Advanced Marine Hydrodynamics NA61202 3-1-0

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Lecture 3: Some elements of kinematics of fluid flows (Part I)

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Spring 2023

Overview

1. What is kinematics ?
2. Idea of a control volume
3. Point of views in Fluid Flow Descriptions
4. Elements of Flow visualizations

Where does the kinematics fit in ?

- Mechanics
  - Statics
  - Dynamics
    - Kinetics
    - Kinematics

- Kinematics is the study of motion without accounting for the causes, i.e., the forcing.
- Example, if we know fluid velocity at one point, how we can predict pressure at the same point and vice-versa ?
- key assumptions that we make to develop theories in wave-body interaction are based on principles of flow kinematics.

A control volume

- What is a control volume ? Typical Representation.
- Why do we need a control volume ?

The Eulerian View point $\mathcal{E}$

- We fix our focus on a particular point in space and time.
- Then we track whatever is happening in this point.

The Lagrangian view point $\mathcal{L}$

- We fix our focus on a particle with an *identity*.
- Then we simply sit over this point and experience the journey with it.

An example

In a 1-D flow field, the velocity at a point may be given in $\mathcal{E}$ by $\mathrm{u} = \mathrm{x} + t$. Determine the displacement of a fluid particle whose initial position is x_0 at initial time t_0 in $\mathcal{L}$.

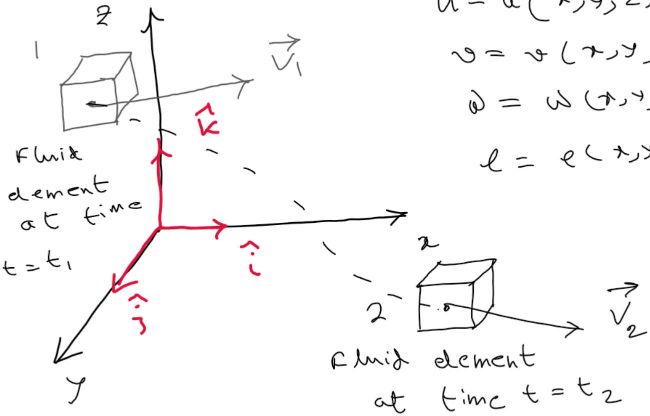
How we can know about something in $\mathcal{E}$ through $\mathcal{L}$?

Total/ substantial derivative operator in the Cartesian Coordinate Frame

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} = \frac{\partial}{\partial t} + \mathrm{V} \cdot \nabla.$$

- Derivation (physical+pure mathematical) in Cartesian coordinate frame.
- The first term is the local derivative showing local rate of change and the second terms is the convective derivative showing rate of change due to change in location.
- The final expression for total/ substantial derivative depends on the base coordinate system.

Physical Derivation of $\frac{D}{Dt}$



$$u = u(x,y,z,t)$$
$$v = v(x,y,z,t)$$
$$w = w(x,y,z,t)$$
$$\rho = \rho(x,y,z,t)$$

Contd..

- since, $\rho = \rho(x, y, z, t)$; we can write ρ_2 as an expansion around ρ_1 using a Taylor series as
$\rho_2 = \rho_1 + \left(\frac{\partial \rho}{\partial x}\right)_1 (x_2 - x_1) + \left(\frac{\partial \rho}{\partial y}\right)_1 (y_2 - y_1) + \left(\frac{\partial \rho}{\partial z}\right)_1 (z_2 - z_1) + \left(\frac{\partial \rho}{\partial t}\right)_1 (t_2 - t_1) + \text{higher order terms}$
- substracting ρ_1 from both sides and then dividing by $t_2 - t_1$, we get
$\frac{\rho_2 - \rho_1}{t_2 - t_1} = \left(\frac{\partial \rho}{\partial x}\right)_1 \frac{x_2 - x_1}{t_2 - t_1} + \left(\frac{\partial \rho}{\partial y}\right)_1 \frac{y_2 - y_1}{t_2 - t_1} + \left(\frac{\partial \rho}{\partial z}\right)_1 \frac{z_2 - z_1}{t_2 - t_1} + \left(\frac{\partial \rho}{\partial t}\right)_1$

Contd..

- $\lim_{t_2 \to t_1} \frac{\rho_2 - \rho_1}{t_2 - t_1} = \frac{D\rho}{Dt}$; $\lim_{t_2 \to t_1} \frac{x_2 - x_1}{t_2 - t_1} = u$; $\lim_{t_2 \to t_1} \frac{y_2 - y_1}{t_2 - t_1} = v$ and $\lim_{t_2 \to t_1} \frac{z_2 - z_1}{t_2 - t_1} = w$.
- Thus, we have $\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z}$.
- $\frac{D\rho}{Dt}$ is the instantaneous time rate of change of density of the fluid element as it moves trough point 1.
- This is different from $\frac{\partial \rho}{\partial t}$ which is the time rate of change of density at a fixed point 1.

Mathematical Derivation of $\frac{D}{Dt}$

- since $\rho = \rho\left(x, y, z, t\right)$ from chain rule we have $\mathrm{d}\rho = \frac{\partial \rho}{\partial x}\mathrm{d}x + \frac{\partial \rho}{\partial y}\mathrm{d}y + \frac{\partial \rho}{\partial z}\mathrm{d}z + \frac{\partial \rho}{\partial t}\mathrm{d}t.$
- which with some manipulations becomes $\frac{\mathrm{d}\rho}{\mathrm{d}t} = \frac{D\rho}{Dt} = \frac{\partial \rho}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial \rho}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t} + \frac{\partial \rho}{\partial z}\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{\partial \rho}{\partial x}u + \frac{\partial \rho}{\partial y}v + \frac{\partial \rho}{\partial z}w + \frac{\partial \rho}{\partial t}.$

State of the terms in total derivative for various flow types

- For a steady flow, $\frac{\partial}{\partial t} = 0$.
- For a uniform flow, any flow variable say $u \neq u\,(x, y, z)$.

Flow Type	Temporal derivative	Convective derivative
steady and uniform	0	0
steady and non-uniform	0	$\neq 0$
Unsteady and uniform	$\neq 0$	0
Unsteady and non-uniform	$\neq 0$	$\neq 0$

A simple example

L3A1 A 2D pressure field $p = 4x^3 - 2y^2$ is associated with a velocity field given by $\mathrm{V} = \left(x^2 - y^2 + x\right)\hat{\imath} - (2xy + y)\hat{\jmath}$. What is the rate of change of pressure at a point $(2, 1)$?

The stream lines

- This is an imaginary line in the flow field where the tangent at any point gives us the velocity at this point.
- Equation of a stream line: $\mathrm{V} \times \delta \mathrm{R}$ for a stream line segment with end-points at R_{i-1} and R_{i}.
- Why don't we use $\mathrm{V} \cdot \delta \mathrm{R}$ as an equation of a stream line ?

The path lines

A path line is the trajectory of a fluid particle for which we capture its location at various instants of time.

The streak lines

A Streak line (at a given instant of time) is the locus of many fluid particles which cross a common point in the flow field at some earlier time.

Example: Streak Lines

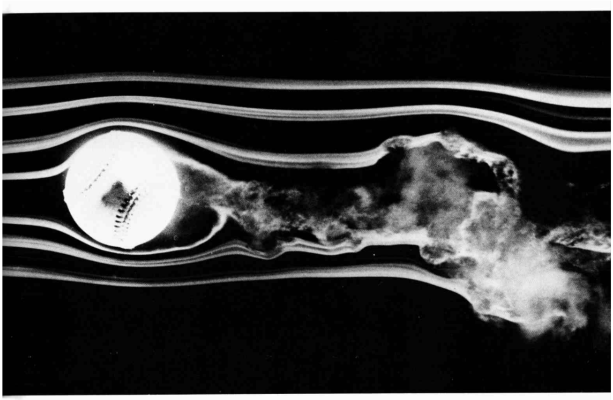


Figure: Streak lines over a spinning (in the same plane as in the image) basket ball rotating at 630 rpm in a outside flow with 77ft/s. Source: An Album of Fluid Motion by Milton Van Dyke.

Example: Path Lines

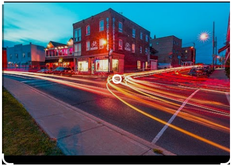


Figure: Photograph taken at a street corner using a DSLR camera and with long exposure. Source YouTube: Visual Art Photography Tutorials-10 Tips For Long Exposure Photography Light Trails.

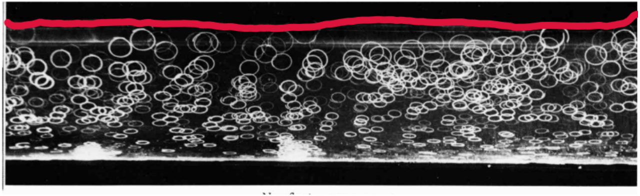


Figure: Pathlines beneath a progressive wave. Source: An Album of Fluid Motion by Milton Van Dyke.

Example: Stream Lines

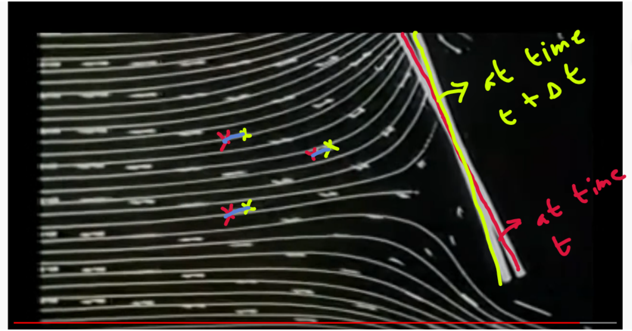


Figure: Streamlines for a flow over an oscillating plate. Source YouTube: Flow Visualization by Stephen J Kline.

Another Question

L3A2 Why for a steady flow, the steam lines, the path lines and the streak lines all has to be the same ?