The Derivative as a Function:

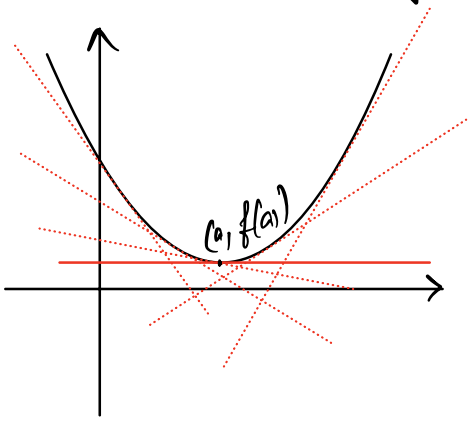
We can take derivative as a function

a ⟿ **Derivative** ⟿ Slope of the tangent line to the function we are dealing with at $x = a$

Notation of derivatives of $y = f(x)$:

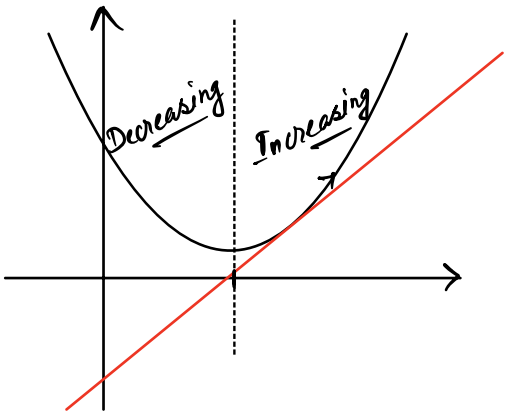
- $f'(x)$
- $\frac{d}{dx}(f(x))$
- y'
- $\frac{dy}{dx} = \frac{d}{dx}(y)$

① $f'(x) = 0$ or Horizontal Tangent.



If $f'(x)$ is 0 for some $x = a$, then the function $f(a)$ changes its direction at $x = a$.

② $f'(x) > 0$ or Increasing function:



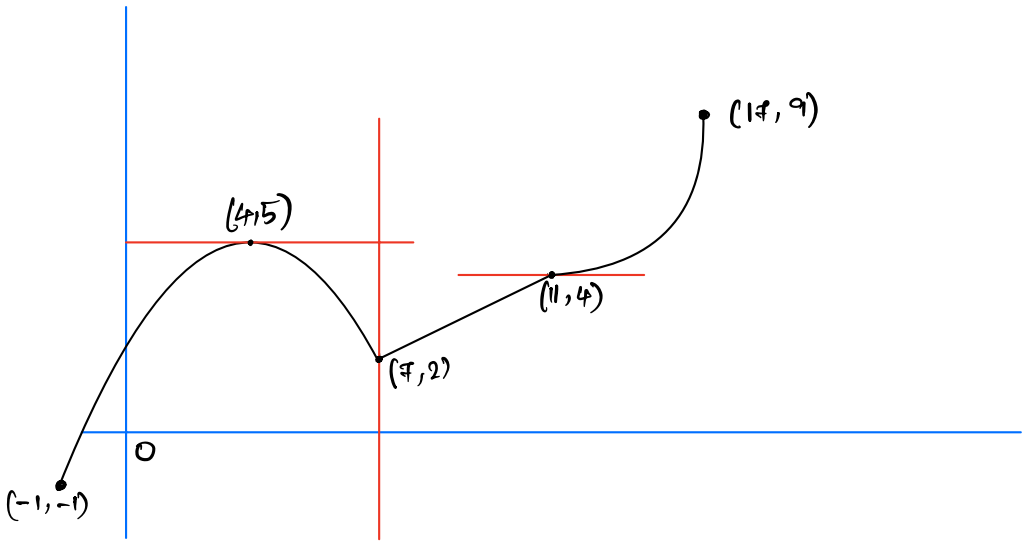
③ $f'(x) < 0$ or Decreasing function.

④ $f'(x)$ is undefined : Vertical

As. $f'(x) = \dfrac{\text{change in } y \text{ at some pt}}{\text{change in } x \text{ at the same pt}}$

$f'(x)$ undefined means no change in x-coord.

Ex. Graph $f'(x)$, where $f(x)$ is given as follows



Note:
- $f(x)$ is increasing between $(-1,-1)$ & $(4,5)$ in a non-linear path (seems like a parabola)

  $\Rightarrow f'(x) > 0$, x in $(-1,4)$ & $f'(x)$ is not constant
- $f(x)$ has a horizontal tangent at $(4,5)$ point.
- $f(x)$ is decreasing between $(4,5)$ & $(7,2)$ in a non-linear path (seems like the same parabola)

$\Rightarrow f'(x) < 0$, x in $(4, 7)$ & $f'(x)$ is not constant

- Between $(7, 2)$ & $(11, 4)$, $f(x)$ is a straight line with fixed slope $= \frac{4-2}{11-7} = \frac{2}{4} = 0.5$
- Between $(11, 4)$ & $(17, 9)$ we have an increasing non-linear (possibly non-parabolic curve)

  $\Rightarrow f'(x) > 0$ in $(11, 17)$, $f'(x)$ is not constant, and possibly not a st. line path (being possibly non-parabolic) ie, any positive curve between $x = 11$ & $x = 17$ will work out (possibly st. line also).