Relations and Functions

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Cartesian Product of Two Sets

The set of ordered pairs of all possible combinations from both sets.

$$A = \{a, b\}$$
$$B = \{1, 2\}$$
$$A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$$

- **Ordered Pair** is a pair of elements in which the order of elements matter.

$$(a_1, a_2) = (b_1, b_2) \text{ if } a_1 = b_1 \text{ and } a_2 = b_2$$

- $n(A \times B) = n(A) \times n(B)$

Relation

A Relation from set A to set B is a relationship between the first and second element in $A \times B$. It is a **subset of** $A \times B$.

$$A = \{1, 2, 3\}$$
$$B = \{a, b, c\}$$
$$R_{AB} = \{(1, a), (2, b), (3, c)\}$$

$$P = \{a, b, c\}$$
$$Q = \{\text{Ali, Bhanu, Binoy, Chandra, Divya}\}$$
$$R_{PQ} = \{(x, y) : x \text{ is the first letter of } y\}$$
$$R = \{(a, \text{Ali}), (b, \text{Bhanu}), (b, \text{Binoy}), (c, \text{Chandra})\}$$

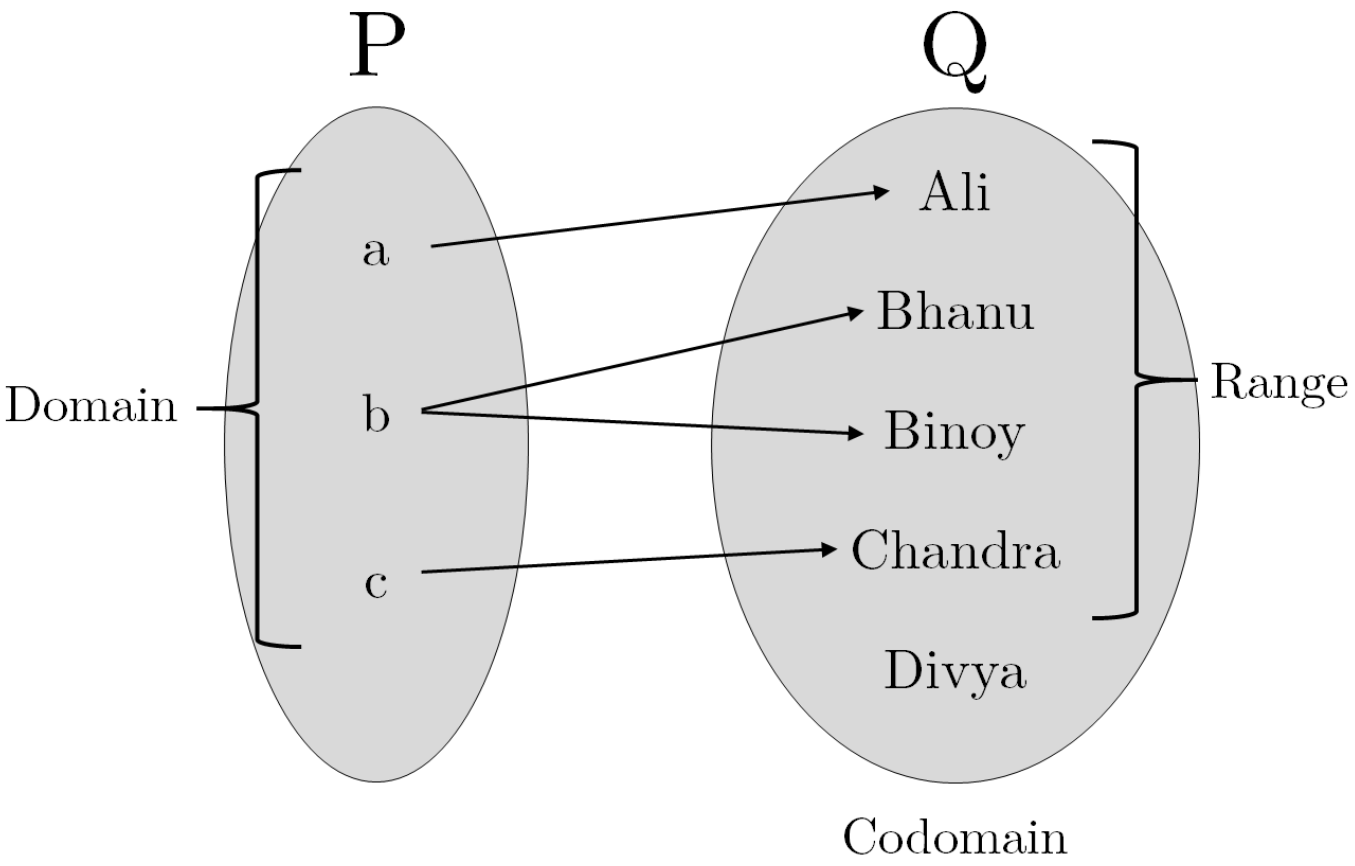

Figure 1: Visual Representation of a Relation

Some Important Terms

Image: The second element in an ordered pair is the image of the first element. (Chandra is the image of C)

Domain: The set of all first elements in the relation. ($D = \{a, b, c\}$)

Range: The set of all second elements in the relation. (R = {Ali, Bhanu, Binoy, Chandra})

Codomain: The entire second set of the relation. (C = {Ali, Bhanu, Binoy, Chandra, Divya})

Functions

A function is a special kind of relation from set A to B, such that:

- Every element in set A has an image in set B
- Every element in set A has only one image

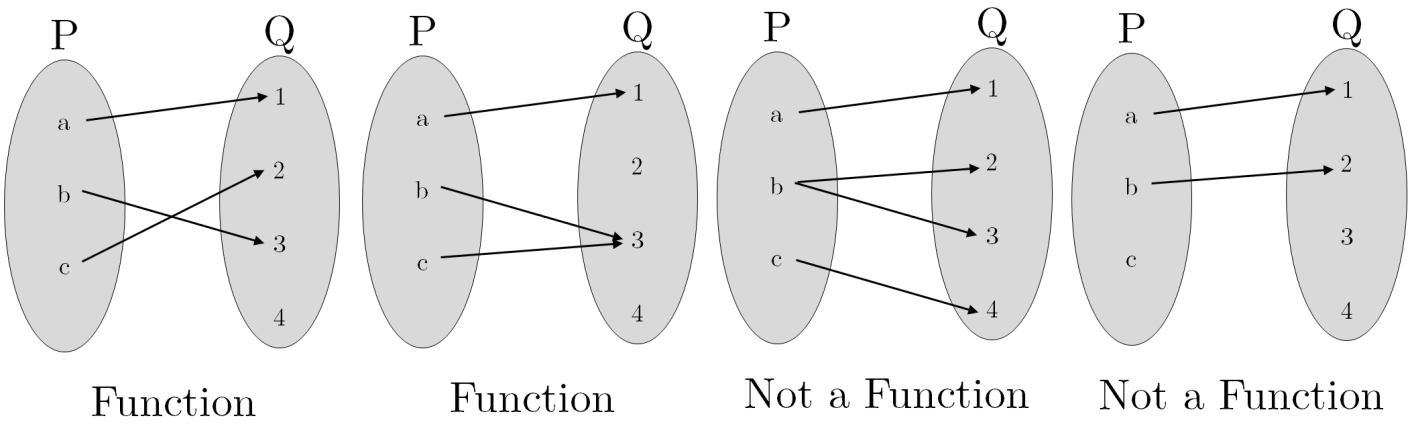

Figure 2: Some Examples of Functions

Real Function: A function where both domain and range are R or its subsets.

Some Important Functions

Identity Function

$$f : R \to R; f(x) = x$$

$$\text{Domain} = R$$

$$\text{Range} = R$$

Constant Function

$$f : R \to R; f(x) = c$$

$$\text{Domain} = R$$

$$\text{Range} = \{c\}$$

Polynomial Function

$$f : R \to R; f(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$$

$$f(x) = x^2$$

$$\text{Domain} = R$$

$$\text{Range} = R^+$$

$$f(x) = \frac{1}{x}$$

$$\text{Domain} = R - \{0\}$$

$$\text{Range} = R - \{0\}$$

$$f(x) = |x|$$

$$\text{Domain} = R$$

$$\text{Range} = [0, \infty)$$

$$\text{Sigmum: } f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ \text{-1}, & \text{if } x < 0 \end{cases}$$

$$\text{Domain} = R$$

$$\text{Range} = \{-1, 0, 1\}$$

Algebra of Real Functions

$$\left.\begin{aligned} f : X \to R \\ g : X \to R \end{aligned}\right\} X \subset R$$

Addition

$$(f + g)(x) = f(x) + g(x)$$

Subtraction

$$(f - g)(x) = f(x) - g(x)$$

Multiplication by a Scalar

$$(f \times \alpha)(x) = f(x) \times \alpha$$

Multiplication

$$(f \times g)(x) = f(x) \times g(x)$$

Quotient

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$