Figure 3. Registered Image, Translated Image, Cross power Spectrum

The inverse Fourier transform of (3) results in a function that is an impulse, i.e., it is approximately zero everywhere except for a small neighbourhood around a single point. This single point is where the absolute value of the inverse Fourier transform of (3) attains its maximum value, and the location of this point is the displacement (x_0, y_0) that is needed to optimally register the two images.

Implementation of the above process uses the Fast Fourier Transform (FFT) to reduce computational costs.

Retrieval of the rotation parameter is as follows :
Formulating the relation between two images, in the spatial domain, where the images differ by a rotation angle θ_0, i.e. $f_2(x, y)$ is a rotated replica of $f_1(x, y)$, we have,

$$f_2(x,y) = f_1(x\cos(\theta_0) + y\sin(\theta_0), -x\sin(\theta_0) + y\cos(\theta_0)) \tag{4}$$

Converting (4) into Fourier space,

$$F_2(\omega_1,\omega_2) = F_1(\omega_1\cos(\theta_0) + \omega_2\sin(\theta_0), -\omega_1\sin(\theta_0) + \omega_2\cos(\theta_0)) \tag{5}$$

If M_1 and M_2 be the magnitudes of F_1 and F_2 respectively in terms of the polar coordinates, we have,

$$M_1(\rho,\theta) = M_2(\rho,\theta - \theta_0) \tag{6}$$

θ_0 is then evaluated using the inverse of the normalized cross – power spectrum equation. In the polar coordinate system, rotation is represented in terms of a translational displacement.

Retrieval of the scale parameter is as follows :
When an image is scaled by a factor a, with respect to another image, let $f_1(x, y)$ be a scaled replica of $f_2(x, y)$, i.e.,

$$f_2(x,y) = f_1(x*a, y*a) \tag{7}$$

The relation in the Fourier space is expressed as

$$F_2(\omega_1,\omega_2) = \frac{1}{a^2}F_1\left({}^{\omega_1}/_{a}, {}^{\omega_2}/_{a}\right) \tag{8}$$

Taking the logarithm, (8) is rewritten as

$$F_2(\omega_1,\omega_2) = F_1(\log\omega_1 - \log a, \log\omega_2 - \log a) \tag{9}$$

The scale factor of ${}^{1}/_{a^2}$ is ignored for simplicity.

As is evident from equation (9), in the logarithmic space, scaling is reduced to a translational displacement.