

Inequalities

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Inequalities are algebraic statements which use the symbols $<,>,\leq,\geq$.

$$5x + 3 \ge 10$$

$$x(x-1) < 0$$

Solution Set: The set of all values of x satisfying the inequality.

Some important points

• Any number can be added/subtracted from LHS and RHS.

$$x + 3 < 0$$

$$x < -3$$
 (subtract 3)

• On multiplying/dividing a **positive number** to both sides, the inequality stays.

$$x > 10$$
 (divide 5)

• On multiplying/dividing a **negative number** to both sides, the inequality inverts.

$$-x > -2$$

$$x < 2$$
 (multiply -1)



Linear Inequalities in One Variable

$$3x - 2 < 0$$

These inequalities can be solved easily by following the rules of inequalities.

Example 1

$$3x - 2 < 0$$

$$3x < 2$$

$$x < \frac{2}{3}$$

$$x \in \left(-\infty, \frac{2}{3}\right)$$

When a pair of linear inequalities are given, we take the intersection of the solution sets of both the inequalities.

Example 2

$$\frac{5x}{4} + \frac{3x}{8} > \frac{39}{8} \text{ and } \frac{2x-1}{12} - \frac{x-1}{3} < \frac{3x+1}{4}$$
Eqn 1: $\frac{5x}{4} + \frac{3x}{8} > \frac{39}{8}$

$$10x + 3x > 39$$

$$\boxed{x > 3}$$
Eqn 2: $\frac{2x-1}{12} - \frac{x-1}{3} < \frac{3x+1}{4}$

$$(2x-1) - 4(x-1) < 3(3x+1)$$

$$2x - 1 - 4x + 4 < 9x + 3$$

$$-2x + 3 < 9x + 3$$

$$0 < 11x$$

$$\boxed{x > 0}$$

The intersection of the solutions x > 0 and x > 3 is x > 3.

$$\therefore x \in (3, \infty)$$

Example 3

$$2 \ge 5x - 2 \ge 4$$



$$4 \ge 5x \ge 6$$

$$\frac{4}{5} \ge x \ge \frac{6}{5}$$

$$x \in \left[\frac{4}{5}, \frac{6}{5}\right]$$

Inequalities of Higher Order

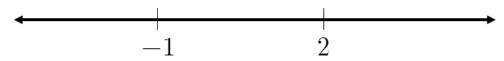
We use the **Wavy Curve Method** to solve general inequalities.

Example 4

$$(x-2)(x+1) > 0$$

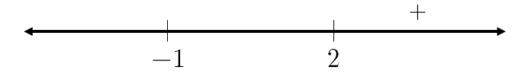
1. We plot the roots of the LHS on a number line.

We plot -1 and 2 on the number line.



2. Starting from the right hand side, we check the sign of the expression at that value x.

When x > 2, then the value of (x - 2)(x + 1) will be +ve.



- 3. Going towards the left, we check the power of the factor.
 - If the power is odd, the opposite sign is indicated on the next interval.
 - If the power is even, the same sign is indicated on the next interval.

The factor (x-2) is raised to an odd power (1). So the next interval (-1 to 2) will be -ve.





4. Continue till all intervals are marked.

The factor (x+1) is raised to an odd power (1). So the next interval $(-\infty \text{ to -1})$ will be +ve.



5. We create the solution set based on the number line.

We want to find all values of x such that (x-2)(x+1) is greater that 0.

$$\therefore x \in (-\infty, -1) \cup (2, \infty)$$

- To use the Wavy Curve Method, we must convert the inequality such that:
 - LHS is a polynomial with linear factors.
 - RHS is 0.
- The roots themselves are included in the solution only if the sign of inequality is \geq or \leq .

Example 5

$$\frac{3x-1}{x-1} \ge 0$$



$$x \in \left(-\infty, \frac{1}{3}\right] \cup (1, \infty)$$

Note that here $\frac{1}{3}$ is included in the solution set because the sign of the inequality is \geq .

+



However, 1 is not included, because the factor (x-1) is in the denominator of the expression. When x=1, the expression becomes *undefined*.

Example 6

$$\frac{2x-3}{3x-5} \ge 3$$

$$\frac{2x-3}{3x-5} - 3 \ge 0$$

$$\frac{2x-3-9x+15}{3x-5} \ge 0$$

$$\frac{-7x+12}{3x-5} \ge 0$$

$$\frac{7x-12}{3x-5} \le 0$$

