Fuzzy Logic

Lecture 3

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Ordinary Fuzzy Set

- Given a relevant universal set X, any arbitrary fuzzy set of this type (e.g. set A) is difined by a function form

  $A : X \rightarrow [0,1]$
- Most common in the literature for various successful applications
- Coined as '***ordinary fuzzy set***'

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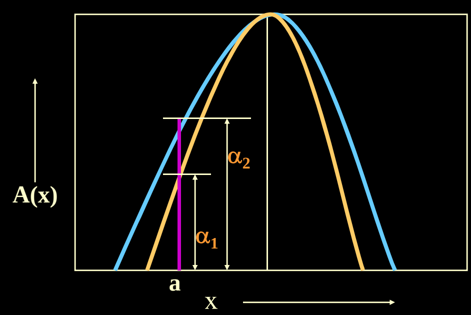
Lower and upper bound of membership grades – Interval Valued Fuzzy sets



An interval valued fuzzy set
$(A(a) = [\alpha_1,\alpha_2])$

$A : X \rightarrow \varepsilon[0,1]$
where $\varepsilon[0,1]$ denotes the family of all closed intervals of real numbers in [0,1]

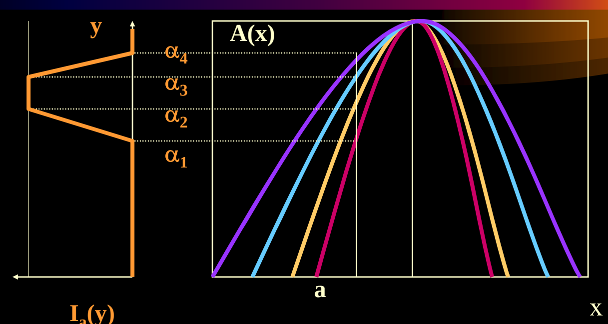
Features of Interval Valued Fuzzy Sets

- Not as specific as ***ordinary fuzzy sets***
- Lack of specificity makes them more realistic
- Allows to express uncertainty in identifying a particular membership function
- Makes results ***less specific*** but ***more credible***
- Computationally more demanding

New Level of Fuzziness

- ***Interval-valued fuzzy sets*** can further be generalized by allowing their intervals to be fuzzy. Each interval now get converted into ordinary fuzzy set.
- They are known as ***Fuzzy Sets of type 2***

Fuzzy Set of Type 2



A : X → Ŧ ([0,1]), where Ŧ ([0,1]) denotes the set of all ordinary fuzzy sets that can be defined within the universal set [0,1].

Ŧ [0,1] is also called a fuzzy power set of [0,1]

Features of Fuzzy Set of Type 2

- Assumed trapezoidal shape
- A great expressive power
- Conceptually quite appealing
- Computationally more demanding than interval-valued fuzzy sets → Hence, never utilized for applications
- Still scope to find higher types of fuzzy set, say ***Fuzzy set of type 3***

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Ordinary (Crisp) Sets to Fuzzy Sets

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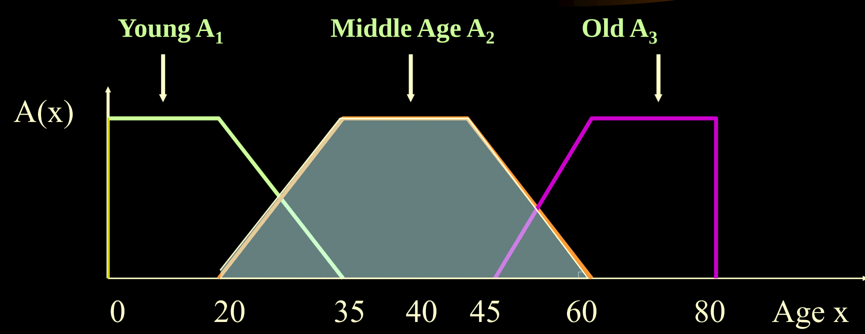
Fuzzy Sets: Basic Concepts Example



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Fuzzy Sets: Basic Concepts Example

Membership Function A_1

$$A_1(x) = \begin{cases} 1 & \text{when } x \le 20 \\ (35-x)/15 & \text{when } 20 < x < 35 \\ 0 & \text{when } x \ge 35 \end{cases}$$

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Fuzzy Sets: Basic Concepts Example

Membership Function A_2

$$A_2(x) = \begin{cases} 0 & \text{when } x \le 20 \text{ or } \ge 60 \\ (x-20)/15 & \text{when } 20 < x < 35 \\ (60-x)/15 & \text{when } 45 < x < 60 \\ 1 & \text{when } 35 \le x \le 45 \end{cases}$$

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Fuzzy Sets: Basic Concepts Example

Membership Function A_3

$$A_3(x) = \begin{cases} 0 & \text{when } x \le 45 \\ (x-45)/15 & \text{when } 45 < x < 60 \\ 1 & \text{when } x \ge 60 \end{cases}$$

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Fuzzy Sets: Basic Concepts Example

Membership Function A_3

$$A_3(x) = \begin{cases} 0 & \text{when } x \le 45 \\ (x-45)/15 & \text{when } 45 < x < 60 \\ 1 & \text{when } x \ge 60 \end{cases}$$

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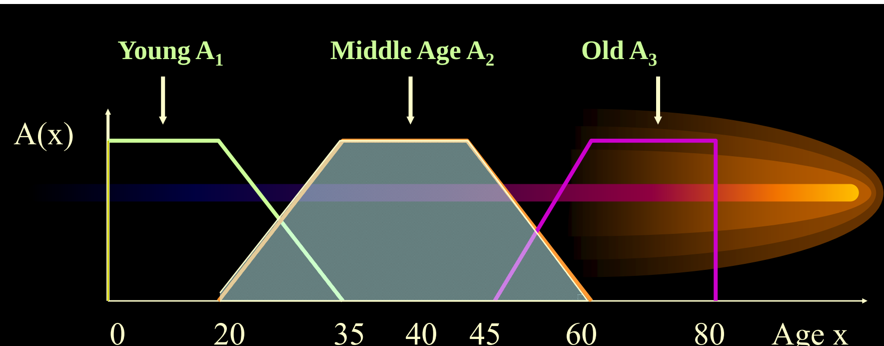

Discrete Approximation of membership function A_2

$D_2 : \{0,2,4,\ldots,80\} \rightarrow [0,1]$

x	$D_2(x)$
$x \notin \{22,24,\ldots,58\}$	0.00
$x \in \{22,58\}$	0.13
$x \in \{24,56\}$	0.27
$x \in \{26,54\}$	0.40
$x \in \{28,52\}$	0.53
$x \in \{30,50\}$	0.67
$x \in \{32,48\}$	0.80
$x \in \{34,46\}$	0.93
$x \in \{36,38,\ldots,44\}$	1.00

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α-Cut Concept

- *α-Cut* and its variant *Strong α-Cut*
- Given a fuzzy set A defined on X and any number $\alpha \in [0,1]$, α-cut , ${}^{\alpha}A$, and strong α-cut, ${}^{\alpha+}A$, are crisp sets

$$ {}^{\alpha}A = \{x \mid A(x) \geq \alpha\} $$

$$ {}^{\alpha+}A = \{x \mid A(x) > \alpha\} $$

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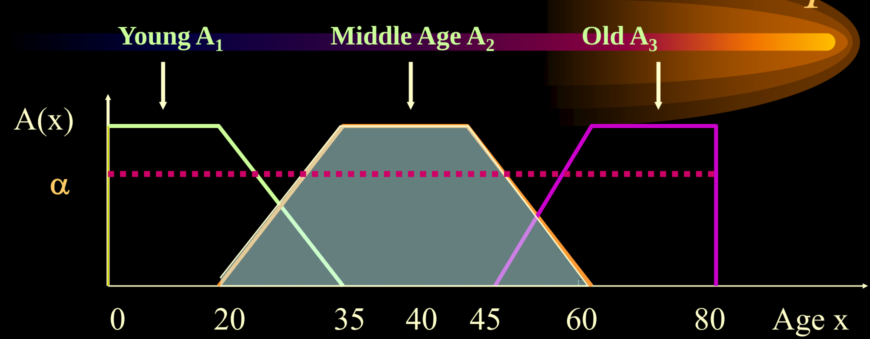
Fuzzy Sets: Basic Concepts Example



${}^{0}A_1 = {}^{0}A_2 = {}^{0}A_3 = [0,80] = X$

${}^{\alpha}A_1 = [0, 35\text{-}15\alpha]$, ${}^{\alpha}A_2 = [15\alpha+20,\ 60\text{-}\ 15\alpha]$, ${}^{\alpha}A_3 = [15\alpha +45,\ 80]$ for all $\alpha \in (0,1]$

${}^{\alpha+}A_1 = (0, 35\text{-}15\alpha)$, ${}^{\alpha+}A_2 = (15\alpha+20,\ 60\text{-}\ 15\alpha)$, ${}^{\alpha+}A_3 = (15\alpha +45,\ 80)$ for all $\alpha \in [0,1)$

${}^{1+}A_1 = {}^{1+}A_2 = {}^{1+}A_3 = \varnothing$

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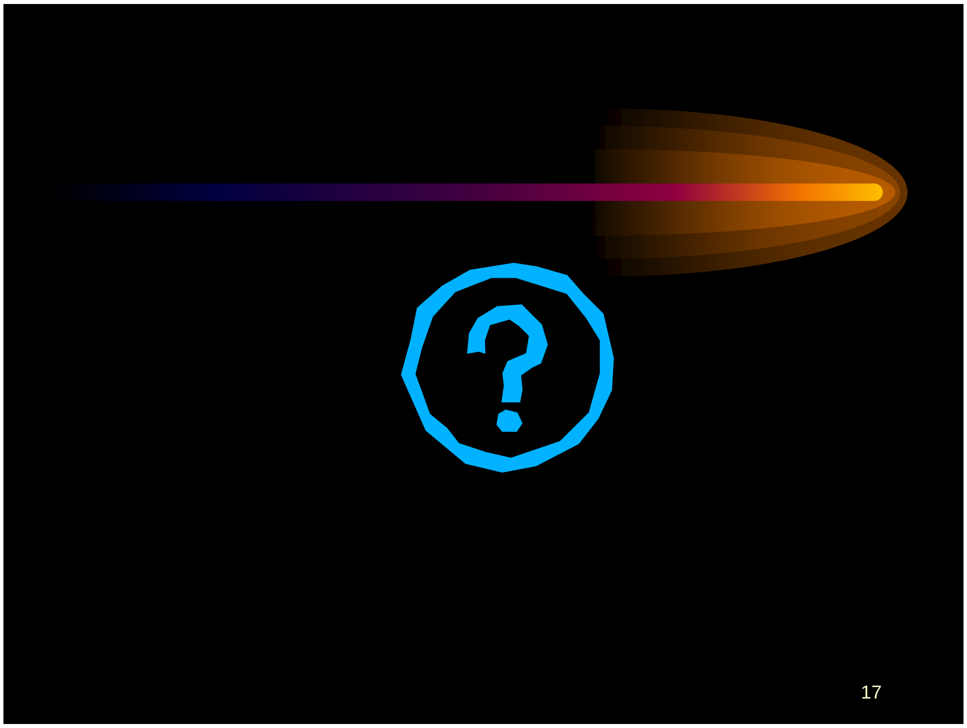
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