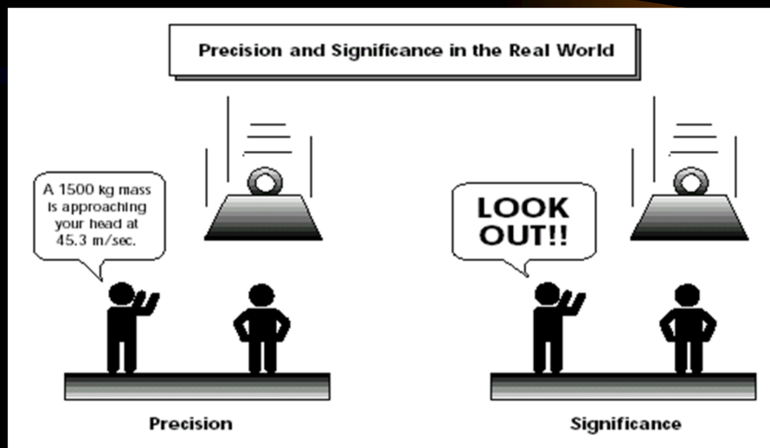
Fuzzy Logic : Introduction

Lecture 2

Lotfi Zadeh

As the complexity of a system increases, our ability to make precise and yet significant statements about its behaviour diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics



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Fuzzy Logic

Consider the following rule:

If the patient has *high* fever
and he is shivering *a lot*
then influenza is *highly likely*

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Fuzzy Rule

The words shown in *italics* are imprecise terms. These are called *fuzzy predicates.*

Such a rule is termed as a *fuzzy rule.*

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Conversion of conventional method into fuzzy method

Conventional crisp method

Natural Language

Recorded performance Numerical data

Mathematical Formula

REPRESENTATION OF A SOLUTUION BEFORE APPLYING FUZZYNESS

Fuzzy Set Theory

Fuzzy IF-THEN rules Language of fuzzy inference

Nonlinguistic Fuzzy Mathematics

REPRESENTATION OF A FUZZIFIED SOLUTION

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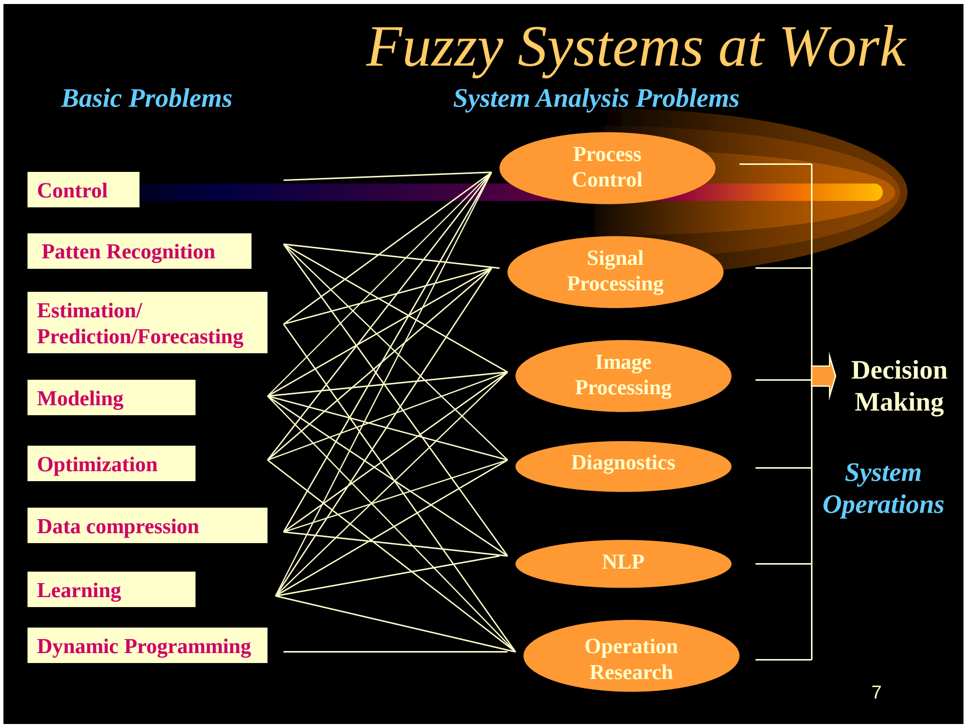

Outline

- Crisp Set and Fuzzy Set
- Membership Function
- Fuzzy Set Operation
- Terminologies

Ordinary (Crisp) Sets to Fuzzy Sets

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World of Difference

- **Crisp Set:** To dichotomize the individuals in some given universe of discourse into two groups: members (those that certainly belong in the set) and non members (those that certainly do not).
- **Fuzzy Set:** mathematically by assigning to each possible individual in the universe of discourse a value representing its grade of membership in the fuzzy set

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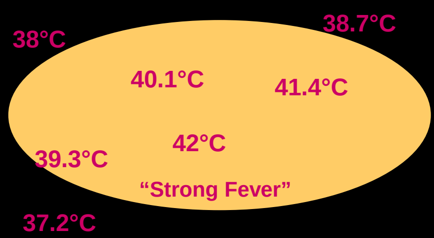
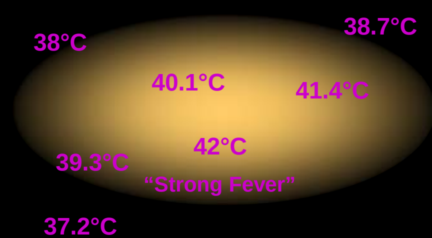
Examples

Crisp Set Theory:



Fuzzy Set Theory:



Crisp Sets: An Overview

- Z = {….,-2,-1,0,1,2,….} – The set of all integers
- N = {1,2,3,…. } – The set of all positive integers or natural numbers
- N_0 ={0,1,2,….} – The set of all non-negative integers
- N_n ={1,2,…,n}
- $N_{0,n}$ ={0,1,2,….,n}
- R: The set of all real numbers
- R^+ : The set of all nonnegative real numbers

Cont…

Crisp Sets: An Overview

- [a, b], (a, b], [a, b), (a,b): Closed, left-open, right open, open interval or real numbers between a and b, respectively.
- $<x_1, x_2, \ldots..x_n>$: ordered n-tuple of elements x_1, x_2, $\ldots..x_n$

Cont…

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Crisp Sets: An Overview

- Three basic Methods to define set within a given universal set X
- List Method: for finite sets

  $A = \{a_1, a_2, \ldots.., a_n\}$
- Rule Method: for property based sets

  $A = \{x \mid P(x)\}$

  P(x) is true or false and $x \in X$

Cont…

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Crisp Sets: An Overview

- Characteristic Function : declares which elements of X are members of the set and which are not.
- Set A is defined by its characteristic function χ_A, as follows:

$$\chi_A(X) = \begin{cases} 1 \text{ for } x \in A \\ 0 \text{ for } x \notin A \end{cases} \qquad \chi_A : X \rightarrow \{0,1\}$$

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Fundamental Properties of Crisp set Operation

- Involution $\overline{\overline{A}} = A$
- Commutative $A \cup B = B \cup A$, $A \cap B = B \cap A$
- Associativity $(A \cup B) \cup C = A \cup (B \cup C)$
  $(A \cap B) \cap C = A \cap (B \cap C)$
- Distributivity $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- ……

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Crisp Sets: An Overview

Homework : General background from recommended text book of ***Klir and Yuan***

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Fuzzy Sets : Basic Types

- The characteristic function of crisp set assigns value of either 1 or 0 to each individual in the universal set.
- Two distinct notations are most commonly used to denote membership functions

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Fuzzy Sets : Basic Types

Membership function of a fuzzy set A is denoted by μ_A; that is

$\mu_A : X \rightarrow [0,1]$

The function is denoted by A and has the same form:

$A : X \rightarrow [0,1]$

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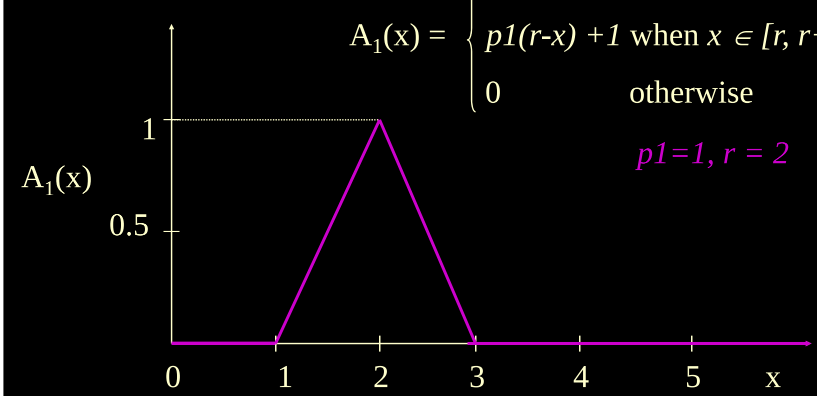
Membership Function

$$A_1(x) = \begin{cases} p1(x-r) + 1 & \text{when } x \in [r-1/p1, r] \\ p1(r-x) + 1 & \text{when } x \in [r, r+1/p1] \\ 0 & \text{otherwise} \end{cases}$$

p1=1, r = 2



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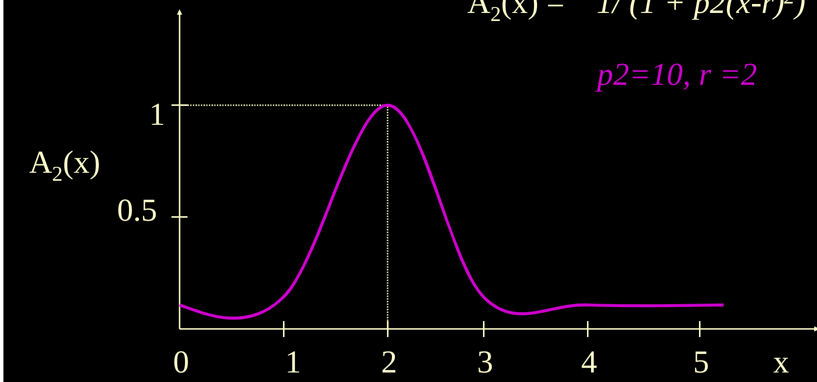
Membership Function

$A_2(x) = \; 1/\,(1 + p2(x\text{-}r)^2)$

p2=10, r =2



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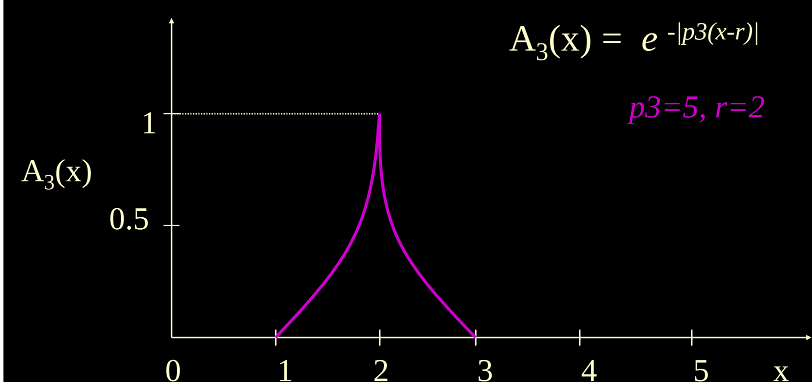
Membership Function

$A_3(x) = \; e^{-|p3(x-r)|}$

p3=5, r=2



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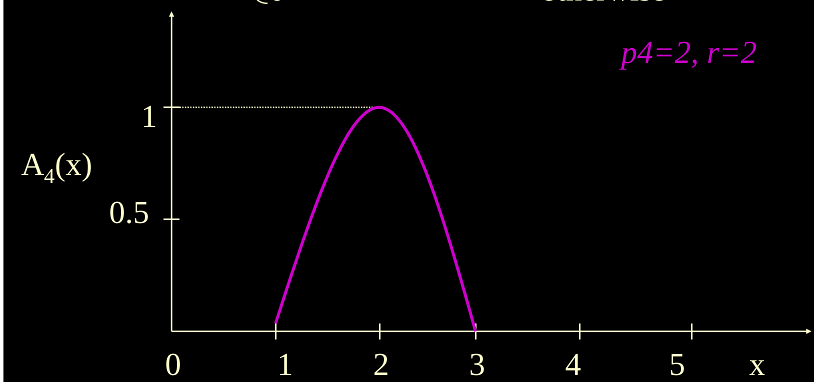
Membership Function

$$A_4(x) = \begin{cases} (1 + \cos(p4\pi(x\text{-}r)))/2 & \text{when } x \in [r\text{-}1/p4,\ r+1/p4] \\ 0 & \text{otherwise} \end{cases}$$

p4=2, r=2



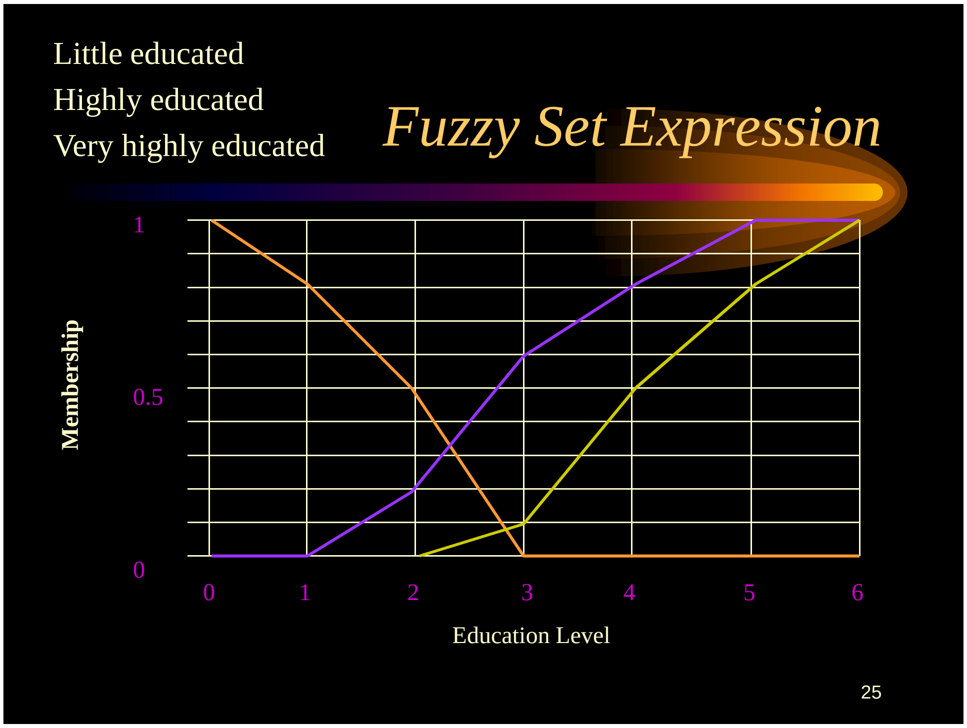
Simple Example

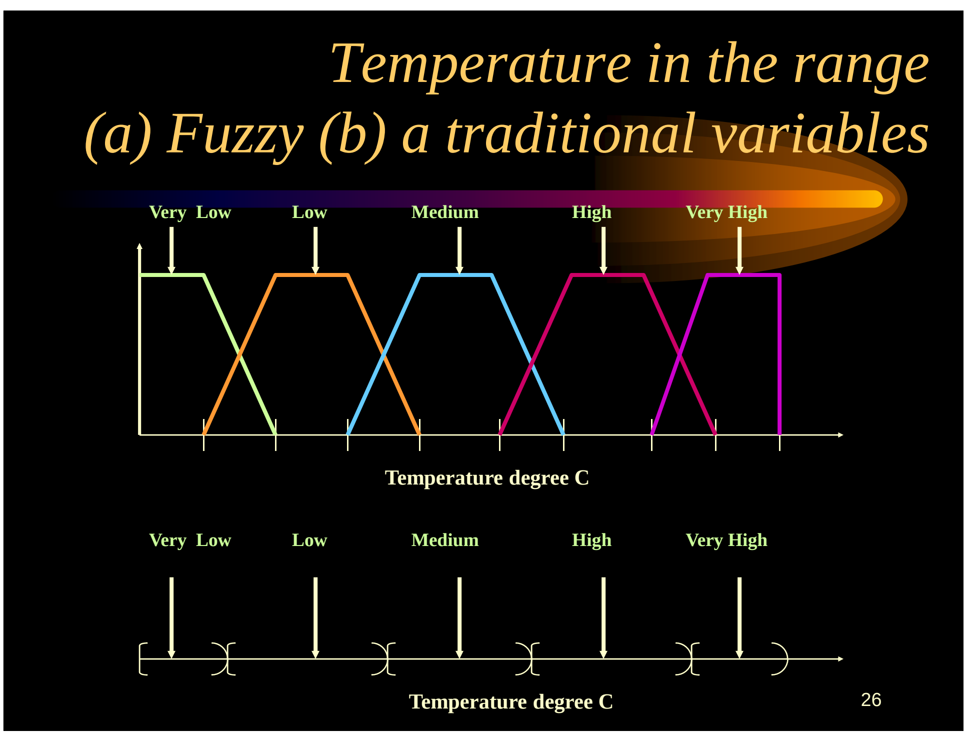
- Fuzzy sets
  (Three Types)

Little educated
Highly educated
Very highly educated

- Finite Universal set consists of seven levels of education

0 – No education
1 – Elementary school
2 – High school
3 - Two-year college degree
4- Bachelor’s degree
5- Master’s degree
6 – Doctoral Degree


Little educated
Highly educated
Very highly educated
Fuzzy Set Expression
1
0.5
0
Membership
0
1
2
3
4
5
6
Education Level
25


Temperature in the range
(a) Fuzzy (b) a traditional variables
Very Low
Low
Medium
High
Very High
Temperature degree C
Very Low
Low
Medium
High
Very High
Temperature degree C
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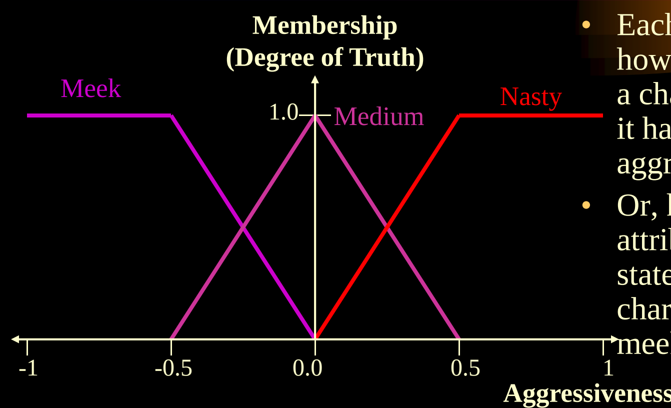
Example Fuzzy Variable

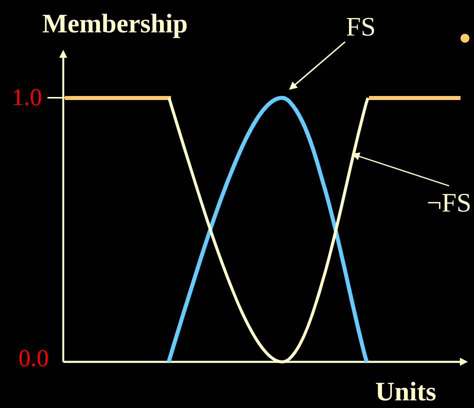


- Each function tells us how much we consider a character in the set if it has a particular aggressiveness value
- Or, how much truth to attribute to the statement: "The character is nasty (or meek, or neither)?"

Basic Operations on Fuzzy Sets

- Complement
- Intersection
- Union

Fuzzy Set Operations: Complement



- The degree to which you believe something is **not** in the set is 1.0 minus the degree to which you believe it is in the set

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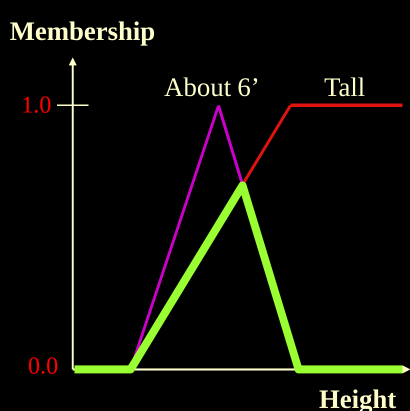
Fuzzy Set Operations: Intersection (AND)



- If you have x degree of faith in statement A, and y degree of faith in statement B, how much faith do you have in the statement A and B?
  - Eg: How much faith in "that person is about **<u>6' high and tall</u>**"
- Does it make sense to attribute **<u>more truth</u>** than you have in one of A or B?

$(A \cap B)(x) = \min[A(x), B(x)]$

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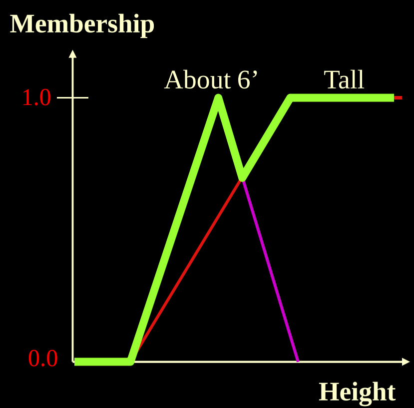
Fuzzy Set Operations: Union (OR)



- If you have x degree of faith in statement A, and y degree of faith in statement B, how much faith do you have in the statement A or B?
  - Eg: How much faith in “that person is about **6’ high or tall**”
- Does it make sense to attribute **less truth** than you have in one of A or B?

$(A \cup B)(x) = \max[A(x), B(x)]$

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CAR EXAMPLE

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Car Example

- "If our distance to the car in front is small, and the distance is decreasing slowly, then decelerate quite hard"
  - Fuzzy variables in blue
  - Fuzzy sets in red
  - Conditions are on membership in fuzzy sets
  - Actions place an output variable (decelerate) in a fuzzy set (the quite hard deceleration set)
- We have a certain belief in the truth of the condition, and hence a certain strength of desire for the outcome
- Multiple rules may match to some degree, so we require a means to arbitrate and choose a particular goal - *defuzzification*

Car Examples

- Rules for controlling a car:
  - Variables are *distance* to car in front and how fast it is changing, *delta*, and *acceleration* to apply
  - Sets are:
    - Very small, small, perfect, big, very big - for distance
    - Shrinking fast, shrinking, stable, growing, growing fast for delta
    - Brake hard, slow down, none, speed up, floor it for acceleration
  - Rules for every combination of distance and delta sets, defining an acceleration set

Car Example

- Assume we have a particular numerical value for distance and delta, and we need to set a numerical value for acceleration
  - Extension: Allow fuzzy values for input variables (degree to which we believe the value is correct)

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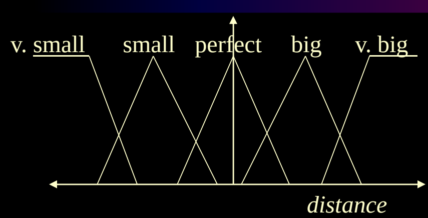
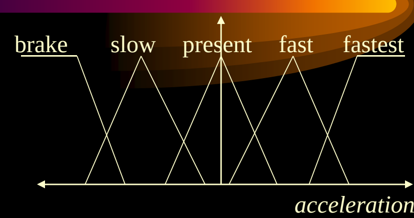
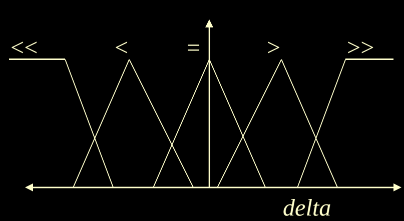
Set Definitions for Example







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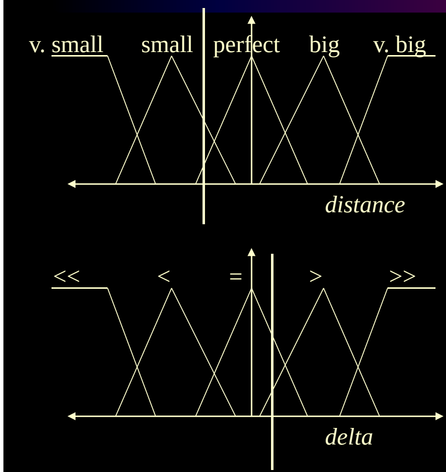
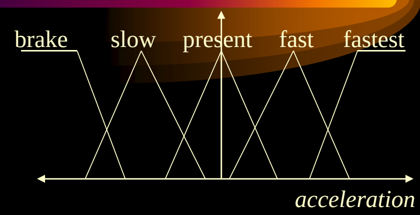
Instance for Example





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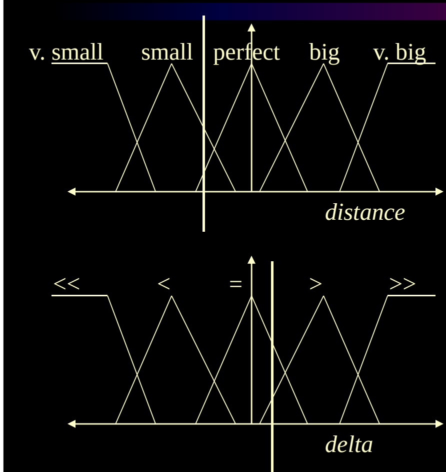
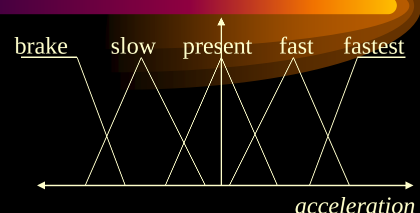
- Distance could be considered small or perfect
- Delta could be stable or growing
- What acceleration?

Instance for Example





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- If distance is small and delta is growing, maintain speed
- If distance is small and delta is stable, slow down
- If distance is perfect and delta is growing, speed up
- If distance is perfect and delta is stable, maintain speed

Matching for Example

- Relevant rules are:
  - If distance is small and delta is growing, maintain speed
  - If distance is small and delta is stable, slow down
  - If distance is perfect and delta is growing, speed up
  - If distance is perfect and delta is stable, maintain speed
- For first rule, distance is small has 0.75 truth, and delta is growing has 0.3 truth
  - So the truth of the **and** is 0.3
- Other rule strengths are 0.6, 0.1 and 0.1

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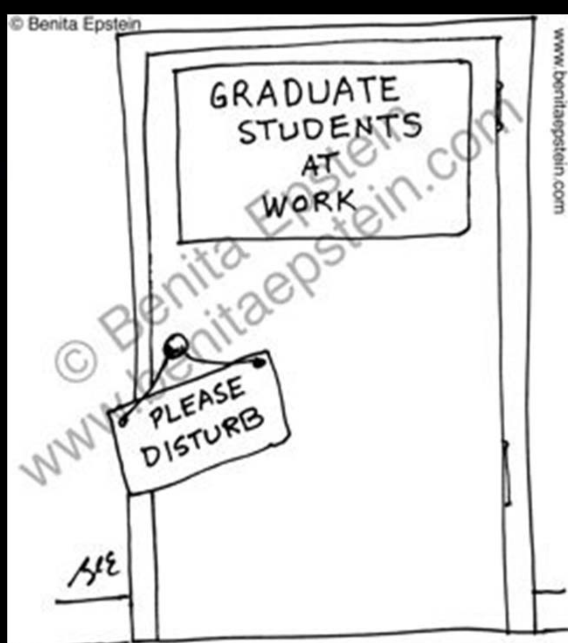

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