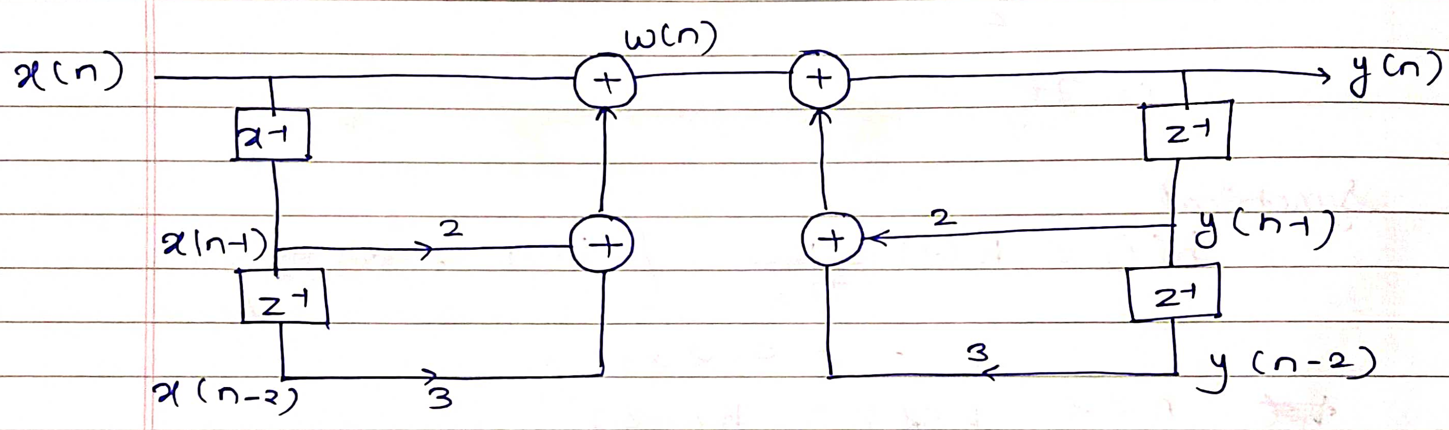
ISE - 3

Q.1 Obtain the direct form - I realization for the following differences equation

$$y(n) = 2y(n-1) + 3y(n-2) + x(n) + 2x(n-1) + 3x(n-2)$$

→

$$y(n) = 2y(n-1) + 3y(n-2) + w(n)$$
$$w(n) = x(n) + 2x(n-1) + 3x(n-2)$$



Q.3 Design an analog Butterworth filter that has 2 dB passband attenuation at a frequency of 20 rad/sec and atleast 10dB stopband attenuation at 30 rad/sec.

→ Given, $\alpha_p = 2\,dB$ $\quad \Omega_p = 20$ rad/sec

$\alpha_s = 10\,dB$ $\quad \Omega_s = 30$ rad/sec

$$N \geq \frac{\log \sqrt{\dfrac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}}}{\log \left(\dfrac{\Omega_s}{\Omega_p}\right)}$$

$$= \frac{\log \sqrt{\dfrac{10^{0.1(10)} - 1}{10^{0.1(2)} - 1}}}{\log \left(\dfrac{30}{20}\right)}$$

$$= \frac{\log \sqrt{\frac{9}{0.584}}}{\log (1.5)}$$

$$= \frac{\log \sqrt{(15.41)}}{0.176} = \frac{0.593}{0.173}$$

$$= \frac{\cancel{1.187}}{\cancel{0.176}}$$

$$N = 3.42$$

$$\boxed{N \approx 4}$$

The normalized T.F. for $N=4$

$$H_a(s) = \frac{1}{(s^2 + 0.7653 s + 1)(s^2 + 1.8477 s + 1)}$$

To find cut-off frequency,

$$\Omega_c = \frac{\Omega_p}{(10^{0.1\alpha_p} - 1)^{1/2N}}$$

$$= \frac{20}{(10^{0.1(20)} - 1)^{1/2\times4}}$$

$$= \frac{20}{(99)^{1/8}}$$

$$= \frac{20}{1.776}$$

(11.26)

$$\Omega_c = 21.38$$

The T.F. for $\Omega_c = 21.38$

$$H(s) = H_a(s) \Big|_{s = \frac{s}{21.38}}$$

Substituting $s = \frac{s}{21.38}$

$$H(s) = \frac{1}{\left(\frac{s}{21.38}\right)^2 + 0.7653\left(\frac{s}{21.38}\right) + 1}$$

$$\times \frac{1}{\left(\frac{s}{21.38}\right)^2 + 1.8477\left(\frac{s}{21.38}\right) + 1}$$

$$H(s) = \frac{0.20921 \times 10^6}{(s^2+16.3686s+457.39)(s^2+39.51s+457.394)}$$

Q.4 For the analog T.F. $H(s) = \frac{2}{(s+1)(s+2)}$.

Determine H(s) using impulsive invarience method.

Assume $T = 1$ sec

→ Given, $H(s) = \frac{2}{(s+1)(s+2)}$

Using partial fraction

$$H(s) = \frac{A}{(s+1)} + \frac{B}{(s+2)}$$

$$H(s) = \frac{A(s+2) + B(s+1)}{(s+1)(s+2)} = \frac{2}{(s+1)(s+2)}$$

$\therefore\ 2 = A(s+2) + B(s+1)$

put $s = -2$

$2 = B(-1)$

$\therefore\ B = -2$

put $s = -1$

$2 = A(-1+2) = A$

$\therefore\ A = 2$

$$\therefore H(s) = \frac{2}{(s+1)} - \frac{2}{(s+2)}$$

$$= \frac{2}{(s-(-1))} + \frac{2}{(s-(+2))}$$

Using impulse varience,

$$H(s) = \sum_{k=1}^{N} \frac{C_k}{s - P_k}$$

$$\therefore H(z) = \sum_{k=1}^{N} \frac{C_k}{1 - e^{P_k T} z^{-1}}$$

$$H(z) = \frac{2}{1 - e^{-T} z^{-1}} + \frac{-2}{1 - e^{-2T} z^{-1}}$$

Here $T = 1$ sec

$$H(z) = \frac{2}{1 - e^{-1} z^{-1}} - \frac{2}{1 - e^{-2} z^{-1}}$$

$$= \frac{2}{1 - 0.3678 z^{-1}} - \frac{2}{1 - 0.1353 z^{-1}}$$

$$H(z) = \frac{0.465 z^{-1}}{1 - 0.503 z^{-1} + 0.04976 z^{-2}}$$

Q5 Realize the cascade form realization of IIR filter for the following difference equation.

$$H = \frac{1 + \frac{1}{3} z^{-1}}{\left(1 - \frac{1}{5} z^{-1}\right)\left(1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}\right)}$$

→

$$H(z) = \frac{1 + \frac{1}{3} z^{-1}}{\left(1 - \frac{1}{5} z^{-1}\right)\left(1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}\right)}$$

Obtain cascade form realization,

$$H(z) = \underbrace{\left(\frac{1 + \frac{1}{3} z^{-1}}{1 - \frac{1}{5} z^{-1}}\right)}_{H_1(z)} \underbrace{\frac{1}{\left(1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}\right)}}_{H_2(z)}$$

Express $H_1(z)$ & $H_2(z)$ in direct form (II)

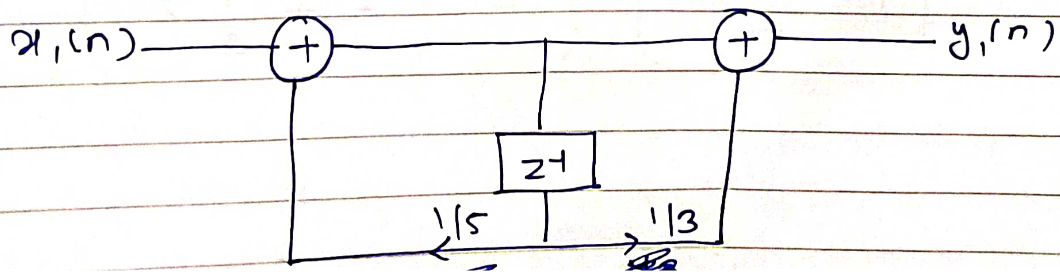
$$H_1(z) = \frac{1 + \frac{1}{3} z^{-1}}{1 - \frac{1}{5} z^{-1}} = \frac{Y_1(z)}{W_1(z)} \cdot \frac{W_1(z)}{X_1(z)}$$

$$\frac{Y_1(z)}{W_1(z)} = 1 + \frac{1}{3} z^{-1}$$

$$\therefore y_1(n) = w_1(n) + \frac{1}{3} w_1(n-1) \quad \text{—— (i)}$$

$$\frac{Y_1(z)}{W_1(z)} = \frac{1}{1 - \frac{1}{5} z^{-1}}$$

$$W_1(n) = x_1(n) + \frac{1}{5} W_1(n-1) \quad \text{—— (ii)}$$



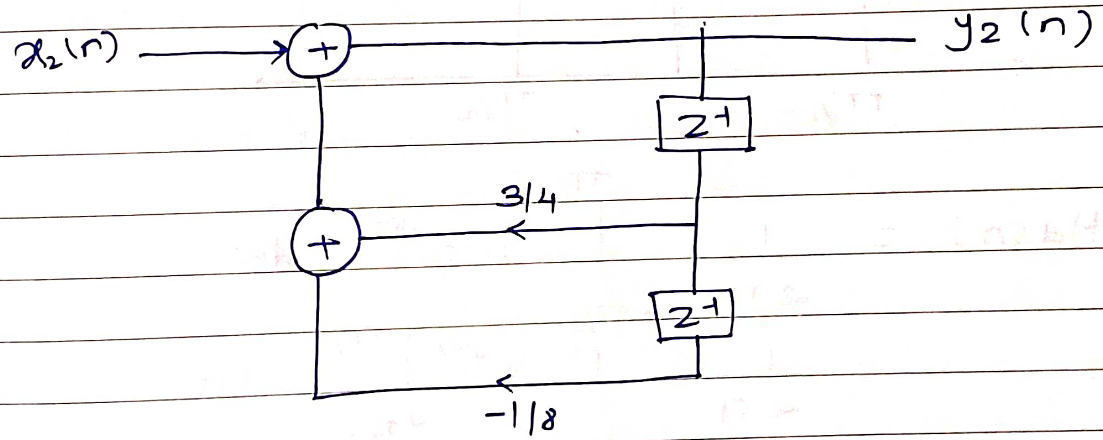
← Direct form II of $H_1(z)$

Similarly $H_2(z) = \dfrac{1}{1 - \frac{3}{4} z^{-1} + \frac{1}{8} z^{-2}}$

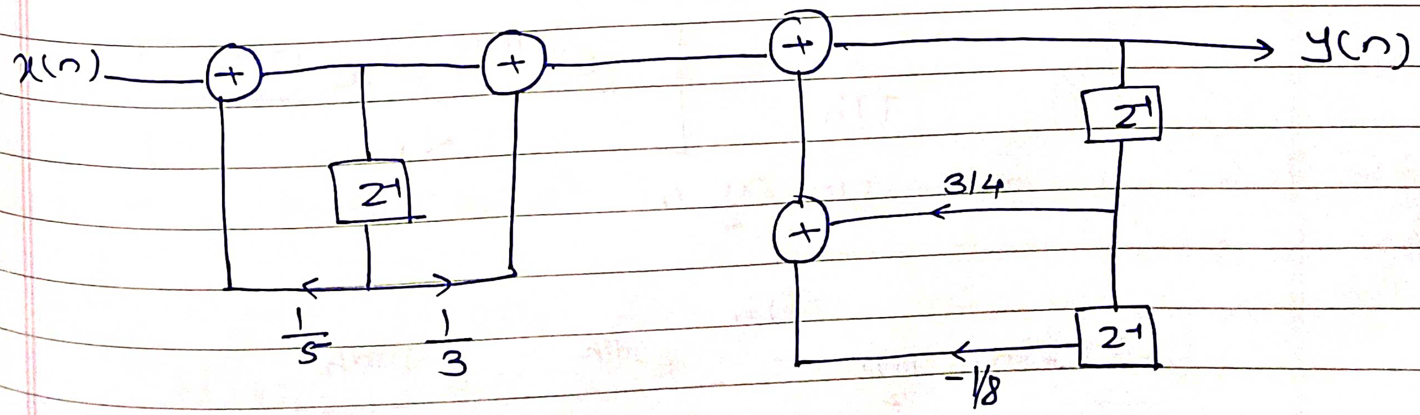
$$= \frac{Y_2(z)}{X_2(z)}$$

$$Y_2(z) - \frac{3}{4} z^{-1} Y_2(z) + \frac{1}{8} z^{-2} Y_2(z) = X_2(z)$$

$$y_2(n) = \frac{3}{4} y_2(n-1) - \frac{1}{8} z^{-2} y_2(n-2) + x_2(n)$$



Cascade form is



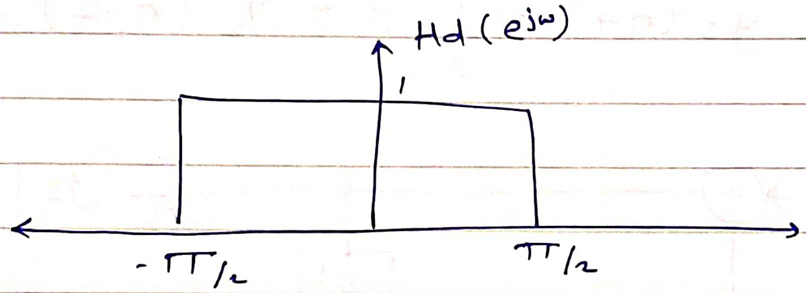
Q.6 Design an ideal LPF with frequency responce

$$H_d(e^{j\omega}) = 1 \quad \text{for} \quad \frac{-\pi}{2} \le \omega \le \frac{\pi}{2}$$

$$= 0 \quad \text{for} \quad \frac{\pi}{2} \le \omega \le \pi$$

→ The desired freq is

$$H_d(n) = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} H_d(e^{j\omega})\, d\omega$$



$$H_d(n) = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} 1 . e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi} \left[e^{j\omega n} \right]_{\pi/2}^{\pi/2}$$

$$= \frac{1}{2\pi j n} \left[e^{j\pi n/2} - e^{-jn\pi/2} \right]$$

$$= \frac{1}{\pi n} \left[\frac{e^{j\pi n/2} - e^{-jn\pi/2}}{2j} \right]$$

$$= \frac{\sin \frac{\pi}{2} n}{\pi n}$$

$$\because \sin \frac{\pi n}{2} = \frac{e^{jn\pi/2} - e^{-j\pi n/2}}{2j}$$

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$$hd(n) = \frac{\sin \frac{\pi n}{2}}{\pi n}$$

Truncating $hd(n)$ to $N = 11$ samples

$$n = \frac{N-1}{2} = \frac{11-1}{2} = 5$$

$$h(n) = \frac{\sin \pi n/2}{\pi n} \quad \text{for } |n| \leq 5$$

$$= 0 \quad \text{otherwise}$$

substitute $n = 0$ to 5 in $h(n)$

$n = 0$: $h(0) = \lim_{n \to 0} \frac{\sin \pi n/2}{2 \times \pi n/2}$

$$= \frac{1}{2} \times 1$$

$n = 1$: $h(1) = \cancel{\lim}\ h(-1) = \frac{\sin \pi/2}{\pi} = \frac{1}{\pi} = 0.3185$

$n = 2$: $h(2) = h(-2) = \frac{\sin \pi}{2\pi} = 0$

$n = 3$: $h(3) = h(-3) = \frac{\sin 3\pi/2}{3\pi} = \frac{-1}{3\pi} = -0.1061$

$n = 4$: $h(4) = h(-4) = \frac{\sin 2\pi}{4\pi} = 0$

$n = 5$: $h(5) = h(-5) = \frac{\sin 5\pi/2}{5\pi} = \frac{1}{5\pi} = 0.063$

The T.F. of the filter is

$$H(z) = h(0) + \sum_{n=1}^{N-1/2} h(n) \left[z^{-n} + z^{n}\right]$$

$$h(z) = h(0) + h(1)\left[z^{1} + z^{-1}\right] + h(2)\left[z^{2}+z^{-2}\right] + h(3)\left[z^{3} + z^{-3}\right] + h(4)\left[z^{4} + z^{-4}\right] + h(5)\left[z^{5}+z^{-5}\right]$$

$$h(z) = \frac{1}{2} + 0.3185\left[z^{1} + z^{-1}\right] + \cancel{h(2)} \{(-0.1061)\left[z^{-3}+z^{3}\right]$$

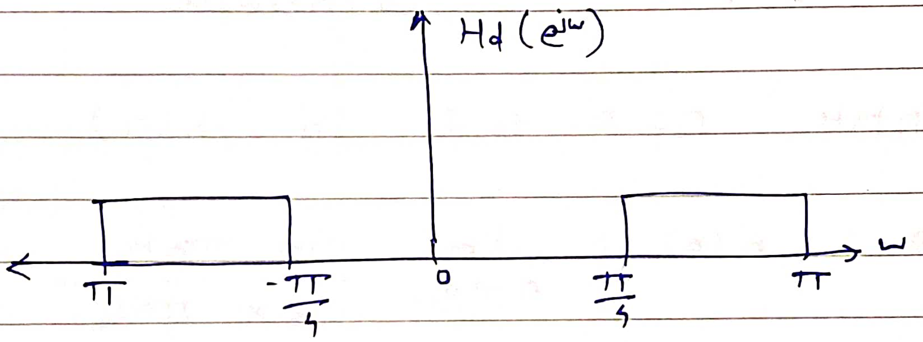
$$+ 0 + 0.063\left(z^{-5} + z^{5}\right]$$

Q.7 Design an ideal HPF with a freq response

$H_d(e^{j\omega}) = 1$ for $\pi/4 \le |\omega| \le \pi$

$= 0$ for $\omega \le \frac{\pi}{4}$

→



$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-\pi/4} e^{j\omega n}\, d\omega + \int_{\pi/4}^{\pi} e^{j\omega n}\, d\omega\right]$$

$$= \frac{1}{2\pi j n}\left\{\left[e^{j\omega n}\right]_{-\pi}^{-\pi/4} + \left[e^{j\omega n}\right]_{\pi/4}^{\pi}\right\}$$

$$= \frac{1}{2\pi j n}\left[e^{-j\pi n/4} - e^{-jn\pi} + e^{jn\pi} - e^{jn\pi/4}\right]$$

$$= \frac{1}{2\pi j n}\left\{\left[\cos\frac{\pi n}{4} - j\sin\frac{n\pi}{4}\right] - \left[\cos \pi n - j\sin \pi n\right] + \left[\cos \pi n + j\sin \pi n\right] - \left[\cos\frac{\pi n}{4} + j\sin\frac{\pi n}{4}\right]\right\}$$

$$= \frac{1}{2\pi j n}\left[2j \sin \pi n - 2j \sin \frac{\pi n}{4}\right]$$

$$= \frac{1}{2\pi}\left[\sin \pi n - \sin \frac{\pi n}{4}\right]$$

Truncating $h_d(n)$ to 11 samples

$$n = \frac{N-1}{2} = \frac{11-1}{2} = 5$$

$$h(n) = h_d(n) \quad \text{for } |n| \leq 5$$

$$= 0 \quad \text{otherwise}$$

substitute $n = 0$ to 5

Q9 Given the specification $\alpha_p = 1$ dB, $\alpha_s = 30$ dB, $\Omega_p = 200$ rad/sec, $\Omega_s = 600$ rad/sec.
Determine the order of filters.

→ The order of filter is

$$N = \frac{\log \sqrt{\dfrac{10^{0.1\alpha_s} - 1}{10^{0.1\alpha_p} - 1}}}{\log \dfrac{\Omega_s}{\Omega_p}}$$

$$= \frac{\log \sqrt{\dfrac{10^{0.1(30)} - 1}{10^{0.1(1)} - 1}}}{\log \left(\dfrac{600}{200}\right)}$$

$$= \frac{\log \sqrt{\dfrac{999}{0.258}}}{\log (3)}$$

$$N = 3.758$$

$$N \approx 4$$