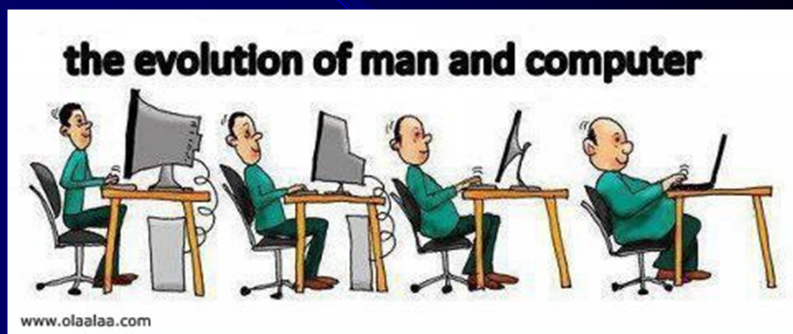
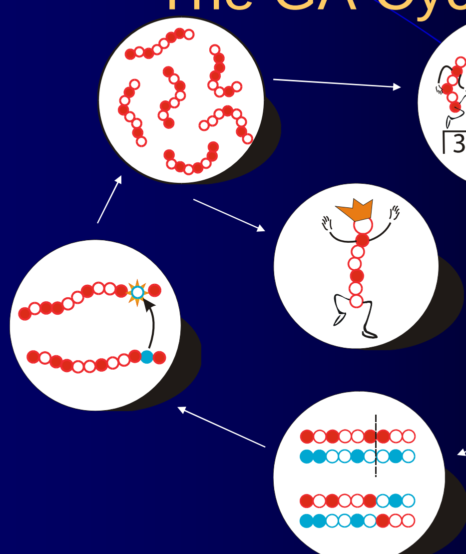
Genetic Algorithm: An Example

Lecture 3



1

The GA Cycle



2

Simple Problem

- The objective is to minimize the function

$$f(x_1, x_2) = (x_1^2 + x_2 - 11)^2 + (x_1 + x_2^2 - 7)^2$$

- In the interval $0 \le x_1, x_2 \le 6$
- True solution is (3,2)

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Step 1

- Choose binary coding for x_1, x_2
- 10-bits are chosen for each variable
- Total string length equal to 20.
- With 10-bits we can get a solution accuracy of $(6-0)/(2^{10} - 1) = 0.006$ in the interval (0,6).
- Chose roulette-wheel selection
- Crossover probability = 0.8
- Mutation probability = 0.05
- Population size = 20
- $t_{max} = 30$

4

	String												Mating Pool	
	Substring 2	Substring 1	x_2	x_1	*f(x)*	F(x)	A	B	C	D	E	F	Substring 2	Substring 1
1	1110010000	1100100000	5.349	4.692	959.680	0.001	0.13	0.007	0.007	0.472	10	0	0010100100	1010101010
2	0001001101	0011100111	0.452	1.355	105.520	0.009	1.10	0.055	0.062	0.108	3	1	1010100001	0111001000
3	1010100001	0111001000	3.947	2.674	126.685	0.008	0.98	0.049	0.111	0.045	2	1	0001001101	0011100111
4	1001000110	1000010100	3.413	3.120	65.026	0.015	1.85	0.093	0.204	0.723	14	2	1110011011	0111000010
5	1100011000	1011100011	4.645	4.334	512.197	0.002	0.25	0.013	0.217	0.536	10	0	0010100100	1010101010
6	0011100101	0011111000	1.343	1.455	70.868	0.014	1.71	0.086	0.303	0.931	19	2	0011100010	1011000011
7	0101011011	0000000111	2.035	0.041	88.273	0.011	1.34	0.067	0.370	0.972	19	1	0011100010	1011000011
8	1110101000	1110101011	5.490	5.507	1436.563	0.001	0.12	0.006	0.376	0.817	17	0	0111000010	1011000110
9	1001111101	1011100111	3.736	4.358	265.556	0.004	0.49	0.025	0.401	0.363	7	1	0101011011	0000000111
10	0010100100	1010101010	0.962	4.000	39.849	0.024	2.96	0.148	0.549	0.189	4	3	1001000110	1000010100
11	1111101001	0001110100	5.871	0.680	814.117	0.001	0.14	0.007	0.556	0.220	6	0	0011100101	0011111000
12	0000111101	0110011101	0.358	2.422	42.598	0.023	2.84	0.142	0.698	0.288	6	3	0011100101	0011111000
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16	0100011111	1100111000	1.683	4.833	197.556	0.005	0.61	0.030	0.777	0.192	4	0	1001000110	1000010100
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20	1011100011	1111010000	4.334	5.724	987.955	0.001	0.13	0.006	1.000	0.413	10	0	0010100100	1010101010

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Step 2

- Evaluate each string in the population
- First sub-string - 1100100000

  → $(2^9 + 2^8 + 2^5)$

  → 800

  → 0 + (6-0) x 800 /(1024-1) = 4.692
- Second sub-string 1110010000 → 5.349
- *f(x)* = 959.680
- Fitness function value at this value using the transformation rule: **F(x) = 1.0/(1.0 + 959.680) =0.001**
- Calculate for other populations

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Step 3

- Since $t < t_{max}$, We proceed to Step 4.

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Step 4

- Picking up good strings – Use roulette-wheel selection procedure.
- Calculate average fitness of the population

$$\underline{F} = \Sigma F_i / n = 0.008$$

- **Column A:** Expected count of each string **F(x)/ $\underline{F}$**
- **Column B:** Probability of each string being copied in the mating pool

  = Value of Column A/ Population size

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Step 4 (Contd.)

- Column C: Cumulative probability of selection
- Column D: In order to form the mating pool, creat random numbers between zero and one and identify the particular string which is specified by each of these random numbers. e.g. for first string it has been 0.472 → that string occupies the interval (0.401,0.549) as shown in Column C → String 10 is identified for Column E

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Step 4 (Contd.)

- Column F : True count in the mating pool (Column E)
- Observation : Columns A and F reveal that the theoretical expected count and the true count of each string more or less agree with each other.

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Step 5

- Mating Pool are used for Crossover.
- In single-point crossover, two strings are selected at random and crossed at a random site.
- e.g. At random select strings 3 and 10 for first cross-over operation.
- First a coin is flipped with a probability p_c = 0.8 to check whether a cross-over is desired or not.
- Let us say – YES.

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Step 5 (Contd.)

- Find the cross-site at random.
- We choose a site by creating a random number between (0, *l-1*) or (0,19)
- In this case, with 10 pairs Crossover operation will be carried out.
- Store the revised strings as **Intermediate Population**

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Step 6

- Mutation process on intermediate population
- For bit-wise mutation, flip the coin with a probability $p_m = 0.05$ for every bit.
- In this case, maximum 0.05 x 20 x 20 =20 bits in population (population size 20, string length 20).

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Step 7

- Resulting population becomes the new population. Carryout steps of 2-6.
- Go on till we find the point at which the fitness value is near to 1.0

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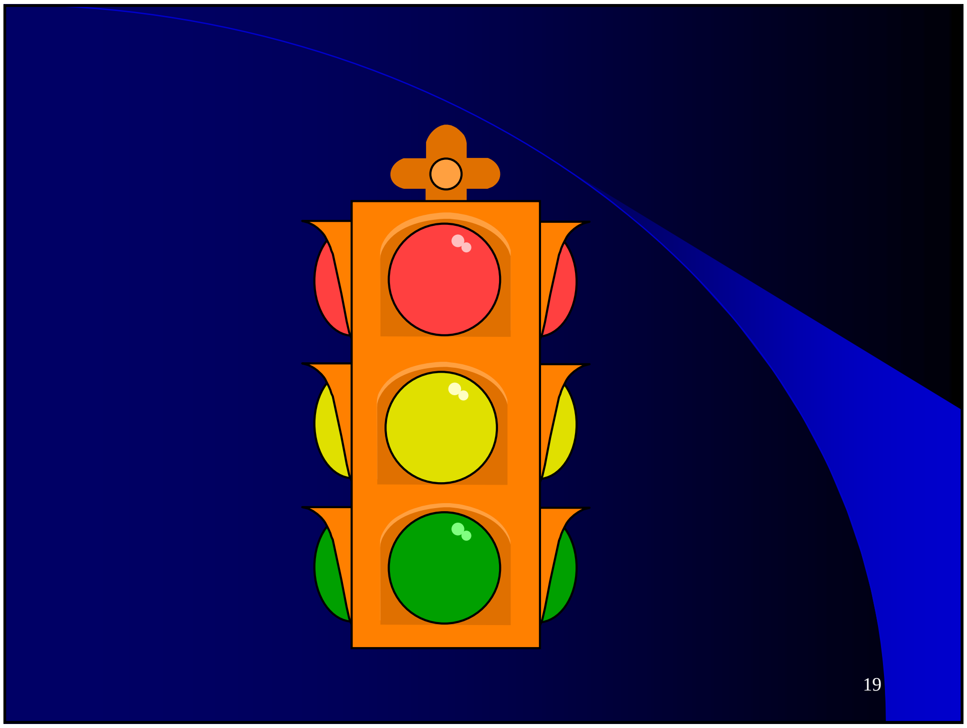
19