Advanced Marine Hydrodynamics NA61202 3-1-0

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Lecture 9: Elementary Flows Part II

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Overview

Doublet Flow

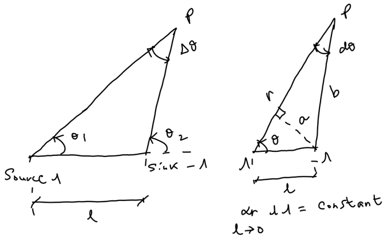

Figure: Schematic of a doublet flow.

- For the source sink pair, $\psi = \frac{\Lambda}{2\pi}(\theta_1 - \theta_2) = -\frac{\Lambda}{2\pi}\Delta\theta$.
- In the limit while $l \to 0$ we obtain a special type of flow called a doubet flow. The strength of the doublet flow is $\kappa = l\Lambda$.
- The stream function is obtained by $\lim_{l\to 0}\left(-\frac{l}{2\pi}\Delta\theta\right)$.

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- From geometry, $a = bd\theta$ and so $d\theta = \frac{a}{b} = \frac{l\sin(\theta)}{r - l\cos(\theta)}$.
- Thus, for the doublet, $\psi = \lim_{l \to 0} \frac{l\sin(\theta)}{r - l\cos(\theta)} = -\frac{\kappa}{2\pi}\frac{\sin(\theta)}{r}$.
- The velocity potential $\phi = \frac{\Lambda}{2\pi}\ln(r) - \frac{\Lambda}{2\pi}\ln(b) = \frac{\Lambda}{2\pi}\ln\left(\frac{r}{b}\right) = \frac{\Lambda}{2\pi}\ln\left(\frac{r}{r - l\cos(\theta)}\right)$.
- The above can be re-expressed as $\phi = -\frac{\Lambda}{2\pi}\ln\left(1 - \frac{l\cos(\theta)}{r}\right)$.
- The natural logarithm now can be expanded in an infinite series. In the limit as $l \to 0$, all terms after first order can be neglected.
- Thus, we have $\phi = \frac{\kappa}{2\pi}\frac{\cos(\theta)}{r}$.

Streamline patterns for the doublet flow

- We recall that the streamfunction for the doublet flow is $\psi = -\frac{\kappa}{2\pi}\frac{\sin(\theta)}{r}$ which we can take as a constant say C for each streamlines.
- We transform this representation into a Cartesian system. So, we substitute $\sin(\theta)$ by $\frac{z}{r}$ and let the negative sign be absorbed by C. Also $r^2 = x^2 + z^2$.
- Therefore, we have for the streamlines $\bar{\kappa}\frac{z}{x^2+z^2} = C$ where $\bar{\kappa} = \frac{\kappa}{2\pi}$.
- This gives, $x^2 + z^2 - \frac{\bar{\kappa}}{C}z = 0$. This represents a family of circles.
- For $x = 0$ there are two values of z one of which is zero. And the centers of the circles fall on the z axis. On the circle, where $z = 0$, x has to be zero for the constant C.
- Thus the family of the circles formed due to different values of C must be tangent at the x axis at the origin.

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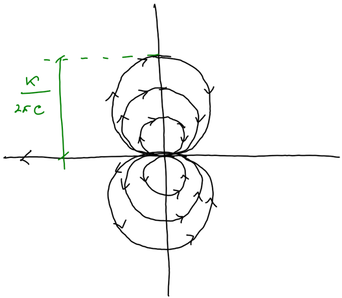

Figure: Streamlines for the doublet flow.

- We put the source to the left of origin and the sink is at the right of the origin. Thus the direction of the flow is out of the origin to the left and then back to sink through right.
- If we swapped the source sink pair, the flow direction would have been opposite.
- The source and the sink although being exactly of opposite strengths, does not annihilate each other while being superimposed. Because we assumed $\kappa = l\Lambda$ is constant. So, while $l \to 0$, $\Lambda \to \infty$. Thus at the origin what we have is merging of two infinite objects leading to a finite value !

Nonlifting flow over a circular cylinder

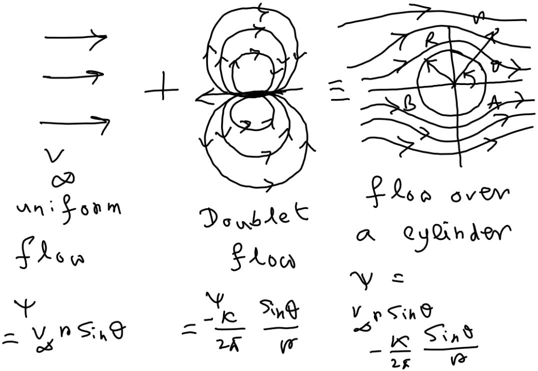

Figure: Superposition of uniform with the doublet flow.

The stream function is given by $\psi = V_\infty r \sin(\theta)\left(1 - \frac{R^2}{r^2}\right)$ where $R^2 = \frac{\kappa}{2\pi V_\infty}$.

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Therefore, the velocity components are $V_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} = V_\infty \cos(\theta)\left(1 - \frac{R^2}{r^2}\right)$ and $V_\theta = -\frac{\partial \psi}{\partial r} = -V_\infty \sin(\theta)\left(1 + \frac{R^2}{r^2}\right)$.

The stagnation point for uniform+ doublet flow

The stagnation point is found by setting $V_r = 0$ and $V_\theta = 0$. These are points A and B in the Figure.

Assignment

Can you show by solving $V_r = V_\infty \cos(\theta)\left(1 - \frac{R^2}{r^2}\right) = 0$ and $V_\theta = -V_\infty \sin(\theta)\left(1 + \frac{R^2}{r^2}\right) = 0$ that the stagnation points are at $(R, 0)$ and (R, π) ?

The stagnation streamline for the uniform+doublet flow

- Substitution for the stagnation points into the equation of the streamline gives $\psi = V_\infty r \sin(\theta)\left(1 - \frac{R^2}{r^2}\right) = 0$ which is true for all values of θ while $r = R$.
- Note $R^2 = \frac{\kappa}{2\pi V_\infty}$ is a constant. In polar coordinates $r = R$ for a constant R gives a circle with it's center at the origin and radius R.
- This equation in also satisfied by setting $\theta = 0$ and $\theta = \pi$ for all values of r.
- Therefore, the entire horizontal axis passing through AB extending infinitely upstream and downstream is also part of the stagnation streamline.

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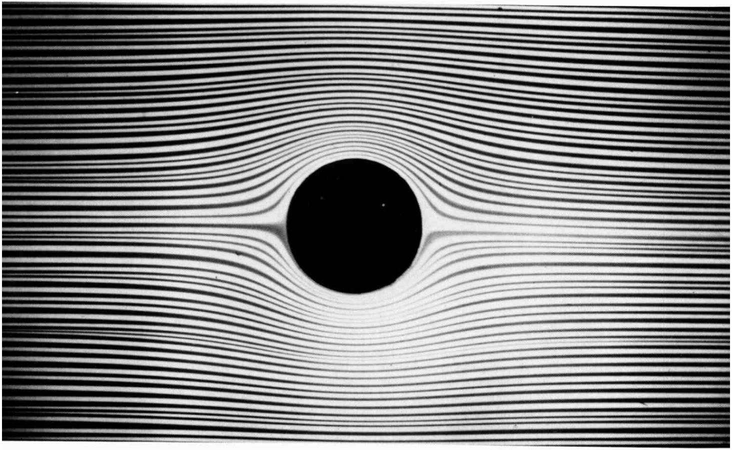
The stagnation streamline divides the flow into two regions:

- The flow inside $\psi = 0$, i.e., inside the circle comes from the doublet;
- whereas, the flow outside the streamline $\psi = 0$ comes from the uniform flow.

Therefore, for an inviscid, irrotational and incompressible flow over a circular cylinder of radius R subjected to an uniform flow of velocity V_∞ can be represented by a superposition of a similar uniform flow and doublet of strength κ where the radius of the cylinder R is $\sqrt{\frac{\kappa}{2\pi V_\infty}}$.

source: An album of Fluid Motion by Milton Van Dyke.



1. Hele-Shaw flow past a circle. Dye shows the streamlines in water flowing at 1 mm per second between glass plates spaced 1 mm apart. It is at first sight paradoxical that the best way of producing the unseparated pattern of plane potential flow past a bluff object, which would be spoiled by separation in a real fluid of even the slightest viscosity, is to go to the opposite extreme of creeping flow in a narrow gap, which is dominated by viscous forces. *Photograph by D. H. Peregrine*

Lift and Drag for the flow past cylinder

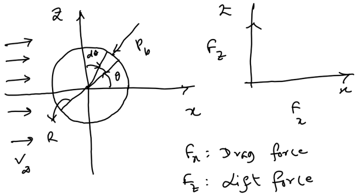


Figure: Pressure on an surface element of the cylinder.

- For any point b on the cylinder, we get from Bernoulli's equation that, $\frac{p_\infty}{\rho g} + \frac{V_\infty^2}{2g} = \frac{p_b}{\rho g} + \frac{V_b^2}{2g}$ where the subscript $'\infty'$ being used to denote properties of the uniform flow.

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- The flow can not penetrate the solid body, i.e., $V_b = V_\theta = -2V_\infty \sin(\theta)$ using $r = R$.
- Thus, $p_b = \rho g \left[\frac{V_\infty^2}{2g} + \frac{p_\infty}{\rho g} - \frac{(2V_\infty \sin(\theta))^2}{2g} \right]$.
- $F_x = -\int_S p_b \cos(\theta) \cdot ds = -\int_0^{2\pi} p_b R \cos(\theta) d\theta$. Clearly, the integration results in zero.
- Similarly, $F_z = -\int_0^{2\pi} p_b R \sin(\theta) d\theta = 0$.
- This is known as *D'Alembert* paradox.

- From Bernoulli's equation we can obtain, $\frac{p_b - p_\infty}{\frac{1}{2}\rho V_\infty^2} = 1 - 4\left(\sin(\theta)\right)^2$.
- The item on the left is known as the pressure coefficient.

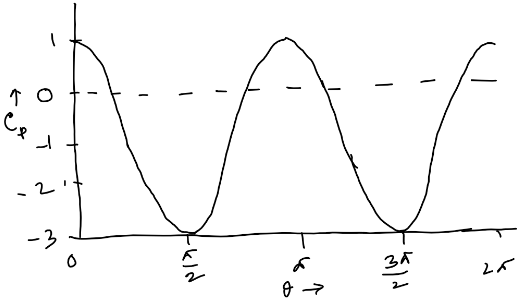

Figure: Variation of C_p over angle along the surface of the cylinder.

Source: Experiments on the flow past a circular cylinder at very high Reynolds number by Anatol Roshko, JFM, 10(3), 1961, 345-356.

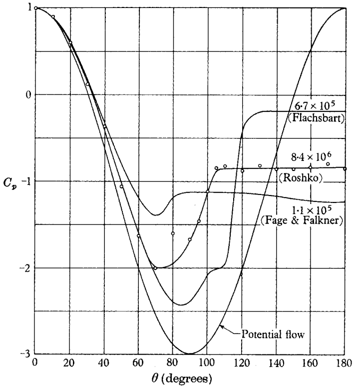

FIGURE 4. Pressure distributions.

Figure: Comparison of the results from the physical experiments with the theoretical results.