(1)

$$x' = \gamma(x - vt) \rightarrow (i)$$

$$y' = y \rightarrow (ii)$$

$$z' = z \rightarrow (iii)$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \rightarrow (iv)$$

$$\frac{x'}{\gamma v} = \frac{x}{v} - t \quad \text{(From (i))} \rightarrow (v)$$

$$\frac{t'}{\gamma} = t - \frac{v}{c^2}x \quad \text{(From (iv))} \rightarrow (vi)$$

(v) + (vi),

$$\frac{1}{\gamma}\left(\frac{x'}{v} + t'\right) = \frac{x}{v} - \frac{v}{c^2}x = \frac{x}{v}\left(1 - \frac{v^2}{c^2}\right)$$

$$\frac{1}{\gamma}\left(\frac{x'}{v} + t'\right) = \frac{x}{v\gamma^2}$$

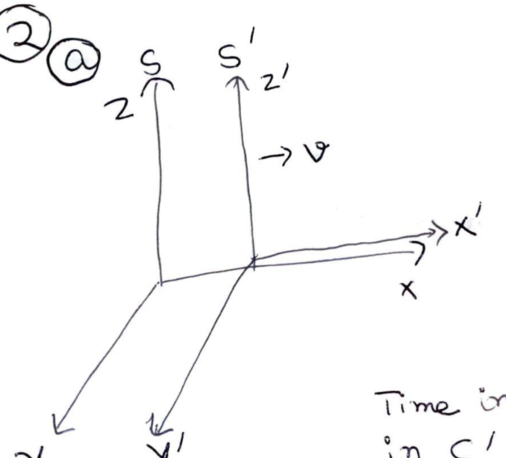
$$x = \gamma(x' + vt')$$

Similarly, $t = \gamma\left(t' + \frac{v}{c^2}x'\right)$

$$y = y'$$

$$z = z'$$

(2) (a)



S' is moving away from S

From the relation,

$$\underbrace{\Delta t'}_{\substack{\text{Time interval}\\ \text{in } S' \text{ betn}\\ \text{occurence of}\\ \text{event A \& B}}} = \gamma\left(\underbrace{\Delta t}_{\substack{\text{Time}\\ \text{interval}\\ \text{in S}}} + \frac{v}{c^2}\underbrace{\Delta x}_{\substack{\text{Position diff}^n\\ \text{betn occurence}\\ \text{of event A \& B.}}}\right)$$

Since, $\underbrace{\Delta t = 0}_{\text{occur at same time}}$, $\Delta x = x_B - x_A$

So, we get

$$t'_A - t'_B = \frac{\gamma v}{c^2}(x_B - x_A)$$

(b) Since, the train is moving towards B, the observer in the train will see event B first.

$$\Delta t = \frac{\gamma v}{c^2}\Delta x$$

$v = \frac{3}{5}c$, $\quad \Delta x = 4\,\text{Km} = 4\times10^3\,m$

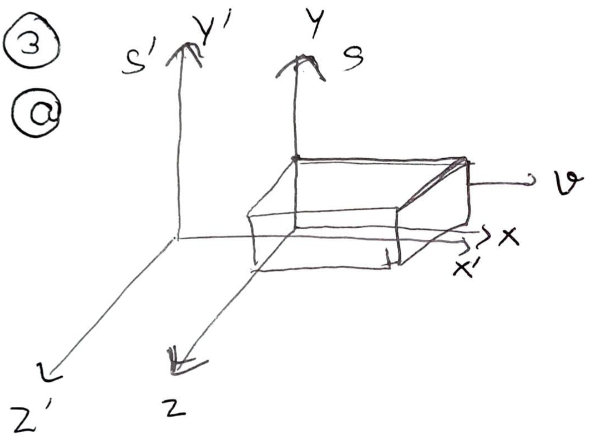
$\gamma = \frac{5}{4}$

$$\Delta t = \frac{5}{4}\times\frac{3/5\,c}{c^2}\times 4\times10^3$$

$$= \frac{3\times4\times10^3}{4\times3\times10^8} = 10^{-5}\,s.$$

(3)

(a)



Suppose l_x, l_y, l_z be dimensions of the box in S.

$V = l_x l_y l_z$

& l_x', l_y', l_z' be dimensions in S', $V' = l_x' l_y' l_z'$

Suppose, the box moves in x-direction.

Dimensions of the box is contracted in the x-direction & all other dimensions remain same.

$$l_x' = \frac{l_x}{\gamma} \qquad l_y' = l_y \qquad l_z' = l_z$$

$$\boxed{V' = l_x' l_y' l_z' = \frac{l_x l_y l_z}{\gamma} = \frac{V}{\gamma}}$$

(b) Since, mass is an invariant quantity (betn ref. frames)

$m' = m$

$\rho' V' = \rho V \Rightarrow \boxed{\rho' = \gamma\rho}$

(4)

(a) From, $x = vt$

$$= 0.998 \times 3 \times 10^{8} \times 2.2 \times 10^{-6} \ m$$

$$\simeq 658 \ m$$

So, according to prerelativistic physics, they would decay way before hitting ground level.

(b) In muon's frame,

$\Delta x = 0$, $\Delta t = 2.2 \times 10^{-6} s$

In lab. frame,

$$\Delta x' = \gamma(\Delta x - v\Delta t)$$

$$|\Delta x'| = \gamma \times v \times \Delta t.$$

$$\gamma = \frac{1}{\sqrt{1-(0.998)^2}} = 15.81$$

$$v = 0.998 \times 3 \times 10^{8} \ m$$

$$|\Delta x'| = 15.81 \times 0.998 \times 3 \times 10^{8} \times 2.2 \times 10^{-6} \ m$$

$$= 15.81 \times 658 \ m$$

$$\approx 10402 \ m$$

So, it reaches ground level.

(c) In frame of the muon, the distance ~~travelled by~~ it from atmosphere to earth is contracted by γ,

$$\Delta l' = \frac{\Delta l}{\gamma} = \frac{8000}{15.81} \approx 506 \ m$$

Time needed to cross earth in muon frame

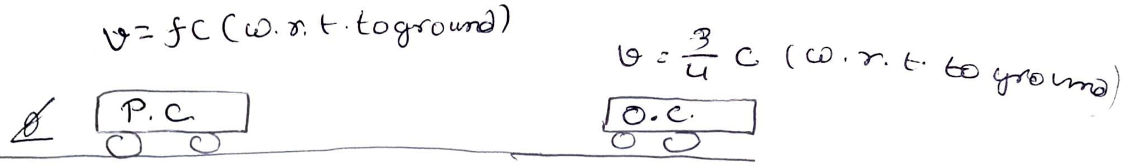
$$= \frac{506}{0.998 \times 3 \times 10^{8}} \ s \approx 1.69 \times 10^{-6} s$$

which is less than its decay time.

(d) Distance travelled by pion in lab frame

$$= \gamma v \frac{\Delta t_{muon}}{100} = \frac{10402}{100} m \approx 104 m$$

(5)



(a) In frame of pursuit car,

$$v(\text{O.C. w.r.t. P.C.}) = \left(\frac{3}{4} - f\right) c.$$

$$v(\text{bullet w.r.t. P.C.}) = \frac{c}{3}.$$

So, for bullet to catch up O.C,

$$\left(\frac{3}{4} - f\right) \cancel{c} < \frac{\cancel{c}}{3}$$

$$\boxed{\Rightarrow f > \frac{3}{4} - \frac{1}{3} = \frac{5}{12}}$$

(b) In relativistic physics,

$$v(\text{O.C. w.r.t. P.C.}) = \frac{\left(\frac{3}{4} - f\right) c}{1 - \frac{\frac{3}{4} f \cancel{c^2}}{\cancel{c^2}}} = \left(\frac{3-4f}{4-3f}\right) c.$$

$$v(\text{bullet w.r.t. P.C.}) = \frac{c}{3}.$$

$\therefore$ So, for bullet to catch up O.C.,

$$\frac{c}{3} > \left(\frac{3-4f}{4-3f}\right) c$$

$$4 - 3f > 9 - 12f$$

$$9f > 5$$

$$\boxed{f > \frac{5}{9}} \quad \text{Also, } f < 1.$$

$$\therefore \boxed{\frac{5}{9} < f < 1}$$

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(7) (6) From problem (1), the inverse transformations are,

$$x^{0'} = \gamma x^0 + \frac{\gamma v}{c} x^1 + 0x^2 + 0x^3$$

$$x^{1'} = \gamma \frac{v}{c} x^0 + \gamma x^1 + 0x^2 + 0x^3$$

$$x^{2'} = 0x^0 + 0x^1 + 1x^2 + 0x^3$$

$$x^{3'} = 0x^0 + 0x^1 + 0x^2 + 0x^3$$

So, the inverse matrix,

$$M = \begin{bmatrix} \gamma & +\frac{\gamma v}{c} & 0 & 0 \\ \frac{\gamma v}{c} & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Also,

$$AM = \begin{bmatrix} \gamma & -\gamma B & 0 & 0 \\ -\gamma B & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \gamma & \gamma B & 0 & 0 \\ \gamma B & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I$$

$$\text{Since, } \gamma^2 - \gamma^2 B^2 = \gamma^2(1 - B^2) = \frac{\gamma^2}{\gamma^2} = 1$$

$$\cancel{\gamma^2 B - \gamma^2 B = 0}.$$

7

$$I = (x^{0'})^2 - (x^{1'})^2 - (x^{2'})^2 - (x^{3'})^2$$

$$= \cancel{\gamma^2 [x/\beta x^0]^2 x - \gamma^2 A}$$

$$= \gamma^2 [x^0 - \beta x^1]^2 - \gamma^2 [x^1 - \beta x^0]^2 - (x^2)^2 - (x^3)^2$$

$$= (x^0)^2 \underbrace{[\gamma^2 - \gamma^2 \beta^2]}_{1} + (x^1)^2 \underbrace{[\gamma^2 \beta^2 - \gamma^2]}_{-1} - (x^2)^2 - (x^3)^2$$

$$-2\gamma^2 \cancel{x^0} \beta x^1 + 2\gamma^2 \cancel{x^1} \beta x^0$$

$$= (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2$$

[~~Invariant~~] So, I is invariant.

8 (a) 10 independent elements [4 diagonal terms + 6 off diagonal terms]

(b) 6 independent ~~elements~~ ~~and~~ elements

[Diagonal terms = 0, so only 6 off diagonal terms]

(c) Since $g_{\mu\nu}$ is symmetric.

$\cancel{S_{\mu\nu} = g_{\mu\lambda} g}$ $\cancel{S_{\mu\nu} = g_{\mu\lambda} S^{\lambda\nu}}$

$$\boxed{S_{\mu\nu} = g_{\mu\rho} g_{\nu\sigma} S^{\rho\sigma}}$$ is also symmetric given $S^{\rho\sigma}$ is symmetric

~~Als~~ And, it $S_{\mu\nu}$ will be antisymmetric if $S^{\rho\sigma}$ is anti-symmetric.

(d) $S^{\mu\nu} \to$ Symmetric $a^{\mu\nu} \to$ Antisymmetric

For, $\mu = \nu$, $a^{\mu\nu} = 0$

$S^{\mu\nu} a_{\mu\nu} = 0$

For $\mu \neq \nu$, $S^{\mu\nu} = S^{\nu\mu}$, $a^{\mu\nu} = -a^{\mu\nu}$

$\therefore S^{\mu\nu} a_{\mu\nu} + S^{\nu\mu} a_{\nu\mu}$

$= S^{\mu\nu} a_{\mu\nu} - S^{\mu\nu} a_{\mu\nu} = 0$

From the above relations, it can be ~~conet~~ concluded that $S^{\mu\nu} a_{\mu\nu} = 0$.

(e) $$t_{\mu\nu} = \underbrace{\frac{t_{\mu\nu} + t_{\nu\mu}}{2}}_{\text{Symmetric } (S^{\mu\nu})} + \underbrace{\frac{t_{\mu\nu} - t_{\nu\mu}}{2}}_{\text{anti-symmetric } (a^{\mu\nu})}$$

(9) $v_x = \frac{3}{5}c$, $v_y = 0$, $v_z = 0$

$$\gamma = \frac{1}{\sqrt{1 - \frac{9}{25}}} = \frac{5}{4}$$

$$\eta^{\mu} = \gamma(c, v_x, v_y, v_z) = \frac{5}{4}\left(c, \frac{3}{5}c, 0, 0\right)$$

$$= \left(\frac{5c}{4}, \frac{3c}{4}, 0, 0\right)$$

(10) From lorentz transformation,

$$p^{\mu'} = \Lambda^{\mu'}_{\ \mu} p^{\mu}$$

$$p^{\mu} = \left(\frac{E}{c}, \vec{p}\right)$$

$$\vec{p}\,' = \gamma(\vec{p} - \beta p^0)$$

$$p^{0'} = \gamma(p^0 - \beta \vec{p})$$

$$\vec{p}_A\,' + \vec{p}_B\,' = \gamma(\vec{p}_A - \beta p^0_A) + \gamma(\vec{p}_B - \beta p^0_B)$$

$$= \gamma(\vec{p}_A + \cancel{\beta}\vec{p}_B) - \gamma\beta(p^0_A + p^0_B)$$

$$= \gamma(\vec{p}_C + \vec{p}_D) - \gamma\beta(p^0_C + p^0_D)$$

$$= \gamma(\vec{p}_C - \beta p^0_C) + \gamma(\vec{p}_D - \gamma\beta p^0_D)$$

$$= \cancel{\gamma}\,\vec{p}_C\,' + \vec{p}_D\,'$$

Similarly it can be proved,

$$p^{0'}_A + p^{0'}_B = p^{0'}_C + p^{0'}_D$$

$$\therefore\ p^{\mu'}_A + p^{\mu'}_B = p^{\mu'}_C + p^{\mu'}_D$$

(11)

$$p^\mu = \left(\frac{E}{c}, p_x, p_y, p_z\right)$$

$$= \gamma m (c, v_x, v_y, v_z)$$

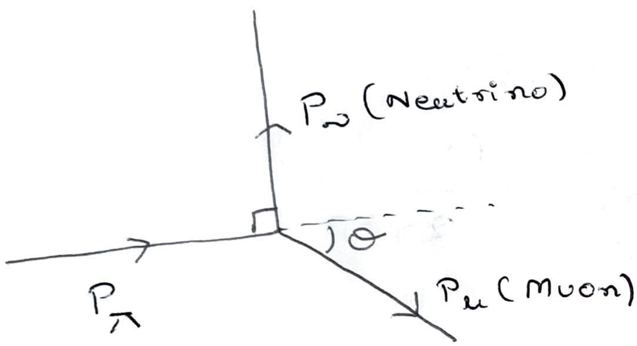
$$P_\mu P^\mu = \frac{E^2}{c^2} - p^2 = m^2 c^2$$

For real particle, $m^2 > 0$, $P_\mu P^\mu > 0$ (Time like)

For massless particle, $m^2 = 0$, $P_\mu P^\mu = 0$ (light like)

For virtual particles, $m^2 < 0$, $P_\mu P^\mu < 0$ (Space-like).

(13)



For WLOG, we can consider initial motion of pion in x direction.

(Conservation of P in x direction) $P_\pi = P_\mu \cos\theta \rightarrow$ (1)

(" " P in y ") $P_\nu = P_\mu \sin\theta \rightarrow$ (2)

(" " E) $E_\pi = E_\mu + E_\nu$

$$\Rightarrow \gamma m_\pi c^{\cancel{2}} = \cancel{c}\sqrt{m_\mu^2 c^2 + P_\mu^2} + P_\nu \cancel{c} \rightarrow (3)$$

From (1) & (2),

$$P_\mu^2 = P_\pi^2 + P_\nu^2$$

From (3),

$$(\gamma m_\pi c - P_\nu)^2 = m_\mu^2 c^2 + P_\mu^2 = P_\pi^2 + P_\nu^2 + m_\mu^2 c^2$$

Since, $P_\lambda = \gamma m_\lambda v$

$$\gamma^2 m_\lambda^2 c^2 + \cancel{P_\omega^2} - 2\gamma m_\lambda c P_\omega = m_{ei}^2 c^2 \pm \cancel{P_\lambda^2} + \cancel{P_\omega^2} + \gamma^2 m_\lambda^2 v^2$$

$$\gamma^2 m_\lambda^2 (c^2 - v^2) - m_{ei}^2 c^2 = 2\gamma m_\lambda c P_\omega$$

$$\Rightarrow P_\omega = \frac{c^{\cancel{2}} (m_\lambda^2 - m_{ei}^2)}{2\gamma m_\lambda \cancel{c}} = \frac{c (m_\lambda^2 - m_{ei}^2)}{2\gamma m_\lambda}$$

From (1) & (2),

$$\tan\theta = \frac{P_\omega}{P_\lambda} = \frac{c (m_\lambda^2 - m_{ei}^2)}{\gamma m_\lambda v (2\gamma m_\lambda)}$$

$$= \frac{(m_\lambda^2 - m_{ei}^2)}{2\gamma^2 m_\lambda^2 (v/c)}$$

$$= \frac{\left(1 - \frac{m_{ei}^2}{m_\lambda^2}\right)}{2\gamma^2 \beta} = \frac{\left(1 - \frac{m_{ei}^2}{m_\lambda^2}\right)}{2\gamma^2 \beta}$$

(14) $A + B \longrightarrow C_1 + C_2 + \cdots + C_n$

Threshold Energy means the total momentum of the particles after collision should be 0 so that no energy is being imparted in the form of K.E.

Before Collision,

In lab frame, $P^\mu_{before} = \left(\frac{E + m_B c^2}{c}, P_A, 0, 0\right)$

After collision,

In C.M. frame $P^\mu_{after} = \left(\sum_i m_i c, 0, 0, 0\right)$

$$= (Mc, 0, 0, 0)$$

$$(P_\mu P^\mu)_{before} = (P_\mu P^\mu)_{after}$$

$$\left(\frac{E}{c} + m_B c\right)^2 - P_A^2 = m^2 c^2$$

Also, $m_A^2 c^4 = E^2 - P_A^2 c^2$

$$\boxed{P_A^2 = \frac{E^2}{c^2} - m_A^2 c^2}$$

$$\therefore \cancel{\frac{E^2}{c^2}} + m_B^2 c^2 + 2 E m_B - \cancel{\frac{E^2}{c^2}} + m_A^2 c^2 = m^2 c^2$$

$$2 E m_B = (m^2 - m_A^2 - m_B^2) c^2$$

$$E = \frac{(m^2 - m_A^2 - m_B^2)}{2 m_B} c^2.$$

15

(a) $E = \frac{[(2m_p)]^2}{}$

(a) $$E = \frac{[(2m_p + m_{\pi^0})^2 - 2m_p^2]\, c^2}{2 m_p}$$

(b) $$E = \frac{[(m_{\pi^+} + m_{\pi^-} + 2m_p)^2 - 2m_p^2]}{2 m_p} c^2$$

(c) $$E = \frac{[(2m_p + m_n)^2 - m_p^2 - m_{\bar{\pi}^-}]}{2m_p} c^2 \quad [m_{\bar{p}} = m_p]$$

(d) $$E = \frac{[(m_{\Sigma^0} + m_{K^0})^2 - m_p - m_{\pi^-}]}{2m_p} c^2$$

(e) $$E = \frac{[(m_p + m_{\Sigma^+} + m_{K^0})^2 - 2m_p^2]}{2m_p} c^2$$

(16) (a) & (b) $A \to B + C$

$P_A = 0$

By conservation of momentum

$P_B + P_C = 0 \Rightarrow P_B = -P_C$

~~$|P_B| = |P_C| = P$~~

By conservation of energy,

$E_A = E_B + E_C$

$m_A c^2 = \sqrt{m_B^2 c^4 + p^2 c^2} + \sqrt{m_C^2 c^4 + p^2 c^2}$ $\quad [|P_B| = |P_C| = P]$

$\Rightarrow m_A c^2 - \sqrt{m_C^2 c^4 + p^2 c^2} = \sqrt{m_B^2 c^4 + p^2 c^2}$

$\Rightarrow m_A^2 c^4 + m_C^2 c^4 + \cancel{p^2 c^2} - 2 m_A c^2 \sqrt{m_C^2 c^4 + p^2 c^2} = m_B^2 c^4 + \cancel{p^2 c^2}$

$\Rightarrow \left(\frac{m_A^2 + m_C^2 - m_B^2}{2 m_A} \right) c = \sqrt{m_C^2 c^2 + p^2}$

$\Rightarrow \left(\frac{m_A^4 + m_C^4 + m_B^4 - 2 m_B^2 m_C^2 - 2 m_A^2 m_B^2 + 2 m_A^2 m_C^2}{4 m_A^2} \right) c^2 = m_C^2 c^2 + p^2$

$\Rightarrow p^2 = \left(\frac{m_A^4 + m_B^4 + m_C^4 - 2 m_B^2 m_C^2 - 2 m_A^2 m_B^2 - 2 m_A^2 m_C^2}{4 m_A^2} \right) c^2$

$\Rightarrow P = \frac{\sqrt{\lambda (m_A^2, m_B^2, m_C^2)}}{2 m_A} c$

~~$|P_B| =$~~

~~$E_B = \frac{}{2 m_A}$~~

$E_B = \sqrt{m_B^2 c^4 + \left(\frac{m_A^4 + m_B^4 + m_C^4 - 2 m_A^2 m_C^2 - 2 m_A^2 m_B^2 - 2 m_B^2 m_C^2}{4 m_A^2} \right) c^4}$

$= \frac{c^2}{2 m_A} \sqrt{m_A^4 + m_B^4 + m_C^4 - 2 m_A^2 m_C^2 - 2 m_B^2 m_C^2 + 2 m_A^2 m_B^2}$

$= \frac{(m_A^2 + m_B^2 - m_C^2)}{2 m_A} c^2$

18 (a) $\pi^- \to \mu^- + \bar{\nu}$

By conservation of momentum,

$$P_{\pi} = 0 = P_{\mu} + P_{\bar{\nu}} \Rightarrow P_{\mu} = -P_{\bar{\nu}}$$

$$|P_{\mu}| = |P_{\bar{\nu}}| = P$$

By conservation of Energy,

$$m_{\pi}c^2 = PC + \sqrt{m_{\mu}^2 c^4 + P^2C^2}$$

$$m_{\pi}^2 c^4 + P^2\cancel{c^2} - 2m_{\pi}Pc^3 = m_{\mu}^2 c^4 + P^2\cancel{c^2}$$

$$\Rightarrow P = \left(\frac{m_{\pi}^2 - m_{\mu}^2}{2m_{\pi}}\right) c$$

Let τ be the decay time. ~~in the par~~
So, in the lab frame, the particle will decay at

$$t' = \gamma\tau$$

Distance travelled; $x' = \gamma \underbrace{v}_{\text{speed in lab frame}} \tau$

$$P = \gamma m_{\mu} v = \left(\frac{m_{\pi}^2 - m_{\mu}^2}{2m_{\pi}}\right) c$$

$$\therefore \gamma v = \left(\frac{m_{\pi}^2 - m_{\mu}^2}{2m_{\pi}m_{\mu}}\right) c$$

$$\therefore x' = \left(\frac{m_{\pi}^2 - m_{\mu}^2}{2m_{\pi}m_{\mu}}\right) c\tau = 186\, m$$

(b) The muon while travelling loses its energy ~~with p~~ due to collision with other particles.

(19) (a) Minimum energy that B can have is $m_B c^2$.

For Maximum energy will be achieved when the all the other particles except B move as a single unit in the direction opp. to B.

From Ques (16),

$$E_{B_{max}} = \left(\frac{m_A^2 + m_B^2 - m^2}{2m_A}\right)c^2 \quad [m = \Sigma m = m_C + m_D + \cdots]$$

(b) $E_{min} = m_e c^2$

$$E_{max} = \left(\frac{m_\mu^2 + m_e^2}{2m_\mu}\right)c^2$$

(20) $\overset{A}{\circ} \longrightarrow v \qquad \overset{B}{\circ}$

(a) Four-p for A, Four p for B,

$p^\mu = \gamma(m_A c^2, m_A v, 0, 0) \qquad p^\mu = \gamma(m_B c^2, 0, 0, 0)$

$|P_{tot}| = \gamma m_A v$

$E_{tot} = \gamma(m_A + m_B)c^2$

$$v_{cm} = \frac{\gamma m_A v c}{\gamma c} = \frac{|P_{tot}|c^2}{E_{tot}} = \frac{\gamma m_A v c^2}{\gamma(m_A + m_B)c^2}$$

$$= \left(\frac{m_A}{m_A + m_B}\right)v = \frac{v}{2}$$

$$v_1(A \text{ w.r.t. CM}) = \frac{v - v_{cm}}{1 - \frac{v v_{cm}}{c^2}} = \frac{v - \frac{v}{2}}{1 - \frac{v^2}{2c^2}} = \frac{v c^2}{2c^2 - v^2}$$

$$v_2(B \text{ w.r.t. CM}) = \frac{v_{cm} - 0}{1 - 0} = v_{cm} = \frac{v}{2}.$$

(b)

$$KE_A = \frac{1}{2} m_A v_A^2 = \frac{1}{2} m \left(\frac{v c^2}{2c^2 - v^2} \right)^2$$

$$K.E._B = \frac{1}{2} m_B v_2^2 = \frac{1}{2} m \left(\frac{v}{2} \right)^2$$