Multistage Graph

A **Multistage graph** is a directed, weighted graph in which the nodes can be divided into a set of stages such that all edges are from a stage to next stage only (In other words there is no edge between vertices of same stage and from a vertex of current stage to previous stage).

The vertices of a multistage graph are divided into n number of disjoint subsets S = { S_1 , S_2 , S_3 .... S_n }, where S_1 is the source and S_n is the sink (destination). The cardinality of S_1 and S_n are equal to 1. i.e., $|S_1| = |S_n| = 1$.
We are given a multistage graph, a source and a destination, we need to find shortest path from source to destination. By convention, we consider source at stage 1 and destination as last stage.
Following is an example graph we will consider

$$1 \overset{1}{-} 2 \overset{4}{-} 5 \overset{20}{-} 8 \qquad \textcircled{25}$$

$$fun(S_i', V_J) = Min \begin{cases} fun(S_{i+1}, K) + C(V_J, K) & \overset{K \neq S_{i+1}}{\forall K \in E} \\ \boxed{C(V_J, D) \overset{S_I = S_{fi-1}}{}} \end{cases}$$

$$fun(2,3)$$

$$1 \overset{1}{-} 2 \overset{4}{-} 5 \overset{20}{-} 8 \qquad \textcircled{25}$$

$$fun(S_i, V_J) = min\{ fun(S_{i+1}, K) + C(V_J, K)\}$$
$$C(V_J, D) \text{ if } S_i = S_{fi-1}$$

$$fun(1,1)$$

$$fun(2,2)+1 \qquad fun(2,3)+2 \qquad fun(2,4)+7$$

$$fun(3,5)+4 \quad fun(3,6)+8 \qquad fun(3,5)+9 \quad fun(3,6)+5 \quad fun(3,7)+17 \qquad fun(3,7)+2$$

"Multistage Graph"

$$1 \;\overset{1}{-}\; 2 \;\overset{4}{-}\; 5 \;\overset{20}{-}\; 8 \quad \boxed{25}$$

$$fun(S_i, V_j) = \min \left\{ fun(S_{i+1}, K) + C(V_j, K) \right\}$$

$$C(V_j, D) \text{ if } S_i = S_{fi-1}$$

$fun(1,1)$ $\boxed{11}$

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         23      24              21              11
        24   fun(2,2)+1      29  fun(2,3)+2        fun(2,4)+7
                    23              20    19           1   4
        fun(3,5)+4  fun(3,6)+8  fun(3,5)+9  fun(3,6)+5  fun(3,7)+17   fun(3,7)+2
         20          15          20          15          2            2
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