## Derivatives & Shapes of Graphs:

- $f$ is increasing on $I \Leftrightarrow f'(x) > 0$, for any $x \in I$

  $\Leftrightarrow$ slope of tangent is +ve.

- $f$ is decreasing on $I \Leftrightarrow f'(x) < 0$, for any $x \in I$

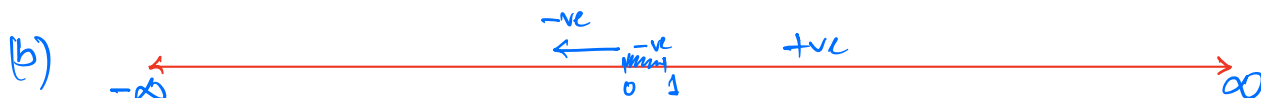  $\Leftrightarrow$ slope of tangent is -ve.

Ex. Let $f(x) = 2x^3 - 3x^2$

    (a) Find the critical pts.

    (b) Find the intervals, where $f$ is increasing/decreasing.

(a) $f'(x) = 0$ (or DNE) implies

$$[2x^3 - 3x^2]' = 0 \quad \text{ie,} \quad 6x^2 - 6x = 0$$
$$\Rightarrow 6x(x-1) = 0$$
$$\Rightarrow x = 0 \text{ or } x = 1.$$

(b)

$$\xleftarrow[\phantom{xxxxx}]{} \quad \underset{0}{\overset{-ve}{\longleftarrow}} \underset{1}{} \quad \overset{+ve}{\longrightarrow}$$

$-\infty \qquad\qquad\qquad\qquad\qquad\qquad\qquad \infty$

So, $f$ is increasing on $(1, \infty)$

$f$ is decreasing on $(-\infty, 0) \cup (0, 1)$
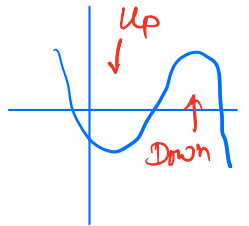
Can we draw the graph of $f$ with the above information?

NO

# Concavity (Up & Down)

$f$ be a differentiable function on $I$. We say

(a) Concave Up : $f'$ is increasing on $I$

(b) Concave Down : $f'$ is decreasing on $I$.

## Working Method :

① $'f'$ Concave Up on $I$, when $f''(x) > 0$, for all $x \in I$

② $'f'$ Concave Down on $I$, when $f''(x) < 0$, for all $x \in I$.

Note :- When $f$ changes Concavity, →(continuous) we call that point ==inflection point== of $f$. $[f''(x) = 0]$

Eg.
$$f(x) = 2x^3 - 3x^2$$
$$f'(x) = 6x^2 - 6x$$
$$f''(x) = 12x - 6$$

So, our pt of inflection is given by $f''(x) = 0$

$$\Rightarrow 12x - 6 = 0$$
$$\Rightarrow x = \tfrac{1}{2}.$$

On left of $x = \tfrac{1}{2}$, ie, on $(-\infty, \tfrac{1}{2})$, $f''(x) < 0$

### Concave Down

On right of $x = \tfrac{1}{2}$, ie, on $(\tfrac{1}{2}, \infty)$, $f''(x) > 0$

### Concave Up.