2

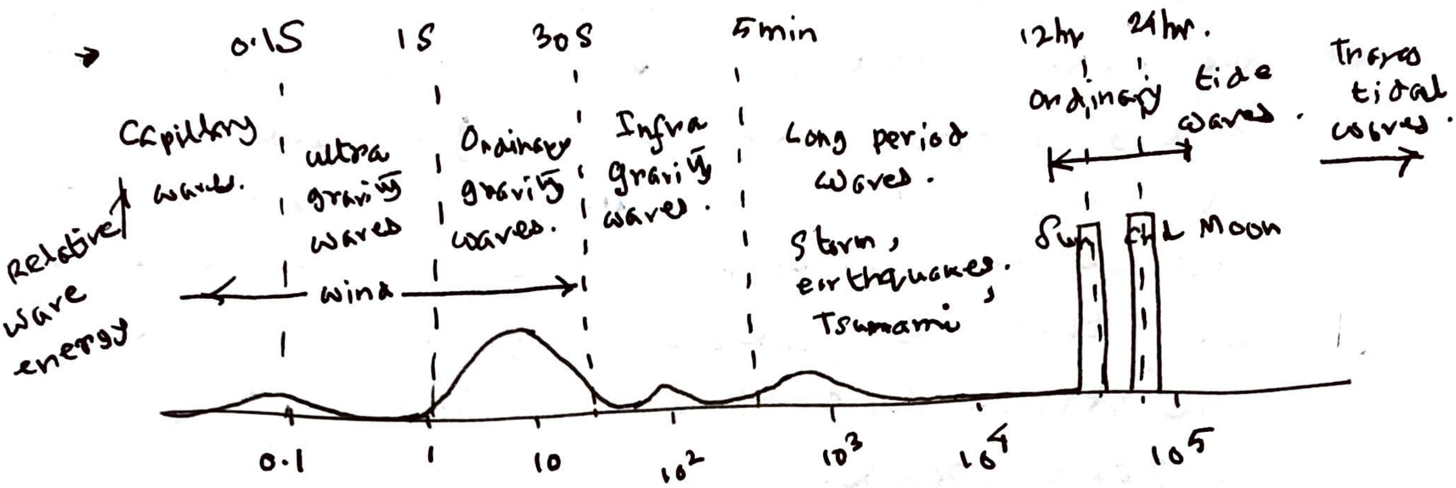
$$\hat{\tau} = \sum_{j=1}^{N} \gamma_j \, ds_j \; ;$$

$$L' = \rho V_\infty \hat{\tau} .$$

→ clearly the solution of γ_j will depend on ~~the~~ angle of attack. Thus, using vortex-panel method we can show that the lift per unit span depend on angle of ~~attack~~ attack.

2m

Introduction to 2D water waves

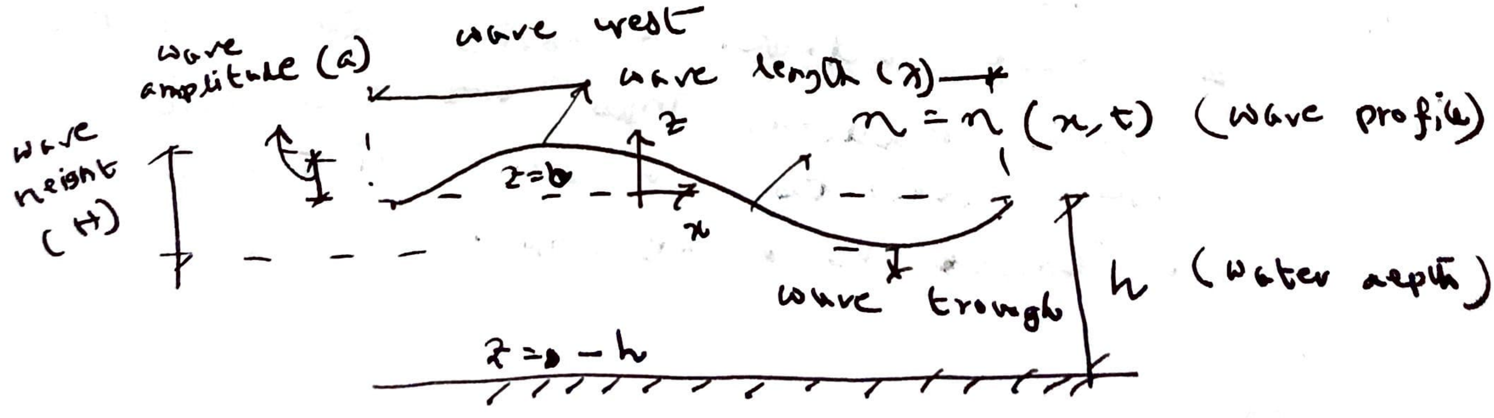


→ wave periods in seconds

- Here we mostly deal with ordinary gravity waves. These type of waves are generated due to wind and propagated under the action of gravity.

- wave period (in s) is a key parameter to ~~speci~~ specify a wave.

- How do we define a regular (2D) wave?



$$\eta = \eta(x,t) = a\cos(kx - \omega t) = \frac{H}{2}\cos(kx - \omega t)$$

where, k is wave number, $= \frac{2\pi}{\lambda}$, and

ω is wave frequency $= \frac{2\pi}{T}$, T is wave period.

- denotes the wave profile ~~varies~~ varies both over space and time. This does not necessarily applies to the fluid body beneath the wave profile.
  A wave profile moving with a speed does not mean that the fluid body also is moving with the same speed in the same direction. The interplay between the wave profile and the underlying flow kinematics is studied in some detail in ~~water wave~~ water wave mechanics.
- And this is fundamentally different from a free stream like river is moving with a constant speed.

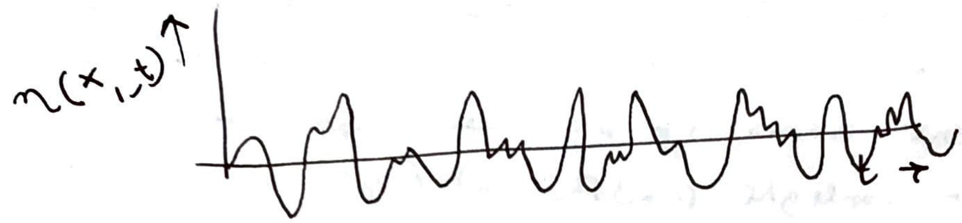
- we need to study water waves because:

(i) movement of sand depends on properties of water waves in a coast.

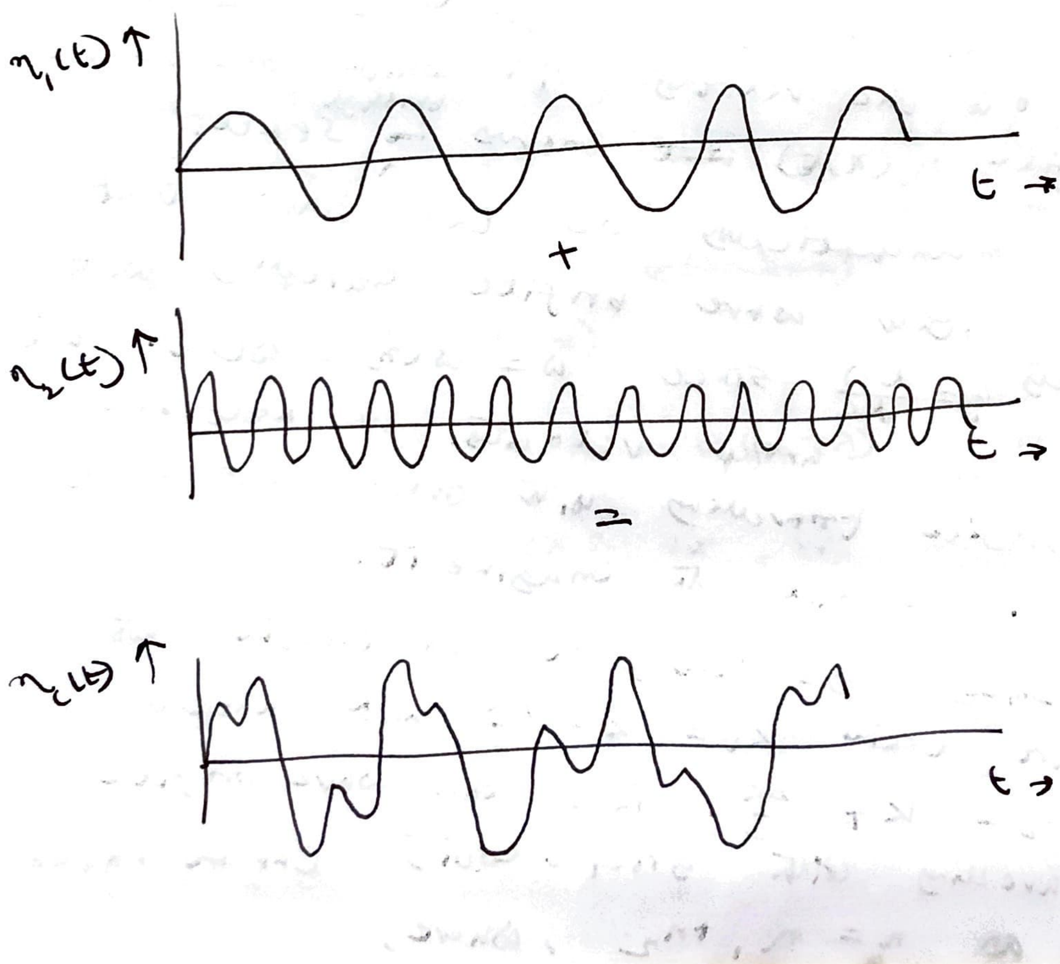
(ii) An offshore platform with a drilling depth nearly around 300 m, is very expensive ~~and~~. It must withstand large wave induced forces during a storm ~~8~~ surge.

(iii) On sea, all ships are subjected to ~~a~~ wave attack. A moving ship generates a wave pattern. Thus ~~the~~ the water wave contributes to the resistance experienced by ~~t~~ the ship.

→ If we measure wave elevation time history at a fixed station we may see that the response is not regular as we use to define key parameters like wave frequency, period or and length.



→ Still, we study regular waves because a ~~[illegible]~~ irregular wave time history that we see above can be obtained by suitable combination of a number of monochromatic (defined by a single amplitude and a single frequency) waves. This follows from the principal of superposition.



→ What is wave dispersion?

$$\lambda = \lambda(\omega) \leftrightarrow \omega = \omega(\lambda)$$

thus waves of different length travels with different velocities. This causes a wave of one length to ~~be~~ disintrigrate from another wave of ~~anoth~~ some other length. Similar property is also observed with light waves if we ~~recall~~ recall what happens when a sunlight passes through a prism.

→ One of the key outcome of dispersion in water waves is that ~~for~~ there may two different velocities for a wave. One is the group velocity and another is phase velocity.

Suppose, we have a wave of the form

$$\eta(x,t) = a\cos(kx - \omega t) \quad \text{——} \; (1)$$

its phase velocity is defined as $c = \frac{\omega}{k}$. This is the velocity with which the wave profile $\eta(x,t)$ ~~with~~ moves ~~in~~ through space.

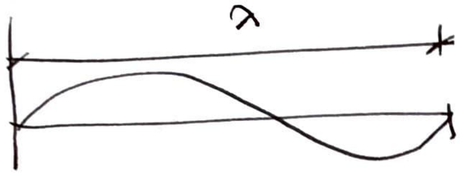
However, mathematically we can show that there is another wave profile travelling with velocity $c_g = \frac{d\omega}{dk}$ since $\omega = \omega(\lambda) \equiv \omega(k)$. c_g is known as group velocity. We don't see this profile travelling with group velocity c_g physically. We need to imagine it.

→ ~~So~~ Since, $\omega = \omega(k)$, we can imagine two wave trains $k_1 = k - \frac{\Delta k}{2}$ and another $k_2 = k + \frac{\Delta k}{2}$. Then the wave profile travelling with group velocity can be shown as $\eta_g = \eta_1 + \eta_2$, where,

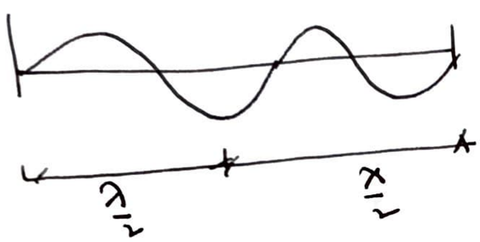
$\eta_1 = a\cos(k_1 x - \omega_1 t)$; and $\eta_2 = a\cos(k_2 x - \omega_2 t)$.

η_g conforms to a modulated wave profile travelling with speed of same as group ~~velocity~~ velocity.

[wave number :



$k = \frac{2\pi}{\lambda}$.



$k = \frac{2\pi}{\left(\frac{\lambda}{2}\right)} = \left(\frac{2\pi}{\lambda}\right) \cdot 2$

- wave number means the number of wave length λ we need to fit into the interval 2π.
If wave number is small, ~~too~~ means wave length is large meaning we can fit the wave form in a few numbers into the interval 2π.
Due to this, we see that, $\cancel{\cos(kx - \omega t)}$

$\cancel{= \cos(k(2\pi + x) - \omega t)}$

So $\cancel{\cos(kx) = \cos(2\pi + kx)}$

$\cancel{\cos(x) = \cos(2\pi + x)}$

$$\cos(kx) = \cos(k(x+\lambda)) = \cos(2\pi + kx)$$

or, $k\lambda = 2\pi$, or, $\boxed{k = \frac{2\pi}{\lambda}}$]

$$\cos(kx - \omega t) = \cos((kx + 2\pi) - \omega t) = \cos(k(x+\lambda) - \omega t)$$

or, $kx + 2\pi = kx + k\cdot\lambda$

or, $k = \frac{2\pi}{\lambda}$.