Jacobian_approach_alt

January 9, 2024

1 Finding Jacobian

What is the change in Intensity for a partial change in any of the TMPs. In this notebook we find an analytical expression(Somewhat) and compare that to a numerical approach. We also look at the dependence of these derivatives on the rest of TMPs and the system itself!

[ ]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from inverse_modelling_tfo.tools.s_based_intensity_datagen import MU_MAP_BASE1,␣
↪MU_MAP_BASE2
from inverse_modelling_tfo.tools.optical_properties import get_tissue_mu_a
from tfo_sensitivity.data import load_raw
from tfo_sensitivity.jacobian import RegularDerivative,␣
↪PartialBloodJacobianMuAEqn, NormalizedDerivative, LogDerivative,␣
↪FullBloodJacobianMuAEqn
import seaborn as sns

plt.style.use('seaborn')

maternal_wall_thickness, uterus_thickness, wave_int = 20, 5, 1
raw_sim_data_path = load_raw(maternal_wall_thickness, uterus_thickness,␣
↪wave_int)
raw_sim_data = pd.read_pickle(raw_sim_data_path)
# Create SDD column!
raw_sim_data['SDD'] = raw_sim_data['X'] - 100
all_sdd = raw_sim_data['SDD'].unique()
```

[ ]:
```
# Preparing the mu_a equations used for Jacobians
maternal_blood_volume_fraction = 0.2
fetal_blood_volume_fraction = 0.22

# Use 1 of theses 2 equations
mu_a_eqn = PartialBloodJacobianMuAEqn(maternal_blood_volume_fraction,␣
↪fetal_blood_volume_fraction)
# mu_a_eqn = FullBloodJacobianMuAEqn()
```

```
jacboian_types = {
    'reg_der': RegularDerivative,
    'norm_der': NormalizedDerivative,
    'log_der': LogDerivative
}
JacobianCalculator = jacboian_types['norm_der']
```

[ ]:
```
# Plotting parameters
# plt.rcParams['figure.dpi'] = 150    # Smaller plot
plt.rcParams['figure.dpi'] = 700    # Paper-ready plots
```

2 Values Ranges Used in Simulation

Variable	Range(lower)	Range(Upper)	Point Count
Materna Sturation	0.9	1.0	5
Maternal Hb Conc	11	15	5
Fetal Saturation	0.2	0.6	5
Fetal Hb Conc	11	15	5
Maternal BVF	0.2		
Fetal BVF	0.22		

2.1 Jacobian For saturation

From my derivations,

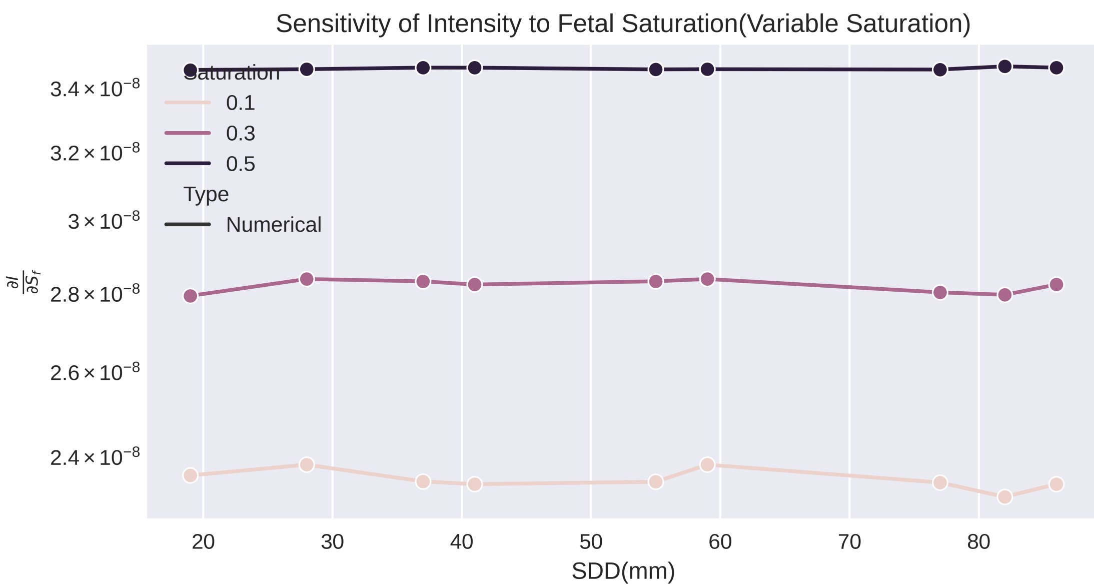
$$\frac{\delta I}{\delta S} = -(\epsilon_{Hbo} - \epsilon_{HHb}) \times c \times \sum(L_i I_i)$$

[ ]:
```
MATERNAL_Hb = 11.0
MATERNAL_SAT = 1.0
FETAL_SAT = 0.60
FETAL_Hb = 11.0
DELTA = 0.00001

data_table = pd.DataFrame(columns=["Saturation", "Derivative", "SDD", "Type"])
all_fetal_sat = np.arange(0.1, 0.65, 0.2)
base_mu_map = MU_MAP_BASE1.copy() if wave_int == 1 else MU_MAP_BASE2

for sdd_index in np.arange(2, 20, 2):
    SDD = all_sdd[sdd_index]
    filtered_photon_data = (raw_sim_data[raw_sim_data["SDD"] == SDD]).copy()
    for fs in all_fetal_sat:
        fs = round(fs, 2)  # np.arange sometimes creates weird numbers... round␣
 ↪to 2 decimal places
        dI = JacobianCalculator(
            raw_sim_data,
```

```
            sdd_index,
            base_mu_map,
            DELTA,
            "FS",
            all_sdd,
            MATERNAL_Hb,
            MATERNAL_SAT,
            FETAL_Hb,
            fs,
            wave_int,
            mu_a_eqn,
        ).calculate_jacobian()

        # Adding to Table
        new_row1 = {"Saturation": fs, "Derivative": dI, "SDD":␣
↪all_sdd[sdd_index], "Type": "Numerical"}
        data_table.loc[len(data_table)] = new_row1
```

[ ]:
```
plt.figure(figsize=(8, 4))
plot = sns.lineplot(data=data_table, x='SDD', y='Derivative', hue='Saturation',␣
 ↪style='Type', marker='o')
plt.title('Sensitivity of Intensity to Fetal Saturation(Variable Saturation)')
plt.yscale('log')
plt.xlabel('SDD(mm)')
plt.ylabel(r'$\frac{\partial I}{\partial S_f} $')
```

[ ]:
```
Text(0, 0.5, '$\\frac{\\partial I}{\\partial S_f} $')
```

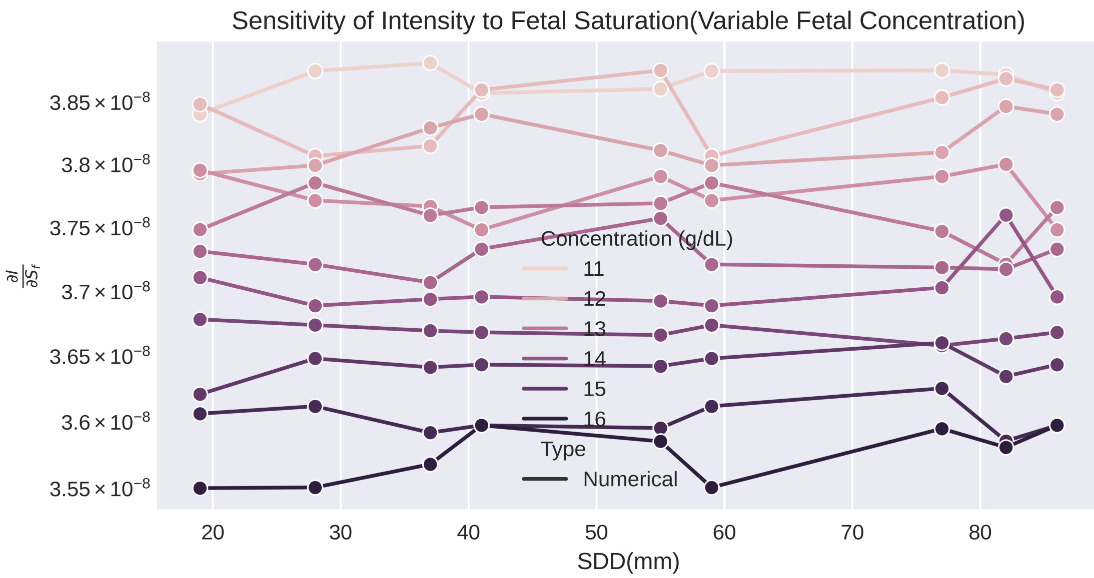


Remarks : This plot is done for a constant c. The partial derivative of the first term(The $\frac{\partial log(|A(c)|)}{\partial c}$) does not have any term related to SDD. According to that part, the derivative is constant. But clearly, the derivative is not. So this would be the effect of the second term.

[ ]:
```python
data_table = pd.DataFrame(columns=["Concentration (g/dL)", "Derivative", "SDD",␣
↪"Type"])
all_fetal_c = np.arange(11.0, 16.1, 0.5)
all_fetal_sat = np.arange(0.1, 0.65, 0.2)
base_mu_map = MU_MAP_BASE1.copy() if wave_int == 1 else MU_MAP_BASE2

for sdd_index in np.arange(2, 20, 2):
    SDD = all_sdd[sdd_index]
    filtered_photon_data = (raw_sim_data[raw_sim_data["SDD"] == SDD]).copy()
    for fc in all_fetal_c:
        dI = JacobianCalculator(
            raw_sim_data,
            sdd_index,
            base_mu_map,
            DELTA,
            "FS",
            all_sdd,
            MATERNAL_Hb,
            MATERNAL_SAT,
            fc,
            FETAL_SAT,
            wave_int,
            mu_a_eqn,
        ).calculate_jacobian()
        # Adding to Table
        new_row1 = {"Concentration (g/dL)": fc, "Derivative": dI, "SDD":␣
↪all_sdd[sdd_index], "Type": "Numerical"}
        data_table.loc[len(data_table)] = new_row1
```

[ ]:
```python
plt.figure(figsize=(8, 4))
plot = sns.lineplot(data=data_table, x='SDD', y='Derivative',␣
↪hue='Concentration (g/dL)', style='Type', marker='o')
plt.title('Sensitivity of Intensity to Fetal Saturation(Variable Fetal␣
↪Concentration)')
plt.yscale('log')
plt.xlabel('SDD(mm)')
plt.ylabel(r'$\frac{\partial I}{\partial S_f} $')
```

[ ]:
```
Text(0, 0.5, '$\\frac{\\partial I}{\\partial S_f} $')
```



2.2 Jacobian For Concentration

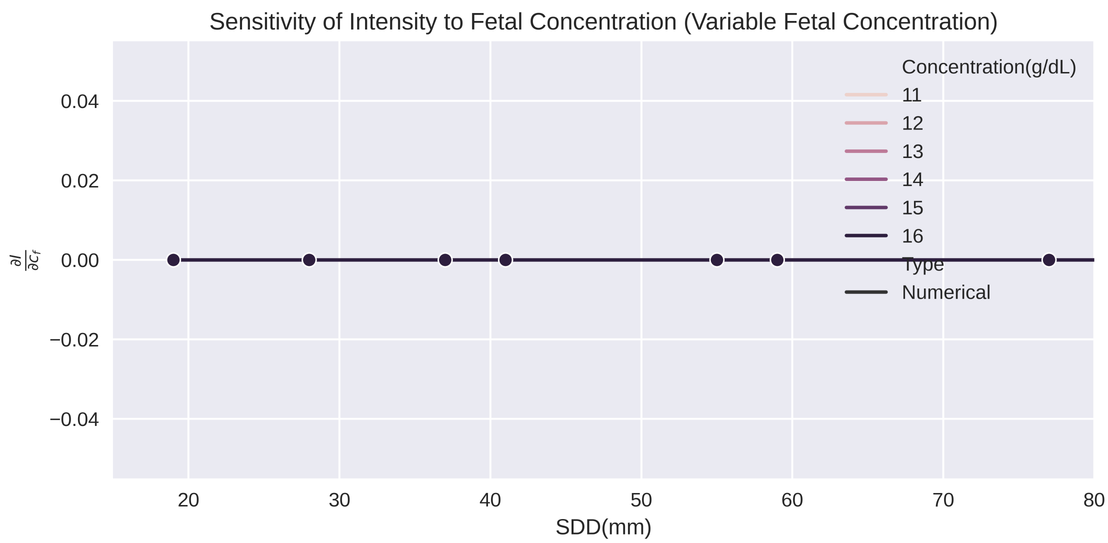
From my derivations,

$$\frac{\delta I}{\delta c} = -\epsilon \times \sum (L_i I_i)$$

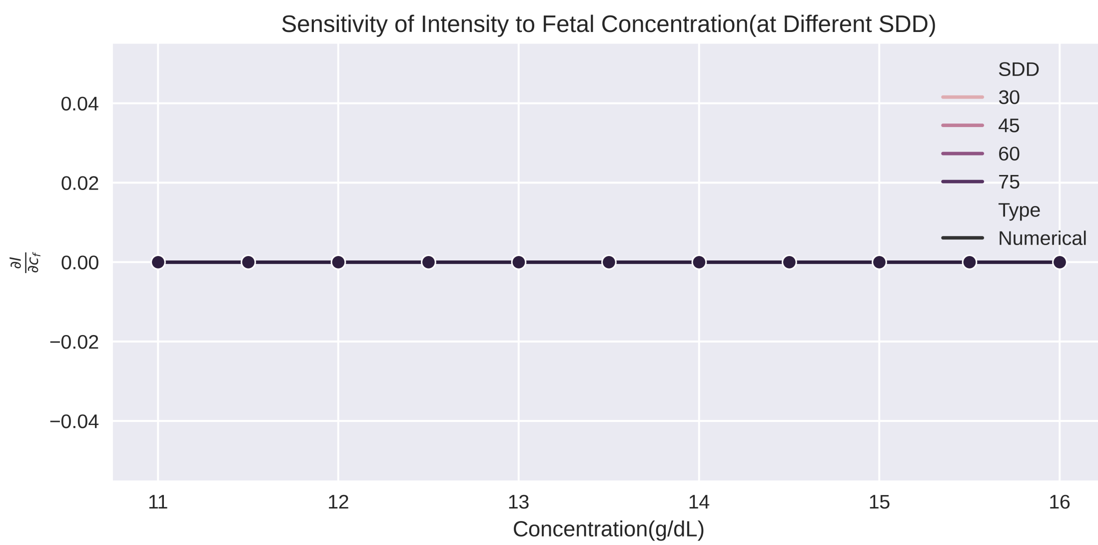
```
[ ]: data_table = pd.DataFrame(columns=["Concentration(g/dL)", "Derivative", "SDD",␣
      ↪"Type"])
     all_fetal_conc = np.arange(11.0, 16.1, 0.5)
     base_mu_map = MU_MAP_BASE1.copy() if wave_int == 1 else MU_MAP_BASE2


     for sdd_index in np.arange(2, 20, 2):
         SDD = all_sdd[sdd_index]
         filtered_photon_data = (raw_sim_data[raw_sim_data["SDD"] == SDD]).copy()
         for fc in all_fetal_conc:
             dI = JacobianCalculator(
                 raw_sim_data,
                 sdd_index,
                 base_mu_map,
                 DELTA,
                 "FC",
                 all_sdd,
                 MATERNAL_Hb,
                 MATERNAL_SAT,
                 fc,
                 FETAL_SAT,
                 wave_int,
```

```
        ).calculate_jacobian()
        new_row1 = {"Concentration(g/dL)": fc, "Derivative": dI, "SDD":␣
↪all_sdd[sdd_index], "Type": "Numerical"}
        data_table.loc[len(data_table)] = new_row1
```

[ ]:
```
plt.figure(figsize=(8, 4))
sns.lineplot(data=data_table, x='SDD', y='Derivative', hue='Concentration(g/
↪dL)', style='Type', marker='o')
plt.title('Sensitivity of Intensity to Fetal Concentration (Variable Fetal␣
↪Concentration)')
plt.legend()
# plt.yscale('log')
plt.xlabel('SDD(mm)')
plt.ylabel(r'$\frac{\partial I}{\partial c_f} $')
plt.xlim([15, 80])    # Avoiding a weird noise artifact
plt.tight_layout()
```



[ ]:
```
plt.figure(figsize=(8, 4))
sns.lineplot(data=data_table, x='Concentration(g/dL)', y='Derivative',␣
↪hue='SDD', style='Type', marker='o')
plt.title('Sensitivity of Intensity to Fetal Concentration(at Different SDD)')
plt.legend()
# plt.yscale('log')
plt.xlabel('Concentration(g/dL)')
plt.ylabel(r'$\frac{\partial I}{\partial c_f} $')
plt.tight_layout()
```



[ ]:
```
data_table = pd.DataFrame(columns=["Saturation", "Derivative", "SDD", "Type"])
all_fetal_sat = np.arange(0.10, 0.60, 0.10)
base_mu_map = MU_MAP_BASE1.copy() if wave_int == 1 else MU_MAP_BASE2

for sdd_index in np.arange(2, 20, 2):
    SDD = all_sdd[sdd_index]
    filtered_photon_data = (raw_sim_data[raw_sim_data["SDD"] == SDD]).copy()
    for fs in all_fetal_sat:
        fs = round(fs, 2)
        dI = JacobianCalculator(
            raw_sim_data,
            sdd_index,
            base_mu_map,
            DELTA,
            "FC",
            all_sdd,
            MATERNAL_Hb,
            MATERNAL_SAT,
            FETAL_Hb,
            fs,
            wave_int,
            mu_a_eqn,
        ).calculate_jacobian()

        # Adding to Table
        new_row1 = {"Saturation": fs, "Derivative": dI, "SDD":␣
↪all_sdd[sdd_index], "Type": "Numerical"}
        data_table.loc[len(data_table)] = new_row1
```

[ ]:
```python
plt.figure(figsize=(8, 4))
sns.lineplot(data=data_table, x='SDD', y='Derivative', hue='Saturation',␣
 ↪style='Type', marker='o')
plt.title('Sensitivity of Intensity to Fetal Concentration(Variable Fetal␣
 ↪Saturation)')
plt.legend()
# plt.yscale('log')
plt.xlabel('SDD(mm)')
plt.ylabel(r'$\frac{\partial I}{\partial c_f} $')
plt.tight_layout()
```