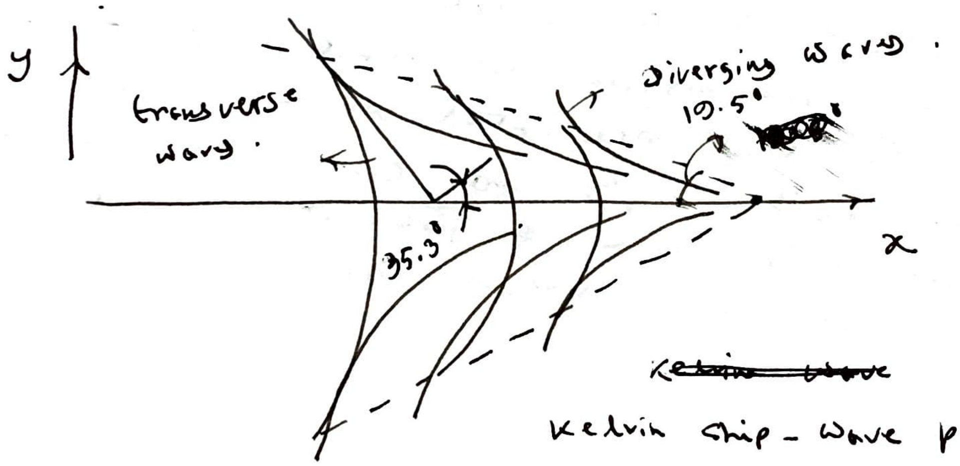
Waves generated due to moving ship: Wave making resistance

As a ship moves over a calm water, it produces wave or disturbance on the free surface. This disturbance is also directly on the pressure on the free surface. In this process of wave generation, energy is transferred from the moving ship to the water. Thus in order to estimate the power required to propell a ship with a prescribed velocity, one must taken into account the energy lost due to waves. In order to know about this energy carried away by the waves, we need to know about the wave pattern created by the moving ship.

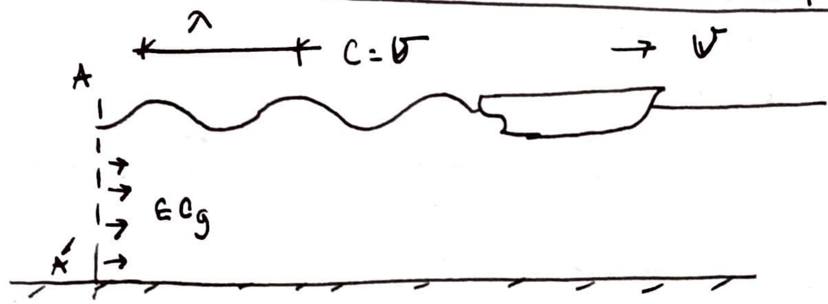


Kelvin ship-wave pattern.

We can view a moving ship as a pressure point source moving at a same speed as the speed of the ship in calm water. The entire wave system due to a moving pressure point source on calm water consists of two wave systems: one diverges out and another transverse wave follow the point source. Kelvin first found out the details about ~~geometry~~ ~~geometry of~~ the geometrical parameters describing these wave systems as shown in the above figure.

$\frac{H}{\cancel{H}}$ H_b^2 - C_{ga} . b_a

- Wave drag due to 2D ship waves in deep water



For a 2D ship moving at a speed v as shown in the above figure, in deep water, the generated wave follows the ship with a phase velocity same as the speed of the ship if we observe the ship from a fixed frame of ~~refer~~ reference. Let us define a fixed section A-A' as shown in the above figure. The energy in the region between the section A-A' and the ship accounts to ~~~~ rate $E.V$. This is balanced by the energy carried away by the ship, i.e., $D.V$, where D is the ~~~~ wave induced drag by the ship.) and the energy flux of the progressive wave. Thus,

$$\cancel{\frac{\rho g a^2}{2}\cdot\frac{V}{2} - D.V + \frac{1}{2}}$$

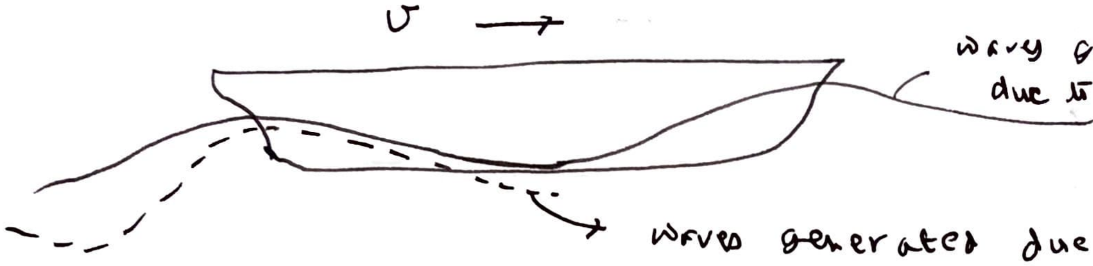
$$E.V = D.V + E.\frac{V}{2} \quad \left(\text{Since at deep water } C_g = \frac{C}{2}\right)$$

$$\text{or, } D = \frac{E}{2} = \frac{1}{4}\rho g a^2$$

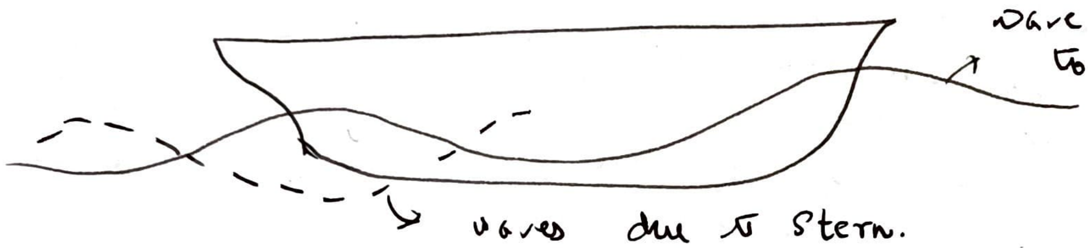
This expression of wave drag is to be understood as force per unit length. Since we are in 2D, we can take this length as ~~a~~ the beam of the ship.

The wave is generated due to a certain cross section of ship enters into free surface with a given velocity. We know there is a continuous change in the geometry of the cross section of a ship across its length. Thus, ~~at each section~~ each section generates waves of different amplitudes. This leads to the idea of constructive and destructive ~~~~ interferences of ~~waves~~ ship generated waves.

Suppose, we have two wave crests: one ~~at~~ at the bow and another at the stern.



in the case as shown above, the two wave system leads to constructive interference. The two wave crests together adds upto the ~~pres~~ hydrodynamic pressure available to move the ship forward.



in this case as shown above, there is a destructive interference between the waves generated due to the bow and the stern. In ~~this~~ this case ~~less for~~. there is ~~to~~ a loss of available pressure to move the ship and thus there is an increase in wave resistance.

Now, in order to ~~accept~~ quantify these interferences we take two wave systems as:

$$\eta_1 = a \cos(kx + \epsilon);$$

$$\eta_2 = a \cos(\cancel{kx}\ k(x + l) + \epsilon)$$

$$= a \cos(kx + kl + \epsilon)$$

Thus,

$$\eta_1 - \eta_2 = a \cos(kx + \epsilon) - a \cos(kx + kl + \epsilon)$$

$$= \text{Re}\{a.\, e^{i(kx+\epsilon)}\} - \text{Re}\{a.\, e^{i(kx+kl+\epsilon)}\}$$

$$= \text{Re}\{a.\, e^{i(kx+\epsilon)}.(1 - e^{ikl})\}$$

$$\therefore |\eta_1 - \eta_2| = a|1 - e^{ikl}| = 2a \sin\frac{kl}{2}$$

$H_c \quad H_b^2 - C_{ga} \,.\, b_a$

Since, $|1-e^{ik\ell}| = |1-\cos(k\ell) - i\sin(k\ell)|$

$$= (\{1-\cos(k\ell)\}^2 + \sin^2(k\ell))^{\frac{1}{2}}$$

$$= \{1 - 2\cos k\ell + \cos^2(k\ell) + \sin^2(k\ell)\}^{\frac{1}{2}}$$

$$= \{2 - 2\cos k\ell\}^{\frac{1}{2}}$$

$$= \{2(1-\cos k\ell)\}^{\frac{1}{2}} = \{2.2\sin^2\frac{k\ell}{2}\}^{\frac{1}{2}}$$

$$= (2.\sin\frac{k\ell}{2}).$$

Thus, the wave drag is $D = \frac{1}{4}\rho g (\eta_1-\eta_2)^2$

$$= \frac{1}{4}\rho g\, 4.a^2\sin^2\left(\frac{k\ell}{2}\right)$$

$$= \rho g a^2 \sin^2\left(\frac{k\ell}{2}\right)$$

for from wave dispersion in deep water, we know,

$\omega^2 = gk$ or,

$c = \frac{\omega}{k}$ or, $\omega = ck$.

or, $c^2k^2 = g.k$ or, $k = \frac{g}{c^2} = \frac{g}{U^2}$

or, $\lambda = \frac{2\pi}{k} = \frac{2\pi U^2}{g}$

Thus, $D = \rho g a^2 \sin^2\left(\frac{g\ell}{2U^2}\right)$

we know, $F_n = \frac{U}{\sqrt{g\ell}}$, thus, $D = \rho g a^2.\left(\frac{1}{2F_n^2}\right)$

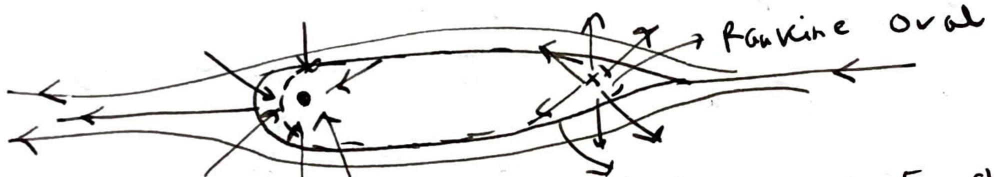
$$\equiv \rho g a^2\left(\frac{1}{2}F_n^{-2}\right)$$

Thus, the wave interferences due to ship generated waves depend on the Froude number, or the length ℓ and the speed U.

The thin ship theory

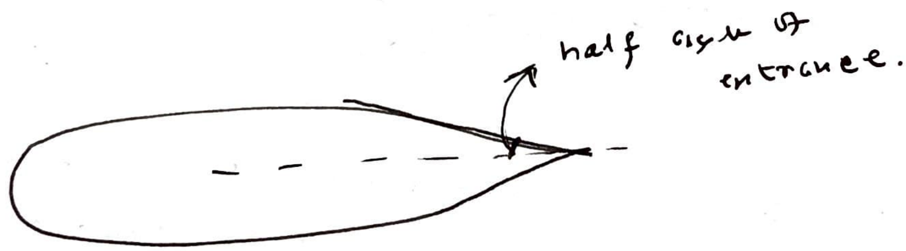
The wave making resistance can be calculated using source distribution along the length of the ship, assuming the beam of the ship is very small compared to the length of the ship. The main difficulty here is the free surface boundary conditions. And thus this problem of finding the particular distribution of source strengths giving the shape of the ship hull as a stagnation stream line is much more involved than finding source strength distributions subjected to uniform flow to find answer for lift.

~~The thin ship theory~~ Still, thin ship theory ~~is devised~~ can be applied to find out the source strengths for a moving ship and is developed by Mitchell.



plan view of the ship

According to this theory, the source strength is proportional to the slope at the water plane area. Thus, by reducing the slope of the line defining the



water plane area at the bow, we can reduce wave making resistance.