**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

**Ans: B)**

work begin after 10 min, so the average time increase from 45min to 55min.

for normal distribution :-

z = (X-μ)/б

= (60-55)/8

= 0.625

Using R software

1-pnorm(0.625)

=0.2659

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

**Ans: TRUE**

*μ* = 38, *σ* =6

Probabolity of employees>44 =P(X>44)=1-P(X<=44)

Z=(x-*μ)/σ = (x-38)/6*

P(X<=44)=P(z<=(44-38)/6)=P(z<=1)=0.8413

Probability that employees will be greater than 44 =1-0.8413=0.158

Probability of employees between 38 & 44= P(X<=44)-P(X>=38)

P(X>=38)P(z>=(38-38)/6)=P(z>=0)=0.5

Therefore, P(x<=44)-P(x>=P(x>=38)=0.84-0.5=0.34

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans: TRUE**

Probability of employees less than 30 =P(X<30)

Z=(x- μ)/ σ = P(z<(30-38)/6)=P(z<-1.3333=0.0917

The number of employees with probability 0.0917 of them being under 30=400\*0.0917=36.68=36

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

The Normal distribution is defined by two parameters, the mean, μ, and the variance σ2

as X~ *N*(μ, σ2).

Given *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables

From the properties of **normal random variables**,

if  *X* ~ *N*(μ1, σ12) and *X*2 ~ *N*(μ2, σ22) and  are two independent identically distributed random variables then

the sum of normal random variables is given by *X1*+ *X*2 ~ *N*(μ1+ μ2, σ12 +σ22)

and the difference of normal random variables is given by *X1*- *X*2 ~ *N*(μ1- μ2, σ12 +σ22)

When  z=ax, the product of X is given by z,

*z* ~ *N*(aμ1, a2σ12)

When  Z=aX+bY, the linear combination of X and Y is given by

z~ *N*(aμ1+b μ2, a2σ12 +b2σ22)

to find 2X1 using property of product

**2X1~ *N*(2μ, 22σ2) ~ N(2μ, 4σ2)**

to find *X*1 + *X*2 using property of addition

***X*1 + *X*2~N(μ+ μ, σ2+ σ2) ~ N(2μ, 2σ2)**

The difference between the two is given by

https://tex.z-dn.net/?f=2X_1-(X_1%2BX_2)%20%5Csim%20N(2%5Cmu%20-%202%5Cmu%2C%202%5Csigma_1%5E2%20%2B%204%5Csigma_2%5E2%20)%5Csim%20N(%200%2C6%20%5Csigma%5E2)

**2X1- *(X*1 + *X*2)~N((2μ-2μ, 2σ2+4σ2)~N(0,6σ2)**

The difference between the two says that the two given variables are **identically** and **independently** distributed

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

**Ans: D**

**from** scipy **import** stats

**from** scipy.stats **import** norm

stats**.**norm**.**interval(0.99,100,20)

(48.48341392902199, 151.516586070978)

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

