Inequalities

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Inequalities are algebraic statements which use the symbols $<$, $>$, $\leq$, $\geq$.

$$5x + 3 \geq 10$$
$$x(x-1) < 0$$

Solution Set: The set of all values of x satisfying the inequality.

Some important points

- Any number can be added/subtracted from LHS and RHS.

$$x + 3 < 0$$
$$x < -3 \qquad \text{(subtract 3)}$$

- On multiplying/dividing a **positive number** to both sides, the inequality stays.

$$5x > 50$$
$$x > 10 \qquad \text{(divide 5)}$$

- On multiplying/dividing a **negative number** to both sides, the inequality inverts.

$$-x > -2$$
$$x < 2 \qquad \text{(multiply } -1\text{)}$$

Linear Inequalities in One Variable

$$3x - 2 < 0$$

These inequalities can be solved easily by following the rules of inequalities.

Example 1

$$3x - 2 < 0$$
$$3x < 2$$
$$x < \frac{2}{3}$$
$$x \in \left(-\infty, \frac{2}{3}\right)$$

When a pair of linear inequalities are given, we take the intersection of the solution sets of both the inequalities.

Example 2

$$\frac{5x}{4} + \frac{3x}{8} > \frac{39}{8} \text{ and } \frac{2x-1}{12} - \frac{x-1}{3} < \frac{3x+1}{4}$$

$$\begin{aligned}
\text{Eqn 1: } & \frac{5x}{4} + \frac{3x}{8} > \frac{39}{8} \\
& 10x + 3x > 39 \\
& 13x > 39 \\
& \boxed{x > 3} \\
\text{Eqn 2: } & \frac{2x-1}{12} - \frac{x-1}{3} < \frac{3x+1}{4} \\
& (2x-1) - 4(x-1) < 3(3x+1) \\
& 2x - 1 - 4x + 4 < 9x + 3 \\
& -2x + 3 < 9x + 3 \\
& 0 < 11x \\
& \boxed{x > 0}
\end{aligned}$$

The intersection of the solutions $x > 0$ and $x > 3$ is $x > 3$.
$\therefore x \in (3, \infty)$

Example 3

$$2 \geq 5x - 2 \geq 4$$

$$4 \geq 5x \geq 6$$
$$\frac{4}{5} \geq x \geq \frac{6}{5}$$
$$x \in \left[\frac{4}{5}, \frac{6}{5}\right]$$

Inequalities of Higher Order

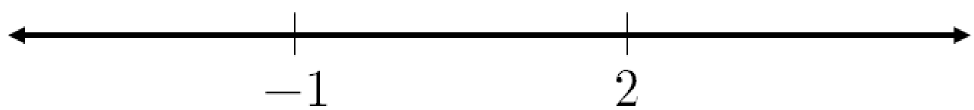
We use the **Wavy Curve Method** to solve general inequalities.

Example 4

$$(x-2)(x+1) > 0$$

1. **We plot the roots of the LHS on a number line.**

   We plot -1 and 2 on the number line.



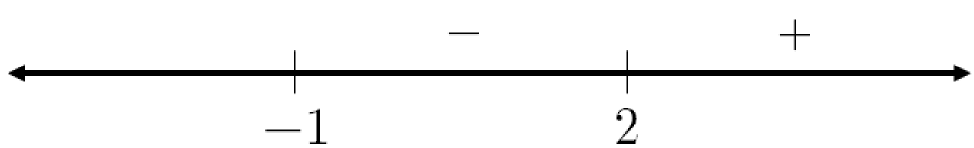
2. **Starting from the right hand side, we check the sign of the expression at that value x.**

   When $x > 2$, then the value of $(x-2)(x+1)$ will be +ve.

3. **Going towards the left, we check the power of the factor.**

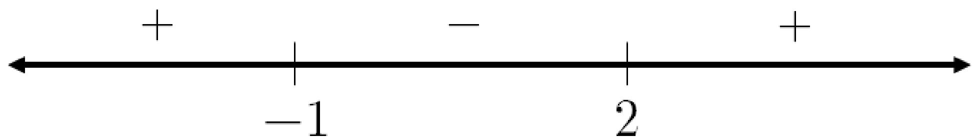
   - If the power is odd, **the opposite sign** is indicated on the next interval.
   - If the power is even, **the same sign** is indicated on the next interval.

   The factor $(x-2)$ is raised to an odd power (1). So the next interval (-1 to 2) will be $-$ve.



4. **Continue till all intervals are marked.**

   The factor $(x+1)$ is raised to an odd power (1). So the next interval ($-\infty$ to -1) will be +ve.



5. **We create the solution set based on the number line.**

   We want to find all values of x such that $(x-2)(x+1)$ is greater that 0.

   $\therefore x \in (-\infty, -1) \cup (2, \infty)$

> - To use the Wavy Curve Method, we must convert the inequality such that:
>   - LHS is a polynomial with linear factors.
>   - RHS is 0.
> - The roots themselves are included in the solution only if the sign of inequality is $\geq$ or $\leq$.

Example 5

$$\frac{3x-1}{x-1} \geq 0$$

$$\boxed{x \in \left(-\infty, \frac{1}{3}\right] \cup (1, \infty)}$$

Note that here $\frac{1}{3}$ is included in the solution set because the sign of the inequality is $\geq$.

However, 1 is not included, because the factor $(x-1)$ is in the denominator of the expression. When $x=1$, the expression becomes *undefined*.

Example 6

$$\frac{2x-3}{3x-5} \geq 3$$

$$\frac{2x-3}{3x-5} - 3 \geq 0$$

$$\frac{2x-3-9x+15}{3x-5} \geq 0$$

$$\frac{-7x+12}{3x-5} \geq 0$$

$$\frac{7x-12}{3x-5} \leq 0$$

$$\boxed{x \in \left(\frac{5}{3}, \frac{12}{7}\right]}$$