Virtual Environments for AUV Development and Ocean Exploration

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Abstract - The MIT Sea Grant College Program Underwater Vehicles Laboratory, in collaboration with elements of the MIT Design Laboratory and the MIT Ocean Seismo-Acoustics Laboratory, is developing technologies for ocean exploration based on autonomous underwater vehicles (AUVs). To support the development of autonomous ocean sampling networks (AOSN), research is being conducted on the creation of an *Ocean Virtual Underwater Environment* (OCEANVUE). OCEANVUE is designed to serve both as a simulation environment for testing pieces of the AOSN and as an operator interface for remote control of AUV field experiments. The goal is a network-distributed software environment allowing a graphical user interface and oceanographic, acoustic, and hydrodynamic simulation models to run in concert on a network of workstations. Work is underway in the areas of geometric representation, vehicle and ocean-acoustic simulation, network distributed control, and visualization. This paper provides an overview of the work being performed at these laboratories to develop such technologies.

Keywords: *virtual environments, autonomous underwater vehicles, visualization, scientific databases*

I. INTRODUCTION

The next generation of oceanographic field programs requires economic access to the ocean. Imperatives include abilities to:

1. obtain spatially distributed, temporally correlated measurements,
2. respond in a timely fashion to episodic events,
3. obtain time series of spatially distributed phenomena,
4. to interact with measurement platforms in the course of observations.

To meet these criteria, the concept of an autonomous ocean sampling network (AOSN) has been proposed [7]. The AOSN is based on a network of small, low cost vehicles supported by a sophisticated communication and control infrastructure. AOSN enables a new paradigm for ocean presence, termed *real-time oceanography*, in which scientists connected via Internet from a laboratory, home, or office can control multiple AUVs and network nodes making observations of complex ocean phenomena in the field.

The realization of AOSN will require advances in many areas, including platforms, communication, navigation, sensors, and information processing technologies. This paper describes our current efforts in the area of information processing technologies. Our work is focusing on the creation of an *Ocean Virtual Underwater Environment* (OCEANVUE) [20]. OCEANVUE serves as both a simulation environment for testing pieces of the AOSN and as an operator interface for remote control of AUV field experiments. The goal is a network-distributed software environment allowing a graphical user interface and oceanographic, acoustic, and hydrodynamic simulation models to run in concert on a network of workstations.

The four critical areas of development include:

Data Representation - nonlinear geometric representation of geophysical features and measurements allowing efficient interrogation and uncertainty management.

Simulation Models - algorithms to simulate both the AUV and the ocean environment (e.g., vehicle hydrodynamics, sensors, environmental acoustics, ocean dynamics).

Network Distributed Control - to enable real (or near real) time mission control from incomplete/sparse information gathered by the AUV(s) and for the coordination and cooperation of multiple vehicles [1].

Visualization to enable the AUV mission to be replayed for efficient analysis of measured data and for planning of future missions.

Advanced multidimensional visualization techniques are a vital part of this work because of the large amount of complex data to be displayed. Simulation provides a tool for mission planning from known information, for the optimal allocation of limited resources, and for the development of new AUV technologies or capabilities. The geometric representation provides a framework for a database of stored maps that compose the unified synthetic environment in which multiple objects reside and interact.

Our work builds on the Odyssey software suite, a constellation of applications which are used to prepare, test, run, and analyze missions [2]. These include: mission configuration software, vehicle control code, data parsing routines, mission data analysis/visualization, simulation models, and data structures. The software suite has been proven reliable in extensive field operations with the Odyssey II AUV over

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the past several years, including under-ice operations in the Arctic and deep-ocean operations in the Pacific.

This paper provides an overview of the work being performed in these areas at the MIT Sea Grant College Program Underwater Vehicles Laboratory in collaboration with the MIT Department of Ocean Engineering Design and Ocean Seismo-Acoustics Laboratories. Section II describes the development of a representation for the environment based on the novel applications of non-linear geophysical databases. The following sections describe vehicle and ocean-acoustic simulation models (Section III), an outline of network distributed control (Section IV) and the visualization of mission and environmental data (Section V). The paper then describes several recent field experiments in which a user at a remote site could monitor and/or control the Odyssey II via a radio modem connection (Section VI). Finally, the paper concludes with a discussion of future research issues (Section VII).

II. DATA REPRESENTATION

Central to the utility of OCEANVUE is the ability to store (represent), interrogate (query) and manipulate ocean data. This includes historical data obtained from previous surveys, simulated data produced by numerical models, and current data that is being collected during a vehicle mission. This ability is commonly realized through the use of a database or database system.

Computer databases have been in use for business and administrative applications for more than twenty years. However, there are major problems in evaluating and interrogating physical data entities to answer important questions using the information encapsuled in those data [11]. What are the reasons for these difficulties? One is that, while in administrative problems, software developed for storing, evaluating, comparing, and sorting discrete records carries very far, this technology is generally not sufficient for more advanced queries on complex physical phenomena [10]. The amount of data is much larger than what is encountered in the business world and the queries are more complicated. Generally, a scientist is not so much interested in sorting and retrieving single data items, but rather in evaluating continuous functions that are defined by the data and by knowledge of the physical laws governing the system. Moreover, since scientific data has an underlying physical meaning, continuous functions (derived from a few basic physical principles) have been extremely successful in describing and predicting physical phenomena.

Therefore, specific objectives of our database are [29]:

1. attacking the problem of multidimensional physical data representation by using advanced higher dimensional geometrical concepts,
2. addressing the problem of uncertainty in data representation and computation by employing the concept of interval B-splines [28] and the theory of interval arithmetic,
3. addressing the problem of geometric and physical information storage, retrieval, reconstruction and interrogation by using methods from algebraic topology and computational geometry,
4. developing new methods for continuous non-linear data representation and interrogation using global topological generalized boundary representation databases based on the cell-tuple structure [3].

Robust and efficient methods exist for the interrogation of B-spline geometries [15, 23]. Analytic geometric descriptions based on piecewise polynomials can efficiently characterize geophysical parameter variations without the combinatorial explosion that would arise through the use of faceted, linear or decompositional models. Further, the use of an explicit rather than a parametric formulation limits the storage needed at a small loss of generality. The introduction of an *interval* representation for the degrees of freedom allows uncertainty in both measured data and in interrogation results to be represented and used in critical applications.

In addition, it is generally not possible to maintain exact fidelity of a data set when using a reduced representation set. Therefore, all methods that reduce data storage requirements are approximations of the original data and have an inherent approximation error. A primary and unique advantage of an interval representation is that it produces the required data reduction with a *guaranteed bounded approximation error*. For example, if there were no uncertainty present in the original data set, then the resulting range in the interval surface would represent a bounded error of the approximation.

The interrogation of geophysical data enables insight into their underlying physical processes. Typically, queries can be cast in the form of a system or systems of nonlinear equations (if the representation is nonlinear). Interval techniques for solving such systems and, specifically, interval Newton methods combined with bisection have been the focus of significant attention [13, 18]. For a review of interval arithmetic, see Moore [17]. Recently, the *projected polyhedron method*, a subdivision-based technique, has been developed by Sherbrooke and Patrikalakis [23] utilizing either floating point or rational arithmetic to solve a system of n nonlinear polynomial equations. This work has been extended to an efficient and robust method utilizing rounded interval arithmetic by Maekawa and Patrikalakis [15]. Such interval methods operate at a very small fraction of the cost of rational arithmetic methods and provide numerical results with certainty and verifiability, thus providing the capability for interrogating the database.

III. SIMULATION MODELS

The Odyssey simulation environment consists of algorithms to model the vehicle's sensors, actuators, and hydrodynamics. A central element is the module which calculates the vehicle state and sensor readings as a result of commands from the control software. This module contains:

- the equations of motion for the vehicle,
- models of the performance of the vehicle thruster and control surface actuators,
- models of sensor system performance, including uncertainty (noise and spurious data).

In the Odyssey software suite, these models replace the driver routines that talk to the various sensors and actuators of the vehicle. The rest of the code (e.g., layered control, dynamic control, data logging) is identical for simulation and real vehicle operations [2].

The importance of simulation as a tool for technology development is widely agreed upon in the AUV community [19, 4, 27]. Simulation has played a vital role because it offers a cost effective alternative to expensive and hazardous field testing. Ideally, a simulator allows for the same source code files to run either in simulation or on the actual AUV hardware. Hybrid simulations connect a real AUV, sitting in the lab, to simulated sensory inputs to allow pseudo-missions to be run with the AUV hardware "in-the-loop". Software bugs can be uncovered without loss of valuable operations time and risk to the AUV.

A virtual environment goes beyond the simulation of the AUV hardware to provide an increasingly realistic model of the AUV's environment. High performance graphics workstations provide a powerful, intuitive interface for the operator to observe and interact with the AUV. Brutzman has pioneered the virtual environment concept for AUVs in the creation of a *virtual underwater world* for the NPS AUV [5]. This work employs state-of-the-art computer graphics to create a visually realistic underwater "world" in which the AUV can operate.

A unique aspect of our virtual ocean environment is that it strives for a close link with ocean science. Specifically, our work is targeted towards dynamic ocean phenomena that are well-suited for observation with an AOSN. Our strategy is to employ state-of-the-art numerical algorithms from the computational acoustic and oceanographic communities [12, 8, 22]. Models of interest include marine hydrodynamic simulation, acoustic propagation and scattering, and ocean transport processes. We envision a structure in which each component of the simulated environment can capitalize on the latest contributions of experts in that particular field.

Tremendous benefits are possible if we can integrate AUV technology development tools with numerical models for environmental acoustics and ocean circulation. Different AUV sampling strategies for mapping dynamic ocean phenomena can be evaluated. The optimal layout and allocation of resources for an AOSN deployment can be determined.

Ultimately, it may be possible to assimilate data from an AUV field mission into environmental simulation models in real-time. Such a task would require tremendous computational resources, and so research is necessary on how to run different pieces of the simulation on different workstations distributed across a network.

IV. NETWORK DISTRIBUTED CONTROL

The central research issues in establishing network distributed control of multiple vehicles and environmental simulation models are:

- bandwidth management,
- robustness,
- reconfigurability.

AOSN presents unique constraints in comparison to more common robotic network control problems. Techniques for coordinating manufacturing robots, for example Integrated Multiple Robot System (IRMS) [24] do not address bandwidth limitations or communication delays. Techniques for space and underwater remote manipulation, such as Teleprogramming [21], do incorporate time delays and will be much more useful. In manipulation, however, there is usually a single, well-defined master-slave relationship, whereas AOSN will involve the coordination of many different types of assets. Research on very large *swarms* of robotic agents has recently been popular [9]. In swarm approaches, random diffusion-like processes are employed rather than direct control of individual agents. However, the AOSN concept derives its strength from the ability to coordinate individual assets under direct shore-station control, keeping the user in-the-loop and maximizing the information gathered by each asset. The *virtual chain* concept [26] for coordinated control is a more appropriate metaphor for AOSN control.

To support AOSN field deployments, we need to establish a robust and reliable software architecture to handle all aspects of distributed control and simulation. Required tasks include mission control of remote experiments, simulation, post-mission data analysis, and training of new scientific users and vehicle operators. The network architecture provides for distributed simulation and control of vehicle operations across a variety of physical links, including acoustic modems, radio ethernet, and Internet. We envision an object-oriented network protocol that can handle the disparate operating situations of

1. shore-station-to-vehicle via acoustic link,
2. shore-station-to-network-node buoy via satellite or radio ethernet,
3. shore station-to-lab-science user via the Internet.

In addition to providing distributed control of actual field missions, the networking protocol will offer the possibility for other laboratories to run hybrid simulations across the Internet in which the core Odyssey simulator (with hydrodynamic model, environmental model, etc.) runs on a workstation at MIT and a model of the new subsystem under development (e.g., a new docking system) runs on a computer at the collaborating institution.

A possible candidate protocol to combine the different simulations and databases used in the AOSN may be developed from the field of Distributed Interactive Simulation (DIS), which is a maturing standard for the interaction of multiple independently computed simulations. The importance of DIS is demonstrated by the support of ARPA and the U.S. Army of such projects as the Combined Arms Tactical Trainer (CATT) and the Louisiana Maneuver Exercises (LAM) (United States Army Posture Statement, 1993) [33].

The primary utility of DIS is to define an infrastructure for linking simulations of various types at multiple locations to create realistic, complex, virtual worlds for the simulation of highly interactive activities. This infrastructure brings together heterogeneous systems and technologies from various vendors and permits their integration. DIS is characterized by the lack of a central database to coordinate simulation activities. Each simulation node broadcasts the status of the entity it is simulating, and the receiving nodes make use of that information as it applies to the entities they are simulating. Otherwise, each node operates its simulation completely independent of the other nodes. This paradigm seems well suited to the operation of the AOSN, and the possible use of DIS for networked simulation should be studied further.

Finally, a concern for utilizing network distributed control is that an acoustic network is a relatively low-bandwidth, long time-delay communication link compared to links typically used to control robots. Layered control provides an attractive framework for remote control of an AUV under these conditions since human intervention can be accomplished with no change at all to the fundamental structure. There are a variety of methods by which operator input can be accepted into the layered control structure, each providing a different level of control of the vehicle. This range of capabilities is critical since an AUV can expect to encounter degraded performance of the acoustic link on occasion during its operation inside a network. At one extreme, it is possible that round-trip communications may take fractions of a minute in a congested network, or even be indefinitely interrupted. On the other extreme, when immediately adjacent to a node, communications will have little delay and have high bandwidth. As the level of sophistication of control increases, the level of abstraction of the commands also increases, decreasing the bandwidth required for command.

Three different levels of control are envisioned:

User override mode: the lowest level of vehicle control would be for the operator to generate commands for the dynamic controller (e.g. heading, depth, velocity, etc.).

Behavior modification mode: the behavior modification mode allows the operator to influence the vehicle by changing the internal setting of vehicle behaviors.

Mission modification mode: the mission modification mode provides the user with the ability to activate and deactivate behaviors, and set their priorities.

V. VISUALIZATION

The most fundamental aspect of mission visualization is parsing the data that has been logged during a mission into a format usable by analyses programs. Parsing of mission data is greatly simplified by the vehicle data structure which formalizes the passing, storing, and retrieving of vehicle relevant data [2]. Currently, we are able to provide quick mission analysis through the use of these parsing routines and standard display objects from MATLAB. The ability to utilize such standardized visualization software means that we already have the ability to perform basic mission analyses on the multitude of platforms supported by this commercial system.

A more sophisticated (but more restrictive) method for mission analysis comes from the development of visualization systems based on the availability of advanced graphics workstations. In essence, this involves not only the display of mission related graphics such as plotted with MATLAB, but to have that data in overlay with the database that composes the synthetic environment. This involves the the construction (or updating) of maps for the database.

Therefore, we are currently developing a computer system, called OCEANVIS [32], that uses advanced computer graphics and methods from computational geometry for the representation, interrogation and visualization of volumetric data collected from AUV missions (e.g. water column data such as conductivity and temperature). A feature of OCEANVIS is that it utilizes the reliable methods for visualizing continuous, volumetric functions described in [32, 30]. It has been developed on Silicon Graphics (SGI) Onyx, Indigo, and Indy workstations running the IRIX operating system and was written in C++ with graphical output utilizing the SGI OpenInventor library.

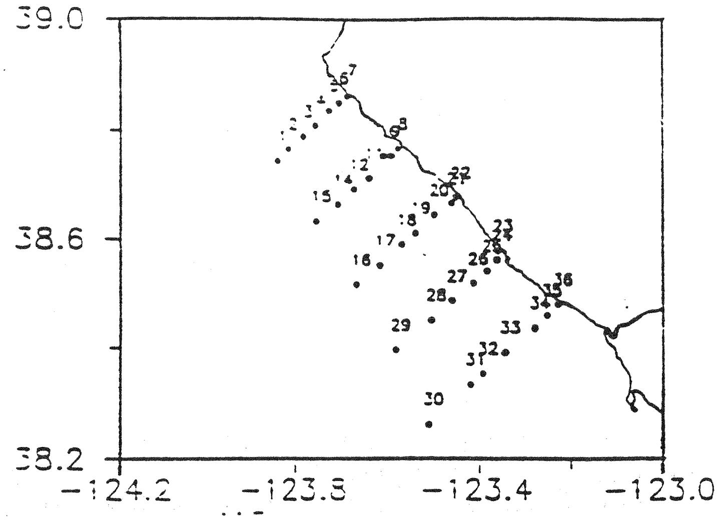

Figure 1. Stations from which CTD data was collected

For a surface example of OCEANVIS, a data set from the Southern Juan de Fuca region off the coast of Washington state [25] was used. Surfaces extending over a region approximately ($10.5km \times 14km$) were constructed and are shown in Figures 4 and 5. The average interval for these surfaces are $O(1m)$ for bathymetry and $O(3\ A/m)$ for magnetization.

For a volumetric example of OCEANVIS, we will use data from the Woods Hole Oceanographic Institution (WHOI), Woods Hole, MA, USA that consists of CTD observations made off the northern coast of California under the Shelf Mixed Layer Experiment (SMILE)[14]. The objective of this hydrographic measurement program was the measurement of temperature, salinity, density, and light transmission fields at various spatial positions in the surface boundary layer along the continental shelf and slope near the SMILE moored array. Long-term (6 months) observations were made in this area from November, 1988 to May, 1989. The spatial distribution of a typical data set is shown in Figure 1. The points numbered from 1 to 36 represent stations from which the data was collected.

In OCEANVIS, the SMILE data set is fit with volumetric splines using methods described in [31]. Figure 2 shows a polygonalization of a volumetric spline function representing temperature. Note that the volumetric splines are fit in parametric space and thus occupy a unit cube volume. Figure 6 shows the ability of OCEANVIS to display iso-valued surfaces corresponding to multiple values of a particular prop-

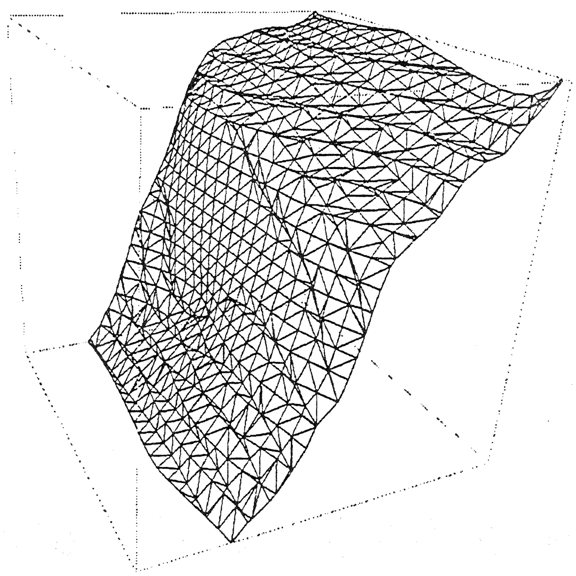

Figure 2. Polygons for B-spline Volume representation for temperature value of 9.73 *degrees* C

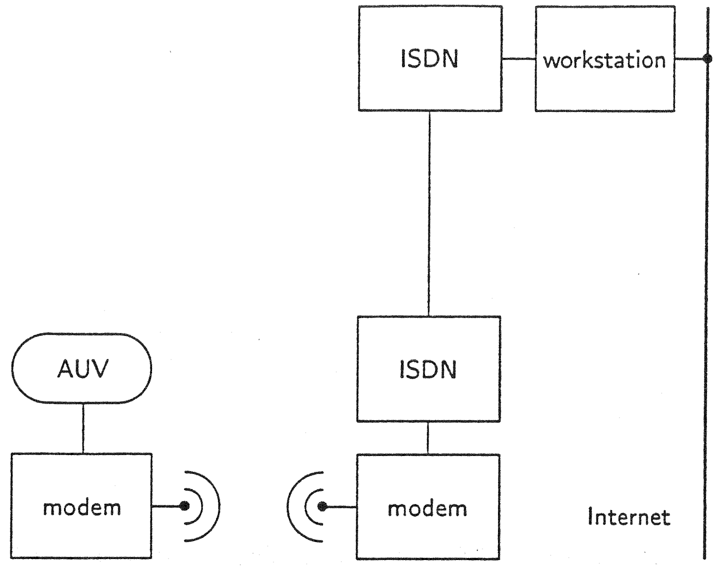

Figure 3. Schematic diagram of radio modem experiment.

erty. Multiple iso-valued surfaces can be rendered interactively and reliably using algorithms described in [30]. Figure 7 shows the ability of OCEANVIS to display not only multiple iso-valued surfaces (as in Figure 6) but also to render contours on isoparametric slices. These contours are computed using a robust algorithm developed by Margetis [16]. Finally, Figure 8 shows the polygons mapped into real space. In this figure, the ocean bottom is shown dramatically scaled in depth for illustrative reasons.

VI. EXPERIMENTAL RESULTS

We are experimenting with methods to view in real-time the data from an ongoing mission of an underwater vehicle. Furthermore, we wish to develop the capability to provide a graphical display of the mission on a workstation in an arbitrary geographic location. Over the past several years, we have conducted a variety of real-time trajectory visualization experiments with the MIT Sea Grant AUV *Odyssey II*.

The first experiment took place in January, 1994 with *Odyssey II* under the ice of Lake Winnepesaukee in New Hampshire. The serial output of an ultra-short baseline tracking system was transmitted to a graphics workstation via a radio modem. In this configuration, a computer operator logged and viewed the progress of various missions from a remote location, using a combination of the locally developed software and the graphics capabilities of the workstation.

The second experiment took place in November, 1994 during operations in the Charles River. This was similar to the first experiment except that an ISDN telephone network replaced the radio modem pair.

The third and most significant experiment took place in May 1995, during operations in the Charles River. For this experiment, we attached a radio modem directly to the main computer of *Odyssey II* via a serial cable. The vehicle towed a float on which rode the modem, enclosed in a splashproof container. On shore, we connected the corresponding modem to an ISDN telephone on some occasions (a remote workstation across campus made a data call to the operations theatre for vehicle communication) and directly to the serial port of a networked workstation on other occasions. Figure 3 depicts this experiment with a block diagram.

Prior experiments demonstrated a remote mission visualization capability limited to the the output of the deck unit of an acoustic tracking system. The direct connection to the vehicle computer of this experiment enables the visualization of data generated on board the vehicle by its oceanographic sensors, fully qualified by its navigation and guidance packages. In addition, the vehicle operator no longer need be on site in the field. We demonstrated the ability for an operator to configure and initiate a mission from the Sea Grant conference room, whereupon actual vehicle communications allowed a graphical display of the trajectory and sensor data on the screen in real time. By taking full advantage of the Internet, we could just as easily have run this mission from *any* computer with TCP/IP and a remote terminal protocol.

This experiment also reduced the number of required onsite personnel to a single person, who ensured a proper start and recovery for each mission. Eventually the AOSN will utilize fully automated docking stations free from required human presence for mission launch and recovery.

Although acceptable for proving the concept of remote vehicle communication, the radio modem represents a transitional step and will be replaced with an acoustic modem [6] in the future.

VII. FUTURE RESEARCH

Our current and future research is aimed at creating a software infrastructure for AOSN to support real-time oceanography. Several complex field deployments are envisioned, entailing collaboration between ocean scientists and vehicle technologists at a variety of institutions. Software will be required for three levels of operators: 1) core science users, 2) vehicle/node programmers, and 3) network level programmers.

The capabilities of the various AOSN elements will not be defined completely for several years to come and will depend on the type of experiment. For example, the performance of the acoustic communication system will have a critical impact on the degree to which high-level decision making can be off-loaded from the vehicle. Also, the bandwidth and cost of the surface-to-shore link will vary dramatically from experiment to experiment, depending on whether the experiment is in the deep ocean or near shore. Communication with an AOSN will be composed of three distinct legs: Internet to radio/satellite communication node, radio/satellite communication with surface node, and acoustic communication between the surface node and sub-sea assets. Within a short distance of shore, usually tens of kilometers, direct radio links can be exploited. For coastal regions off the United States cellular telephones provide a simple communication solution with well supported hardware and a well defined cost structure. Sub-sea fiber-optic cables offer the highest data rates, and the prospect of power transmission as well, however the costs of deployment are high, as are maintenance costs in the event of damage to the cable. For the larger part of the ocean satellites are the only communication option. In this deep-water scenario, a satellite link will in most cases be necessary and may provide more of a bandwidth bottleneck than the acoustic communications.

Finally, the demands of the sampling strategy of a particular experiment will strongly drive the desirability of different aspects of the overall AOSN configuration. Thus our work will focus on ease of reconfigurability rather than optimized performance for a single operating scenario. Our goal is to develop the right set of tools to let us capitalize on a number of upcoming AOSN deployment opportunities. Fortunately, ease of reconfiguration is one of the distinguishing features of our current software architecture, based on layered control [2]. This has been demonstrated by its performance in a variety of challenging AUV experiments over the last few years.

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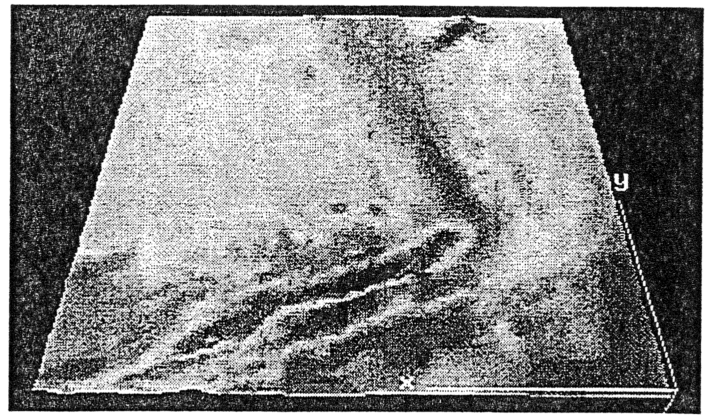

Figure 4. Interval B-spline surface representing bathymetry in units of m.

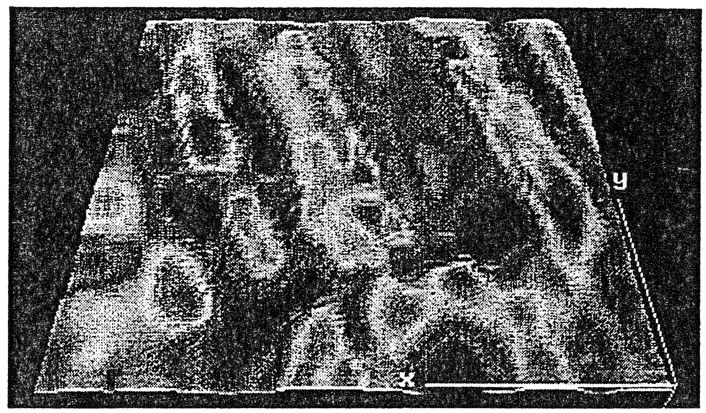

Figure 5. Interval B-spline surface representing magnetization in units of A/m.

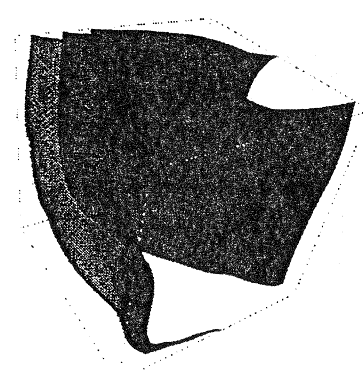

Figure 6. Multiple isosurfaces for B-spline volume representation of salinity values of 33.31 (green), 33.63 (blue), and 33.88 (violet) (all in *practical salinity units*, or *psu*).

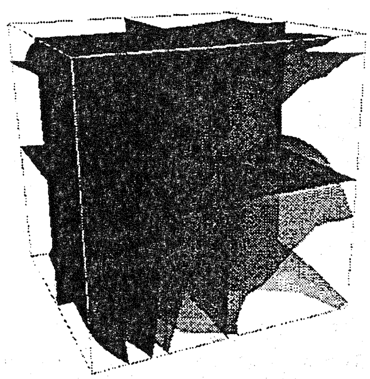

Figure 7. Simultaneous visualization of contour slices and isosurfaces for B-spline volume representation of temperature values (red represents low values and violet represents high values)

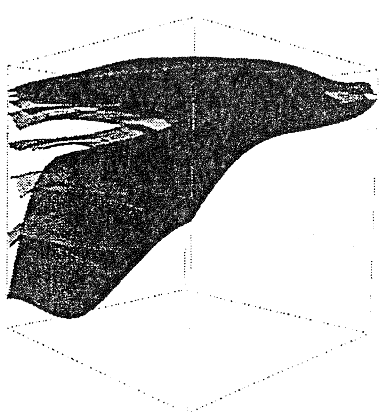

Figure 8. Real space visualization for B-spline volume representation of temperature values in *degrees* C (red represents low values, violet represents high values, and the bottom is displayed in brown)