(21) $A + B \rightarrow A + B$

In CM frame,

Before Collision,

$$p^{\mu}_{cm} = \left(\frac{E_A}{c}, \vec{P}_1\right)$$

After Collision,

$$p^{\mu'}_{cm} = \left(\frac{E_A}{c}, \vec{P}_2\right)$$

[Since energy remains conserved before & after collision]

In Breit frame

Before Collision,

$$p^{\mu}_{breit} = \left(\frac{E_A'}{c}, \vec{P}_3\right)$$

After Collision,

$$p^{\mu}_{breit} = \left(\frac{E_A'}{c}, \vec{P}_4\right)$$

[Since energy remains conserved]

Since, dot products of 2-vectors is invariant of frame.

$$(P_{cm})^{\mu}(P'_{cm})_{\mu} = (P_{breit})^{\mu}(P'_{breit})_{\mu}$$

$$\frac{E_A^2}{c^2} - \vec{P}_1 \cdot \vec{P}_2 = \frac{E_A'^2}{c^2} - \vec{P}_3 \cdot \vec{P}_4$$

$$\frac{E_A^2}{c^2} - p^2 \cos\theta = \frac{E_A'}{c^2} + (p')^2$$

Now,

$$\frac{E_A^2}{c^2} - P^2 = m_A^2 c^2$$

$$p^2 = \frac{E_A^2}{c^2} - m_A^2 c^2$$

Similarly,

$$(p')^2 = \left(\frac{E_A'}{c}\right)^2 - m_A^2 c^2$$

Therefore, we get -

$$\left(\frac{E_A}{c}\right)^2 - \left(\frac{E_A}{c}\right)^2 \cos\theta + m_A^2 c^2 \cos\theta = \left(\frac{E_A'}{c}\right)^2 + \left(\frac{E_A'}{c}\right)^2 - m_A^2 c^2$$

$$E_A^2 (1-\cos\theta) + m_A^2 c^4 (1+\cos\theta) = \cancel{E_A}\ 2(E_A')^2$$

$$\cancel{E_A' = \sqrt{E_A^2 \cos^2(\theta/2)}}$$

$$E_A' = \sqrt{E_A^2 \sin^2(\theta/2) + m_A^2 c^4 \cos^2(\theta/2)}$$

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(a) $S + t + u = \frac{1}{c^2}\left[P_A^2 + P_B^2 + 2P_A P_B + P_A^2 + P_C^2 - 2P_A P_C + P_A^2 + P_D^2 - 2P_A P_D\right]$

$$= \frac{1}{c^2}\left[P_A^2 + P_B^2 + P_C^2 + P_D^2 + 2P_A \underbrace{(P_A + P_B - P_C - P_D)}_{0}\right]$$

$$= m_A^2 + m_B^2 + m_C^2 + m_D^2 \qquad [\because P_A + P_B = P_C + P_D]$$

(b) In cm frame,

$$P_A^\mu = \left(\frac{E_A}{c}, \vec{P}_A\right) \qquad P_B^\mu = \left(\frac{E_B}{c}, \cancel{0}\ \vec{P}_B\right)$$

$$\vec{P}_B = -\vec{P}_A$$

$$P_A^2 = (P_A)^\mu (P_A)_\mu = \frac{E_A^2}{c^2} - \vec{P}_A^2$$

$$\Rightarrow \vec{P}_A^2 = \frac{E_A^2}{c^2} - P_A^2 = -\vec{P}_A \cdot \vec{P}_B$$

$$P_B^2 = (P_B)^\mu (P_B)_\mu = \frac{E_B^2}{c^2} - \vec{P}_B^2 = \frac{E_B^2}{c^2} - (\vec{P}_A)^2$$

$$\frac{E_B}{c} = \sqrt{P_B^2 + \vec{P}_A^2} = \sqrt{P_B^2 + \frac{E_A^2}{c^2} - P_A^2}$$

$$c^2 s = P_A^2 + 2P_A \cdot P_B + P_B^2$$

$$= P_A^2 + 2\left(\frac{E_A}{c}\frac{E_B}{c} - \vec{P}_A \cdot \vec{P}_B\right) + P_B^2$$

$$= P_A^2 + 2\left(\frac{E_A}{c}\sqrt{P_B^2 + \frac{E_A^2}{c^2} - P_A^2} + \frac{E_A^2}{c^2} - P_A^2\right) + P_B^2$$

$$c^2 s + P_A^2 - P_B^2 - \frac{2E_A^2}{c^2} = 2\frac{E_A}{c}\sqrt{P_B^2 + \frac{E_A^2}{c^2} - P_A^2}$$

$$\Rightarrow (c^2 s + P_A^2 - P_B^2)^2 - \frac{4}{c^2}(c^2 s + \cancel{P_A^2} - \cancel{P_B^2})E_A^2 + \cancel{\frac{4E_A^4}{c^4}}$$

$$= \frac{4E_A^2}{c^2}\left(\cancel{P_B^2} + \cancel{\frac{E_A^2}{c^2}} - \cancel{P_A^2}\right)$$

$$\Rightarrow (c^2 s + P_A^2 - P_B^2)^2 = \frac{4E_A^2 s}{\cancel{\quad}}$$

$$\Rightarrow (E_A^{cm})^2 = \frac{(c^2 s + P_A^2 - P_B^2)^2}{4s}$$

$$\Rightarrow E_A^{cm} = \frac{(s + m_A^2 - m_B^2)c^2}{2\sqrt{s}}$$

(c) $\vec{P}_B = 0$, $E_B = m_B c^2$

$$c^2 s = P_A^2 + 2\vec{P}_A \cdot \vec{P}_B + P_B^2$$

$$= m_A^2 c^4 + 2$$

(d) $\dfrac{E^{cm\,2}_{tot}}{c^2} = \left(\dfrac{E_1 + E_2}{c}\right)^2$

$$\boxed{E^{cm}_{tot} = \sqrt{s}\, c^2}$$

(23)

$s = \frac{1}{c^2}$ In cm frame,

$\vec{P}_A + \vec{P}_B = 0$ & $E_A = E_B = \sqrt{\vec{p}^2 c^2 + m^2 c^4}$ $\left[\begin{array}{l} m_A = m_B \\ \cancel{\vec{P}_A = \vec{P}_B} \\ \vec{P}_A = -\vec{P}_B \end{array}\right]$

$$S = \frac{1}{c^2}(P_A + P_B)^2$$

$$= \frac{1}{c^2}\left[\left(\frac{E_A + E_B}{c}\right)^2 - (\overset{0}{\cancel{\vec{P}_A + \vec{P}_B}})\right]$$

$$= \frac{1}{c^2} \times \frac{4(\vec{p}^2 c^2 + m^2 c^4)}{c^2} = \frac{4(\vec{p}^2 + m^2 c^2)}{c^2}$$

Again, $E_A = E_C = \sqrt{\vec{p}^2 c^2 + m^2 c^4}$, $\vec{P}_A \neq$

$$t = \frac{1}{c^2}(P_A - P_C)^2$$

$$= \frac{1}{c^2}(P_A^2 + 2P_A P_C + P_C^2)$$

$$= \frac{1}{c^2}\left[(\overset{0}{\cancel{E_A - E_C}})^2 - (\vec{P}_A - \vec{P}_C)^2\right]$$

$$= -\frac{1}{c^2}\left[\vec{P}_A^2 + \vec{P}_C^2 - 2\vec{P}_A \cdot \vec{P}_C\right]$$

$$= -\frac{1}{c^2} \times 2p^2(1 - \cos\theta) = \frac{-2\vec{p}^2}{c^2}(1 - \cos\theta)$$

$$U = \frac{1}{c^2}(P_A - P_D)^2$$

$$= \frac{1}{c^2}\left[\cancel{(E_A - E_D)^2}^{\,0} - (\vec{P}_A - \vec{P}_D)^2\right]$$

$$= -\frac{1}{c^2}\left[\vec{P}_A^{\,2} + \vec{P}_D^{\,2} - 2\vec{P}_A \cdot \vec{P}_D\right]$$

$$= -\frac{1}{c^2}\left[2\vec{P}_A^{\,2} + 2\vec{P}_A^{\,2}\cos\theta\right]$$

$$= -\frac{2\vec{P}_A^{\,2}}{c^2}(1+\cos\theta)$$

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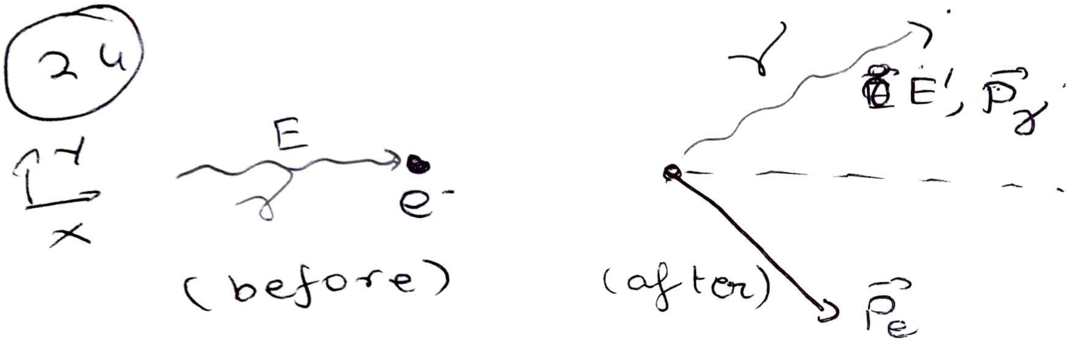

<u>Conservation of momentum</u>

$$P_e \sin\phi = P_\gamma \sin\theta \qquad (y\text{-dir}^n)$$

$$\frac{E}{c} = P_\gamma \cos\theta + P_e \cos\phi = \frac{E'}{c}\cos\theta + P_e\sqrt{1 - \left(\frac{P_\gamma \sin\theta}{P_e}\right)^2}$$

$$E - E'\cos\theta = \sqrt{P_e^2 c^2 - (E'\sin\theta)^2}$$

$$\Rightarrow E^2 - 2EE'\cos\theta + E'^2\cos^2\theta + E'^2\sin^2\theta = P_e^2 c^2$$

$$\Rightarrow E^2 - 2EE'\cos\theta + E'^2 = P_e^2 c^2$$

Conservation of Energy

$$E + mc^2 = E' + \sqrt{m^2c^4 + P_e^2c^2}$$

$$\Rightarrow (E + mc^2 - E')^2 = m^2c^4 + E^2 - 2EE'\cos\theta + E'^2$$

$$\Rightarrow \cancel{E^2} + \cancel{E'^2} + \cancel{m^2c^4} - 2EE' + 2Emc^2 - 2E'mc^2$$

$$= \cancel{m^2c^4} + \cancel{E^2} - 2EE'\cos\theta + \cancel{E'^2}$$

$$\Rightarrow \cancel{2}mc^2(E - E') = \cancel{2}EE'(1 - \cos\theta)$$

$$E' = \frac{hc}{\lambda'} \quad ; \quad E = \frac{hc}{\lambda}$$

$$\Rightarrow m\cancel{c}^2 \times \cancel{h}c\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = \frac{h^{\cancel{2}}c^{\cancel{2}}}{\lambda\lambda'}(1 - \cos\theta)$$

$$\Rightarrow \cancel{m}mc\frac{(\lambda' - \lambda)}{\lambda'\cancel{\lambda}} = \frac{h}{\lambda\cancel{\lambda'}}(1 - \cos\theta)$$

$$\Rightarrow \lambda' = \lambda + \frac{h}{mc}(1 - \cos\theta)$$