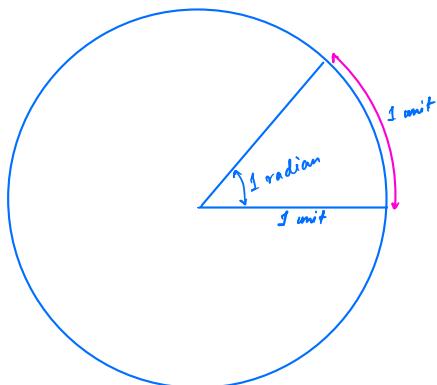


Radians : The trigonometric functions are measured with radians.

$$1 \text{ radian} = \frac{1 \text{ unit arc length}}{1 \text{ unit radius length}} \quad \left. \right\} \text{ for an unit circle.}$$



$$\begin{aligned} \text{Circumference} &= 2\pi \\ \text{Radius} &= 1. \end{aligned}$$

$$360^\circ = \frac{2\pi}{1} \text{ rad}$$

$$\Rightarrow 180^\circ = \pi \text{ rad}$$

$$1 \text{ radian} = 57.2958\ldots^\circ$$

Degree	Radian
--------	--------

0	0
---	---

30	$\frac{\pi}{6}$
----	-----------------

45	$\frac{\pi}{4}$
----	-----------------

60	$\frac{\pi}{3}$
----	-----------------

90	$\frac{\pi}{2}$
----	-----------------

120	$\frac{2\pi}{3}$
-----	------------------

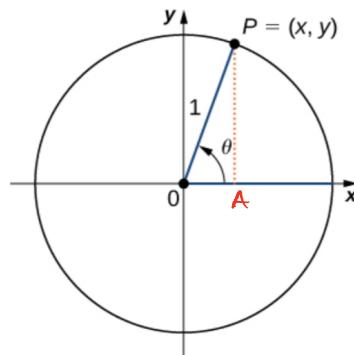
135	$\frac{3\pi}{4}$
-----	------------------

150	$\frac{5\pi}{6}$
-----	------------------

180	π
-----	-------

Basic Trigonometric Functions:

Let $P(x, y)$ be a point on a unit circle, centred at origin $(0, 0)$.



$OA = x = \text{Adjacent Side}$
 $AP = y = \text{Opposite side}$
 $OP = \text{Hypotenuse}$

$\triangle OAP$ is right angle
 \Downarrow
 $OA^2 + AP^2 = OP^2$

Sine function

$$\sin \theta = \frac{OPP}{HYP} = \frac{y}{1} = y$$

Cosine function

$$\cos \theta = \frac{ADJ}{HYP} = \frac{x}{1} = x$$

Tangent function

$$\tan \theta = \frac{OPP}{ADJ} = \frac{y}{x} = \frac{\sin \theta}{\cos \theta}$$

not defined
for $x=0$

Cosecant function

$$\csc \theta = \frac{\text{HYP}}{\text{OPP}} = \frac{1}{y} = \frac{1}{\sin \theta}$$

not defined
for $y=0$

Secant function

$$\sec \theta = \frac{\text{HYP}}{\text{ADJ}} = \frac{1}{x} = \frac{1}{\cos \theta}$$

not defined
for $x=0$

Cotangent function

$$\cot \theta = \frac{\text{ADJ}}{\text{OPP}} = \frac{x}{y} = \frac{1}{\tan \theta}$$

not defined
for $y=0$

Sin & Cos Value-table

θ	$\sin \theta$	$\cos \theta$
0	0	1
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{2}$	1	0

Pythagorean Identities:

$$\textcircled{1} \quad \sin^2\theta + \cos^2\theta = 1$$

$$\textcircled{2} \quad 1 + \tan^2\theta = \sec^2\theta$$

$$\textcircled{3} \quad 1 + \cot^2\theta = \csc^2\theta$$

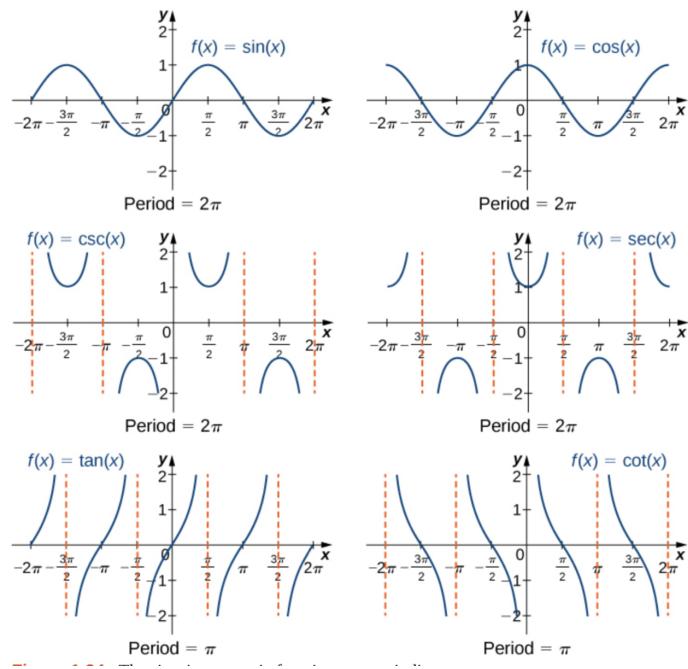
Addition/Subtraction formulae:

$$\sin(\alpha \pm \beta) = \sin\alpha \cos\beta \pm \cos\alpha \sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \sin\alpha \sin\beta.$$

Cor: $\sin 2\alpha = 2 \sin\alpha \cos\alpha$

$$\begin{aligned}\cos 2\alpha &= \cos^2\alpha - \sin^2\alpha \\ &= 2\cos^2\alpha - 1 \\ &= 1 - 2\sin^2\alpha\end{aligned}$$



A Particular form of trigonometric functions.

$$f(x) = A \sin(B(x - \alpha)) + C$$

$\alpha \rightarrow$ phase shift

$|A| \rightarrow$ Amplitude of f

$C \rightarrow$ Vertical shift

$B \rightarrow$ factor