Relations and Functions

Relations

- Relations are a subset of the **cardinal product of two sets.**

A relation between set A and B can be denoted as:

$$R_{AB} \subseteq \{(x, y) : x \in A, y \in B\}$$

Functions

- Functions are Relations of whom:
    - Every element of first set should have an image in second set.
    - No $x \in A$ can have more than one image in B

Types of Relations

1. Void Relation

Let A be a set. Then $\phi \subseteq A \times A$, so it is a relation on A.

2. Universal Relation

Let A be a set. Then $A \times A \subseteq A \times A$

3. Identity Relation

Let A be a set. Then the Relation $I_A = \{(a, a) : a \in A\}$

4. Reflexive Relation

A relation R on a set A is said to be reflexive if every element of A is related to itself. The R is reflexive $\Leftrightarrow$ $(a, a) \in R$ for all $a \in A$

Eg:

$$A = \{1, 2, 3\}$$

$$R_1 = \{(1, 1), (2, 2), (3, 3), (1, 3), (1, 2)\}$$

R_1 is reflexive.

$$R_2 = \{(1,1),(2,2)\}$$

R_2 is not reflexive.

- Symmetric Relations

A relation R on a set A is symmetric iff:

$$(a,b) \in R \implies (b,a) \in R$$

for all $a, b \in A$

- Transitive Relations

A relation R on a set A is transitive iff:

$$(a,b) \in R \wedge (b,c) \in R \implies (a,c) \in R$$

for all $a, b, c \in A$

Notes:

- R_1 is Symmetric and Transitive $\implies$ R_1 is Reflexive.

- Equivalence relation:

Iff:

- it is reflexive ((a,a) is present for all a)
- it is symmetric ((a,b) implies (b,a))
- it is transitive ((a,b) and (b,c) implies (a,c))