid move of blank tile

From state [1,2,3,4,5,6,7,8,0], I can move blank left to get [1,2,3,4,5,6,7,0,8]
Or move blank up to get [1,2,3,4,0,6,7,8,5]

Find path from initial state to goal state [1,2,3,4,5,6,7,8,0]

Why implicit graphs matter: You don't store them explicitly (too much memory). Instead, you compute neighbors on-the-fly during traversal.

✅ Key Insights

1. **Representation choice affects performance:** Adjacency list is most common for sparse graphs.
2. **Traversal takes O(V + E) time** regardless of representation (assuming you visit each node/edge once).
3. **Implicit graphs are huge and undefined by storage**—they're defined by rules.
4. **Understanding the graph structure** is the first step before choosing an algorithm.

Day 2: Breadth-First Search (BFS) – Finding Shortest Paths

▯ Mental Model: Level-by-Level Exploration

BFS is like **dropping a stone in water**—the ripples spread level by level, equidistant from the source.

Imagine you're at a **train station** and want to visit all stations reachable within 2 stops:

- **Level 0 (distance 0):** You're at station A
- **Level 1 (distance 1):** All stations directly connected to A
- **Level 2 (distance 2):** All new stations connected to Level 1 stations
- **Level 3 (distance 3):** And so on...

Why this matters: In an **unweighted graph**, BFS guarantees you reach each node via the shortest path. The first time you visit a node is always at minimum distance from source.

▯ BFS Visualization

```
Graph: BFS from node 0:
1 ─── 3
/ \ / Queue operations:
0 2 ──
\ Initial: queue = [0], visited = {0}
4 Step 1: dequeue 0, add neighbors 1,4
queue = [1, 4], visited = {0, 1, 4}
Step 2: dequeue 1, add neighbors 2,3
queue = [4, 2, 3], visited = {0, 1, 4, 2, 3}
Step 3: dequeue 4, no new neighbors
queue = [2, 3], visited = {0, 1, 4, 2, 3}
Step 4: dequeue 2, no new neighbors
queue = [3], visited = {0, 1, 4, 2, 3}
Step 5: dequeue 3, no new neighbors
queue = [], visited = {0, 1, 4, 2, 3}
```

Order of exploration: 0 → 1 → 4 → 2 → 3 (level by level)
Distances from 0: {0: 0, 1: 1, 4: 1, 2: 2, 3: 2}

BFS Algorithm (Pseudocode)

BFS(graph, start):
queue ← Queue()
visited ← Set()
distance ← Map()

```
queue.enqueue(start)
visited.add(start)
distance[start] ← 0

while queue is not empty:
    node ← queue.dequeue()
    for neighbor in graph[node]:
        if neighbor not in visited:
            visited.add(neighbor)
            distance[neighbor] ← distance[node] + 1
            queue.enqueue(neighbor)

return distance
```

C# Implementation

///

/// Breadth-First Search (BFS) for unweighted shortest paths
/// Mental Model: Explore level by level, like ripples in water
/// Time: O(V + E), Space: O(V)
///

public class BFSSolver
{
// MENTAL MODEL: Queue ensures we explore all nodes at distance d
// before any node at distance d+1
public Dictionary<int, int> BFS(Dictionary<int, List<int>> graph, int start)
{
// STEP 1: Guard clauses - handle edge cases first
if (graph == null || !graph.ContainsKey(start))
return new Dictionary<int, int>();

```
    // STEP 2: Initialize data structures
    // Queue stores nodes to explore in FIFO order (level by level)
    Queue<int> queue = new Queue<int>();
```

```
        HashSet<int> visited = new HashSet<int>();
        Dictionary<int, int> distance = new Dictionary<int, int>();

        // Mark start node: visited and at distance 0
        queue.Enqueue(start);
        visited.Add(start);
        distance[start] = 0;

        // STEP 3: Core BFS loop - process nodes level by level
        while (queue.Count > 0)
        {
            int node = queue.Dequeue();

            // Explore all neighbors of current node
            foreach (int neighbor in graph[node])
            {
                // Only process unvisited neighbors
                if (!visited.Contains(neighbor))
                {
                    visited.Add(neighbor);
                    distance[neighbor] = distance[node] + 1;
                    queue.Enqueue(neighbor);
                }
            }
        }

        return distance;
    }
```

}

Key BFS Properties

1. **Shortest Path Guarantee:** First time you reach a node is at minimum distance (only for unweighted graphs)
2. **Time Complexity:** O(V + E)—visit each node once, traverse each edge once
3. **Space Complexity:** O(V)—queue can hold at most V nodes
4. **Connected Components:** Can find all reachable nodes from a source
5. **Single-Source Shortest Path:** Answers "how many stops to reach node X?"

✅ Common Applications

Problem	How BFS Helps	Example
Shortest path (unweighted)	Explores level by level	Minimum hops in network
Level-order traversal	BFS on trees	Tree serialization
Nearest neighbor	Stop when found	Closest friend in social network
Connected components	Count disconnected regions	Island counting problems
Bipartite testing	2-coloring during BFS	Checking if graph is 2-colorable

---

Day 3: DFS & Topological Sort – Depth-First Structure Exploration

Mental Model: Going Deep, Then Backtracking

DFS is like **exploring a maze** by going as far as possible, then backtracking when you hit a dead end.

Imagine exploring a **cave system**:

- Start at entrance
- Go down one passage as far as possible
- When stuck, backtrack and try another passage
- Mark every passage you've explored to avoid revisiting

Unlike BFS (which explores level-by-level), **DFS dives deep** first.

DFS Visualization

Graph: DFS from node 0 (using recursion):
1 —— 3
/ \ / Call stack:
0 2 ——
\ dfs(0) → visit 0
4 dfs(1) → visit 1
dfs(2) → visit 2
(dead end, backtrack to 1)
(dead end, backtrack to 0)
dfs(4) → visit 4
(dead end, backtrack to 0)

(back to dfs(0))
dfs(3) → visit 3
(dead end, backtrack)

Order of exploration: 0 → 1 → 2 → 4 → 3 (depth first)
Post-order: 2 → 1 → 4 → 3 → 0 (finish times)

DFS Algorithm (Recursive)

DFS(node, graph, visited):
visited.add(node)
for neighbor in graph[node]:
if neighbor not in visited:
DFS(neighbor, graph, visited)

```
# After processing all descendants, do post-order work
postOrder.add(node)
```

C# Implementation (Recursive)

///

/// Depth-First Search (DFS) - Recursive implementation
/// Mental Model: Go deep first, backtrack when stuck
/// Time: O(V + E), Space: O(V) call stack
///

public class DFSSolver
{
// MENTAL MODEL: Recursion naturally handles the backtracking
// We go as deep as possible before exploring other paths
public void DFS(int node, Dictionary<int, List<int>> graph,
HashSet<int> visited, List<int> order)
{
// STEP 1: Guard clauses
if (graph == null || !graph.ContainsKey(node))
return;

```
    // STEP 2: Mark current node as visited
    if (visited.Contains(node))
        return;
    visited.Add(node);

    // Pre-order work (process node before children)
    order.Add(node);
```

```
        // STEP 3: Recursively explore all neighbors
        // This naturally creates depth-first behavior
        foreach (int neighbor in graph[node])
        {
            if (!visited.Contains(neighbor))
            {
                DFS(neighbor, graph, visited, order);
            }
        }

        // Post-order work (process node after children)
        // Used for topological sort and finish times
    }
```

}

C# Implementation (Iterative with Stack)

///

/// Depth-First Search (DFS) - Iterative implementation using stack
/// Useful when recursion depth might exceed stack limit
/// Time: O(V + E), Space: O(V)
///

public class DFSIterativeSolver
{
// MENTAL MODEL: Stack simulates the recursion call stack
// Top of stack is the "current" node we're exploring
public void DFSIterative(int start, Dictionary<int, List<int>> graph,
List<int> postOrder)
{
// STEP 1: Guard clauses
if (graph == null || !graph.ContainsKey(start))
return;

```
        // STEP 2: Initialize stack and tracking structures
        Stack<int> stack = new Stack<int>();
        HashSet<int> visited = new HashSet<int>();
        HashSet<int> finished = new HashSet<int>();

        stack.Push(start);
        // STEP 3: Process nodes using stack
        while (stack.Count > 0)
        {
            int node = stack.Peek();

            // If we've already processed this node's children, mark finished
            if (finished.Contains(node))
            {
                stack.Pop();
                postOrder.Add(node);
                continue;
            }

            if (!visited.Contains(node))
            {
                visited.Add(node);
            }

            // Push all unvisited neighbors
            bool hasUnvisited = false;
            foreach (int neighbor in graph[node])
            {
                if (!visited.Contains(neighbor))
                {
                    stack.Push(neighbor);
                    hasUnvisited = true;
                }
            }

            if (!hasUnvisited)
            {
                finished.Add(node);
            }
        }
    }
```

}

DFS Tree Edge Types (For Directed Graphs)

When we run DFS on a directed graph, we can classify edges:

Edge Type	Definition	Example	Significance
Tree Edge	Edge to unvisited node	First time we follow an edge	Part of DFS tree structure
Back Edge	Edge to ancestor in DFS tree	Indicates a cycle	**Cycle exists if back edge present**
Forward Edge	Edge to descendant (non-child)	Jump forward in tree	Rare in simple DFS
Cross Edge	Edge to neither ancestor nor descendant	Between branches of tree	Common in directed graphs

Example:
DFS tree structure:
0
/ |
1 2 3
|
4

Back edge 4 → 1 creates cycle: 1 → 4 → 1
Forward edge 0 → 4 (jump over children)
Cross edge 3 → 2 (between branches)

♻ Cycle Detection Using DFS

Key insight: A cycle exists if we ever find a **back edge** (edge to ancestor).

public bool HasCycle(int node, Dictionary<int, List<int>> graph,
HashSet<int> inStack, HashSet<int> visited)
{
// MENTAL MODEL: inStack tracks current path in recursion
// If we reach a node already in current path, it's a back edge (cycle)

```
if (!visited.Contains(node))
{
    visited.Add(node);
    inStack.Add(node);
    foreach (int neighbor in graph[node])
    {
        if (!visited.Contains(neighbor))
        {
            if (HasCycle(neighbor, graph, inStack, visited))
                return true;
        }
        else if (inStack.Contains(neighbor))
        {
            // Back edge found! Node is ancestor in current path
            return true;
        }
    }
}

inStack.Remove(node);
return false;
```

}

Topological Sort – Ordering Dependencies

Definition: A topological sort is a linear ordering of nodes such that for every directed edge u → v, u comes before v.

Only works on DAGs (Directed Acyclic Graphs). If the graph has cycles, topological sort is impossible.

Real-world examples:

- Task scheduling with dependencies
- Build system dependency resolution
- Course prerequisites
- Spreadsheet cell computation order

Method 1: DFS Post-Order

Post-order traversal naturally produces topological sort:

public List<int> TopologicalSortDFS(Dictionary<int, List<int>> graph)
{
// MENTAL MODEL: Nodes that finish processing come later in ordering
// We want to process a node AFTER all nodes that depend on it

```
// STEP 1: Guard clauses
if (graph == null || graph.Count == 0)
    return new List<int>();

// STEP 2: Initialize tracking structures
HashSet<int> visited = new HashSet<int>();
Stack<int> finishOrder = new Stack<int>();

// STEP 3: Run DFS from all unvisited nodes
foreach (int node in graph.Keys)
{
    if (!visited.Contains(node))
    {
        TopologicalDFSHelper(node, graph, visited, finishOrder);
    }
}

// STEP 4: Return finish order (which is topological sort)
return new List<int>(finishOrder);
```

}

private void TopologicalDFSHelper(int node, Dictionary<int, List<int>> graph, HashSet<int> visited, Stack<int> finishOrder)
{
visited.Add(node);

```
foreach (int neighbor in graph[node])
{
    if (!visited.Contains(neighbor))
    {
        TopologicalDFSHelper(neighbor, graph, visited, finishOrder);
    }
}

// Post-order: add to result after processing children
finishOrder.Push(node);
```

}

Why it works: If edge u → v exists, v is visited before u finishes. So v is pushed onto finish stack before u. When we pop stack, u comes before v. ✓

Method 2: Kahn's Algorithm (BFS-based)

Use in-degree counts:

```
public List<int> TopologicalSortKahn(Dictionary<int, List<int>> graph)
{
// MENTAL MODEL: Always process nodes with no incoming edges
// When we process a node, decrease in-degree of its neighbors
// This reveals new nodes with no incoming edges
```

```
// STEP 1: Guard clauses
if (graph == null || graph.Count == 0)
    return new List<int>();

// STEP 2: Calculate in-degrees
Dictionary<int, int> inDegree = new Dictionary<int, int>();
foreach (int node in graph.Keys)
{
    if (!inDegree.ContainsKey(node))
        inDegree[node] = 0;

    foreach (int neighbor in graph[node])
    {
        if (!inDegree.ContainsKey(neighbor))
            inDegree[neighbor] = 0;
        inDegree[neighbor]++;
    }
}

// STEP 3: Find all nodes with in-degree 0
Queue<int> queue = new Queue<int>();
foreach (var node in inDegree)
{
    if (node.Value == 0)
        queue.Enqueue(node.Key);
}

// STEP 4: Process nodes, decreasing in-degrees
```

```
List<int> result = new List<int>();
while (queue.Count > 0)
{
    int node = queue.Dequeue();
    result.Add(node);

    foreach (int neighbor in graph[node])
    {
        inDegree[neighbor]--;
        if (inDegree[neighbor] == 0)
            queue.Enqueue(neighbor);
    }
}

// STEP 5: Check for cycles (if result doesn't contain all nodes)
if (result.Count != graph.Count)
    return new List<int>(); // Cycle detected

return result;
```

}

Comparison:

- **DFS post-order:** Intuitive, naturally produces topological order
- **Kahn's algorithm:** Explicitly finds nodes with in-degree 0, can detect cycles

✅ Key Insights

1. **DFS explores deeply** before backtracking (unlike BFS level-by-level)
2. **Post-order traversal** produces topological sort on DAGs
3. **Back edges indicate cycles** in directed graphs
4. **Topological sort only exists for DAGs**
5. Both DFS post-order and Kahn's algorithm solve topological sort

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Day 4: Connectivity & Bipartite Graphs

Connected Components

Problem: Given a graph, find all **isolated groups** of nodes (components).

Real-world: Social network analysis—find clusters of interconnected users not connected to other clusters.

Algorithm: Run BFS/DFS from each unvisited node:

public List<List<int>> FindConnectedComponents(Dictionary<int, List<int>> graph)
{
// MENTAL MODEL: Each component is an isolated subgraph
// We find one component, mark all nodes as visited,
// then find next component in remaining unvisited nodes

```
// STEP 1: Guard clauses
if (graph == null || graph.Count == 0)
    return new List<List<int>>();

// STEP 2: Initialize tracking
HashSet<int> visited = new HashSet<int>();
List<List<int>> components = new List<List<int>>();

// STEP 3: For each unvisited node, explore its component
foreach (int node in graph.Keys)
{
    if (!visited.Contains(node))
    {
        List<int> component = new List<int>();
        BFSComponent(node, graph, visited, component);
        components.Add(component);
    }
}

return components;
```

}

private void BFSComponent(int start, Dictionary<int, List<int>> graph,
HashSet<int> visited, List<int> component)
{
Queue<int> queue = new Queue<int>();
queue.Enqueue(start);
visited.Add(start);

```
while (queue.Count > 0)
{
    int node = queue.Dequeue();
    component.Add(node);
    foreach (int neighbor in graph[node])
    {
        if (!visited.Contains(neighbor))
        {
            visited.Add(neighbor);
            queue.Enqueue(neighbor);
        }
    }
}
```

}

Time: O(V + E)
Space: O(V)
Application: Island counting, social clustering, network analysis

⚖ Bipartite Graphs – Two-Coloring

Definition: A graph is **bipartite** if nodes can be divided into two groups where edges only connect between groups (no edges within groups).

Visual:
Bipartite (valid 2-coloring): Not Bipartite (cycle of odd length):
Group A: {0, 2} 0 ─── 1
Group B: {1, 3} / \ /
2 3
0 ─── 1 └────┘
│ │
2 ─── 3

All edges cross groups Edge 0-1, 1-3, 3-0 form triangle
(odd cycle) - can't 2-color

Algorithm: 2-Coloring with BFS

public bool IsBipartite(Dictionary<int, List<int>> graph)
{
// MENTAL MODEL: Try to color graph with 2 colors
// Color node 0 with color A, all neighbors with B,
// all their neighbors with A, etc.
// If we ever need to color a node with both colors, it's not bipartite

```
// STEP 1: Guard clauses
if (graph == null)
    return true; // Empty graph is bipartite
```

```
// STEP 2: Initialize color map (-1 = uncolored, 0 = colorA, 1 = colorB)
Dictionary<int, int> color = new Dictionary<int, int>();
foreach (int node in graph.Keys)
    color[node] = -1;

// STEP 3: Check each connected component
foreach (int node in graph.Keys)
{
    if (color[node] == -1)
    {
        if (!BFSCheckBipartite(node, graph, color))
            return false;
    }
}

return true;
```

}

private bool BFSCheckBipartite(int start, Dictionary<int, List<int>> graph,
Dictionary<int, int> color)
{
Queue<int> queue = new Queue<int>();
queue.Enqueue(start);
color[start] = 0; // Color start node with 0

```
while (queue.Count > 0)
{
    int node = queue.Dequeue();
    int currentColor = color[node];
    int neighborColor = 1 - currentColor; // Opposite color

    foreach (int neighbor in graph[node])
    {
        if (color[neighbor] == -1)
        {
            // Uncolored: color with opposite
            color[neighbor] = neighborColor;
            queue.Enqueue(neighbor);
        }
        else if (color[neighbor] != neighborColor)
        {
            // Already colored but wrong color: conflict!
            return false;
        }
    }
}

return true;
```

}

Time: O(V + E)
Space: O(V)
Key insight: Graph is bipartite iff no odd-length cycles exist

✅ Applications of Bipartite Testing

Problem	Nodes	Edges	Why Bipartite?
Job matching	People & Jobs	Person can do job	Check if valid assignment exists
Actor-Movie network	Actors & Movies	Actor in movie	Partition by node type
Scheduling conflicts	Time slots & Events	Can't overlap	2-coloring = conflict-free schedule

---

Day 5 [OPTIONAL]: Strongly Connected Components (SCC)

♻️ What Are Strongly Connected Components?

Definition: In a directed graph, an SCC is a **maximal set of nodes** where every node can reach every other node.

Example:
Graph:
0 ──→ 1 ──→ 2
↑ ↓
└────────┘

SCCs: {0}, {1, 2}

- Node 0 can't reach 1, 2 (no incoming edges)
- Nodes 1 and 2 reach each other: 1→2→1

Real-world: Web graph analysis—find tightly interconnected websites that form clusters.

🔲 Kosaraju's Algorithm (Conceptual)

Two-pass algorithm:

1. **Pass 1:** Run DFS on original graph, record finish order
2. **Pass 2:** Run DFS on transposed graph in reverse finish order

Graph: Transpose:
0 → 1 1 ← 0
↓ ↓ ↑ ↑
2 → 3 2 ← 3

Pass 1 DFS from 0: finish order = [3, 2, 1, 0]
Pass 2 DFS in reverse on transpose from 3: finds SCC {3}, {2}, {1}, {0}

Why it works: Finish order from original graph processes nodes with no outgoing edges to other SCCs first. Reverse graph reveals incoming edges, helping us identify complete SCCs.

✅ Key Insights

1. SCCs partition directed graphs into components
2. DAG of SCCs reveals overall graph structure
3. Kosaraju and Tarjan algorithms both solve SCC in O(V + E)
4. Applications: compiler optimization, web analysis, recommendation systems

---

🔲 SECTION 4: MECHANICS & TRADEOFFS

Real-World System Implementations

Google Maps: Road Network as Graph

Problem: Find shortest route from point A to B

Graph structure:

- **Nodes:** Intersections, landmarks
- **Edges:** Roads with weights (distance or time)
- **Type:** Weighted, directed (one-way streets), dynamic (traffic)

Algorithm choice:

- Can't use BFS (weighted graph)
- Use Dijkstra's (Week 9) for single-source shortest path
- But BFS is used for preprocessing: group nearby nodes, create graph hierarchy

Real-world constraint: Graph has billions of nodes. Can't explore all in real-time. Solution: Use **contraction hierarchies** and **precomputed distance tables** alongside BFS/Dijkstra.

LinkedIn: Social Network Analysis

Problem: Find shortest connection path between two users

Graph structure:

- **Nodes:** Users
- **Edges:** Mutual connections
- **Type:** Undirected (connections are mutual), unweighted (distance = hop count)

Algorithm: BFS is perfect!

- Unweighted graph → BFS guarantees shortest path
- First time you reach destination is the minimum hops
- O(V + E) is efficient for billions of nodes (sparse graph)

Real-world optimization:

- Don't traverse entire graph
- Use **bidirectional BFS**: start from both source and destination, meet in middle
- Cuts search space by ~50%

Compiler: Task Dependency Resolution

Problem: Compile modules in correct order respecting dependencies

Graph structure:

- **Nodes:** Source files/modules
- **Edges:** Dependency (A depends on B means B → A edge)
- **Type:** Directed, acyclic (DAG—circular dependencies = error)

Algorithm: Topological Sort

- DFS post-order or Kahn's algorithm
- O(V + E) finds valid compilation order
- Kahn's algorithm can detect cycles (circular dependencies)

Real-world optimization:

- Parallel compilation: nodes with no incoming edges can compile in parallel
- Incremental builds: only recompile changed files and dependents

---

Performance Comparison

Algorithm	Use Case	Time	Space	Strength	Weakness
BFS	Unweighted shortest path	O(V+E)	O(V)	Optimal for unweighted	Inefficient for weighted
DFS	Structure exploration	O(V+E)	O(V)	Cache-friendly, less memory	Not guaranteed shortest path
Topological Sort (DFS)	Task scheduling	O(V+E)	O(V)	Intuitive, finds cycles	Need cycle check
Topological Sort (Kahn)	Task scheduling	O(V+E)	O(V)	Explicit cycle detection	Slightly more code
SCC (Kosaraju/Tarjan)	Component analysis	O(V+E)	O(V)	Optimal, reveals structure	Conceptually complex

---

✅ SECTION 5: INTEGRATION & MASTERY

Key Concepts to Internalize

1. Graph Representation Choice

Decision Matrix:

Scenario	Choose	Reason
Sparse graph ($E << V^2$)	Adjacency List	Memory efficient O(V+E)
Dense graph ($E \approx V^2$)	Adjacency Matrix	Fast edge lookup O(1)
Need edge weights	Edge List	Most flexible
Interview coding	Adjacency List + Dictionary	Most common, clean

2. BFS vs DFS Selection

Criterion	BFS	DFS
Goal	Shortest path (unweighted)	Structure, cycles, topological
Data structure	Queue (FIFO)	Stack/recursion (LIFO)
Memory	May use more (level-by-level)	Often less (recursive stack)
Discovery order	Level-by-level	Depth-first
Best for	Social networks, recommendation	Compilers, web analysis

3. When Each Algorithm Shines

```
Problem: Social network analysis
├─ Find shortest path between users → BFS
├─ Find all users reachable from user → Both (BFS simpler)
├─ Find groups of users → Connected components (BFS/DFS)
└─ Detect if network has cycles → DFS cycle detection

Problem: Build system
├─ Find compilation order → Topological sort (Kahn's)
├─ Detect circular dependency → Kahn's (explicit detection)
├─ Parallel compile groups → Topological + in-degree groups
└─ Incremental compile → BFS/DFS on dependency DAG

Problem: Web graph (100 billion pages)
├─ Crawl reachable pages → BFS (level-by-level)
├─ Find tightly linked clusters → SCC (Kosaraju)
├─ Shortest path between pages → BFS or Week 9: SSSP
└─ PageRank computation → Iterative BFS on DAG of SCCs
```

Cognitive Lenses: 5 Perspectives

1. The Traversal Lens

All Week 8 algorithms are traversals. BFS and DFS are the **foundation**:

- BFS = "explore all nodes at distance d before distance d+1"
- DFS = "explore one path fully before trying another"
- Topological sort = DFS variant that orders by finish time
- Connected components = multiple DFS/BFS calls

Key: Understand BFS/DFS deeply, and other algorithms follow naturally.

2. The Cache-Locality Lens

Memory access pattern matters:

- BFS: Queue access pattern is irregular (cache-unfriendly)
- DFS: Recursion stack is local memory (cache-friendly)
- In practice: Iterative DFS with explicit stack often outperforms BFS by 2-3x due to cache locality

Take-away: For performance-critical applications, consider DFS despite BFS's theoretical advantage.

3. The Cycle Detection Lens

Cycles appear everywhere:

- Directed graph: back edge during DFS → cycle
- Undirected graph: revisit parent during DFS → cycle (mostly)
- DAG: no cycles; topological sort proves it
- Most real-world graphs have cycles (social networks, web)

Intuition: Cycles often represent constraints. Task scheduling needs DAG (no cycles). Social networks can have cycles (mutual friends).

4. The Problem Modeling Lens

Many problems are secretly graph problems:

- Shortest path: graph with weighted edges
- Sudoku: graph with constraint edges (cells that can't have same value)
- N-queens: graph of valid placements
- State-space search: implicit graph of states and transitions

Skill: Recognize hidden graphs, model correctly, apply standard algorithm.

5. The Scalability Lens

Week 8 algorithms handle real-world scale:

- O(V + E) is efficient even for billions of nodes
- BFS uses O(V) space: feasible (but need careful implementation)
- Graph compression (SCCs, hierarchies) reveals structure
- Parallel BFS/DFS possible on multi-core systems

Practical: Week 8 patterns are production-ready for internet-scale systems.

Practice Problems Ladder

Stage 1: Canonical Problems (Master Core)

BFS Problems:

1. **LeetCode 102 - Binary Tree Level Order Traversal** (Easy)
    - **Concept:** BFS on tree (implicit graph)
    - **Challenge:** Distinguish levels, track level boundaries
    - **Why:** Core BFS pattern, teaches level-by-level thinking
2. **LeetCode 200 - Number of Islands** (Medium)
    - **Concept:** Connected components on grid (implicit graph)
    - **Challenge:** Model grid as graph, track visited
    - **Why:** Implicit graph + DFS/BFS connected components
3. **LeetCode 286 - Walls and Gates** (Medium)
    - **Concept:** Multi-source BFS
    - **Challenge:** BFS from multiple starting points simultaneously
    - **Why:** Extends BFS to multiple sources

DFS Problems:

4. **LeetCode 200 - Number of Islands** (Medium)
    - **Concept:** Connected components on grid (with DFS instead)
    - **Challenge:** DFS recursion vs BFS queue
    - **Why:** Compare BFS and DFS approaches
5. **LeetCode 207 - Course Schedule** (Medium)
    - **Concept:** Cycle detection in directed graph
    - **Challenge:** Model prerequisites as edges, detect cycles
    - **Why:** Real-world dependency problem

Topological Sort Problems:

6. **LeetCode 207 - Course Schedule II** (Medium)
    - **Concept:** Topological sort (return order if possible)
    - **Challenge:** Kahn's algorithm or DFS post-order
    - **Why:** Direct topological sort application

Stage 2: Variations (Recognize Pattern Boundaries)

Boundary 1: Multiple Starting Points

7. **LeetCode 1091 - Shortest Path in Binary Matrix** (Medium)
    - **Twist:** Start from multiple cells (multi-source BFS)
    - **When Pattern Breaks:** Standard BFS assumes single source
    - **Why:** Tests understanding of BFS flexibility

Boundary 2: Weighted Graphs (Preview)

8. **LeetCode 743 - Network Delay Time** (Medium)
    - **Twist:** Weighted directed graph (shortest path)
    - **When Pattern Breaks:** BFS doesn't guarantee shortest path
    - **Why:** Foreshadows Week 9 (Dijkstra), shows BFS limitation

Boundary 3: Bipartite Testing

9. **LeetCode 785 - Is Graph Bipartite?** (Medium)
    - **Twist:** 2-coloring bipartite check
    - **When Pattern Breaks:** Need to track coloring during traversal
    - **Why:** Tests understanding of graph coloring invariant

Stage 3: Integration (Combine Patterns)

Integration 1: BFS + Cycle Detection

10. **LeetCode 701 - Course Schedule III** (Hard)
    - **Patterns Required:** Cycle detection + topological sort + greedy
    - **Challenge:** Combine multiple concepts
    - **Why:** Real-world: course planner with prerequisites and schedule constraints

Integration 2: SCC + Graph Coloring

11. **LeetCode 1192 - Critical Connections in Network** (Hard)
    - **Patterns Required:** Graph structure + bridge finding + DFS
    - **Challenge:** Find critical edges (bridges) in graph
    - **Why:** Real-world: network reliability analysis

Common Mistakes & How to Fix Them

Mistake 1: Forgetting to Mark Visited

```
// ❌ WRONG: Infinite loop if graph has cycles
Queue<int> queue = new Queue<int>();
queue.Enqueue(start);

while (queue.Count > 0)
{
int node = queue.Dequeue();
foreach (int neighbor in graph[node])
{
queue.Enqueue(neighbor); // Never marks visited!
}
}

// ✅ CORRECT: Track visited nodes
HashSet<int> visited = new HashSet<int>();
queue.Enqueue(start);
visited.Add(start);

while (queue.Count > 0)
{
int node = queue.Dequeue();
foreach (int neighbor in graph[node])
{
if (!visited.Contains(neighbor))
{
visited.Add(neighbor);
queue.Enqueue(neighbor);
}
}
}
```

Why: Without tracking, we revisit nodes infinitely in cyclic graphs. Visited set prevents cycles.

Mistake 2: Using DFS for Shortest Path (Unweighted)

```
// ✖ WRONG: DFS doesn't guarantee shortest path
public int ShortestPathDFS(int node, int target, /* ... */)
{
if (node == target) return distance;
```

```
int shortest = int.MaxValue;
foreach (int neighbor in graph[node])
{
    shortest = Math.Min(shortest, ShortestPathDFS(neighbor, target, /* ... */));
}
return shortest;
```

```
}
// Problem: Explores ALL paths (exponential time)

// ✅ CORRECT: Use BFS
public int ShortestPathBFS(int start, int target)
{
if (start == target) return 0;
```

```
Queue<int> queue = new Queue<int>();
HashSet<int> visited = new HashSet<int>();
Dictionary<int, int> distance = new Dictionary<int, int>();

queue.Enqueue(start);
visited.Add(start);
distance[start] = 0;

while (queue.Count > 0)
{
    int node = queue.Dequeue();
    if (node == target) return distance[node];

    foreach (int neighbor in graph[node])
    {
        if (!visited.Contains(neighbor))
        {
            visited.Add(neighbor);
            distance[neighbor] = distance[node] + 1;
            queue.Enqueue(neighbor);
        }
    }
}

return -1; // Not reachable
```

}

Why: BFS explores level-by-level, so first time reaching target is shortest path. DFS might explore deep wrong paths.

Mistake 3: Forgetting Edge Cases

// ✖ INCOMPLETE: Misses edge cases
public List<List<int>> FindConnectedComponents(Dictionary<int, List<int>> graph)
{
HashSet<int> visited = new HashSet<int>();
List<List<int>> components = new List<List<int>>();

```
foreach (int node in graph.Keys)
{
    if (!visited.Contains(node))
    {
        List<int> component = new List<int>();
        BFS(node, graph, visited, component);
        components.Add(component);
    }
}

return components;
```

}

// ✅ COMPLETE: Handles all cases
public List<List<int>> FindConnectedComponents(Dictionary<int, List<int>> graph)
{
if (graph == null || graph.Count == 0)
return new List<List<int>>();

```
HashSet<int> visited = new HashSet<int>();
List<List<int>> components = new List<List<int>>();

// Handle nodes with no outgoing edges (isolated nodes)
HashSet<int> allNodes = new HashSet<int>(graph.Keys);
foreach (var neighbors in graph.Values)
    foreach (int neighbor in neighbors)
        allNodes.Add(neighbor);

foreach (int node in allNodes)
{
    if (!visited.Contains(node))
    {
        List<int> component = new List<int>();
        if (graph.ContainsKey(node))
            BFS(node, graph, visited, component);
        else
            component.Add(node); // Isolated node
        components.Add(component);
    }
}

return components;
```

}

Why: Isolated nodes (no outgoing edges) might not appear in graph dictionary. Must handle separately.

Mastery Checklist

Conceptual Understanding

- [ ] Explain why BFS finds shortest paths in unweighted graphs
- [ ] Describe the difference between DFS recursion and iterative (stack)
- [ ] Define topological sort and when it's applicable (DAG)
- [ ] Explain cycle detection using DFS (back edges)
- [ ] Know when to use 2-coloring (bipartite testing)
- [ ] Understand connected components and when they're useful
- [ ] Define SCCs and their role in web graph analysis (optional)

Implementation Fluency

- [ ] Code BFS with queue (5 minutes, no notes)
- [ ] Code DFS with recursion (3 minutes)
- [ ] Code iterative DFS with stack (5 minutes)
- [ ] Implement topological sort (both methods) (10 minutes)
- [ ] Code cycle detection (5 minutes)
- [ ] Implement connected components (5 minutes)
- [ ] Code 2-coloring bipartite test (5 minutes)

Problem-Solving

- [ ] Recognize BFS problems (shortest path signals)
- [ ] Recognize DFS problems (cycle, structure signals)
- [ ] Model implicit graphs (grids, state spaces)
- [ ] Choose appropriate representation (list vs matrix)
- [ ] Identify and solve dependency problems (topological sort)
- [ ] Test bipartite property (2-coloring)
- [ ] Solve connected component problems

Interview Readiness

- [ ] Solve LeetCode 200 (Islands) in < 10 minutes
- [ ] Solve LeetCode 207 (Course Schedule) in < 15 minutes
- [ ] Explain graph choice and algorithm clearly
- [ ] Handle all edge cases (empty graph, isolated nodes, cycles)
- [ ] Write clean, readable code with comments
- [ ] Discuss time/space complexity confidently
- [ ] Know when each algorithm is better than alternatives

Quick Reference: Problem Categories

When to Use BFS

- ✓ Shortest path in unweighted graph
- ✓ Level-order traversal
- ✓ Nearest neighbor search
- ✓ Connected components (simpler than DFS)
- ✓ Bipartite testing (simpler than DFS)
- ✓ Multi-source shortest path

When to Use DFS

- ✓ Topological sort (post-order)
- ✓ Cycle detection
- ✓ Path existence check
- ✓ Permutation/combination generation
- ✓ Backtracking problems
- ✓ SCC finding
- ✓ Cache-locality performance

When to Use Topological Sort

- ✓ Task scheduling with dependencies
- ✓ Build system compilation
- ✓ Course prerequisites
- ✓ Dependency resolution
- ✓ Spreadsheet cell computation order

When to Use Connected Components

- ✓ Find isolated groups
- ✓ Social network clustering
- ✓ Island/region counting
- ✓ Network segmentation

Integration with Rest of Curriculum

Week 08 → Week 09 Bridge

Week 08 (Today): Understand graph structure and traversal

Week 09 (Next): Shortest paths in weighted graphs

- **Dijkstra:** Needs BFS understanding, adds priority queue
- **Bellman-Ford:** Needs DFS/cycle detection
- **Floyd-Warshall:** All-pairs shortest path

Connection: Week 08 is foundation. Week 09 extends to weighted graphs using these traversals.

Week 08 → Week 10+ Bridge

DAG DP (Week 11):

- Requires topological sort (Week 08) as preprocessing
- Many DP problems are disguised as DAGs

Example: Longest path in DAG

1. Topological sort (Week 08)
2. DP: dp[v] = max(dp[u] + weight(u,v)) for all u→v

Supplementary Materials

5 Mental Models to Remember

1. **BFS** = **Ripples in Water**
    - Start at source, expand outward level by level
    - First reach = shortest distance (unweighted)
2. **DFS** = **Maze Exploration**
    - Go deep one path, backtrack when stuck
    - Recursive stack naturally handles backtracking
3. **Topological Sort** = **Layering**
    - Layer nodes by dependency depth
    - Process no dependencies first, then dependents
4. **Bipartite** = **2-Coloring**
    - Color nodes with 2 colors, opposite colors at edges
    - If conflict (same color at edge endpoints), not bipartite
5. **SCC** = **Mutual Reachability**
    - Nodes that reach each other form component
    - SCCs partition directed graph structure

Resources for Deep Learning

Videos:

- MIT 6.006 Lecture 13: Breadth-First Search
- MIT 6.006 Lecture 14: Depth-First Search

Interactive Tools:

- VisuAlgo - Graph Algorithms Visualizer
- Graph Online - Drawing & Algorithm Simulation

Books:

- CLRS "Introduction to Algorithms" Chapters 22-23 (Graph algorithms)
- DPV "Algorithms" Chapter 3 (Graphs)

Advanced Topics (Optional Exploration)

1. Bidirectional BFS

Idea: Start BFS from both source and target, meet in middle

Benefit: Reduces search space by ~50%

Use case: Navigation apps with fixed start/end

2. Iterative Deepening DFS

Idea: DFS with increasing depth limits

Benefit: Uses less memory than BFS, finds shortest path eventually

Use case: Memory-constrained systems

3. Graph Compression

Idea: Collapse SCCs to single super-nodes

Benefit: Reveals DAG structure, enables DP on original graph

Use case: Large graphs with many cycles

Week 08 Mastery Rubric (Self-Assessment)

Rate yourself 1-5 on each dimension (1 = beginner, 5 = expert):

Dimension	Rating	Checkpoint
Terminology	[ ]	Can explain directed/undirected, weighted/unweighted, DAG, SCC
BFS Implementation	[ ]	Code BFS from memory in < 5 minutes, no errors
DFS Implementation	[ ]	Code DFS (recursive + iterative) in < 5 minutes, no errors
Traversal Tracing	[ ]	Trace BFS/DFS on paper for 8-node graph without errors
Topological Sort	[ ]	Both DFS post-order and Kahn's algorithm, understand cycle detection
Problem Recognition	[ ]	Instantly recognize when to use BFS vs DFS vs topological sort
Edge Cases	[ ]	Handle isolated nodes, disconnected components, cycles, self-loops
Performance	[ ]	Confident explaining O(V+E) time and O(V) space for all algorithms

Target: Score 4+ in most dimensions for mastery

✅ FINAL CHECKLIST: Ready for Week 09?

Before moving to shortest paths, verify:

Conceptual:

- [ ] Understand BFS level-by-level traversal
- [ ] Understand DFS depth-first with backtracking
- [ ] Know topological sort and cycle detection
- [ ] Understand connected components and bipartite

Implementation:

- [ ] BFS coded and tested
- [ ] DFS (recursive and iterative) coded and tested
- [ ] Cycle detection works
- [ ] Topological sort produces correct order

Problem-Solving:

- [ ] Solved 5+ BFS problems (LeetCode 102, 200, 286, etc.)
- [ ] Solved 5+ DFS problems (LeetCode 200, 207, etc.)
- [ ] Solved topological sort problem (LeetCode 207, 210)
- [ ] Solved bipartite problem (LeetCode 785)

Performance:

- [ ] Code runs in < 1ms on graphs with 10K nodes
- [ ] Memory usage is reasonable (not storing redundant data)
- [ ] No unnecessary traversals or O(V^2) operations

If all checked: **Ready for Week 09 (Shortest Paths)!**

Learning Path Recommendations

Path 1: Quick Review (1 hour)

1. Skim Section 1 (Context & Motivation)
2. Read Section 3 Mental Models only (skip implementations)
3. Study the Quick Reference section
4. Review mastery checklist

Outcome: Understand concepts, not ready for interviews

Path 2: Focused Learning (6 hours)

1. Read Sections 1-3 completely
2. Code all 6 C# implementations yourself
3. Solve Stage 1 problems (LeetCode Easy/Medium)
4. Self-check with mastery rubric

Outcome: Solid understanding, ready for interviews

Path 3: Deep Mastery (15+ hours)

1. Work through Sections 1-5 sequentially
2. Code implementations multiple times until muscle memory
3. Solve all Stage 1-3 problems
4. Study real-world applications (Google Maps, LinkedIn, compiler)
5. Explore optional advanced (SCCs, bidirectional BFS)
6. Rate yourself 4+ on mastery rubric

Outcome: Expert-level understanding, ready to teach others

Path 4: Interview Prep (8 hours)

1. Rapid review Section 3 implementations
2. Solve representative problems: LeetCode 200, 207, 785
3. Time yourself: should solve in < 30 minutes
4. Practice explaining algorithm choice and complexity
5. Review edge cases and common mistakes

Outcome: Interview-ready, confident in technical interviews

Conclusion

Week 08 is the foundation of graph algorithms. BFS and DFS are the two pillars on which all other graph algorithms build. By mastering these patterns, you unlock:

- **Week 09:** Shortest paths (Dijkstra, Bellman-Ford)
- **Week 09:** Minimum spanning trees
- **Week 11:** DAG DP (dynamic programming on directed acyclic graphs)
- **Interviews:** 30% of graph questions are Week 8 algorithms

Your Action: Pick a learning path above, commit to it, and code through the implementations. Don't just read—write code. The difference between understanding and mastery is the code you write.

By end of this week, you should feel:

- Comfortable modeling problems as graphs
- Confident coding BFS and DFS without notes
- Able to recognize which algorithm solves which problem
- Ready for Week 09 (weighted graphs)

Next week: Dijkstra's algorithm and shortest paths. Everything you learn here applies directly there.

Happy coding!

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- **Prerequisite:** Week 01-07 completed
- **Next:** Week 09 (Shortest Paths & MST)