$f(1) = 2$

$f(5) = 2$

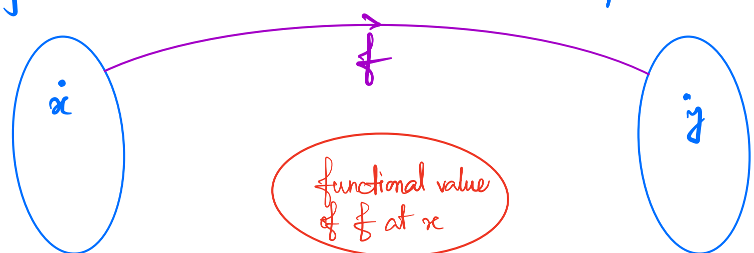
⊛ A function is a relation between a set of inputs & a set of permissible outputs.

meaning every element in the input set has one & only one image in the output set

In case you have $f(1)=2$ & $f(1)=7$ in a list of data, then f is not a function.

Note: We call functions as mappings too, as it maps one element of Input set to one-unique element in the Output set.

Notation :- ① Input Set Output Set

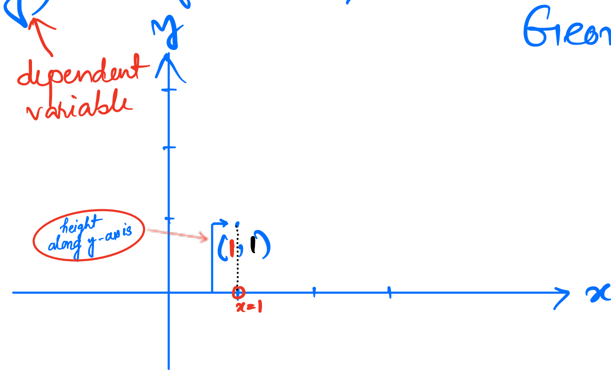


$y = f(x)$ ⟿ 'y' is equal to 'f' of 'x'.

② (x, y) or $(x, f(x))$ ⟿ More used for Geometry.

independent variable

dependent variable



⊛ The set of inputs is called the Domain & the set of outputs is called the Range of the function.

Note: There is something called Codomain, it contains the Range of the function as a set.

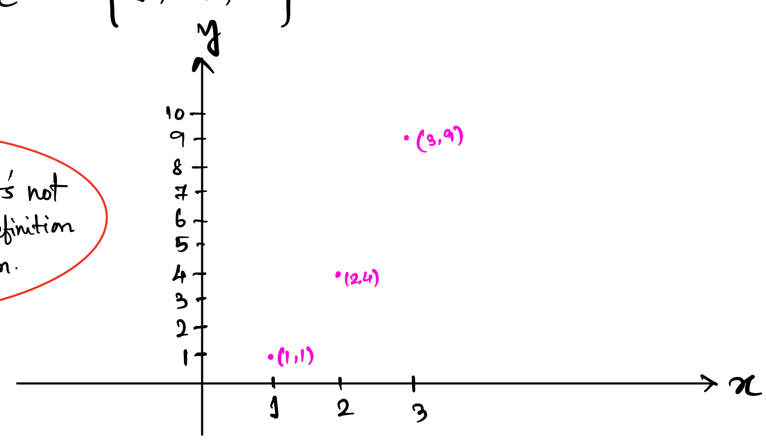
⊛ Graph of a Function: It's the geometry associated to the function.

Say, Domain = $\{1, 2, 3\}$ ⇝ discrete, finite domain.

Codomain = the set of real numbers.

Range = $\{1, 4, 9\}$

In this notation it's not clear what's the definition or rule of the function.



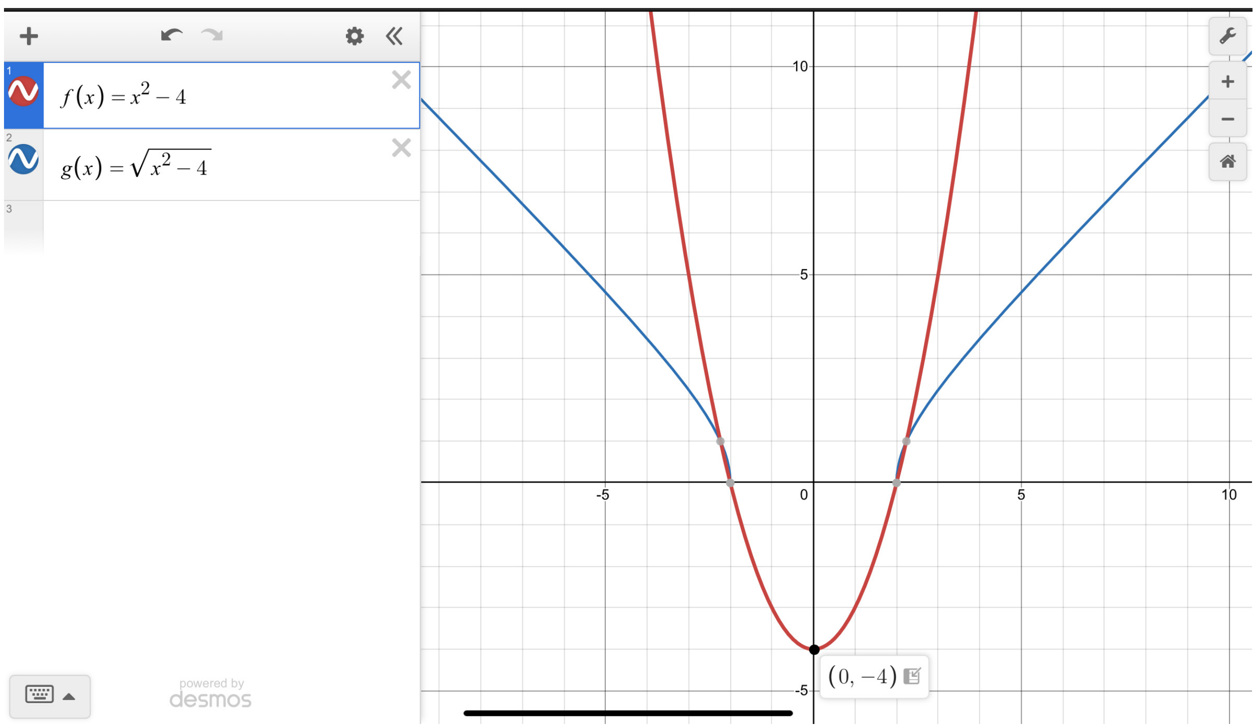
If we write $(1,1)$, $(2,4)$, $(3,9)$, then it makes complete sense to draw, as we don't know the exact function.

Say, Domain = the set of real numbers $\equiv \mathbb{R}$

Codomain = the set of real numbers.

function is defined as ① $f(x) = x^2 - 4$

② $g(x) = \sqrt{x^2 - 4}$



What we can say about range?

Range of function f is all real numbers greater than or equal to -4. ($[-4, \infty)$ or $y \geq -4$)

Range of function g is all real numbers greater than or equal to 0. ($[0, \infty)$ or $y \geq 0$)

⊛ Set Builder Notation:- $\{x \mid x \text{ has some property}\}$

'x' such that ' x has some prop.'

Example: ① $\{x \mid 1 < x < 5\} = (1,5)$

② $\{x \mid 1 \leq x \leq 5\} = [1,5]$

③ $\{x \mid x \geq 0\} = [0, \infty)$

④ $\{x \mid x \text{ is any real number}\}$

$\{x \mid x \in \mathbb{R}\}$

$\in$ → Belongs to

⊛ Interval notation: $(a,b) = \{x \mid a < x < b\}$ → Open Interval

$(a,b] = \{x \mid a < x \leq b\}$

$[a,b) = \{x \mid a \leq x < b\}$

$[a,b] = \{x \mid a \leq x \leq b\}$ → Closed Interval.

⊛ Evaluating Functions: If $f(x) =$ some formula of x, then to evaluate, replace x by the given point where you are asked to evaluate.

Eg: $f(x) = x^2 + 4x + 3$, evaluate at $x=1$, a, $b+1$

$f(1) = 1^2 + 4 \cdot 1 + 3 = 1 + 4 + 3 = 8$

$f(a) = a^2 + 4a + 3$

$$f(b+1) = (b+1)^2 + 4(b+1) + 3$$
$$= (b^2+2b+1) + (4b+4) + 3$$
$$= b^2 + 6b + 8$$

$(b+1)^2 = (b+1)(b+1) = b(b) + b(1) + 1(b) + 1(1)$
$= b^2 + b + b + 1$
$= b^2 + 2b + 1$

⊛ Finding Domain.

① $f(x) = \dfrac{3}{x^2-1}$

We cannot divide by 0

So, Domain$(f) = \{x \mid x^2 - 1 \neq 0\} = \{x \mid x \neq \pm 1\}$

② $f(x) = \dfrac{2x+5}{3x^2+4}$

$x^2 \geqslant 0$, for all real x

Since $3x^2+4 \geqslant 4$, for all real values of x.

So, Domain$(f) = (-\infty, \infty) = \mathbb{R}$

③ $f(x) = \sqrt{4-3x}$

Part under the square root symbol must be non-negative.

So, Domain$(f) = \{x \mid 4-3x \geqslant 0\}$
$= \{x \mid 4 \geqslant 3x\}$
$= \{x \mid \frac{4}{3} \geqslant x\}$

or

$\{x \mid x \leqslant \frac{4}{3}\}$

⊛ Function Notation:

– by table

Hours after Midnight	Temperature (°F)	Hours after Midnight	Temperature (°F)
0	58	12	84
1	54	13	85
2	53	14	85
3	52	15	83
4	52	16	82
5	55	17	80
6	60	18	77
7	64	19	74
8	72	20	69
9	75	21	65
10	78	22	60
11	80	23	58

Table 1.1 Temperature as a Function of Time of Day

– by graph

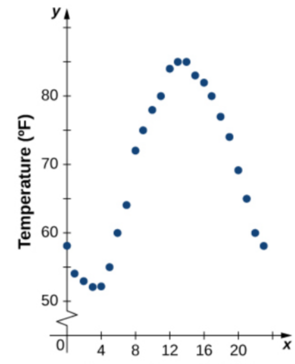


Figure 1.6 The graph of the data from Table 1.1 shows temperature as a function of time.

– by formula.

$$f(x) = \sqrt{x+3} + 1$$

Practice: Find the $\begin{cases} \text{domain of } f. \\ \text{zeros of } f \text{ (if any).} \end{cases}$

& try sketching it.

⊛ Piecewise-defined function.

$$f(x) = \begin{cases} \text{Rule 1 for } x, & \text{when } x \text{ is in some set } A_1 \\ \text{Rule 2 for } x, & \text{when } x \text{ is in some other set } A_2 \\ \vdots & \end{cases}$$

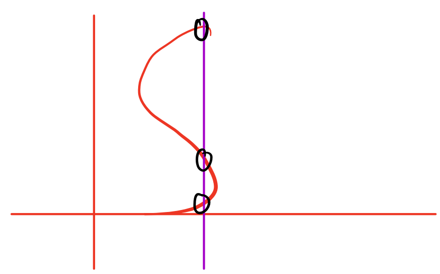
& so on.

Eg: $$f(x) = \begin{cases} -x, & \text{when } x < 4 \\ 0, & \text{when } x = 4 \\ x^2, & \text{when } x > 4 \end{cases}$$

Try to sketch.

⊛ Vertical Line Test: Given a function f, every vertical line that may be drawn intersects the graph of f, no more than once.

If it intersects more than once, then the set of points doesn't represent a function.



⊛ Zeros or x-intercepts & y-intercepts of a function.

$f(x) = -x+2$ ⇝ only one zero at $x=2$.
⇝ y-intercept is given by $(0, f(0))$
ie, $(0, 2)$

Eg. Let $f(x) = \sqrt{x^2-1}$

Zero or x-intercept: We need to find such x for which $f(x)=0$.

Note: $\sqrt{x} = 0$ is only possible if $x=0$ & $x^2=a^2 \Rightarrow x=\pm a$

Using this note we get, $f(x)=0 \Rightarrow \sqrt{x^2-1}=0 \Rightarrow x^2-1=0$
$\Rightarrow x^2=1 \Rightarrow x=\pm 1$

So, x-intercepts are $x=-1$ & $x=1$.

y-intercept: we need to find $f(0)$.

Now, $f(0) = \sqrt{0^2-1} = \sqrt{0-1} = \sqrt{-1}$, DNE $\Rightarrow$ No y-intercept in this case.