Fuzzy Logic

Lecture 4

Outline

- Typical Car Example
- Fuzzy Set: Basic Concepts Example
- α-Cut and Strong α-cut Concept
- The Support and Core of a Fuzzy Set
- Height of Fuzzy Set A
- Important Property of Fuzzy Set –Convexity
- Scalar Cardinality

CAR EXAMPLE

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Car Example

- "If our distance to the car in front is small, and the distance is decreasing slowly, then decelerate quite hard"
  - Fuzzy variables in blue
  - Fuzzy sets in red
  - Conditions are on membership in fuzzy sets
  - Actions place an output variable (decelerate) in a fuzzy set (the quite hard deceleration set)
- We have a certain belief in the truth of the condition, and hence a certain strength of desire for the outcome
- Multiple rules may match to some degree, so we require a means to arbitrate and choose a particular goal - *defuzzification*

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Car Examples

- Rules for controlling a car:
  - Variables are *distance* to car in front and how fast it is changing, *delta*, and *acceleration* to apply
  - Sets are:
    - Very small, small, perfect, big, very big - for distance
    - Shrinking fast, shrinking, stable, growing, growing fast for delta
    - Brake hard, slow down, none, speed up, floor it for acceleration
  - Rules for every combination of distance and delta sets, defining an acceleration set

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Car Example

- Assume we have a particular numerical value for distance and delta, and we need to set a numerical value for acceleration
  - Extension: Allow fuzzy values for input variables (degree to which we believe the value is correct)

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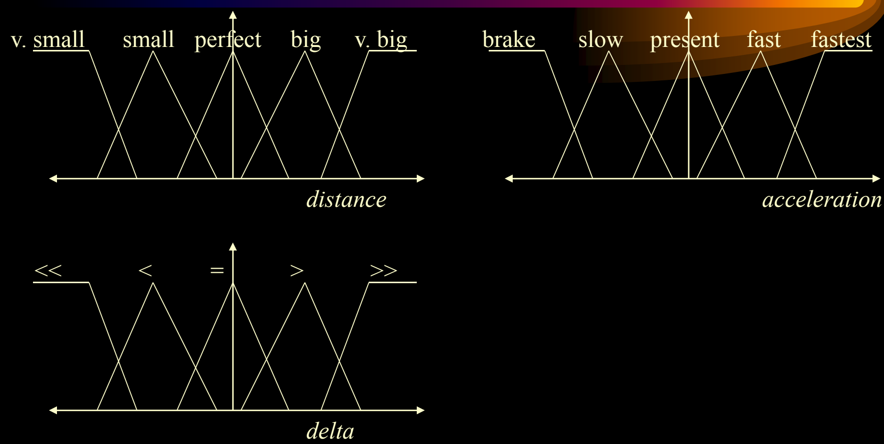
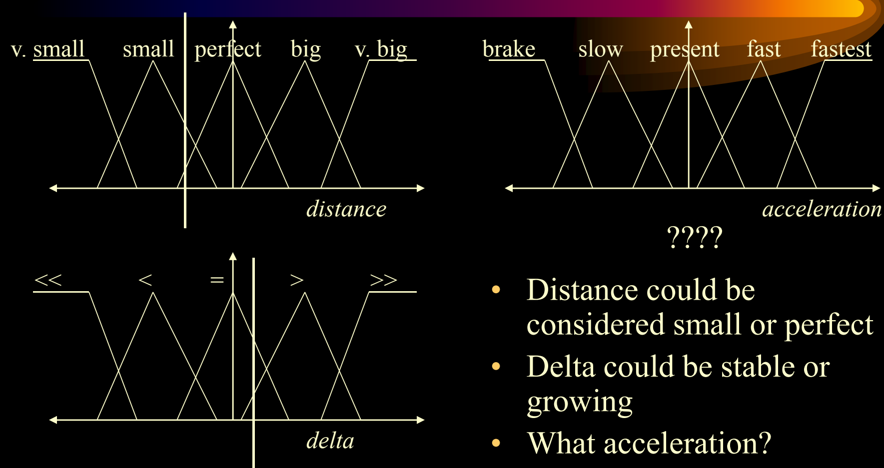
Set Definitions for Example

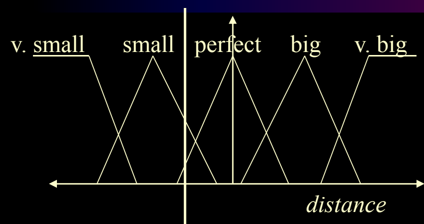


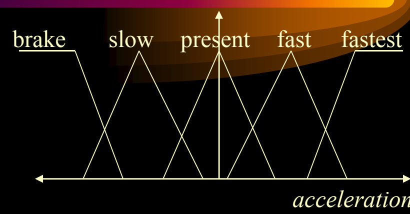
Instance for Example



- Distance could be considered small or perfect
- Delta could be stable or growing
- What acceleration?

Instance for Example





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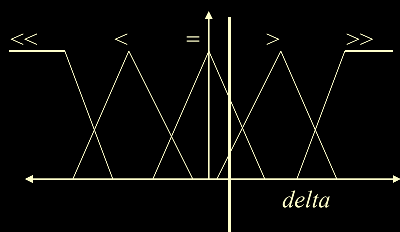

- If distance is small and delta is growing, maintain speed
- If distance is small and delta is stable, slow down
- If distance is perfect and delta is growing, speed up
- If distance is perfect and delta is stable, maintain speed

Matching for Example

- Relevant rules are:
  - If distance is small and delta is growing, maintain speed
  - If distance is small and delta is stable, slow down
  - If distance is perfect and delta is growing, speed up
  - If distance is perfect and delta is stable, maintain speed
- For first rule, distance is small has 0.75 truth, and delta is growing has 0.3 truth
  - So the truth of the **and** is 0.3
- Other rule strengths are 0.6, 0.1 and 0.1

Do you have further interest in operations of Fuzzy Set?

- Please refer the Chapter 2-3 of Klir and Yuan's book for details

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Ordinary Fuzzy Set

- Given a relevant universal set X, any arbitrary fuzzy set of this type (e.g. set A) is difined by a function form

  $A : X \rightarrow [0,1]$
- Most common in the literature for various successful applications
- Coined as '***ordinary fuzzy set'***

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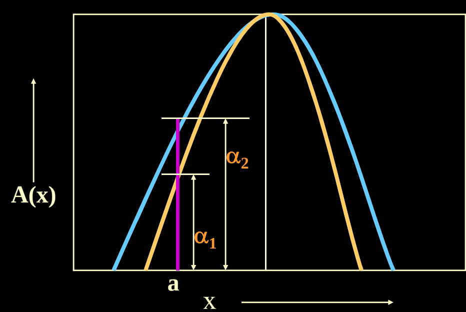
Lower and upper bound of membership grades – Interval Valued Fuzzy sets



An interval valued fuzzy set
$(A(a) = [\alpha_1, \alpha_2])$

$A : X \rightarrow \varepsilon[0,1]$

where $\varepsilon[0,1]$ denotes the family of all closed intervals of real numbers in [0,1]

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Features of Interval Valued Fuzzy Sets

- Not as specific as ***ordinary fuzzy sets***
- Lack of specificity makes them more realistic
- Allows to express uncertainty in identifying a particular membership function
- Makes results ***less specific*** but ***more credible***
- Computationally more demanding

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New Level of Fuzziness

- ***Interval-valued fuzzy sets*** can further be generalized by allowing their intervals to be fuzzy. Each interval now get converted into ordinary fuzzy set.
- They are known as ***Fuzzy Sets of type 2***

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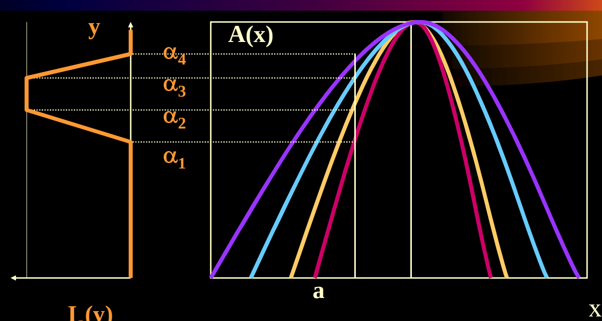
Fuzzy Set of Type 2



A : X → Ŧ ([0,1]), where Ŧ ([0,1]) denotes the set of all ordinary fuzzy sets that can be defined within the universal set [0,1].

Ŧ [0,1] is also called a fuzzy power set of [0,1]

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Features of Fuzzy Set of Type 2

- Assumed trapezoidal shape
- A great expressive power
- Conceptually quite appealing
- Computationally more demanding than interval-valued fuzzy sets → Hence, never utilized for applications
- Still scope to find higher types of fuzzy set, say ***Fuzzy set of type 3***

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Ordinary (Crisp) Sets to Fuzzy Sets

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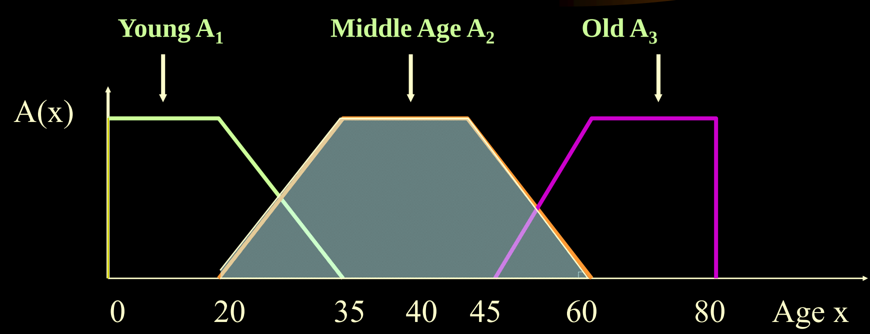
Fuzzy Sets: Basic Concepts Example



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Fuzzy Sets: Basic Concepts Example

Membership Function A_1

$$A_1(x) = \begin{cases} 1 & \text{when } x \leq 20 \\ (35-x)/15 & \text{when } 20 < x < 35 \\ 0 & \text{when } x \geq 35 \end{cases}$$

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Fuzzy Sets: Basic Concepts Example

Membership Function A_2

$$A_2(x) = \begin{cases} 0 & \text{when } x \le 20 \text{ or } \ge 60 \\ (x-20)/15 & \text{when } 20 < x < 35 \\ (60-x)/15 & \text{when } 45 < x < 60 \\ 1 & \text{when } 35 \le x \le 45 \end{cases}$$

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Fuzzy Sets: Basic Concepts Example

Membership Function A_3

$$A_3(x) = \begin{cases} 0 & \text{when } x \le 45 \\ (x-45)/15 & \text{when } 45 < x < 60 \\ 1 & \text{when } x \ge 60 \end{cases}$$

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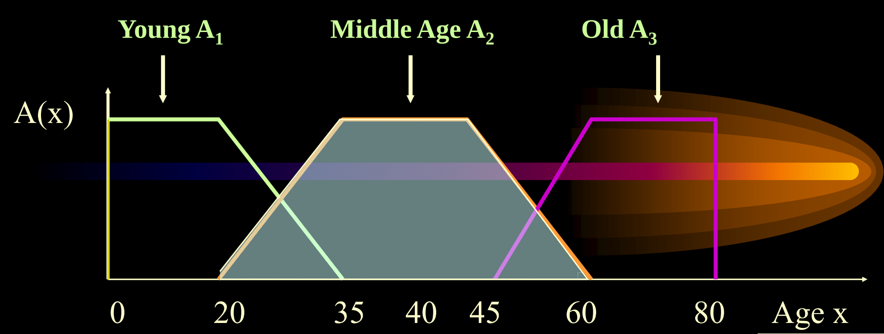

x	$D_2(x)$
$x \notin \{22,24,\ldots,58\}$	0.00
$x \in \{22,58\}$	0.13
$x \in \{24,56\}$	0.27
$x \in \{26,54\}$	0.40
$x \in \{28,52\}$	0.53
$x \in \{30,50\}$	0.67
$x \in \{32,48\}$	0.80
$x \in \{34,46\}$	0.93
$x \in \{36,38,\ldots,44\}$	1.00

Discrete Approximation of membership function A_2

$D_2 : \{0,2,4,\ldots,80\} \rightarrow [0,1]$

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α-Cut Concept

- *α-Cut* and its variant *Strong α-Cut*
- Given a fuzzy set A defined on X and any number $\alpha \in [0,1]$, α-cut , ${}^{\alpha}A$, and strong α-cut, ${}^{\alpha+}A$, are crisp sets

$${}^{\alpha}A = \{x \mid A(x) \geq \alpha\}$$

$${}^{\alpha+}A = \{x \mid A(x) > \alpha\}$$

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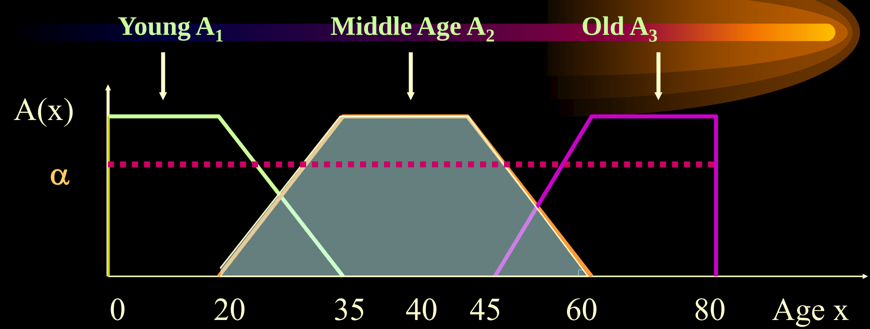
Fuzzy Sets: Basic Concepts Example



${}^{0}A_1 = {}^{0}A_2 = {}^{0}A_3 = [0,80] = X$

${}^{\alpha}A_1 = [0, 35-15\alpha]$, ${}^{\alpha}A_2 = [15\alpha+20,\ 60-15\alpha]$, ${}^{\alpha}A_3 = [15\alpha+45,\ 80]$ for all $\alpha \in (0,1]$

${}^{\alpha+}A_1 = (0, 35-15\alpha)$, ${}^{\alpha+}A_2 = (15\alpha+20,\ 60-15\alpha)$, ${}^{\alpha+}A_3 = (15\alpha+45,\ 80)$ for all $\alpha \in [0,1)$

${}^{1+}A_1 = {}^{1+}A_2 = {}^{1+}A_3 = \varnothing$

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Fuzzy Sets: Basic Concepts Example

- The set of all levels $\alpha \in [0,1]$, that represent distinct α - cuts of a given fuzzy set A is called a ***level set of A***. Formally
- $\Lambda(A) = \{\alpha \mid A(x) = \alpha$ for some $x \in X\}$
- Where A denotes the *level set of fuzzy set A* defined on X.
- $\Lambda(A_1) = \Lambda(A_2) = \Lambda(A_3) = [0,1]$ and
- $\Lambda(D_2) = \{0, 0.13, 0.27, 0.4, 0.53, 0.67, 0.8, 0.93, 1\}$

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Characteristics of α-cut and Strong α-cut

- For any fuzzy set A and pair $\alpha_1, \alpha_2 \in [0,1]$ of distinct values such that $\alpha_1 < \alpha_2$, We have

$$^{\alpha_1}A \supseteq {}^{\alpha_2}A \quad \text{and} \quad {}^{\alpha_1+}A \supseteq {}^{\alpha_2+}A$$

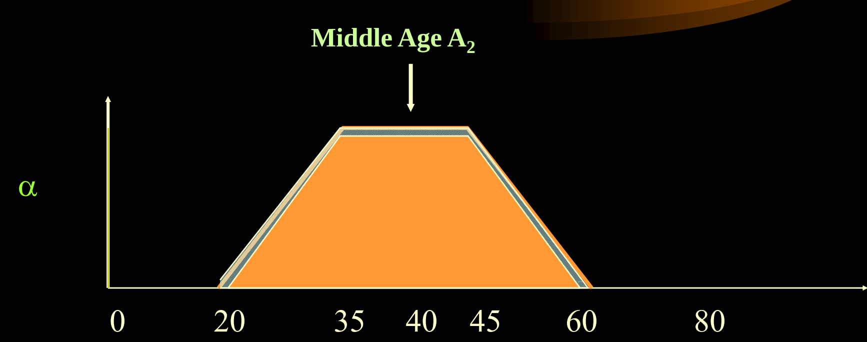
- This property can be expressed by the equations

$$^{\alpha_1}A \cap {}^{\alpha_2}A = {}^{\alpha_2}A \text{ and } {}^{\alpha_1}A \cup {}^{\alpha_2}A = {}^{\alpha_1}A$$

$$^{\alpha_1+}A \cap {}^{\alpha_2+}A = {}^{\alpha_2+}A \text{ and } {}^{\alpha_1+}A \cup {}^{\alpha_2+}A = {}^{\alpha_1+}A$$

- For α-cuts and Strong α-cuts of any fuzzy set form two distinct families of *nested crisp sets*

Nested Family Example



α - cuts and strong α - cuts of D_2

The Support and Core of a Fuzzy Set A

- The support of fuzzy set A within a universal set X is the Crisp set that contains all the elements of X that have non-zero membership grade in A
- Same as the strong α-cut of A for $\alpha = 0$.
- Notation – ***S(A)*** or ***supp(A)* –** Denotes Support of A - Natural symbol – ^{0+}A
- The 1-cut, ^{1}A, is known as the ***core of A***

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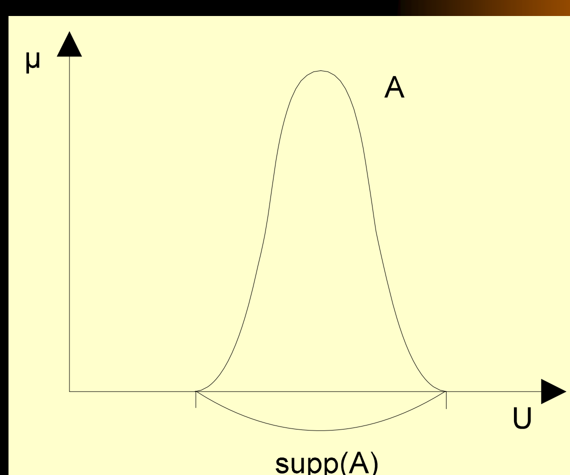
Support of Fuzzy Set A



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Height of Fuzzy Set A

- The height, h(A), of a fuzzy set A is the largest membership grade obtained by any element in the set.
- $h(A) = \sup_{x \in X} A(x)$
- A fuzzy set A is called **normal** when h(A) = 1
- A fuzzy set A is called **subnormal** when h(A) < 1

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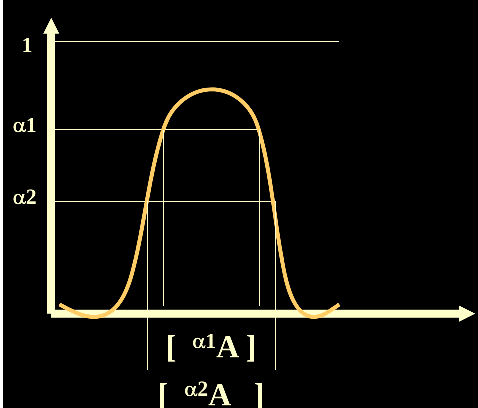
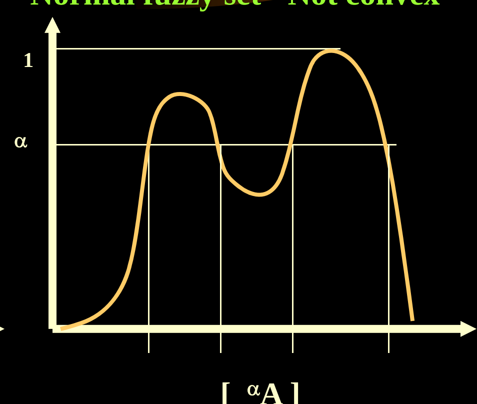
Important Property of Fuzzy Set - Convexity

Convex subnormal fuzzy set



Normal fuzzy set -Not convex



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Scalar Cardinality

- For fuzzy set A defined on a finite universal set X, scalar cardinality $|A|$ is expressed as
- $|A| = \sum_{x \in X} A(x)$

Scalar cardinality of fuzzy set D_2 is given by

$|D_2| = 2(0.13 + 0.27 + 0.4 + 0.53 + 0.67 + 0.8 + 0.93) + 5 (1) = 12.46$

x	$D_2(x)$
$x \notin \{22,24,\ldots,58\}$	0.00
$x \in \{22,58\}$	0.13
$x \in \{24,56\}$	0.27
$x \in \{26,54\}$	0.40
$x \in \{28,52\}$	0.53
$x \in \{30,50\}$	0.67
$x \in \{32,48\}$	0.80
$x \in \{34,46\}$	0.93
$x \in \{36,38,\ldots,44\}$	1.00

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Fuzzy set with finite support – Special notation

- For a fuzzy set A defined on a finite universal set X, $x_1, x_2, \ldots, x_n$ denote elements of the support ${}^{0+}A$ of A and let a_i denote the grade of membership of x_i in A for all $i \in N_n$. Then, A is expressed as
- $A = a_1/x_1 + a_2/x_2 + \ldots + a_n/x_n$

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Extension Principle for Fuzzy Sets

- Crisp function f: X → Y
- Fuzzified function for which the same symbol f is usually used

  f : F(X) → F(Y)

  and its inverse function f^{-1}, has the form

  f^{-1}:F(Y) → F(X)

A principle for fuzzyfying crisp function is called an extension principle.

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Simple example

- Let X = {a,b,c} and Y = {1,2} and let us consider the function

  f : a→1

  b→ 1

  c→2

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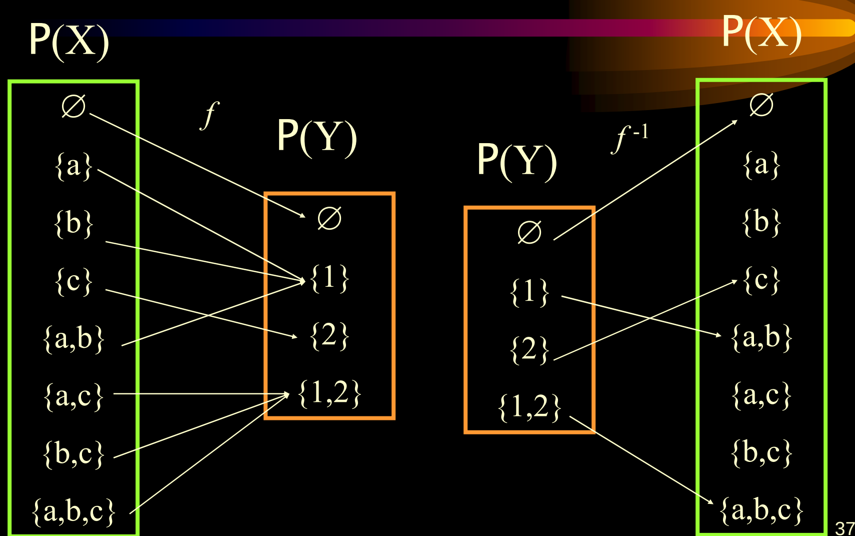
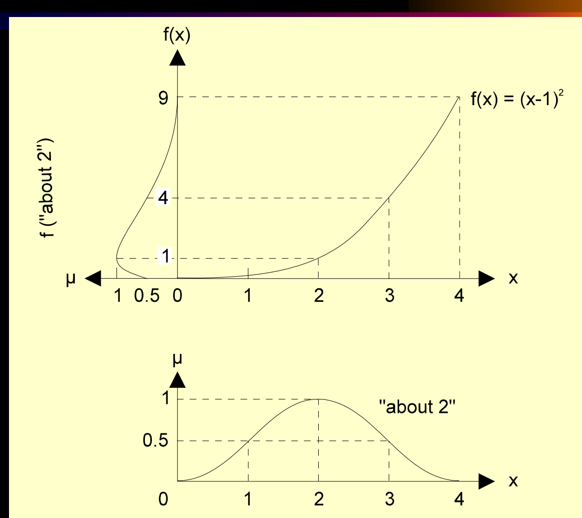
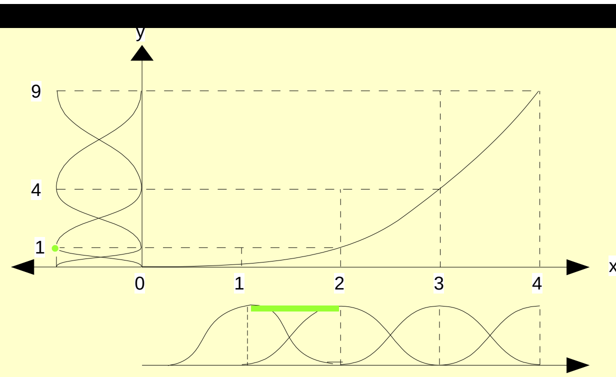
Classical extensions of function $f: a \rightarrow 1, b \rightarrow 1, c \rightarrow 2$ and its inverse f^{-1}



An illustration of the extension principle in fuzzy theory: an example of transforming a fuzzy set "x is about 2" into a fuzzy set "f is about f(2)" for $f(x)= (x-1)^2$





Exact rules:

IF x = 1 THEN y = 0
IF x = 2 THEN y = 1
IF x = 3 THEN y = 4
IF x = 4 THEN y = 9

Interval rules:

IF $x \le 1$ THEN y = 0
IF $1 < x \le 2$ THEN y = 1
IF $2 < x \le 3$ THEN y = 4
IF $3 < x \le 4$ THEN y = 9

Fuzzy rules:

IF x is "about 1" THEN y is "about 0"
IF x is "about 2" THEN y is "about 1"
IF x is "about 3" THEN y is "about 4"
IF x is "about 4" THEN y is "about 9"

Inferencing with Rules

- *Backward Chaining:* [Goal Driven]
  - Start with a goal. Look for a rule that has that goal as its conclusion. Check the premise of this first rule. Check the assertions (facts) first. If that fails, find another rule whose conclusion is the same as the premise of the first rule. Try to satisfy this second rule. Continue until the first rule is satisfied, or all possibilities have been checked.

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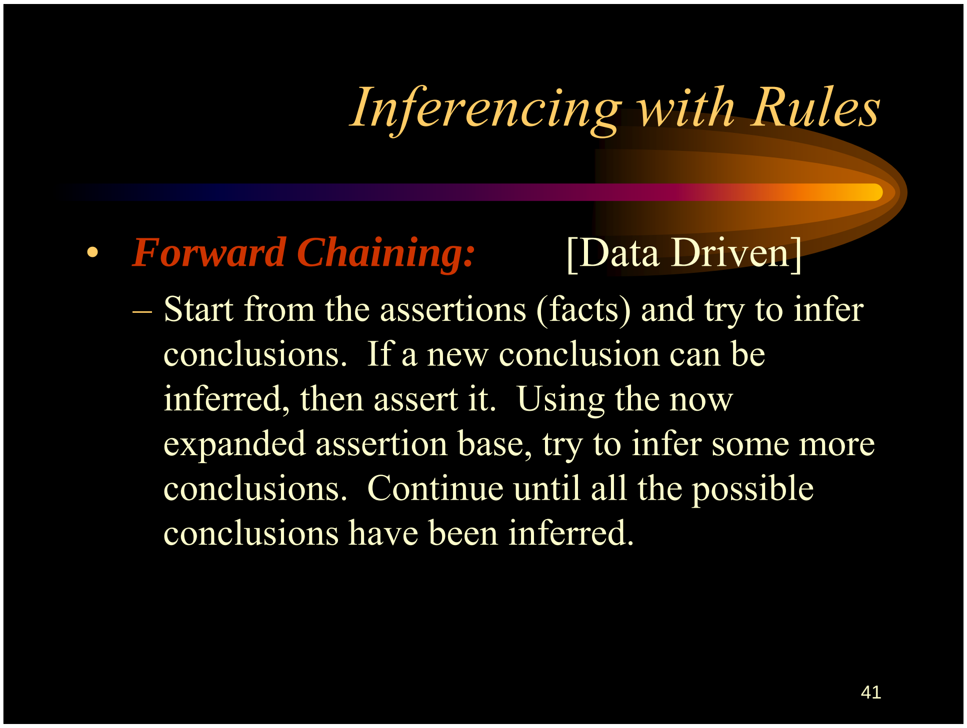
Inferencing with Rules
• Forward Chaining: [Data Driven]
– Start from the assertions (facts) and try to infer conclusions. If a new conclusion can be inferred, then assert it. Using the now expanded assertion base, try to infer some more conclusions. Continue until all the possible conclusions have been inferred.
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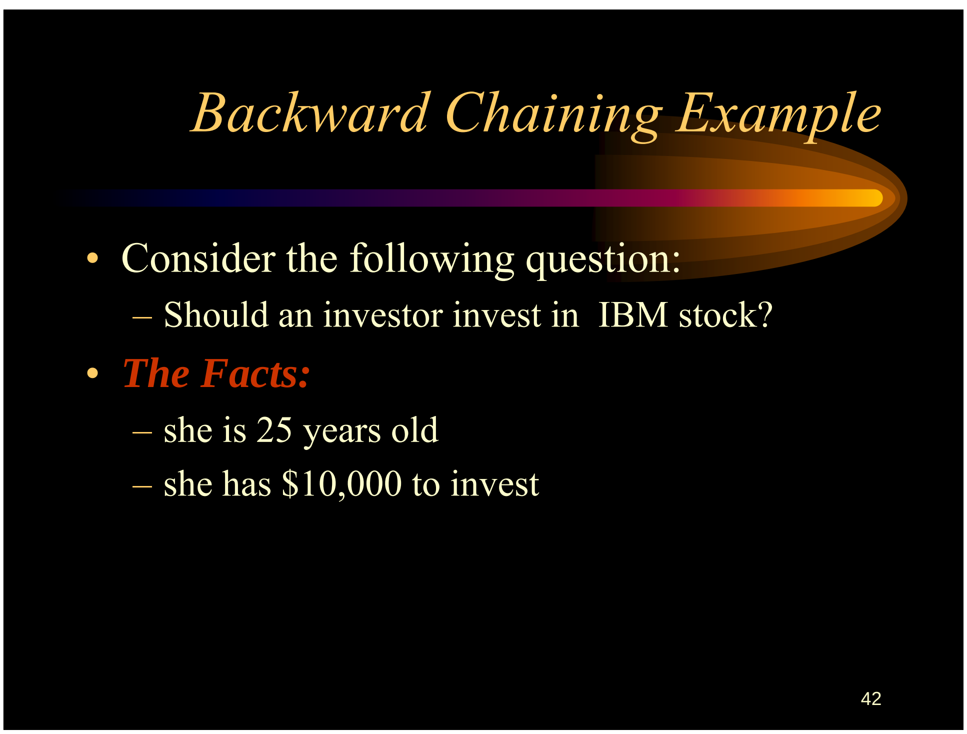
Backward Chaining Example
• Consider the following question:
– Should an investor invest in IBM stock?
• The Facts:
– she is 25 years old
– she has $10,000 to invest
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Backward Chaining Example (Contd.)

- The following variables are involved:
  - A. Have \$10,000
  - B. Younger than 30
  - C. Education at college level
  - D. Annual income > \$40,000
  - E. Invest in securities
  - F. Invest in growth stocks
  - G. Invest in IBM stock

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Backward Chaining Example (Contd.)

- ***The Rules:***
  - R1: IF she has \$10,000
    and has a college degree,
    THEN she should invest in securities
  - R2: IF her annual income is at least \$40,000
    and she has a college degree,
    THEN she should invest in growth stocks
  - R3: IF she is younger than 30
    and invests in securities,
    THEN she should invest in growth stocks
  - R4: IF she is younger than 30,
    THEN she has a college degree
  - R5: IF she wants to invest in a growth stock,
    THEN the stock should be IBM

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Backward Chaining Example (Contd.)

The Starting Point:

- Goal: Should she invest in IBM stock?
- only rule R5 satisfies the goal as its conclusion

R1: IF she has $10,000
and has a college degree,
THEN she should invest in securities

R2: IF her annual income is at least $40,000
and she has a college degree,
THEN she should invest in growth stocks

R3: IF she is younger than 30
and invests in securities,
THEN she should invest in growth stocks

R4: IF she is younger than 30,
THEN she has a college degree

R5: IF she wants to invest in a growth stock,
THEN the stock should be IBM

given facts
- **she is 25 years old**
- **she has $10,000 to invest**

asserted facts

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Backward Chaining Example (Contd.)

Step 1:

- try to *accept* or *reject* our Goal
  - establish if "she wants to invest in a growth stock" is true
- if we check our assertion base (ie. the given and asserted facts), we see that:
  - "she is 25 years old" and "she has $10,000 to invest" are both true
  - rule R5 cannot be fired (invoked) yet

R1: IF she has $10,000
and has a college degree,
THEN she should invest in securities

R2: IF her annual income is at least $40,000
and she has a college degree,
THEN she should invest in growth stocks

R3: IF she is younger than 30
and invests in securities,
THEN she should invest in growth stocks

R4: IF she is younger than 30,
THEN she has a college degree

R5: IF she wants to invest in a growth stock,
THEN the stock should be IBM

given facts
- **she is 25 years old**
- **she has $10,000 to invest**

asserted facts

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Backward Chaining Example (Contd.)

Step 2:

- to establish if "she wants to invest in a growth stock" is true:
- note that "she wants to invest in a growth stock" is the conclusion of both rule R2 and rule R3
- suppose we look at the premises of rule R2 (arbitrarily) first

R1: IF she has $10,000
and has a college degree,
THEN she should invest in securities

R2: IF her annual income is at least $40,000
and she has a college degree,
THEN she should invest in growth stocks

R3: IF she is younger than 30
and invests in securities,
THEN she should invest in growth stocks

R4: IF she is younger than 30,
THEN she has a college degree

R5: IF she wants to invest in a growth stock,
THEN the stock should be IBM

given facts

- **she is 25 years old**
- **she has $10,000 to invest**

asserted facts

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Backward Chaining Example (Contd.)

Step 3:

- to fire rule R2 requires both premises be true
- but "her annual income is at least $40,000" is neither a given fact nor is it the conclusion of any rule
- either more information is required (ie a fact about "her annual income is at least $40,000" must be established)
- or we can return to rule R3
- [this backing away from one rule and looking at an alternative is called backtracking]

R1: IF she has $10,000
and has a college degree,
THEN she should invest in securities

R2: IF her annual income is at least $40,000
and she has a college degree,
THEN she should invest in growth stocks

R3: IF she is younger than 30
and invests in securities,
THEN she should invest in growth stocks

R4: IF she is younger than 30,
THEN she has a college degree

R5: IF she wants to invest in a growth stock,
THEN the stock should be IBM

given facts

- **she is 25 years old**
- **she has $10,000 to invest**

asserted facts

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Backward Chaining Example (Contd.)

Step 4:

- to fire rule R3 requires both “she is younger than 30” and “invests in securities” be true
- we know “she is younger than 30” is true - it is a given fact (“she is 25 years old”)
- to establish if “invests in securities” is true, we note that it is the conclusion for rule R1

R1: IF she has $10,000
and has a college degree,
THEN she should invest in securities

R2: IF her annual income is at least $40,000
and she has a college degree,
THEN she should invest in growth stocks

R3: IF she is younger than 30
and invests in securities,
THEN she should invest in growth stocks

R4: IF she is younger than 30,
THEN she has a college degree

R5: IF she wants to invest in a growth stock,
THEN the stock should be IBM

given facts
- **she is 25 years old**
- **she has $10,000 to invest**

asserted facts

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Backward Chaining Example (Contd.)

Step 5:

- to fire rule R1, we require “she has $10,000” and “has a college degree” to be true
- “she has $10,000” is true - it is a given fact
- to test “has a college degree” , we note that it is the conclusion of rule R4

R1: IF she has $10,000
and has a college degree,
THEN she should invest in securities

R2: IF her annual income is at least $40,000
and she has a college degree,
THEN she should invest in growth stocks

R3: IF she is younger than 30
and invests in securities,
THEN she should invest in growth stocks

R4: IF she is younger than 30,
THEN she has a college degree

R5: IF she wants to invest in a growth stock,
THEN the stock should be IBM

given facts
- **she is 25 years old**
- **she has $10,000 to invest**

asserted facts

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Backward Chaining Example (Contd.)

Step 6:

- to fire rule R4, we require "she is younger than 30" to be true
- "she is younger than 30" is true - it is a given fact ("she is 25 years old")
- therefore rule R4 fires and "has a college degree" becomes a fact and is added to the assertion base

R1: IF she has $10,000
and has a college degree,
THEN she should invest in securities

R2: IF her annual income is at least $40,000
and she has a college degree,
THEN she should invest in growth stocks

R3: IF she is younger than 30
and invests in securities,
THEN she should invest in growth stocks

R4: IF she is younger than 30,
THEN she has a college degree

R5: IF she wants to invest in a growth stock,
THEN the stock should be IBM

given facts

- **she is 25 years old**
- **she has $10,000 to invest**

asserted facts

- **she has a college degree**

Backward Chaining Example (Contd.)

Step 7:

- moving forward again along the rule chain:
- we can use our newly asserted fact to fire rule R1 to establish "she should invest in securities" as a fact

R1: IF she has $10,000
and has a college degree,
THEN she should invest in securities

R2: IF her annual income is at least $40,000
and she has a college degree,
THEN she should invest in growth stocks

R3: IF she is younger than 30
and invests in securities,
THEN she should invest in growth stocks

R4: IF she is younger than 30,
THEN she has a college degree

R5: IF she wants to invest in a growth stock,
THEN the stock should be IBM

given facts

- **she is 25 years old**
- **she has $10,000 to invest**

asserted facts

- **she has a college degree**
- **she should invest in securities**

Backward Chaining Example (Contd.)

Step 8:

- moving forward again along the rule chain:
- we can now fire rule R3 to assert "she should invest in growth stocks" as a fact

R1: IF she has $10,000
and has a college degree,
THEN she should invest in securities

R2: IF her annual income is at least $40,000
and she has a college degree,
THEN she should invest in growth stocks

R3: IF she is younger than 30
and invests in securities,
THEN she should invest in growth stocks

R4: IF she is younger than 30,
THEN she has a college degree

R5: IF she wants to invest in a growth stock,
THEN the stock should be IBM

given facts
- **she is 25 years old**
- **she has $10,000 to invest**

asserted facts
- **she has a college degree**
- **she should invest in securities**
- **she should invest in growth stocks**

Backward Chaining Example (Contd.)

Step 9:

- moving forward again along the rule chain:
- we can now fire rule R5 to assert "she should invest in IBM stock" as a fact

R1: IF she has $10,000
and has a college degree,
THEN she should invest in securities

R2: IF her annual income is at least $40,000
and she has a college degree,
THEN she should invest in growth stocks

R3: IF she is younger than 30
and invests in securities,
THEN she should invest in growth stocks

R4: IF she is younger than 30,
THEN she has a college degree

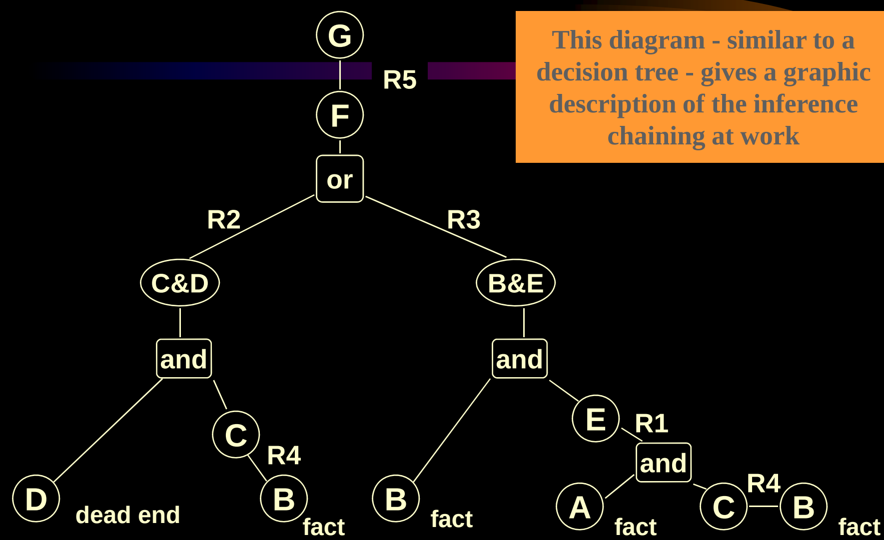
R5: IF she wants to invest in a growth stock,
THEN the stock should be IBM

given facts
- **she is 25 years old**
- **she has $10,000 to invest**

asserted facts
- **she has a college degree**
- **she should invest in securities**
- **she should invest in growth stocks**
- **the stock should be IBM**

Inference Tree

This diagram - similar to a decision tree - gives a graphic description of the inference chaining at work



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Forward Chaining Example

- The Facts:
  - she is 25 years old
  - she has $10,000 to invest
- The Rules:
  - R1: IF she has $10,000
and has a college degree,
THEN she should invest in securities
  - R2: IF her annual income is at least $40,000
and she has a college degree,
THEN she should invest in growth stocks
  - R3: IF she is younger than 30
and invests in securities,
THEN she should invest in growth stocks
  - R4: IF she is younger than 30,
THEN she has a college degree
  - R5: IF she wants to invest in a growth stock,
THEN the stock should be IBM
- **What conclusions can we draw?**

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Forward Chaining Example (Contd.)

The Starting Point:

- Known: "she has $10,000" and "she is younger than 30" are true
- rule R1 (arbitrarily) contains "she has $10,000" in its IF side
- rule R1 includes "she should invest in securities" in its conclusion (THEN side)

R1: IF she has $10,000
and has a college degree,
THEN she should invest in securities

R2: IF her annual income is at least $40,000
and she has a college degree,
THEN she should invest in growth stocks

R3: IF she is younger than 30
and invests in securities,
THEN she should invest in growth stocks

R4: IF she is younger than 30,
THEN she has a college degree

R5: IF she wants to invest in a growth stock,
THEN the stock should be IBM

given facts

- **she is 25 years old**
- **she has $10,000 to invest**

asserted facts

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Forward Chaining Example (Contd.)

Step 1:

- attempt to verify "she should invest in securities"
- since "she has $10,000" is known (fact)
- try to find "she has a college degree" in the assertion base - it's not there
- because "she has a college degree" is not asserted, find a rule with "she has a college degree" in the THEN side
- this is rule R4

R1: IF she has $10,000
and has a college degree,
THEN she should invest in securities

R2: IF her annual income is at least $40,000
and she has a college degree,
THEN she should invest in growth stocks

R3: IF she is younger than 30
and invests in securities,
THEN she should invest in growth stocks

R4: IF she is younger than 30,
THEN she has a college degree

R5: IF she wants to invest in a growth stock,
THEN the stock should be IBM

given facts

- **she is 25 years old**
- **she has $10,000 to invest**

asserted facts

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Forward Chaining Example (Contd.)

Step 2:

- test rule R4
- "she has a college degree" is true because "she is younger than 30" (fact) is in the assertion base
- assert "she has a college degree" as true

R1: IF she has $10,000
and has a college degree,
THEN she should invest in securities

R2: IF her annual income is at least $40,000
and she has a college degree,
THEN she should invest in growth stocks

R3: IF she is younger than 30
and invests in securities,
THEN she should invest in growth stocks

R4: IF she is younger than 30,
THEN she has a college degree

R5: IF she wants to invest in a growth stock,
THEN the stock should be IBM

given facts
- **she is 25 years old**
- **she has $10,000 to invest**

asserted facts
- **she has a college degree**

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Forward Chaining Example (Contd.)

Step 3:

- rule R1 now fires and "she should invest in securities" is asserted
- this leads to rule R3 where "she should invest in securities" is in the IF side

R1: IF she has $10,000
and has a college degree,
THEN she should invest in securities

R2: IF her annual income is at least $40,000
and she has a college degree,
THEN she should invest in growth stocks

R3: IF she is younger than 30
and invests in securities,
THEN she should invest in growth stocks

R4: IF she is younger than 30,
THEN she has a college degree

R5: IF she wants to invest in a growth stock,
THEN the stock should be IBM

given facts
- **she is 25 years old**
- **she has $10,000 to invest**

asserted facts
- **she has a college degree**
- **she should invest in securities**

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Forward Chaining Example (Contd.)

Step 4:

- since "she is younger than 30" (fact) is in the assertion base
- and "she should invest in securities" (asserted) is in the the assertion base
- rule R3 fires and "she should invest in growth stocks" is asserted

R1: IF she has $10,000
and has a college degree,
THEN she should invest in securities

R2: IF her annual income is at least $40,000
and she has a college degree,
THEN she should invest in growth stocks

R3: IF she is younger than 30
and invests in securities,
THEN she should invest in growth stocks

R4: IF she is younger than 30,
THEN she has a college degree

R5: IF she wants to invest in a growth stock,
THEN the stock should be IBM

given facts

- **she is 25 years old**
- **she has $10,000 to invest**

asserted facts

- **she has a college degree**
- **she should invest in securities**
- **she should invest in growth stocks**

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"Fuzzy Logic is basically a multivalued logic that allows intermediate values to be defined between conventional evaluations like yes/no, true/false, black/white, etc. Notions like rather warm or pretty cold can be formulated mathematically and processed by computers."

http://www.aaai.org/aitopics/html/fuzzy.html

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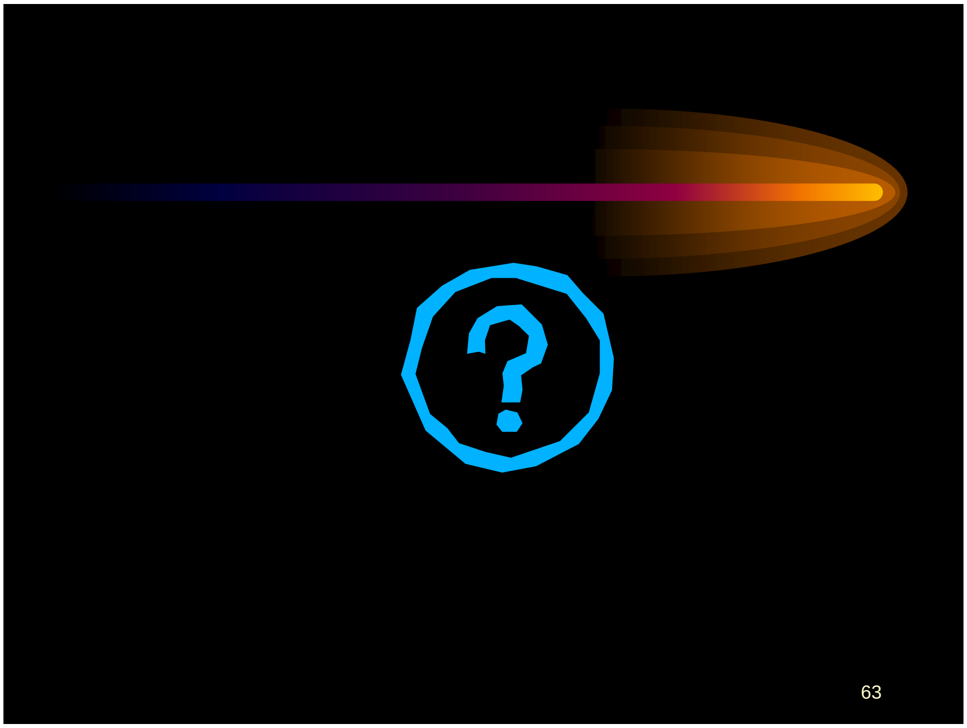
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