Sets

Dipam Sen (Fun Planet)

A set is a **well defined collection of objects.**

The following are some examples of valid sets.

$$N = \{1, 2, 3, 4, 5, \cdots\}$$

$$V = \{A, E, I, O, U\}$$

$$A = \{\text{all students in a class}\}$$

The following examples are not well defined, thus they cannot be called sets.

$$\text{F} = \{\text{toughest chapters in JEE Mains}\}$$

Representation of a Set

A Set can be represented in two forms:

1. **Roster form:** All the elements are listed in curly brackets ({ }). Multiple elements can be shown using $\cdots$.

$$A = \{3, 6, 9, 12, \cdots\}$$

$$B = \{\text{A, E, I, O, U}\}$$

2. **Set builder form:** Each element of the set is represented by a symbol (x). After the colon (:), the characteristic properties of the elements are mentioned.

$$A = \{x : x \text{ is divisible by } 3; x \in N\}$$

$$B = \{x : x \text{ is a vowel in the English alphabet}\}$$

- The order in which elements are listed in a set do not matter.

$$\{a, b, c\} = \{c, a, b\}$$

- All elements are counted only once in a set, i.e. repetition of elements in a set do not change the set.

$$A = \{x : x \text{ is a letter in the word PLANE}\}$$

$$B = \{x : x \text{ is a letter in the word PLAN}\}$$

$$A = B = \{\text{A, L, N, P}\}$$

Common Sets

$$\begin{aligned} N &= \text{natural numbers} \\ Z &= \text{integers} \\ Q &= \text{rational numbers} \\ R &= \text{real numbers} \end{aligned}$$

Belongingness

$\in$ = belongs to
$\notin$ = does not belong to

$$A = \{x : x \text{ is a factor of } 30\}$$

$$5 \in A$$

$$7 \notin A$$

Null Set

A set which does not contain any element.

$$\phi = \{\ \}$$

Two sets are said to be **equal** iff all the elements in both sets are same.

$$A = \{x : 0 \leq x \leq 5; x \in Z\}$$

$$B = \{0, 1, 2, 3, 4, 5\}$$

$$A = B$$

Subsets

A set X is the **subset of** set A iff all elements of X are in A

$$X = \{a, b\}$$

$$A = \{a, b, c, d, e\}$$

$$X \subset A$$

$\subset$ = proper subset of
$\subseteq$ = subset of

$$A \subset B \text{ if } x \in A \implies x \in B$$

- ϕ is a subset of every set.
- Every set is a subset of itself. ($A \subseteq A$)
- $N \subset Z, Z \subset Q, Q \subset R$

Intervals

An interval is a **subset of** R which denotes a range of real numbers.

Notation:
[= closed end (included)
(= open end (excluded)

$$[\mathbf{a},\ \mathbf{b}] = \{x : a \leq x \leq b; x \in R\}$$

$$[\mathbf{a},\ \mathbf{b}) = \{x : a \leq x < b; x \in R\}$$
$$(\mathbf{a},\ \mathbf{b}] = \{x : a < x \leq b; x \in R\}$$
$$(\mathbf{a},\ \mathbf{b}) = \{x : a < x < b; x \in R\}$$

- $(-\infty, \infty)$ is the set of all real numbers.
- **Length** of an interval (a, b), $[a, b)$, $(a, b]$, $[a, b]$ is $(b - a)$

Cardinal Number

The number of elements in a set.

$$A = \{a, b, c\}$$
$$n(A) = 3$$
$$n(\phi) = 0$$

Power Set

The set of all subsets.

$$A = \{a, b, c\}$$
$$P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$$

- The number of subsets $= 2^n$

$$n(A) = m \implies n\left(P(A)\right) = 2^m$$

Universal Set

The set containing all the possible elements in a particular context.

$$U = R$$
$$A = \{x : x^2 - 6x + 5 = 0; x \in R\}$$

- All sets are subset of U.

Set Operations

Union $(\cup)$

Combination of all elements in the sets

$$A = \{a, b, c\}$$
$$B = \{b, c, d, e\}$$
$$A \cup B = \{a, b, c, d, e\}$$

$$A \cup B = \{x : x \in A \text{ or } x \in B\}$$

Intersection $(\cap)$

Elements which are common in both the sets

$$A = \{a, b, c\}$$
$$B = \{b, c, d, e\}$$
$$A \cap B = \{b, c\}$$

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

Difference $(-)$

Elements which are only present in the first set and not in the second set.

$$A = \{a, b, c\}$$
$$B = \{b, c, d, e\}$$
$$A - B = \{a\}$$

$$A - B = \{x : x \in A \text{ and } x \notin B\}$$

Complement ($'$)

Elements which are not present in the set. (Inverse of the set)

$$U = \{a, b, c, d, e, f, g\}$$
$$A = \{a, b, c\}$$
$$A' = \{d, e, f, g\}$$

$$A' = U - A = \{x : x \notin A; x \in U\}$$

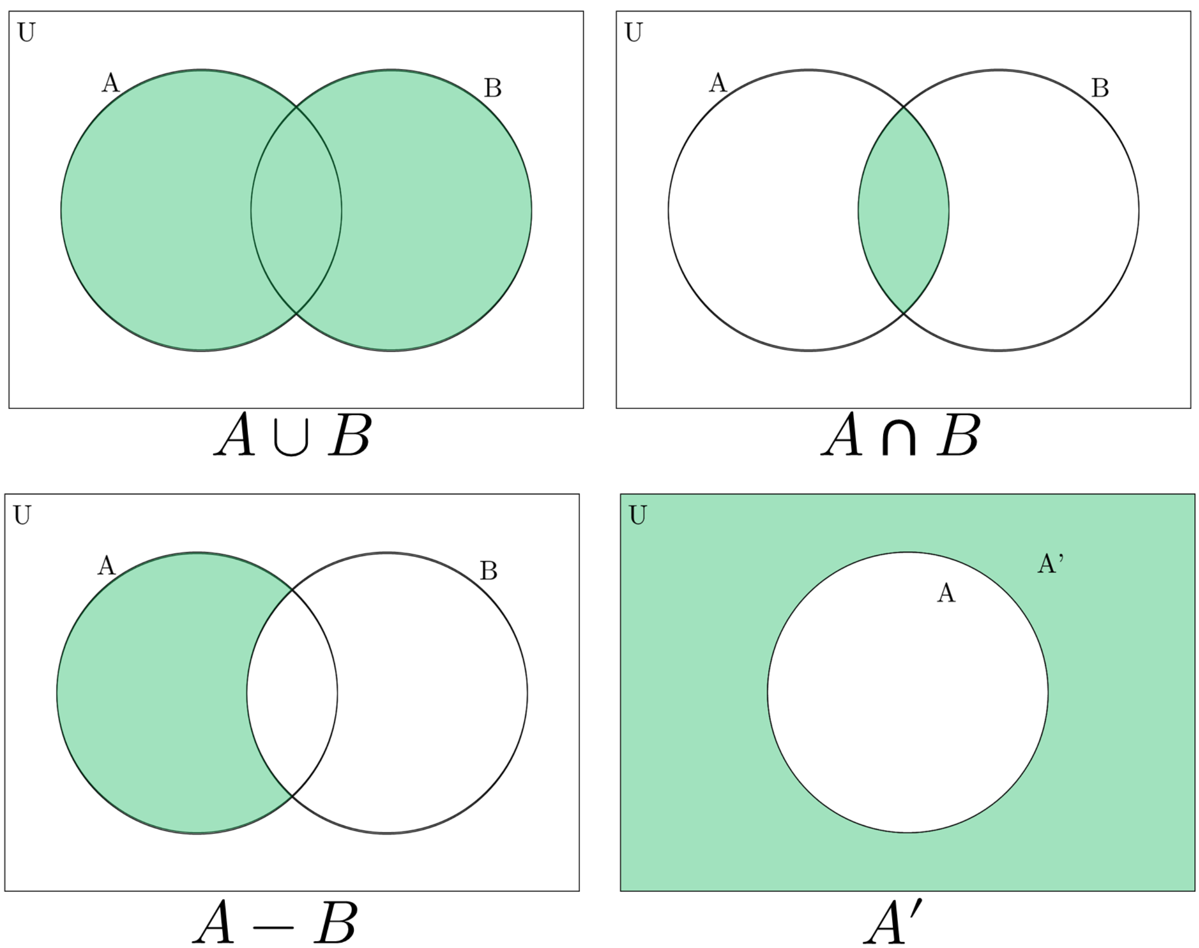

Figure 1: Operations on Sets

Algebra Laws of Sets

1. **Idempotent Law**

- $A \cup A = A$
- $A \cap A = A$

2. **Identity Law**

- $A \cap U = A$
- $A \cup \phi = A$

3. **Commutative Law**

- $A \cup B = B \cup A$
- $A \cap B = B \cap A$

4. **Associative Law**

- $(A \cup B) \cup C = A \cup (B \cup C)$
- $(A \cap B) \cap C = A \cap (B \cap C)$

5. **Distributive Law**

- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

6. **De-Morgan's Law**

- $(A \cup B)' = A' \cap B'$
- $(A \cap B)' = A' \cup B'$

7. **Formulas in Practical Application**

- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$