Fuzzy Logic

Lecture 6

1

General Approach

Linguistic Variables

Convert into Fuzzy Functions

Fuzzy Arithmetic Operations

Map Fuzzy Functions on Final Fuzzy Template

Apply Criteria

Final Results in Linguistic Form

Basic Operations – Addition, subtraction, multiplication, division

Different operations involving multi-parameters

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Demonstrative Example

- Consider Two triangular-shape fuzzy set numbers A and B defined as

$$A(x) = \begin{cases} 0 & \text{for } x \le -1 \text{ and } x > 3 \\ (x+1)/2 & \text{for } -1 < x \le 1 \\ (3-x)/2 & \text{for } 1 < x \le 3 \end{cases}$$

$$B(x) = \begin{cases} 0 & \text{for } x \le 1 \text{ and } x > 5 \\ (x-1)/2 & \text{for } 1 < x \le 3 \\ (5-x)/2 & \text{for } 3 < x \le 5 \end{cases}$$

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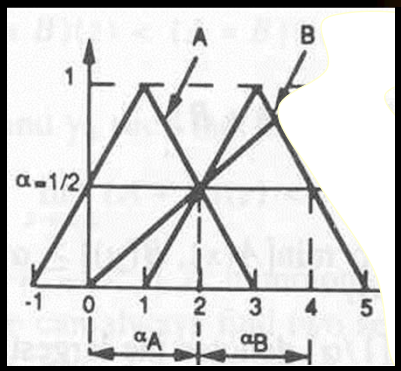
Demonstrative Example



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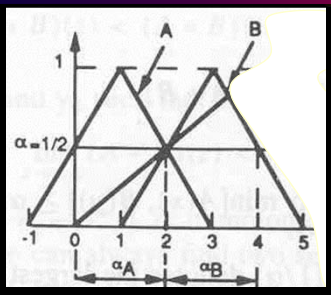
Demonstrative Example



α - cuts are:

$^{\alpha}A = [\,2\alpha - 1,\ 3 - 2\alpha]$

$^{\alpha}B = [2\alpha + 1,\ 5 - 2\alpha]$

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Demonstrative Example

$^{\alpha}(A + B)$ = $[4\alpha,\ 8 - 4\alpha]$ for $\alpha \in (0,1]$

$^{\alpha}(A - B)$ = $[4\alpha - 6,\ 2 - 4\alpha]$ for $\alpha \in (0,1]$

$^{\alpha}(A \,.\, B)$ = $[-4\alpha^2 + 12\alpha - 5,\ 4\alpha^2 - 16\alpha + 15]$ for $\alpha \in (0,0.5]$
$[-4\alpha^2 - 1,\ 4\alpha^2 - 16\alpha + 15]$ for $\alpha \in (0.5,\ 1]$

$^{\alpha}(A / B)$ = $[(2\alpha - 1)/(2\alpha + 1),\ (3 - 2\alpha)/(2\alpha + 1)]$ for $\alpha \in (0,0.5]$
$[(2\alpha - 1)/(5 - 2\alpha),\ (3 - 2\alpha)/(2\alpha + 1)]$ for $\alpha \in (0.5,1]$

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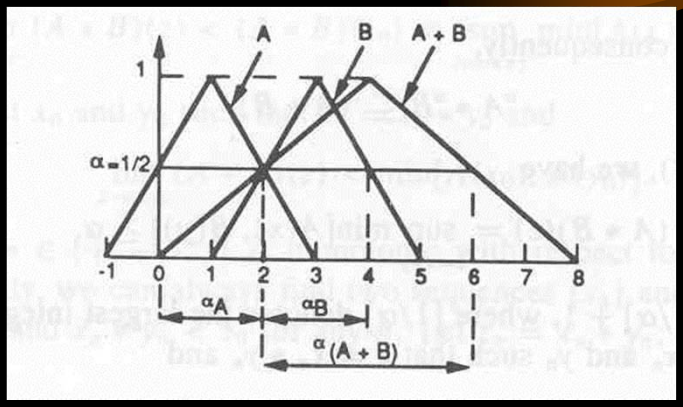
A +B


A
B
A + B
1
α=1/2
-1 0 1 2 3 4 5 6 7 8
αA
αB
α(A + B)

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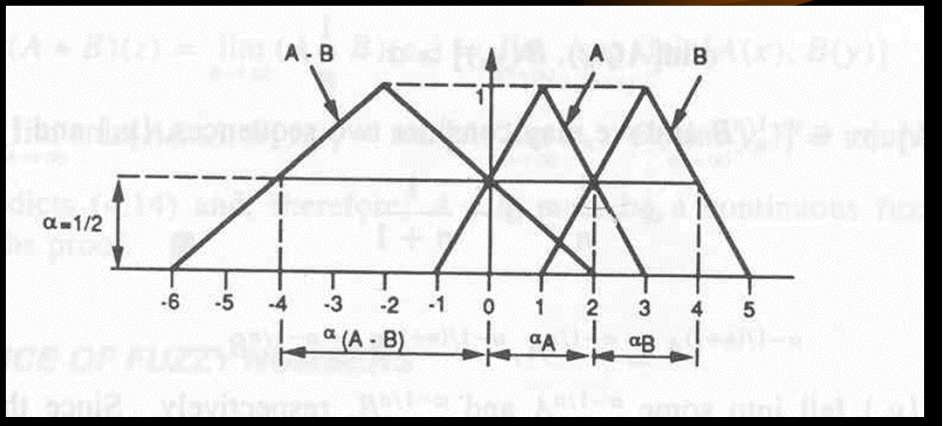
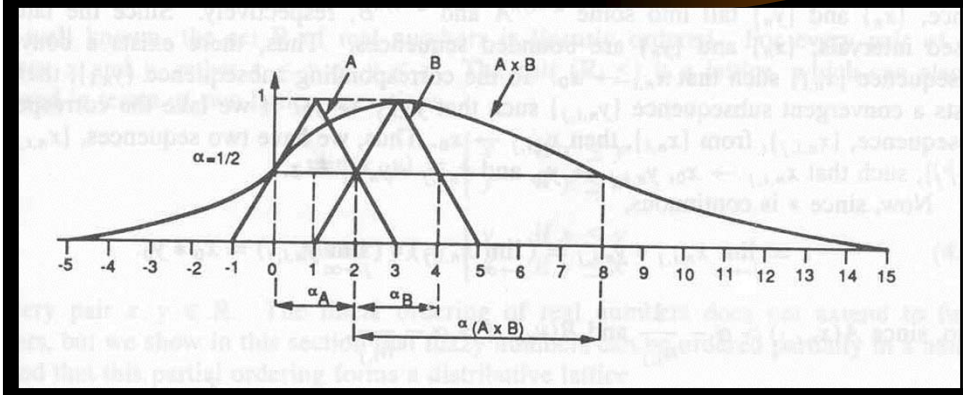
A - B


A - B
A
B
1
α=1/2
-6 -5 -4 -3 -2 -1 0 1 2 3 4 5
α(A - B)
αA
αB

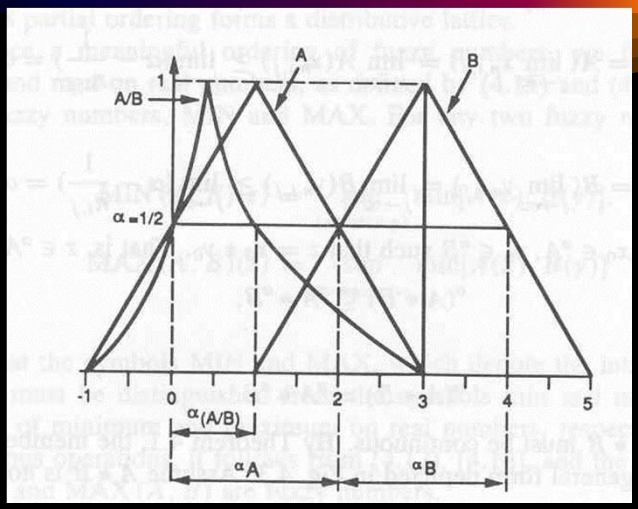
8

A . B


A
B
A x B
1
α=1/2
-5 -4 -3 -2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
αA
αB
α(A x B)

9

A/B


A
B
1
A/B
α=1/2
-1 0 1 2 3 4 5
α(A/B)
αA
αB

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Fuzzy Equation

$A + X = B$

and

$A. X = B$

Home-work

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General Approach

Linguistic Variables

Convert into Fuzzy Functions

Fuzzy Arithmetic Operations

Map Fuzzy Functions on Final Fuzzy Template

Apply Criteria

Final Results in Linguistic Form

Basic Operations – Addition, subtraction, multiplication, division

Different operations involving multi-parameters

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The Criteria

- No single criteria is satisfactory for all situations
- The choice of criteria is context dependent

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List of Criteria

- The pseudo-expectation criterion
- The gravity center criterion
- The most possible criterion
- The pessimistic criterion
- The optimistic criterion
- The α-pessimistic criterion
- The α-optimistic criterion
- The average mean criterion
- The nearest to the ideal optimum criterion
- The dominance criterion
- The classification criterion
- The four point average criterion

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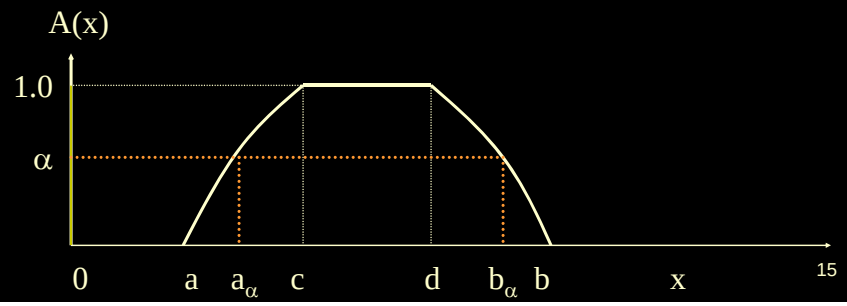
Notation

- A Fuzzy Number
- A_α α-Cut of Fuzzy number A
- a the left of *supp(A)*
- b the right of *supp(A)*
- c the left end of *core(A)*
- d the right end of *core (A)*
- a_α the left end of A_α
- b_α the right end of A_α



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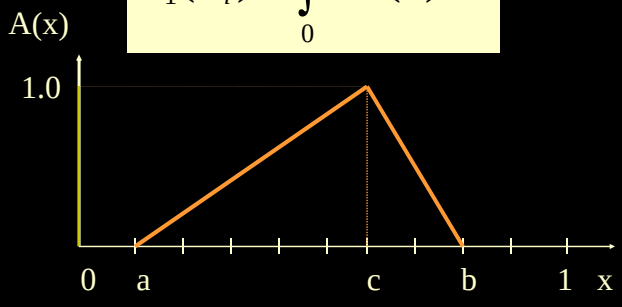
The Pseudo-expectation criterion

$$D_1(A_i) = \int_0^1 x.A(x).dx$$



$$D_1(A_i) = \frac{(b-a)(a+b+c)}{6}$$

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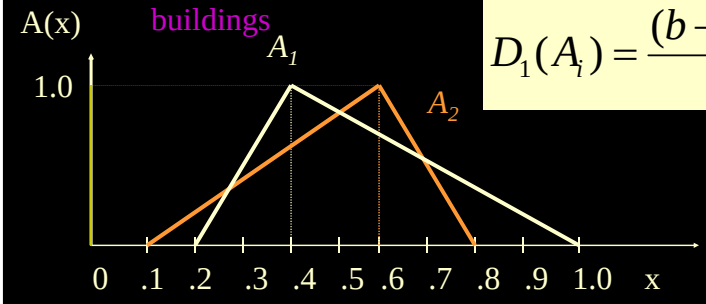
The Pseudo-expectation criterion Building Risk Problem

Risk values for buildings



$$D_1(A_i) = \frac{(b-a)(a+b+c)}{6}$$

$D_1(A_1) = 0.213$

$D_1(A_2) = 0.175$

A1* has higher risk than *A2

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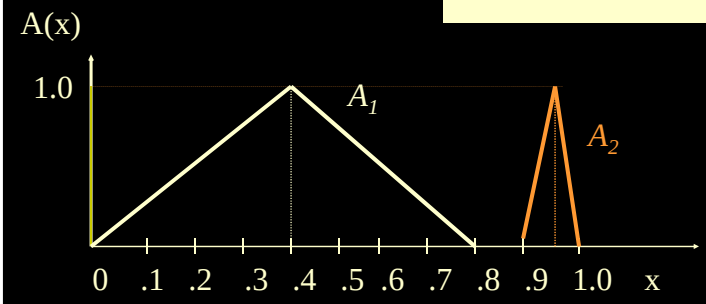
The Pseudo-expectation criterion Pathological Case

$$D_1(A_i) = \frac{(b-a)(a+b+c)}{6}$$



$D_1(A_1) = 0.16$

$D_1(A_2) = 0.048$

A1* has higher risk than *A2

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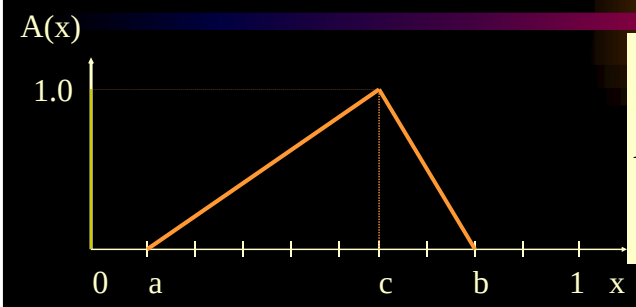
The gravity center criterion



$$D_2(A_i) = \frac{\int_0^1 xA(x).dx}{\int_0^1 A(x).dx}$$

$$D_2(A_i) = \frac{(a+b+c)}{3}$$

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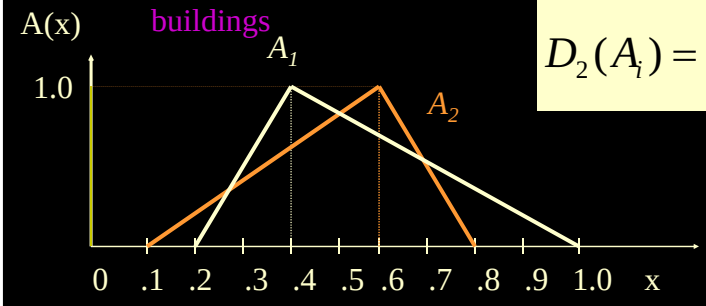
The gravity center criterion
Building Risk Problem

Risk values for buildings



$$D_2(A_i) = \frac{(a+b+c)}{3}$$

$D_1(A_1) = 0.53$

$D_1(A_2) = 0.50$

A1* has higher risk than *A2

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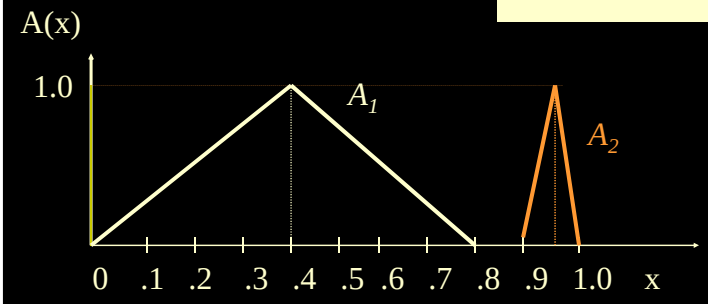
The gravity center criterion Pathological Case

$$D_2(A_i) = \frac{(a+b+c)}{3}$$



$D_1(A_1) = 0.40$

$D_1(A_2) = 0.95$

A2* has higher risk than *A1

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The Most Possible Criterion

$$D_3(A_i) = x_0$$

$$Where, \mu_{Ai}(x_0) = \max_x (\mu_{Ai}(x))$$

For Triangle Function

$$D_3(A_i) = c$$

For Trapezoidal Function

$$D_3(A_i) = \frac{c_i + d_i}{2}$$

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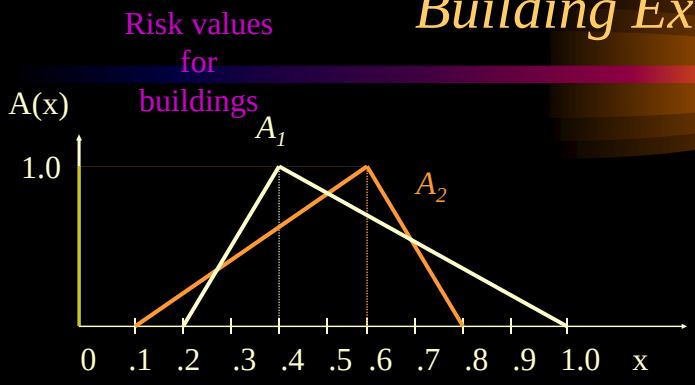
The Most Possible Criterion – Building Example



$D_3(A_1) = 0.4$ $D_3(A_2) = 0.6$

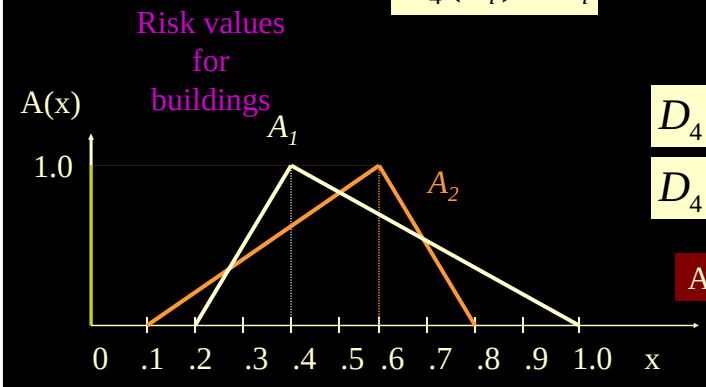
A2 has higher Risk

Approach considers only the risk with the maximum membership value

The Pessimistic Criterion Building Example

- Maximize the possible minimum risk

$$D_4(A_i) = a_i$$



$D_4(A_1) = 0.2$

$D_4(A_2) = 0.1$

A1 has higher Risk

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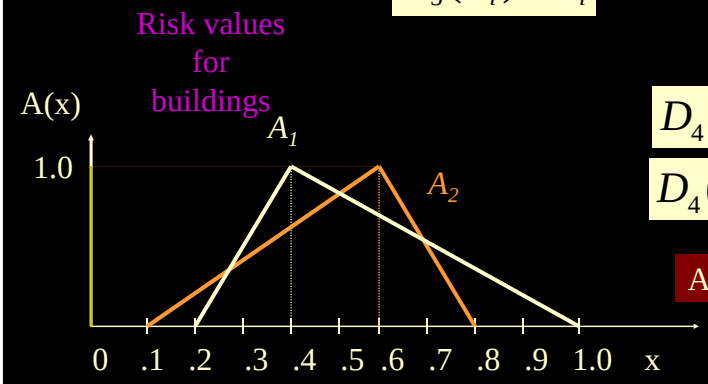
The Optimistic Criterion Building Example

- Maximize the possible maximum risk

$$D_5(A_i) = b_i$$



$$D_4(A_1) = 1.0$$

$$D_4(A_2) = 0.8$$

A1 has higher Risk

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List of Criteria

- ✓ The pseudo-expectation criterion
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- The α-optimistic criterion
- The average mean criterion
- The nearest to the ideal optimum criterion
- The dominance criterion
- The classification criterion
- The four point average criterion

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Refer following paper for details

- ***Bortolan G. and Degani R (1985) – A Review of some methods for ranking fuzzy subsets, Journals of Fuzzy Sets and Systems, 15, pp:1-19***

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Industrial Production Problem

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Industrial Production Problem

- Production Process – Involves three features
  - ***Pressure***
  - ***Temperature***
  - ***Flow Rate***

  Pattern Recognition task – The system reads sensor indicators of each features as crisp read-out values – Determine the current mode of operation

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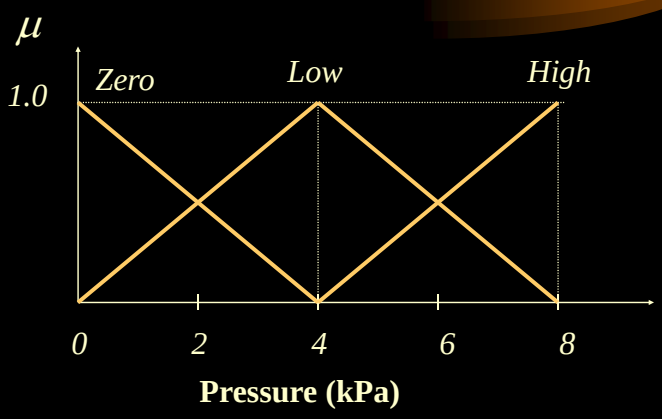
Linguistic Representation of Pressure



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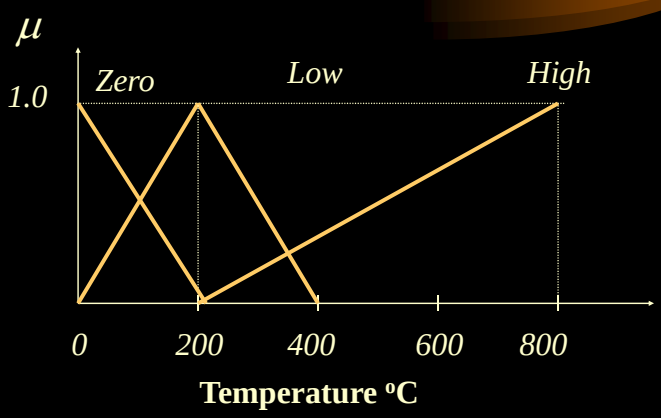
Linguistic Representation of Temperature



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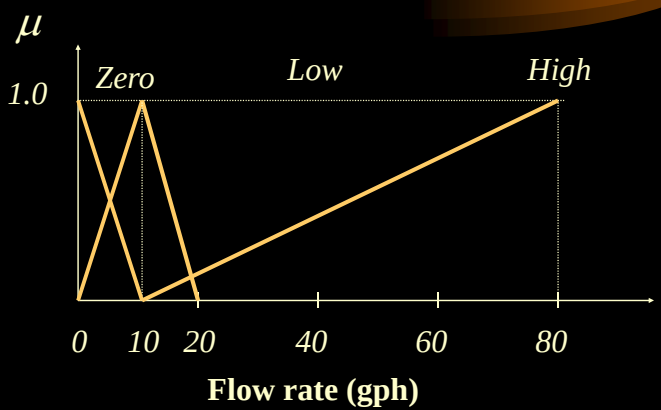
Linguistic Representation of Flow rate



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Relationship between operation mode and feature values

Mode (Pattern)	Pressure	Temperature	Flow Rate
Autoclaving	High	High	Zero
Annealing	High	Low	Zero
Sintering	Low	Zero	Low
Transport	Zero	Zero	High

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Problem Objective

- System reads from a set of sensors a set of Crisp readings
- Pressure = 5 kPa
- Temperature = 150 ^{o}C
- Flow rate = 5 gph (gallon per hour)

Find Mode of Operation

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Assumption

- Let us decide to apply weights to each features
- Feature - Pressure can be more hazardous in comparison to other features, hence provide higher weights to it.
  - **$W_{pressure} = 0.5$**
  - **$W_{temperature} = 0.25$**
  - **$W_{flow} = 0.25$**

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We have

- X = { 5 kPa, 150 ^{o}C, 5 gph}
- W = {0.5, 0.25, 0.25}

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Computation Membership Values for Each Operations

$\mu_{autoclaving}$ = (0.5).(0.25) + (0.25).(0) + (0.25).(0.5)/1= 0.25

$\mu_{annealing}$ = (0.5).(0.25) + (0.25).(0.75) + (0.25).(0.5)/1 = 0.4375

$\mu_{sintering}$ = (0.5).(0.75) + (0.25).(0.25) + (0.25).(0.5)/1 = ***0.5625***

$\mu_{transport}$ = (0.5).(0) + (0.25).(0.25) + (0.25).(0)/1 = 0.0625

Therefore, X = { 5 kPa, 150 ^{o}C, 5 gph} matches most closely with ***Sintering***

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Problem Solving

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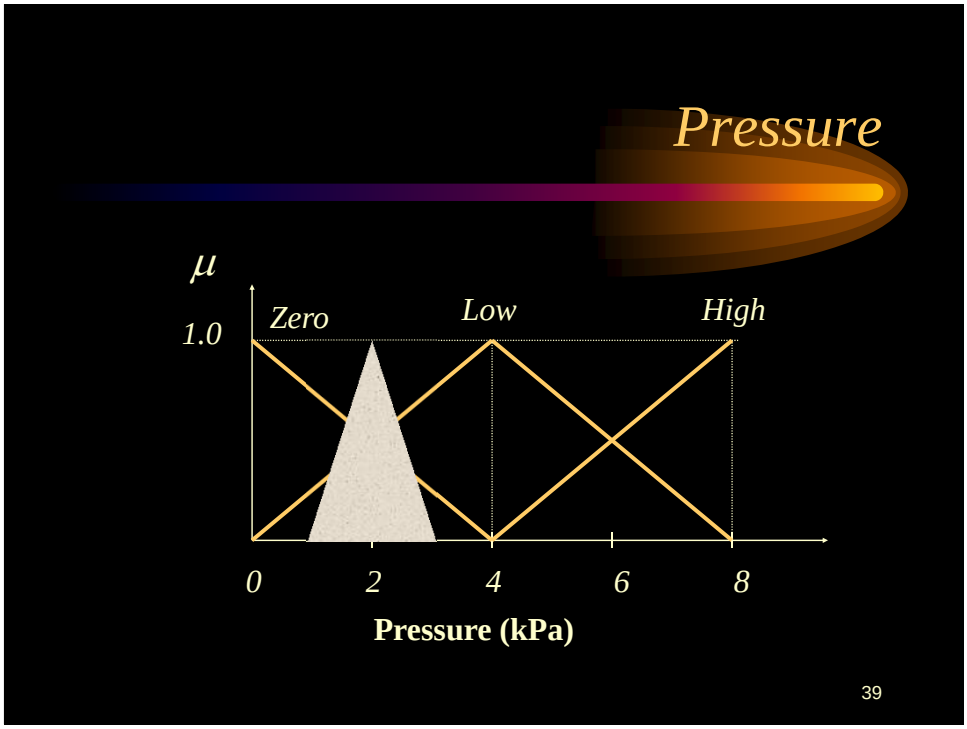
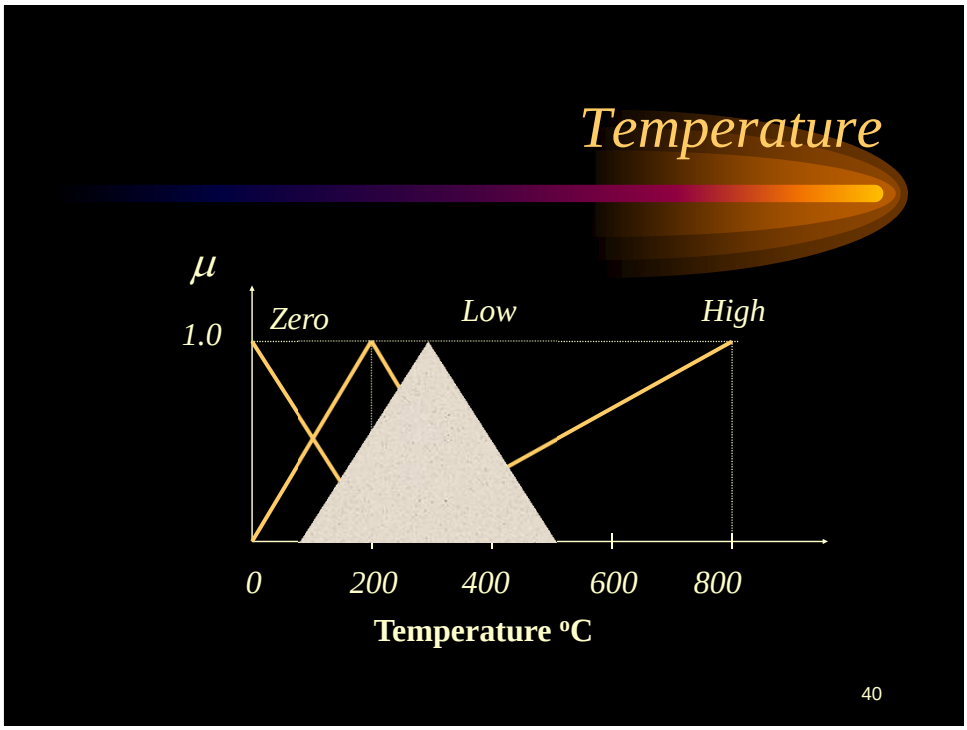
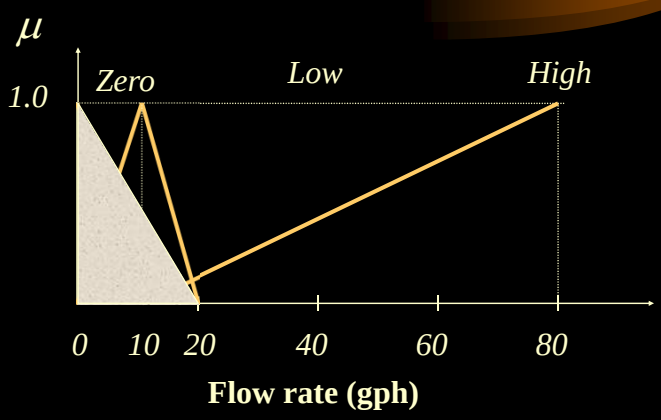

Pressure
μ
1.0
Zero
Low
High
0
2
4
6
8
Pressure (kPa)
39


Temperature
μ
1.0
Zero
Low
High
0
200
400
600
800
Temperature °C
40

Flow rate



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Assumption

- $W_{pressure} = 0.5$
- $W_{temperature} = 0.25$
- $W_{flow} = 0.25$

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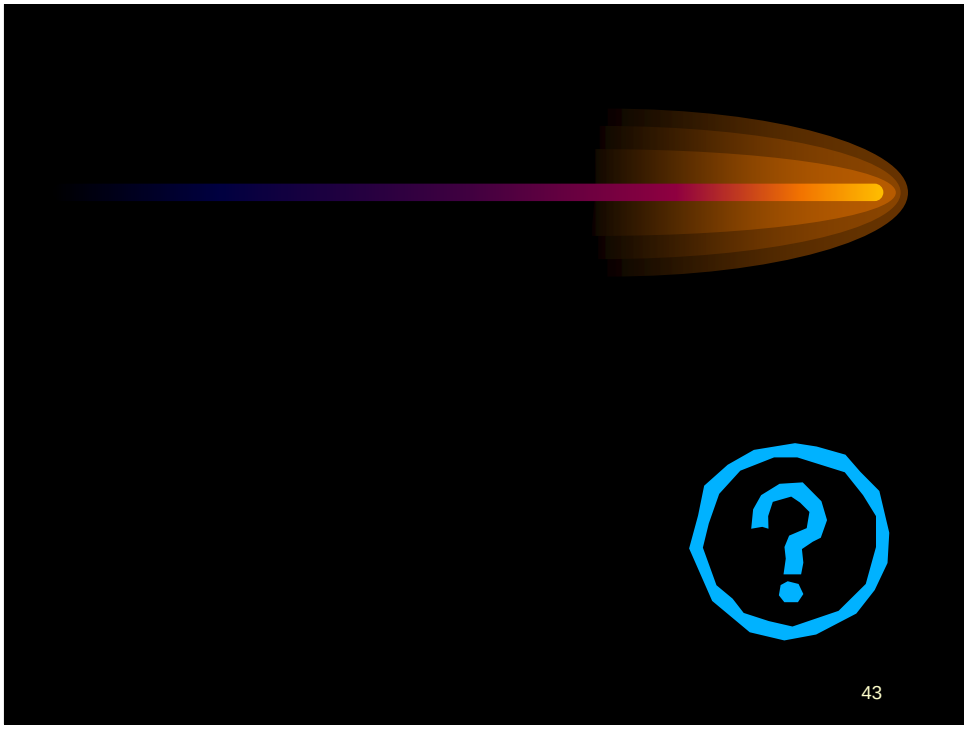
43