Dynamic Programming

Dynamic programming is a technique that breaks the problems into sub-problems, and saves the result for future purposes so that we do not need to compute the result again. The subproblems are optimized to optimize the overall solution is known as optimal substructure property. The main use of dynamic programming is to solve optimization problems. Here, optimization problems mean that when we are trying to find out the minimum or the maximum solution of a problem. The dynamic programming guarantees to find the optimal solution of a problem if the solution exists.

The definition of dynamic programming says that it is a technique for solving a complex problem by first breaking into a collection of simpler subproblems, solving each subproblem just once, and then storing their solutions to avoid repetitive computations.

Consider an example of the Fibonacci series. The following series is the Fibonacci series:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ,…

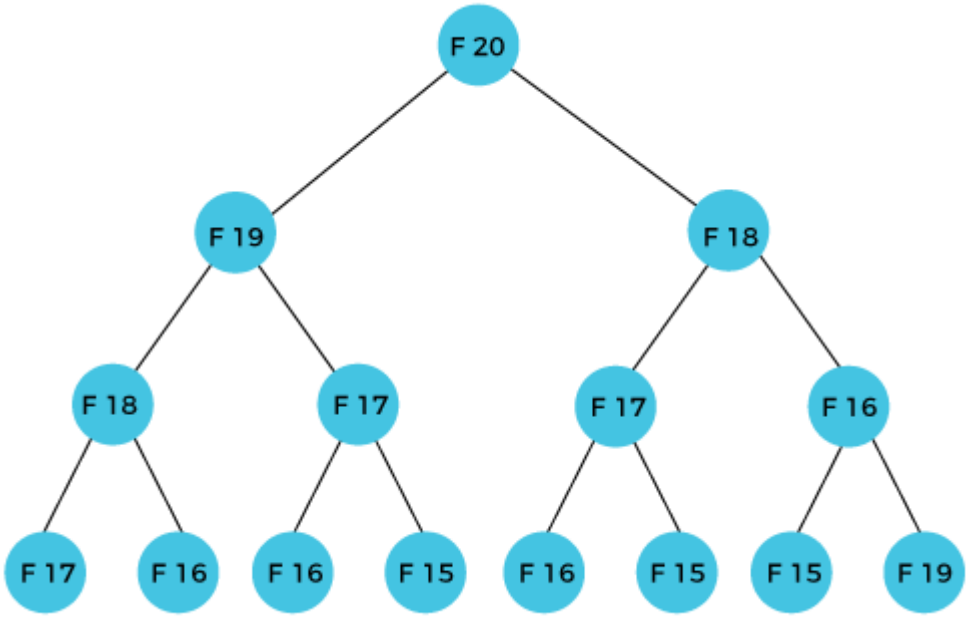
The numbers in the above series are not randomly calculated. Mathematically, we could write each of the terms using the below formula:

F(n) = F(n-1) + F(n-2),

With the base values F(0) = 0, and F(1) = 1. To calculate the other numbers, we follow the above relationship. For example, F(2) is the sum **f(0)** and **f(1),** which is equal to 1.

How can we calculate F(20)?

The F(20) term will be calculated using the nth formula of the Fibonacci series. The below figure shows that how F(20) is calculated.



As we can observe in the above figure that F(20) is calculated as the sum of F(19) and F(18). In the dynamic programming approach, we try to divide the problem into the similar subproblems. We are following this approach in the above case where F(20) into the similar subproblems, i.e., F(19) and F(18). If we recap the definition of dynamic programming that it says the similar subproblem should not be computed more than once. Still, in the above case, the subproblem is calculated twice. In the above example, F(18) is calculated two times; similarly, F(17) is also calculated twice. However, this technique is quite useful as it solves the similar subproblems, but we need to be cautious while storing the results because we are not particular about storing the result that we have computed once, then it can lead to a wastage of resources.

In the above example, if we calculate the F(18) in the right subtree, then it leads to the tremendous usage of resources and decreases the overall performance.

The solution to the above problem is to save the computed results in an array. First, we calculate F(16) and F(17) and save their values in an array. The F(18) is calculated by summing the values of F(17) and F(16), which are already saved in an array. The computed value of F(18) is saved in an array. The value of F(19) is calculated using the sum of F(18), and F(17), and their values are already saved in an array. The computed value of F(19) is stored in an array. The value of F(20) can be calculated by adding the values of F(19) and F(18), and the values of both F(19) and F(18) are stored in an array. The final computed value of F(20) is stored in an array.

How does the dynamic programming approach work?

The following are the steps that the dynamic programming follows:

- It breaks down the complex problem into simpler subproblems.
- It finds the optimal solution to these sub-problems.

- It stores the results of subproblems (memoization). The process of storing the results of subproblems is known as memorization.
- It reuses them so that same sub-problem is calculated more than once.
- Finally, calculate the result of the complex problem.

The above five steps are the basic steps for dynamic programming. The dynamic programming is applicable that are having properties such as:

Those problems that are having overlapping subproblems and optimal substructures. Here, optimal substructure means that the solution of optimization problems can be obtained by simply combining the optimal solution of all the subproblems.

In the case of dynamic programming, the space complexity would be increased as we are storing the intermediate results, but the time complexity would be decreased.

Approaches of dynamic programming

There are two approaches to dynamic programming:

- Top-down approach
- Bottom-up approach

Top-down approach

The top-down approach follows the memorization technique, while bottom-up approach follows the tabulation method. Here memorization is equal to the sum of recursion and caching. Recursion means calling the function itself, while caching means storing the intermediate results.

Advantages

- It is very easy to understand and implement.
- It solves the subproblems only when it is required.
- It is easy to debug.

Disadvantages

It uses the recursion technique that occupies more memory in the call stack. Sometimes when the recursion is too deep, the stack overflow condition will occur.

It occupies more memory that degrades the overall performance.

Let's understand dynamic programming through an example.

```
1. int fib(int n)
2. {
3.    if(n<0)
4.    error;
5.  if(n==0)
6.  return 0;
```

```
7.  if(n==1)
8.  return 1;
9.  sum = fib(n-1) + fib(n-2);
10. }
```

In the above code, we have used the recursive approach to find out the Fibonacci series. When the value of 'n' increases, the function calls will also increase, and computations will also increase. In this case, the time complexity increases exponentially, and it becomes 2^n.

One solution to this problem is to use the dynamic programming approach. Rather than generating the recursive tree again and again, we can reuse the previously calculated value. If we use the dynamic programming approach, then the time complexity would be O(n).

When we apply the dynamic programming approach in the implementation of the Fibonacci series, then the code would look like:

```
1.  static int count = 0;
2.  int fib(int n)
3.  {
4.  if(memo[n]!= NULL)
5.  return memo[n];
6.  count++;
7.    if(n<0)
8.    error;
9.   if(n==0)
10.  return 0;
11.  if(n==1)
12. return 1;
13. sum = fib(n-1) + fib(n-2);
14. memo[n] = sum;
15. }
```

In the above code, we have used the memorization technique in which we store the results in an array to reuse the values. This is also known as a top-down approach in which we move from the top and break the problem into sub-problems.

Bottom-Up approach

The bottom-up approach is also one of the techniques which can be used to implement the dynamic programming. It uses the tabulation technique to implement the dynamic programming approach. It solves the same kind of problems but it removes the recursion. If we remove the recursion, there is no stack overflow issue and no overhead of the recursive functions. In this tabulation technique, we solve the problems and store the results in a matrix.

There are two ways of applying dynamic programming:

- **Top-Down**
- **Bottom-Up**

The bottom-up is the approach used to avoid the recursion, thus saving the memory space. The bottom-up is an algorithm that starts from the beginning, whereas the recursive algorithm starts from the end and works backward. In the bottom-up approach, we start from the base case to find the answer for the end. As we know, the base cases in the Fibonacci series are 0 and 1. Since the bottom approach starts from the base cases, so we will start from 0 and 1.

Key points

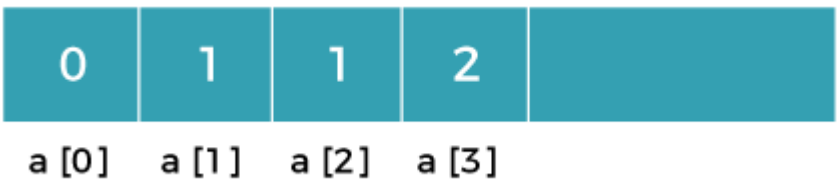
- We solve all the smaller sub-problems that will be needed to solve the larger sub-problems then move to the larger problems using smaller sub-problems.
- We use for loop to iterate over the sub-problems.
- The bottom-up approach is also known as the tabulation or table filling method.

Let's understand through an example.

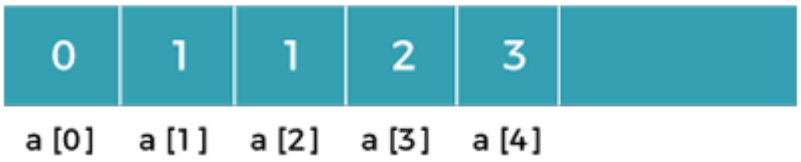
Suppose we have an array that has 0 and 1 values at a[0] and a[1] positions, respectively shown as below:

Since the bottom-up approach starts from the lower values, so the values at a[0] and a[1] are added to find the value of a[2] shown as below:

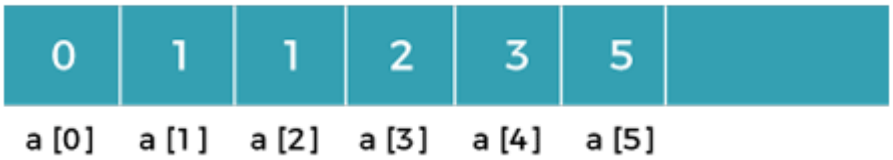
The value of a[3] will be calculated by adding a[1] and a[2], and it becomes 2 shown as below:



The value of a[4] will be calculated by adding a[2] and a[3], and it becomes 3 shown as below:



The value of a[5] will be calculated by adding the values of a[4] and a[3], and it becomes 5 shown as below:



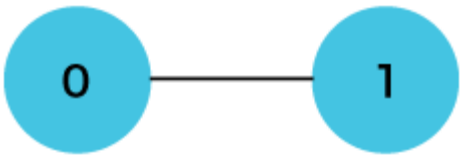
The code for implementing the Fibonacci series using the bottom-up approach is given below:

```
1.  int fib(int n)
2.  {
3.      int A[];
4.     A[0] = 0, A[1] = 1;
5.      for( i=2; i<=n; i++)
6.     {
7.          A[i] = A[i-1] + A[i-2]
8.     }
9.      return A[n];
10. }
```

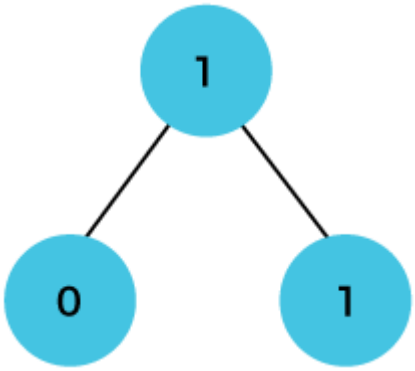
In the above code, base cases are 0 and 1 and then we have used for loop to find other values of Fibonacci series.

Let's understand through the diagrammatic representation.

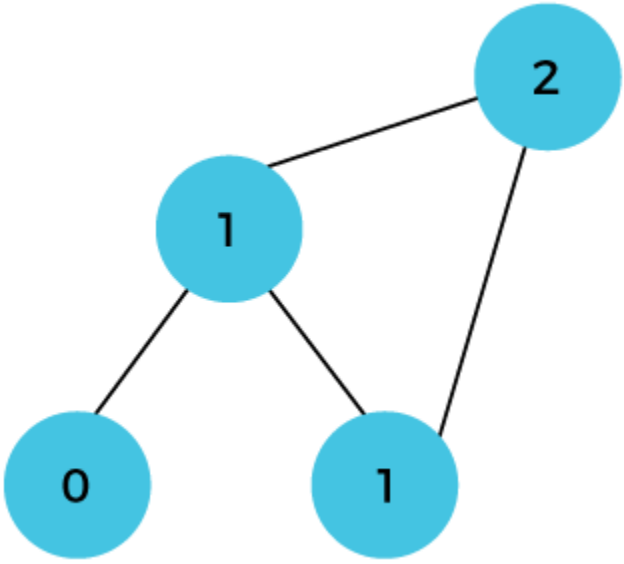
Initially, the first two values, i.e., 0 and 1 can be represented as:



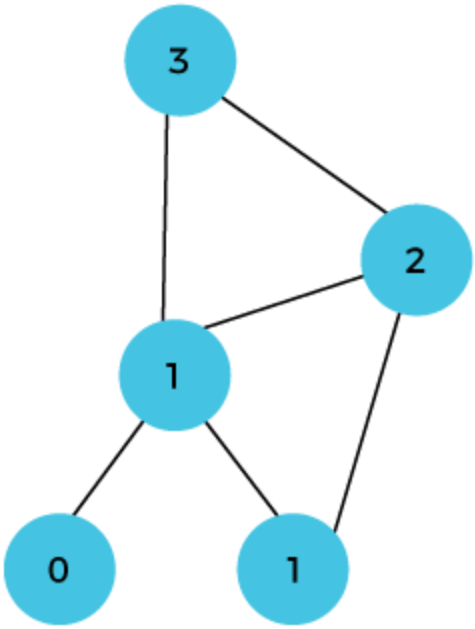
When i=2 then the values 0 and 1 are added shown as below:



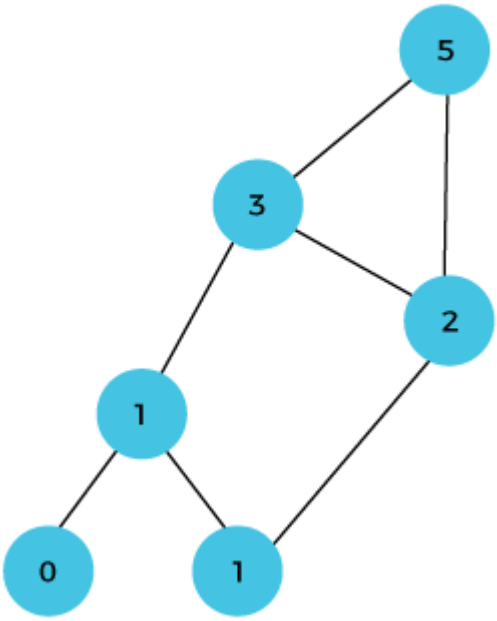
When i=3 then the values 1and 1 are added shown as below:



When i=4 then the values 2 and 1 are added shown as below:



When i=5, then the values 3 and 2 are added shown as below:



0/1 Knapsack problem

Here knapsack is like a container or a bag. Suppose we have given some items which have some weights or profits. We have to put some items in the knapsack in such a way total value produces a maximum profit.

For example, the weight of the container is 20 kg. We have to select the items in such a way that the sum of the weight of items should be either smaller than or equal to the weight of the container, and the profit should be maximum.

There are two types of knapsack problems:

- 0/1 knapsack problem
- Fractional knapsack problem

We will discuss both the problems one by one. First, we will learn about the 0/1 knapsack problem.

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DurationÂ 18:10
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What is the 0/1 knapsack problem?

The 0/1 knapsack problem means that the items are either completely or no items are filled in a knapsack. For example, we have two items having weights 2kg and 3kg, respectively. If we pick the 2kg item then we cannot pick 1kg item from the 2kg item (item is not divisible); we have to pick the 2kg item completely. This is a 0/1 knapsack problem in which either we pick the item completely or we will pick that item. The 0/1 knapsack problem is solved by the dynamic programming.

What is the fractional knapsack problem?

The fractional knapsack problem means that we can divide the item. For example, we have an item of 3 kg then we can pick the item of 2 kg and leave the item of 1 kg. The fractional knapsack problem is solved by the Greedy approach.

Example of 0/1 knapsack problem.

Consider the problem having weights and profits are:

Weights: {3, 4, 6, 5}

Profits: {2, 3, 1, 4}

The weight of the knapsack is 8 kg

The number of items is 4

The above problem can be solved by using the following method:

x_i = {1, 0, 0, 1}

= {0, 0, 0, 1}

= {0, 1, 0, 1}

The above are the possible combinations. 1 denotes that the item is completely picked and 0 means that no item is picked. Since there are 4 items so possible combinations will be:

2^4 = 16; So. There are 16 possible combinations that can be made by using the above problem. Once all the combinations are made, we have to select the combination that provides the maximum profit.

Another approach to solve the problem is dynamic programming approach. In dynamic programming approach, the complicated problem is divided into sub-problems, then we find the solution of a sub-problem and the solution of the sub-problem will be used to find the solution of a complex problem.

How this problem can be solved by using the Dynamic programming approach?

First,

we create a matrix shown as below:

	0	1	2	3	4	5	6	7	8
0									
1									
2									
3									
4									

In the above matrix, columns represent the weight, i.e., 8. The rows represent the profits and weights of items. Here we have not taken the weight 8 directly, problem is divided into sub-problems, i.e., 0, 1, 2, 3, 4, 5, 6, 7, 8. The solution of the sub-problems would be saved in the cells and answer to the problem would be stored in the final cell. First, we write the weights in the ascending order and profits according to their weights shown as below:

w_i = {3, 4, 5, 6}

p_i = {2, 3, 4, 1}

The first row and the first column would be 0 as there is no item for w=0

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0								
2	0								
3	0								
4	0								

When i=1, W=1

w_1 = 3; Since we have only one item in the set having weight 3, but the capacity of the knapsack is 1. We cannot fill the item of 3kg in the knapsack of capacity 1 kg so add 0 at M[1][1] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0							
2	0								
3	0								
4	0								

When i = 1, W = 2

w_1 = 3; Since we have only one item in the set having weight 3, but the capacity of the knapsack is 2. We cannot fill the item of 3kg in the knapsack of capacity 2 kg so add 0 at M[1][2] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0						
2	0								
3	0								
4	0								

When i=1, W=3

w_1 = 3; Since we have only one item in the set having weight equal to 3, and weight of the knapsack is also 3; therefore, we can fill the knapsack with an item of weight equal to 3. We put profit corresponding to the weight 3, i.e., 2 at M[1][3] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2					
2	0								
3	0								
4	0								

When i=1, W = 4

w_1 = 3; Since we have only one item in the set having weight equal to 3, and weight of the knapsack is 4; therefore, we can fill the knapsack with an item of weight equal to 3. We put profit corresponding to the weight 3, i.e., 2 at M[1][4] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2				
2	0								
3	0								
4	0								

When i=1, W = 5

w_1 = 3; Since we have only one item in the set having weight equal to 3, and weight of the knapsack is 5; therefore, we can fill the knapsack with an item of weight equal to 3. We put profit corresponding to the weight 3, i.e., 2 at M[1][5] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2			
2	0								
3	0								

4	0								

When i =1, W=6

w_1 = 3; Since we have only one item in the set having weight equal to 3, and weight of the knapsack is 6; therefore, we can fill the knapsack with an item of weight equal to 3. We put profit corresponding to the weight 3, i.e., 2 at M[1][6] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2		
2	0								
3	0								
4	0								

When i=1, W = 7

w_1 = 3; Since we have only one item in the set having weight equal to 3, and weight of the knapsack is 7; therefore, we can fill the knapsack with an item of weight equal to 3. We put profit corresponding to the weight 3, i.e., 2 at M[1][7] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2	2	
2	0								
3	0								
4	0								

When i =1, W =8

w_1 = 3; Since we have only one item in the set having weight equal to 3, and weight of the knapsack is 8; therefore, we can fill the knapsack with an item of weight equal to 3. We put profit corresponding to the weight 3, i.e., 2 at M[1][8] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2	2	2
2	0								
3	0								
4	0								

Now the value of 'i' gets incremented, and becomes 2.

When i =2, W = 1

The weight corresponding to the value 2 is 4, i.e., $w_2 = 4$. Since we have only one item in the set having weight equal to 4, and the weight of the knapsack is 1. We cannot put the item of weight 4 in a knapsack, so we add 0 at M[2][1] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2	2	2
2	0	0							
3	0								
4	0								

When i =2, W = 2

The weight corresponding to the value 2 is 4, i.e., $w_2 = 4$. Since we have only one item in the set having weight equal to 4, and the weight of the knapsack is 2. We cannot put the item of weight 4 in a knapsack, so we add 0 at M[2][2] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2	2	2
2	0	0	0						
3	0								
4	0								

When i =2, W = 3

The weight corresponding to the value 2 is 4, i.e., $w_2 = 4$. Since we have two items in the set having weights 3 and 4, and the weight of the knapsack is 3. We can put the item of weight 3 in a knapsack, so we add 2 at M[2][3] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2	2	2
2	0	0	0	2					
3	0								
4	0								

When i =2, W = 4

The weight corresponding to the value 2 is 4, i.e., $w_2 = 4$. Since we have two items in the set having weights 3 and 4, and the weight of the knapsack is 4. We can put item of weight 4 in a

knapsack as the profit corresponding to weight 4 is more than the item having weight 3, so we add 3 at M[2][4] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2	2	2
2	0	0	0	2	3				
3	0								
4	0								

When i = 2, W = 5

The weight corresponding to the value 2 is 4, i.e., $w_2 = 4$. Since we have two items in the set having weights 3 and 4, and the weight of the knapsack is 5. We can put item of weight 4 in a knapsack and the profit corresponding to weight is 3, so we add 3 at M[2][5] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2	2	2
2	0	0	0	2	3	3			
3	0								
4	0								

When i = 2, W = 6

The weight corresponding to the value 2 is 4, i.e., $w_2 = 4$. Since we have two items in the set having weights 3 and 4, and the weight of the knapsack is 6. We can put item of weight 4 in a knapsack and the profit corresponding to weight is 3, so we add 3 at M[2][6] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2	2	2
2	0	0	0	2	3	3	3		
3	0								
4	0								

When i = 2, W = 7

The weight corresponding to the value 2 is 4, i.e., $w_2 = 4$. Since we have two items in the set having weights 3 and 4, and the weight of the knapsack is 7. We can put item of weight 4 and 3 in a knapsack and the profits corresponding to weights are 2 and 3; therefore, the total profit is 5, so we add 5 at M[2][7] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2	2	2
2	0	0	0	0	3	3	3	5	
3	0								
4	0								

When i = 2, W = 8

The weight corresponding to the value 2 is 4, i.e., $w_2 = 4$. Since we have two items in the set having weights 3 and 4, and the weight of the knapsack is 7. We can put item of weight 4 and 3 in a knapsack and the profits corresponding to weights are 2 and 3; therefore, the total profit is 5, so we add 5 at M[2][7] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2	2	2
2	0	0	0	2	3	3	3	5	5
3	0								
4	0								

Now the value of 'i' gets incremented, and becomes 3.

When i = 3, W = 1

The weight corresponding to the value 3 is 5, i.e., $w_3 = 5$. Since we have three items in the set having weights 3, 4, and 5, and the weight of the knapsack is 1. We cannot put neither of the items in a knapsack, so we add 0 at M[3][1] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2	2	2
2	0	0	0	2	3	3	3	5	5
3	0	0							
4	0								

When i = 3, W = 2

The weight corresponding to the value 3 is 5, i.e., $w_3 = 5$. Since we have three items in the set having weight 3, 4, and 5, and the weight of the knapsack is 1. We cannot put neither of the items in a knapsack, so we add 0 at M[3][2] shown as below:

	0	1	2	3	4	5	6	7	8

0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2	2	2
2	0	0	0	2	3	3	3	5	5
3	0	0	0						
4	0								

When i = 3, W = 3

The weight corresponding to the value 3 is 5, i.e., $w_3 = 5$. Since we have three items in the set of weight 3, 4, and 5 respectively and weight of the knapsack is 3. The item with a weight 3 can be put in the knapsack and the profit corresponding to the item is 2, so we add 2 at M[3][3] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2	2	2
2	0	0	0	2	3	3	3	5	5
3	0	0	0	2					
4	0								

When i = 3, W = 4

The weight corresponding to the value 3 is 5, i.e., $w_3 = 5$. Since we have three items in the set of weight 3, 4, and 5 respectively, and weight of the knapsack is 4. We can keep the item of either weight 3 or 4; the profit (3) corresponding to the weight 4 is more than the profit corresponding to the weight 3 so we add 3 at M[3][4] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2	2	2
2	0	0	0	2	3	3	3	5	5
3	0	0	0	1	3				
4	0								

When i = 3, W = 5

The weight corresponding to the value 3 is 5, i.e., $w_3 = 5$. Since we have three items in the set of weight 3, 4, and 5 respectively, and weight of the knapsack is 5. We can keep the item of either weight 3, 4 or 5; the profit (3) corresponding to the weight 4 is more than the profits corresponding to the weight 3 and 5 so we add 3 at M[3][5] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2	2	2
2	0	0	0	2	3	3	3	5	5
3	0	0	0	1	3	3			
4	0								

When i =3, W = 6

The weight corresponding to the value 3 is 5, i.e., $w_3 = 5$. Since we have three items in the set of weight 3, 4, and 5 respectively, and weight of the knapsack is 6. We can keep the item of either weight 3, 4 or 5; the profit (3) corresponding to the weight 4 is more than the profits corresponding to the weight 3 and 5 so we add 3 at M[3][6] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2	2	2
2	0	0	0	2	3	3	3	5	5
3	0	0	0	1	3	3	3		
4	0								

When i =3, W = 7

The weight corresponding to the value 3 is 5, i.e., $w_3 = 5$. Since we have three items in the set of weight 3, 4, and 5 respectively, and weight of the knapsack is 7. In this case, we can keep both the items of weight 3 and 4, the sum of the profit would be equal to (2 + 3), i.e., 5, so we add 5 at M[3][7] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2	2	2
2	0	0	0	2	3	3	3	5	5
3	0	0	0	1	3	3	3	5	
4	0								

When i = 3, W = 8

The weight corresponding to the value 3 is 5, i.e., $w_3 = 5$. Since we have three items in the set of weight 3, 4, and 5 respectively, and the weight of the knapsack is 8. In this case, we can keep both the items of weight 3 and 4, the sum of the profit would be equal to (2 + 3), i.e., 5, so we add 5 at M[3][8] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2	2	2
2	0	0	0	2	3	3	3	5	5
3	0	0	0	1	3	3	3	5	5
4	0								

Now the value of 'i' gets incremented and becomes 4.

When i = 4, W = 1

The weight corresponding to the value 4 is 6, i.e., $w_4 = 6$. Since we have four items in the set of weights 3, 4, 5, and 6 respectively, and the weight of the knapsack is 1. The weight of all the items is more than the weight of the knapsack, so we cannot add any item in the knapsack; Therefore, we add 0 at M[4][1] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2	2	2
2	0	0	0	2	3	3	3	5	5
3	0	0	0	1	3	3	3	5	5
4	0	0							

When i = 4, W = 2

The weight corresponding to the value 4 is 6, i.e., $w_4 = 6$. Since we have four items in the set of weights 3, 4, 5, and 6 respectively, and the weight of the knapsack is 2. The weight of all the items is more than the weight of the knapsack, so we cannot add any item in the knapsack; Therefore, we add 0 at M[4][2] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2	2	2
2	0	0	0	2	3	3	3	5	5
3	0	0	0	1	3	3	3	5	5
4	0	0	0						

When i = 4, W = 3

The weight corresponding to the value 4 is 6, i.e., $w_4 = 6$. Since we have four items in the set of weights 3, 4, 5, and 6 respectively, and the weight of the knapsack is 3. The item with a weight 3 can be put in the knapsack and the profit corresponding to the weight 4 is 2, so we will add 2 at M[4][3] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2	2	2
2	0	0	0	2	3	3	3	5	5
3	0	0	0	1	3	3	3	5	5
4	0	0	0	2					

When i = 4, W = 4

The weight corresponding to the value 4 is 6, i.e., $w_4 = 6$. Since we have four items in the set of weights 3, 4, 5, and 6 respectively, and the weight of the knapsack is 4. The item with a weight 4 can be put in the knapsack and the profit corresponding to the weight 4 is 3, so we will add 3 at M[4][4] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2	2	2
2	0	0	0	2	3	3	3	5	5
3	0	0	0	1	3	3	3	5	5
4	0	0	0	2	3				

When i = 4, W = 5

The weight corresponding to the value 4 is 6, i.e., $w_4 = 6$. Since we have four items in the set of weights 3, 4, 5, and 6 respectively, and the weight of the knapsack is 5. The item with a weight 4 can be put in the knapsack and the profit corresponding to the weight 4 is 3, so we will add 3 at M[4][5] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2	2	2
2	0	0	0	2	3	3	3	5	5
3	0	0	0	1	3	3	3	5	5
4	0	0	0	2	3	3			

When i = 4, W = 6

The weight corresponding to the value 4 is 6, i.e., $w_4 = 6$. Since we have four items in the set of weights 3, 4, 5, and 6 respectively, and the weight of the knapsack is 6. In this case, we can put the items in the knapsack either of weight 3, 4, 5 or 6 but the profit, i.e., 4 corresponding to the weight 6 is highest among all the items; therefore, we add 4 at M[4][6] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2	2	2
2	0	0	0	2	3	3	3	5	5
3	0	0	0	1	3	3	3	5	5
4	0	0	0	2	3	3	4		

When i = 4, W = 7

The weight corresponding to the value 4 is 6, i.e., $w_4 = 6$. Since we have four items in the set of weights 3, 4, 5, and 6 respectively, and the weight of the knapsack is 7. Here, if we add two items of weights 3 and 4 then it will produce the maximum profit, i.e., (2 + 3) equals to 5, so we add 5 at M[4][7] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2	2	2
2	0	0	0	2	3	3	3	5	5
3	0	0	0	2	3	3	3	5	5
4	0	0	0	2	3	3	4	5	

When i = 4, W = 8

The weight corresponding to the value 4 is 6, i.e., $w_4 = 6$. Since we have four items in the set of weights 3, 4, 5, and 6 respectively, and the weight of the knapsack is 8. Here, if we add two items of weights 3 and 4 then it will produce the maximum profit, i.e., (2 + 3) equals to 5, so we add 5 at M[4][8] shown as below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2	2	2
2	0	0	0	2	3	3	3	5	5
3	0	0	0	2	3	3	3	5	5
4	0	0	0	2	3	3	4	5	5

As we can observe in the above table that 5 is the maximum profit among all the entries. The pointer points to the last row and the last column having 5 value. Now we will compare 5 value with the previous row; if the previous row, i.e., i = 3 contains the same value 5 then the pointer will shift upwards. Since the previous row contains the value 5 so the pointer will be shifted upwards as shown in the below table:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2	2	2
2	0	0	0	2	3	3	3	5	5
3	0	0	0	2	3	3	3	5	5
4	0	0	0	2	3	3	4	5	5

Again, we will compare the value 5 from the above row, i.e., i = 2. Since the above row contains the value 5 so the pointer will again be shifted upwards as shown in the below table:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2	2	2
2	0	0	0	2	3	3	3	5	5
3	0	0	0	2	3	3	3	5	5
4	0	0	0	2	3	3	4	5	5

Again, we will compare the value 5 from the above row, i.e., i = 1. Since the above row does not contain the same value so we will consider the row i=1, and the weight corresponding to the row is 4. Therefore, we have selected the weight 4 and we have rejected the weights 5 and 6 shown below:

x = { 1, 0, 0}

The profit corresponding to the weight is 3. Therefore, the remaining profit is (5 - 3) equals to 2. Now we will compare this value 2 with the row i = 2. Since the row (i = 1) contains the value 2; therefore, the pointer shifted upwards shown below:

	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	0	2	2	2	2	2	2
2	0	0	0	2	3	3	3	5	5
3	0	0	0	2	3	3	3	5	5
4	0	0	0	2	3	3	4	5	5

Again we compare the value 2 with a above row, i.e., i = 1. Since the row i =0 does not contain the value 2, so row i = 1 will be selected and the weight corresponding to the i = 1 is 3 shown below:

X = {1, 1, 0, 0}

The profit corresponding to the weight is 2. Therefore, the remaining profit is 0. We compare 0 value with the above row. Since the above row contains a 0 value but the profit corresponding to this row is 0. In this problem, two weights are selected, i.e., 3 and 4 to maximize the profit.