

Simple Problem

The objective is to minimize the function

$$f(x_1, x_2) = (x_1^2 + x_2 - 11)^2 + (x_1 + x_2^2 - 7)^2$$

- In the interval $0 \le x_b, x_2 \le 6$
- True solution is (3,2)

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Step 1

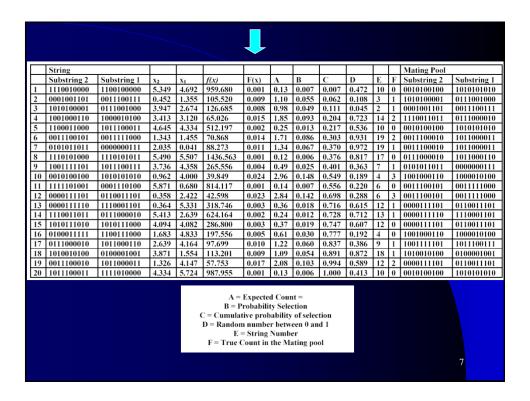
- Choose binary coding for x_0, x_2
- 10-bits are chosen for each variable
- Total string length equal to 20.
- With 10-bits we can get a solution accuracy of $(6-0)/(2^{10}-1) = 0.006$ in the interval (0,6).
- Chose roulette-wheel selection
- Crossover probability = 0.8
- Mutation probability = 0.05
- Population size = 20
- $t_{max} = 30$

	Ctuina												Mating Pool	
⊩	String	C., b., 4., !	-		(Y-1)	F(x)	A	В	С	D	Е	F		C., b., t., !
I .	Substring 2	Substring 1	5.349	X ₁	f(x) 959.680		0.13	0.007	~	0.472	10	0	- and a triang	Substring 1
1	1110010000 0001001101	1100100000 0011100111	0.452	4.692	105.520	0.001	1.10	0.007	0.007	0.472	3	1	0010100100 1010100001	1010101010 0111001000
3	101010001101	011100111	3.947	1.355 2.674	126.685	0.009	0.98	0.055	0.062	0.108	2	1	0001001101	00111001000
4	100100001	1000010100	3.413	3.120	65.026	0.008	1.85	0.049	0.111	0.723	14	2	1110011011	011100011
5	1100011000110	10111100011	4.645	4.334	512.197	0.015	0.25	0.093	0.204	0.723	10	0	001010011011	1010101010
	0011100101		1.343		70.868		1.71	0.013	0.217	0.536	19	2	0010100100	
6		0011111000		1.455		0.014				0.931	19			1011000011
7	0101011011	0000000111	2.035	0.041	88.273	0.011	1.34	0.067	0.370	0.15		1	0011100010	1011000011
8	1110101000	1110101011	5.490	5.507	1436.563	0.001	0.12	0.006	0.376	0.817	17	_	0111000010	1011000110
9	1001111101	1011100111	3.736	4.358	265.556	0.004	0.49	0.025	0.401	0.363	7	1	0101011011	0000000111
10	0010100100	1010101010	0.962	4.000	39.849	0.024	2.96	0.148	0.549	0.189	4	3	1001000110	1000010100
11	1111101001	0001110100	5.871	0.680	814.117	0.001	0.14	0.007	0.556	0.220	6	0	0011100101	0011111000
12	0000111101	0110011101	0.358	2.422	42.598	0.023	2.84	0.142	0.698	0.288	6	3	0011100101	0011111000
13	0000111110	1110001101	0.364	5.331	318.746	0.003	0.36	0.018	0.716	0.615	12	1	0000111101	0110011101
14	1110011011	0111000010	5.413	2.639	624.164	0.002	0.24	0.012	0.728	0.712	13	1	0000111110	1110001101
15	1010111010	1010111000	4.094	4.082	286.800	0.003	0.37	0.019	0.747	0.607	12	0	0000111101	0110011101
16	0100011111	1100111000	1.683	4.833	197.556	0.005	0.61	0.030	0.777	0.192	4	0	1001000110	1000010100
17	0111000010	1011000110	2.639	4.164	97.699	0.010	1.22	0.060	0.837	0.386	9	1	1001111101	1011100111
18	1010010100	0100001001	3.871	1.554	113.201	0.009	1.09	0.054	0.891	0.872	18	1	1010010100	0100001001
19	0011100010	1011000011	1.326	4.147	57.753	0.017	2.08	0.103	0.994	0.589	12	2	0000111101	0110011101
20	1011100011	1111010000	4.334	5.724	987.955	0.001	0.13	0.006	1.000	0.413	10	0	0010100100	1010101010
A = Expected Count = B = Probability Selection C = Cumulative probability of selection D = Random number between 0 and 1 E = String Number F = True Count in the Mating pool												5		

- Evaluate each string in the population
- First sub-string 1100100000

$$\rightarrow$$
 (2⁹ + 2⁸ + 2⁵)

- → 800
- \rightarrow 0 + (6-0) x 800 /(1024-1) = 4.692
- Second sub-string 1110010000 → 5.349
- f(x) = 959.680
- Fitness function value at this value using the transformation rule: F(x) = 1.0/(1.0 + 959.680) = 0.001
- Calculate for other populations





- Picking up good strings Use roulette-wheel selection procedure.
- Calculate average fitness of the population

$$\underline{\mathbf{F}} = \mathbf{\Sigma} \; \mathbf{F_i} / \, \mathbf{n} = 0.008$$

- Column A: Expected count of each string F(x)/F
- Column B: Probability of each string being copied in the mating pool
 - = Value of Column A/ Population size

							<u> </u>	<u> </u>						
	String												Mating Pool	
	Substring 2	Substring 1	X ₂	X ₁	f(x)	F(x)	A	В	C	D	E	F	Substring 2	Substring 1
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Step 4 (Contd.)

- Column C: Cumulative probability of selection
- Column D: In order to form the mating pool, creat random numbers between zero and one and identify the particular string which is specified by each of these random numbers. e.g. for first string it has been 0.472 → that string occupies the interval (0.401,0.549) as shown in Column C → String 10 is identified for Column E

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Step 4 (Contd.)

- Column F: True count in the mating pool (Column E)
- Observation: Columns A and F reveal that the theoretical expected count and the true count of each string more or less agree with each other.

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						Expecte								
	B = Probability Selection													
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	$\mathbf{D} = \mathbf{Random\ number\ between\ 0\ and\ 1}$													
	E = String Number													
	F = True Count in the Mating pool													
														14

- Mating Pool are used for Crossover.
- In single-point crossover, two strings are selected at random and crossed at a random site.
- e.g. At random select strings 3 and 10 for first cross-over operation.
- First a coin is flipped with a probability $p_c = 0.8$ to check whether a cross-over is desired or not.
- Let us say YES.

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Step 5 (Contd.)

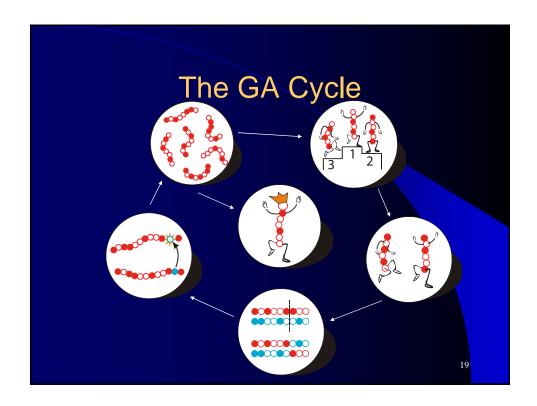
- Find the cross-site at random.
- We choose a site by creating a random number between (0, l-1) or (0,19)
- In this case, with 10 pairs Crossover operation will be carried out.
- Store the revised strings as Intermediate Population

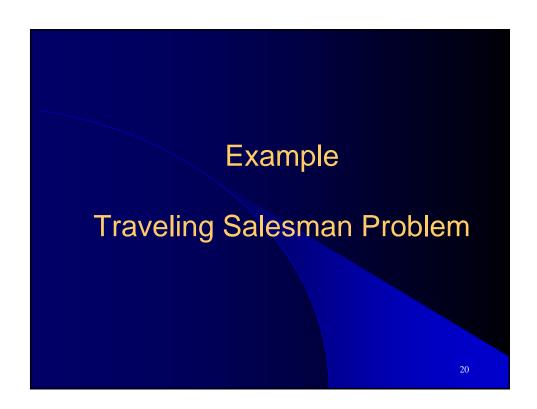
- Mutation process on intermediate population
- For bit-wise mutation, flip the coin with a probability $p_m = 0.05$ for every bit.
- In this case, maximum 0.05 x 20 x 20 = 20 bits in population (population size 20, string length 20).

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Step 7

- Resulting population becomes the new population. Carryout steps of 2-6.
- Go on till we find the point at which the fitness value is near to 1.0

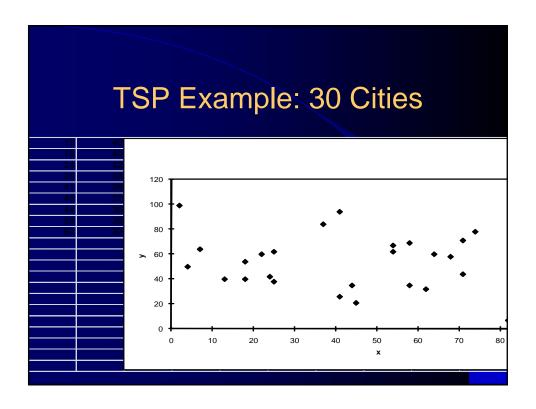


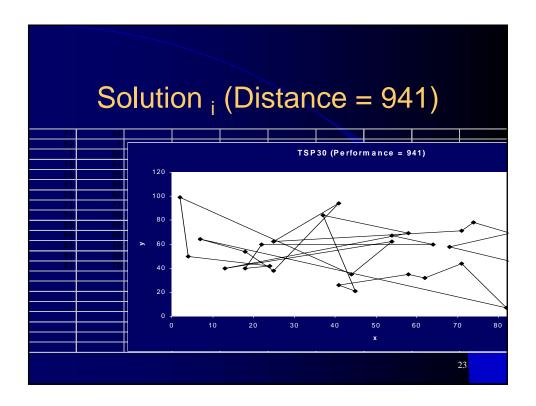


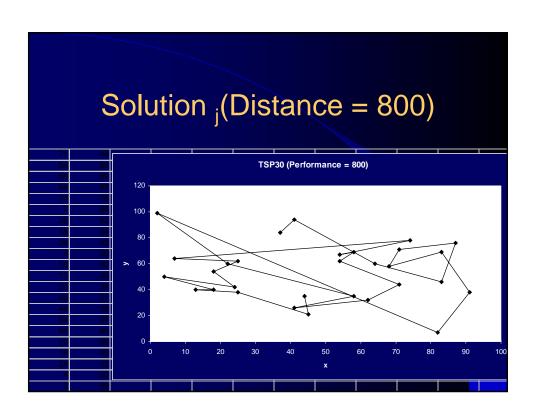
Traveling Salesman Problem

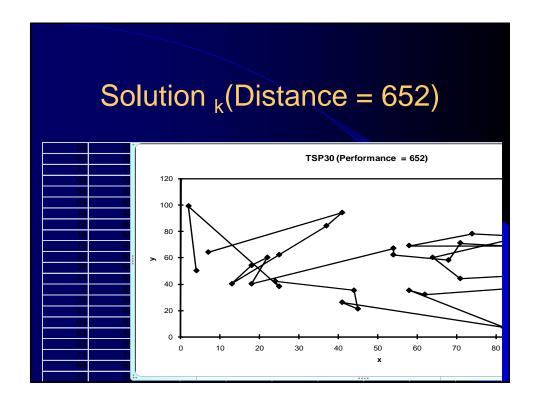
Find a tour of a given set of cities so that

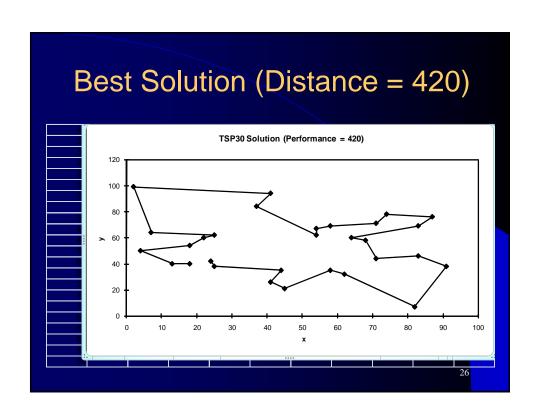
- each city is visited only once
- the total distance traveled is minimized

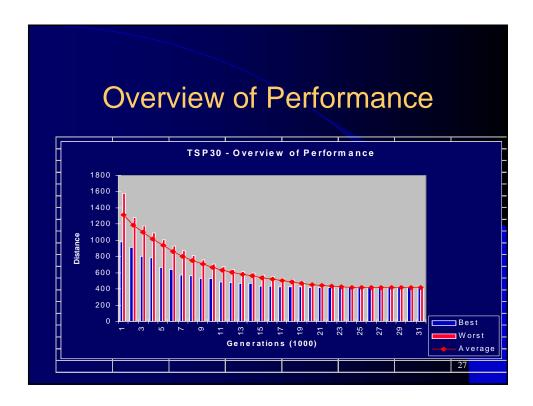








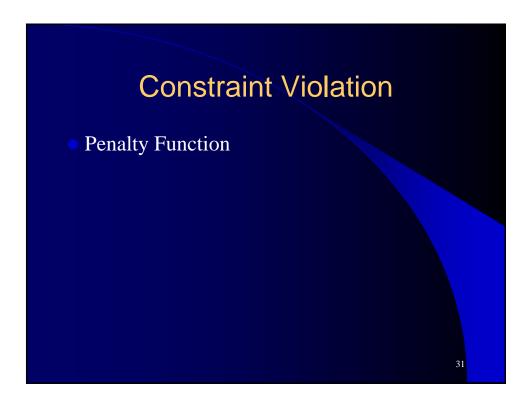


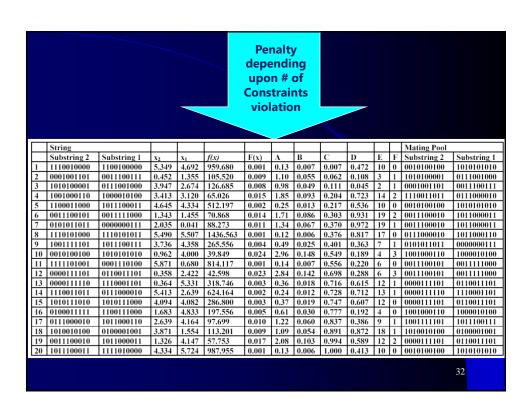




Discussion on Issues Constraints Violation Selection Cross over Mutation Debate – Cross over Vs Mutation Coding Approaches

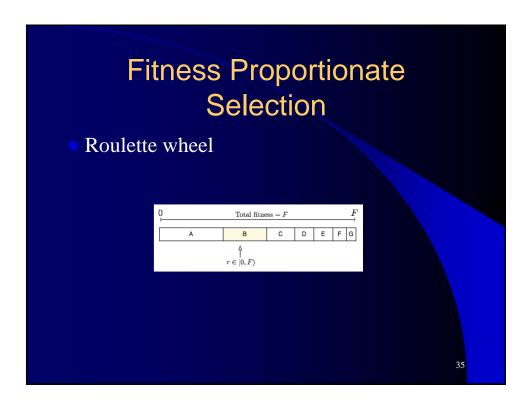


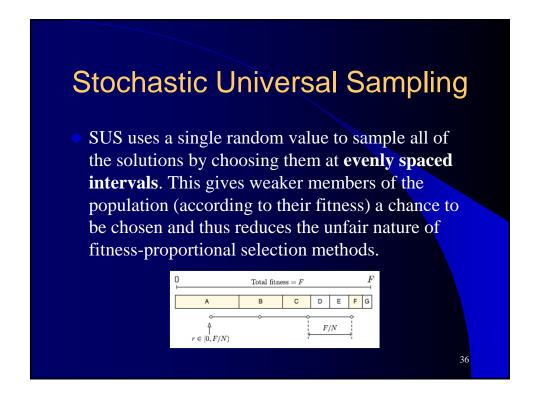












Tournament Selection

- **Tournament selection** is a method of selecting an individual from a population of individuals in a genetic algorithm.
- Tournament selection involves running several "tournaments" among a few individuals chosen at random from the population.
- The winner of each tournament (the one with the best fitness) is selected for <u>crossover</u>.
- Selection pressure is easily adjusted by changing the tournament size.
- If the tournament size is larger, weak individuals have a smaller chance to be selected.

Reward Based Selection

- Reward-based selection is a technique used in <u>evolutionary algorithms</u> for selecting potentially useful solutions for recombination.
- The probability of being selected for an individual is proportional to the cumulative reward, obtained by the individual.
- The cumulative reward can be computed as a sum of the individual reward and the reward, inherited from parents.



Cross Over Approaches One-point crossover Two-point crossover "Cut and splice" Uniform Crossover and Half Uniform Crossover Three parent crossover Crossover for Ordered Chromosomes

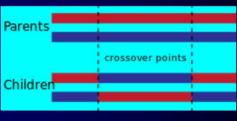
One Point Crossover

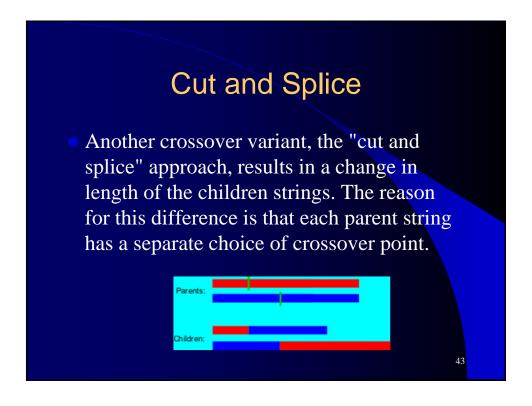
A single crossover point on both parents' organism strings is selected. All data beyond that point in either organism string is swapped between the two parent organisms. The resulting organisms are the children:

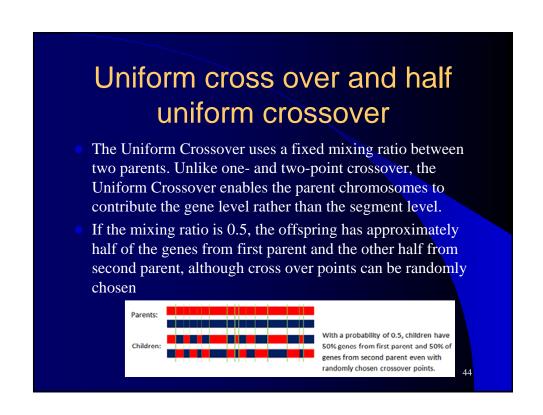
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Two Point Crossover

Two-point crossover calls for two points to be selected on the parent organism strings.
 Everything between the two points is swapped between the parent organisms, rendering two child organisms:







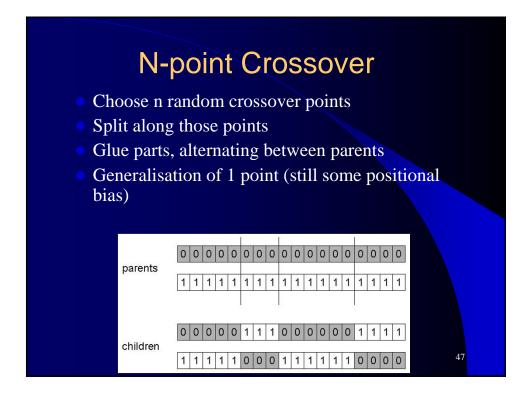
Three Parent Crossover

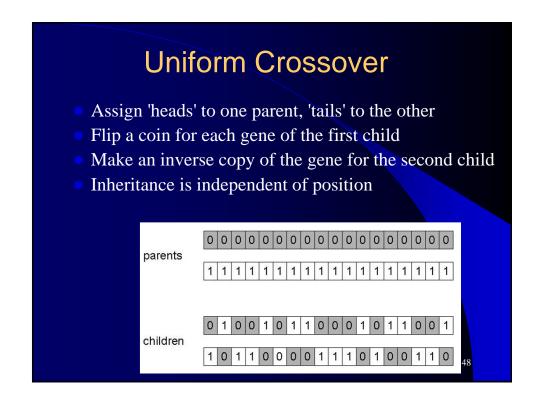
- In this technique, the child is derived from three parents. They are randomly chosen. Each bit of first parent is checked with bit of second parent whether they are same. If same then the bit is taken for the offspring otherwise the bit from the third parent is taken for the offspring. For example, the following three parents:
- parent1 1 1 0 1 0 0 0 1 0
- parent2 0 1 1 0 0 1 0 0 1
- parent3 1 1 0 1 1 0 1 0 1produces the following offspring:
- offspring 1 1 0 1 0 0 0 0 1

4:

Alternative Crossover Operators

- Performance with 1 Point Crossover depends on the order that variables occur in the representation
 - more likely to keep together genes that are near each other







Mutation

- Flip Bit This mutation operator takes the chosen genome and inverts the bits. (i.e. if the genome bit is 1,it is changed to 0 and vice versa)
- Boundary This mutation operator replaces the genome with either lower or upper bound randomly. This can be used for integer and float genes.

Mutation

Non-Uniform - The probability that amount of mutation will go to 0 with the next generation is increased by using non-uniform mutation operator. It keeps the population from stagnating in the early stages of the evolution. It tunes solution in later stages of evolution. This mutation operator can only be used for integer and float genes.

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Mutation

Uniform – This operator replaces the value of the chosen gene with a uniform random value selected between the user-specified upper and lower bounds for that gene. This mutation operator can only be used for integer and float genes.

Mutation

Gaussian - This operator adds a unit Gaussian distributed random value to the chosen gene. If it falls outside of the user-specified lower or upper bounds for that gene, the new gene value is clipped. This mutation operator can only be used for integer and float genes.



Crossover OR Mutation?

- Decade long debate: which one is better / necessary / main-background
- Answer (at least, rather wide agreement):
 - it depends on the problem, but
 - in general, it is good to have both
 - both have another role
 - mutation-only-EA is possible, xover-only-EA would not work

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Crossover OR Mutation?

Exploration: Discovering promising areas in the search space, i.e. gaining information on the problem

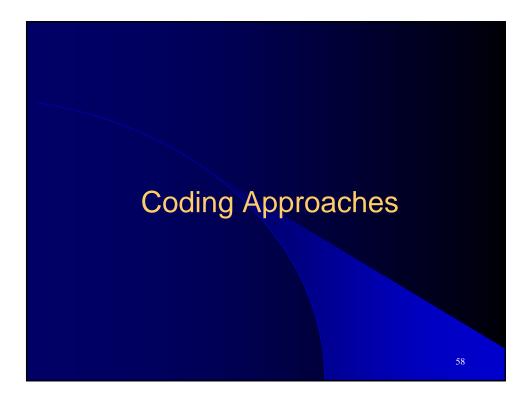
Exploitation: Optimising within a promising area, i.e. using information

There is co-operation AND competition between them

- Crossover is explorative, it makes a *big* jump to an area somewhere "in between" two (parent) areas
- Mutation is exploitative, it creates random *small* diversions, thereby staying near (in the area of) the parent

Crossover OR Mutation?

- Only crossover can combine information from two parents
- Only mutation can introduce new information
- To hit the optimum you often need a 'lucky' mutation



Coding Representations Binary Coding Real Coding

