Discrete Event Simulation

Monte Carlo Simulation

DES

- Introduction:
  - System
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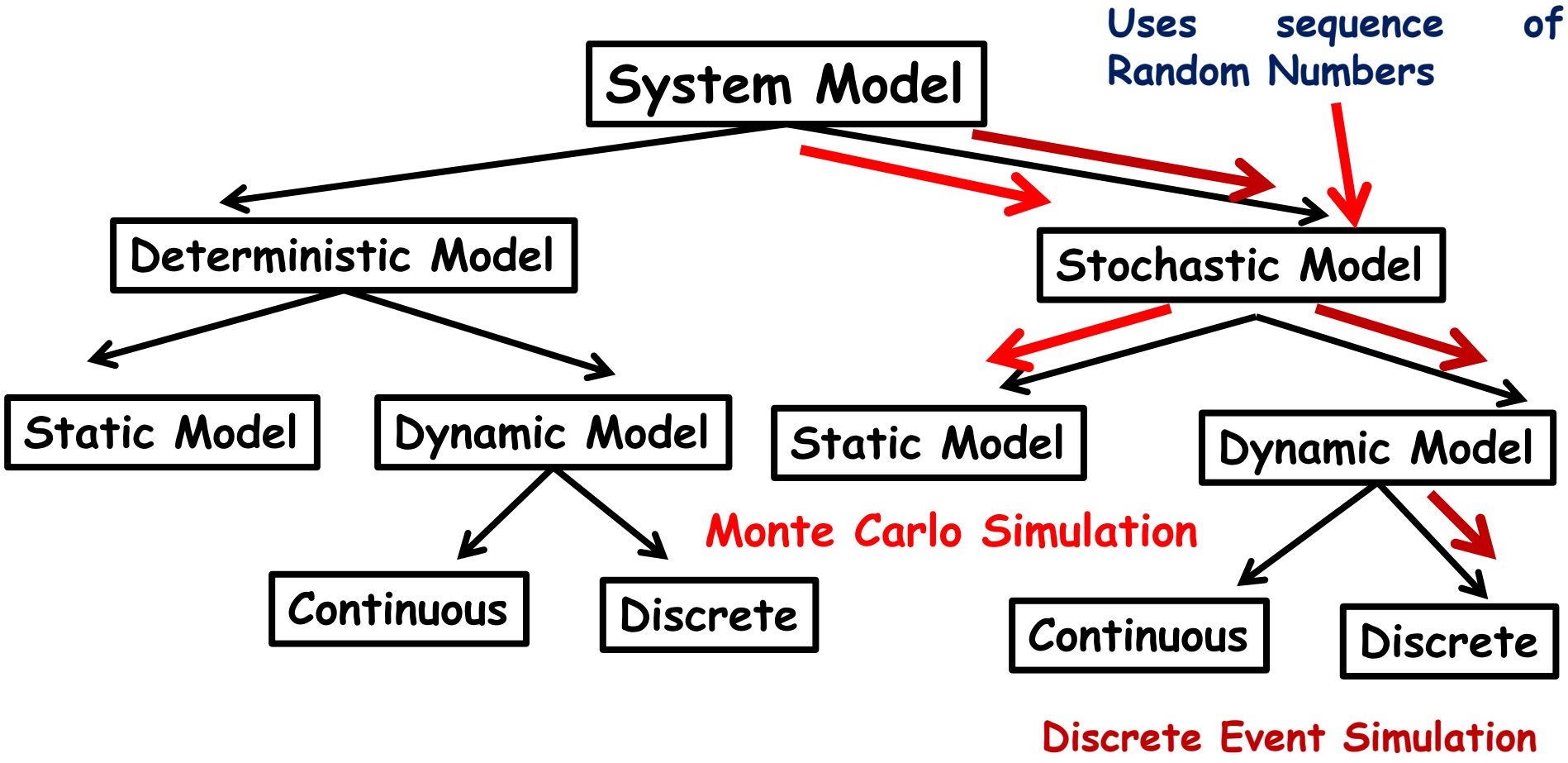
  - Discrete event simulation and
  - Continuous simulation
- Discrete Event Simulation:
  - Time-Advance Mechanisms
  - Event Modeling of discrete dynamic systems
  - Single-Server Single-Queue Model
  - Event graphics
- Monte Carlo Simulation

Models Taxonomy



What is Monte Carlo Simulation ?

Monte Carlo Simulation refers to
any method that
makes use of random numbers

A Monte Carlo simulation is a
statistical simulation technique
that provides approximate solutions
to problems expressed mathematically.

What is Monte Carlo Simulation used ?

Monte Carlo simulation is

more specifically used to

describe a method

for propagating uncertainties in model inputs

into uncertainties in model outputs (results).

Monte Carlo Simulation: Example

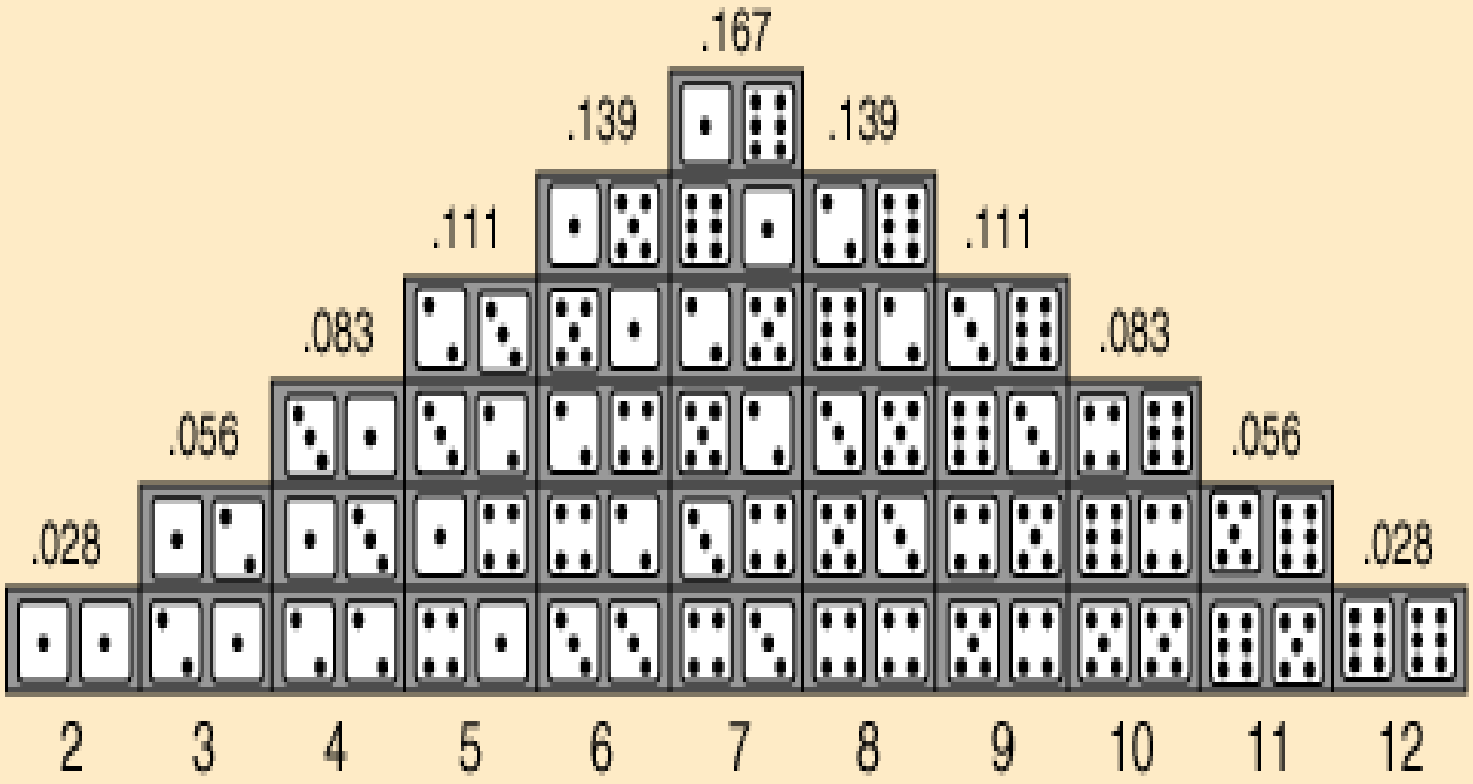
Two Dice throw

Calculate the probability of a particular sum of the throw of two dice (with each die having values one through six).

In this particular case, there are 36 combinations of dice rolls:

Monte Carlo Simulation: Example

Two Dice throw



Total number of states: 36

Monte Carlo Simulation: Example

Two Dice throw

Outcome(x_i)	Probability $p(x_i)=f(x_i)$	Cumulative Probability , F(x)	U[1,100]Random Number Assignment
2	(1/36)=0.028	0.028	[1-2]
3	(2/36)=0.056	0.084	(2-8]
4	(3/36)=0.083	0.167	(8-16]
5	(4/36)=0.111	0.278	(16-27]
6(Then x=6)	(5/36)=0.139	0.417	(27-41]=U(Let u=30)
7	(6/36)=0.167	0.584	(41-58]
8	(5/36)=0.139	0.723	(58-72]
9	(4/36)=0.111	0.834	(72-83]
10	(3/36)=0.083	0.917	(83-91]
11	(2/26)=0.056	0.973	(91-97]
12	(1/36)=0.028	1.001	(97-100]

Monte Carlo Simulation: Example

Two Dice throw

Based on this, you can manually compute the probability of a particular outcome.

For example, there are six different ways that the dice could sum to seven.

Hence, the probability of rolling seven is equal to 6 divided by 36 = 0.167.

Instead of computing the probability in this way, however, we could instead throw the dice a hundred times and record how many times each outcome occurs.

Monte Carlo Simulation: Example

Two Dice throw

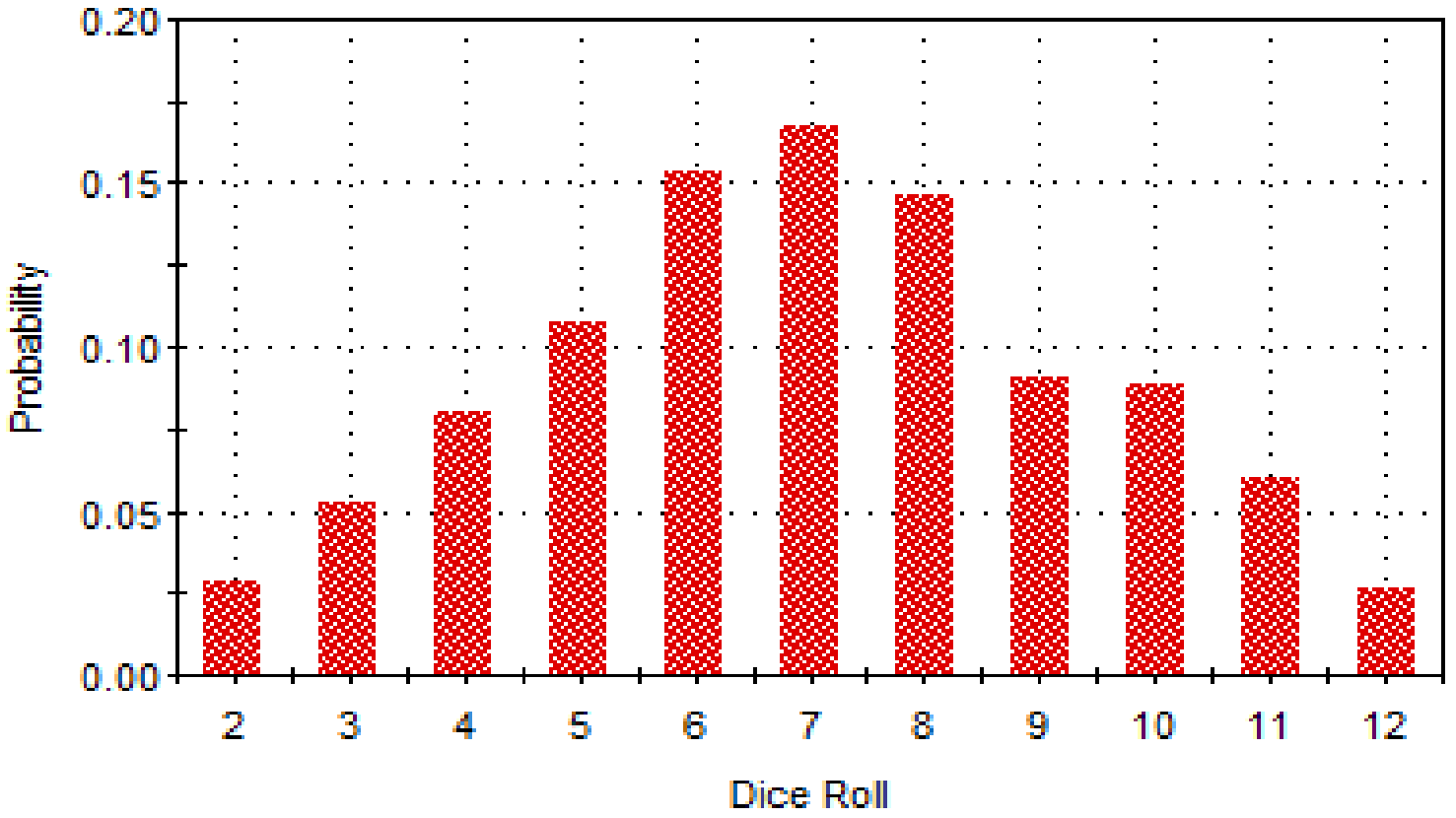
If the dice totaled seven 18 times (out of 100 rolls), we would conclude that the probability of rolling seven is approximately 0.18 (18%).

Obviously, the more times we rolled the dice, the less approximate our result would be.

Better than rolling dice a hundred times, we can easily use a computer to simulate rolling the dice 10,000 times (or more).

The output of 10,000 realizations (using GoldSim software):

Monte Carlo Simulation: Example
Two Dice throw



General Algorithm of Monte Carlo Simulation

Step 1: Set up probability distributions

Step 2: Build cumulative probability distributions

Step 3: Establish an interval of random numbers for each variable

Step 4: Generate random numbers: only accept numbers that satisfies a given condition.

Step 4: Simulate a series of trials

Example . HERFY Cake Shop

The cake seller HERFY shop sells a random number of cakes each day.

No of Times Observed: Frequency

Demand	Frequency	Probability for cakes
0	10	0.05
1	20	0.1
2	40	0.2
3	60	0.3
4	40	0.2
5	30	0.15
	200 cakes	1.00

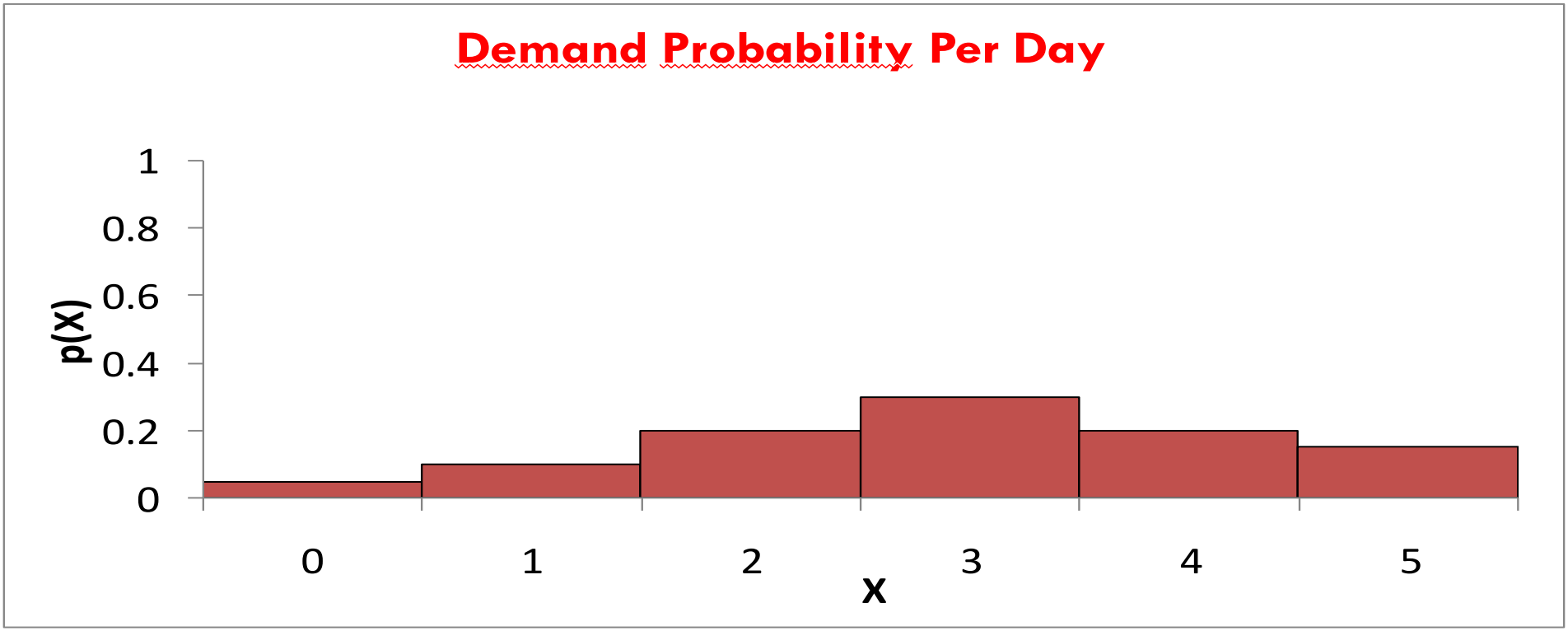
=10/200

HERFY manager wants to determine a policy for managing his inventory of cakes (e.g. how many cakes should he prepare in 10 days).

Example . HERFY Cake Shop

Step 1: Set up the probability distribution for cake sales.

Using historical data HERFY Shop determined that 5% of the time 0 cakes were demanded, 10% of the time 1 cake was demanded, etc…



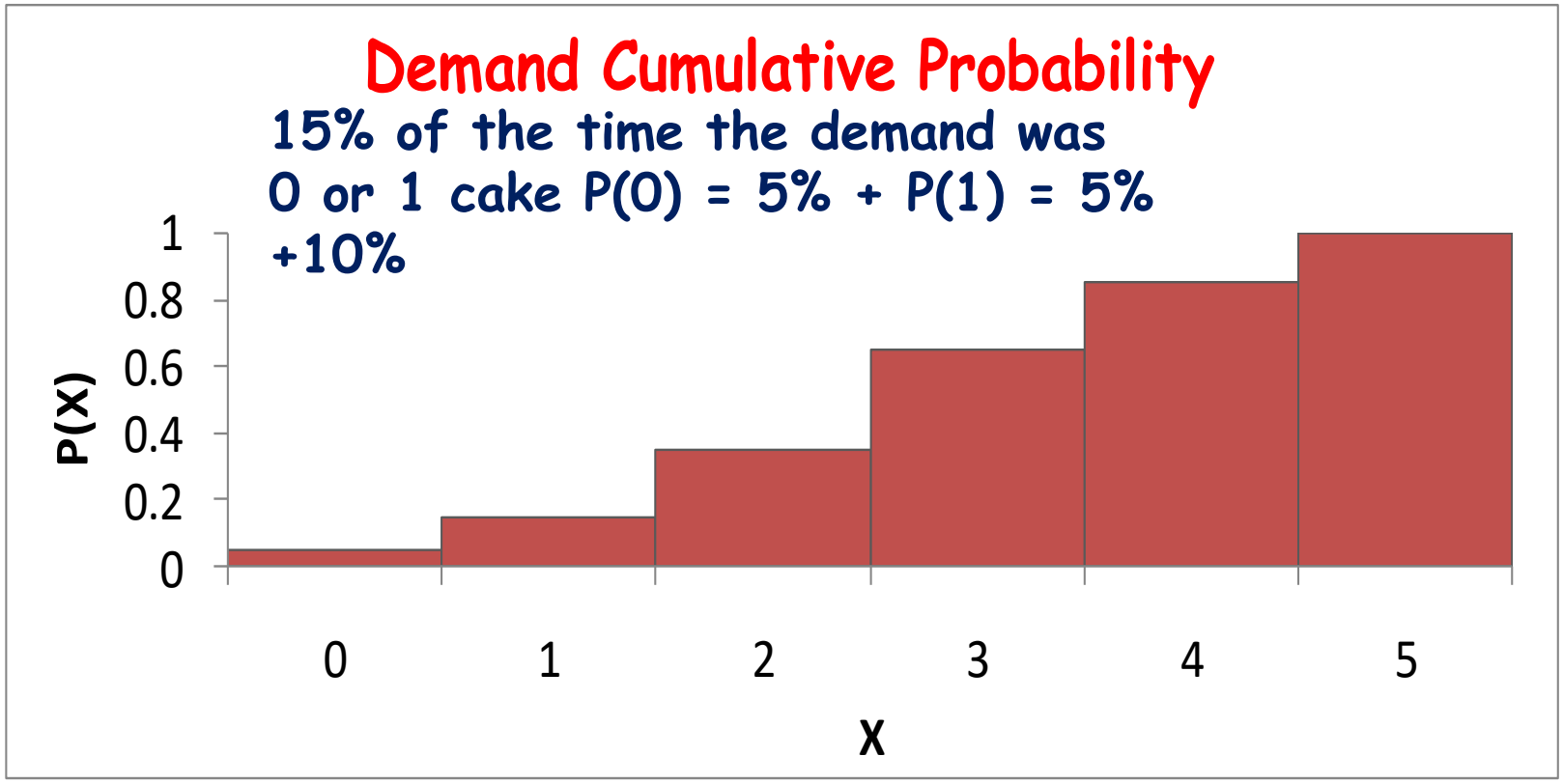
Example . HERFY Cake Shop

Step 2: Build a Cumulative Probability Distribution

Demand	Frequency	Probability	Cumulative Probability
0	10	0.05	0.05
1	20	0.10	0.15
2	40	0.20	0.35
3	60	0.30	0.65
4	40	0.20	0.85
5	30	0.15	1.00

Example . HERFY Cake Shop

Step 2: Build a Cumulative Probability Distribution



Example . HERFY Cake Shop

Step 3: Establish an interval of random numbers.

Demand	Frequency	Probability	Cumulative Probability	Random Number Interval
0	10	0.05	0.05	01 - 05
1	20	0.10	0.15	06 - 15
2	40	0.20	0.35	16 - 35
3	60	0.30	0.65	36 - 65
4	40	0.20	0.85	66 - 85
5	30	0.15	1.00	86 -100

Note: 5% of the time 0 cakes are demanded, so the random number interval contains 5% of the numbers between 1 and 100

Must be in correct proportion

Example . HERFY Cake Shop

Step 4: Generate random numbers: only accept numbers that satisfies a given condition.

52	06	50	88	53	30	10	47	99	37
37	63	28	02	74	35	24	03	29	60
82	57	68	28	05	94	03	11	27	79
69	02	36	49	71	99	32	10	75	21
98	94	90	36	06	78	23	67	89	85
96	52	62	87	49	56	59	23	78	71
33	69	27	21	11	60	95	89	68	48
50	33	50	95	13	44	34	62	64	39
88	32	18	50	62	57	34	56	62	31
90	30	36	24	69	82	51	74	30	35
50	48	61	18	85	23	08	54	17	12
27	88	21	62	69	64	48	31	12	73
45	14	46	32	13	49	66	62	74	41
81	02	01	78	82	74	97	37	45	31
66	83	14	74	27	76	03	33	11	97
74	05	82	82	93	09	96	33	52	78
30	34	87	01	74	11	46	82	59	94

Example . HERFY Cake Shop

Step 5: Simulate a series of trials.

Using random number table on previous slide, simulated demand for 10 days is:

Cake Demanded	Interval of Random Numbers
0	01 - 05
1	06 - 15
2	16 - 35
3	36 - 65
4	66 - 85
5	86 - 100

Generate a random Number and look it up in the table.

Random number:	52	06	50	88	53	30	10	47	99	37
Simulated demand:	3	2	5	3	1	4	2	1	2	3

Example . HERFY Cake Shop

Step 5: Simulate a series of trials.

Day	1	2	3	4	5	6	7	8	9	10
Random No:	52	06	50	88	53	30	10	47	99	37
Simulated Demand	3	2	5	3	1	4	2	1	2	3

What is difference between Simulation, Monte Carlo Method and Monte Carlo Simulation?

A simulation is a fictitious representation of reality.

A Monte Carlo method is a technique that can be used to solve statistical problem.

A Monte Carlo simulation uses repeated sampling to determine the properties of some phenomenon (or behavior).

What is difference between Simulation, Monte Carlo Method and Monte Carlo Simulation?

Simulation: Example

Drawing one pseudo-random uniform variable from the interval (0,1] can be used to simulate the tossing of a coin:

> If the value is less than or equal to 0.50 designate the outcome as heads, but if the value is greater than 0.50 designate the outcome as tails.

This is a simulation, but not a Monte Carlo simulation

What is difference between Simulation, Monte Carlo Method and Monte Carlo Simulation?

Monte Carlo method: Example

Pouring out a box of coins on a table, and then computing the ratio of coins that land heads versus tails is a Monte Carlo method of determining the behavior of repeated coin tosses, but it is not a simulation.

What is difference between Simulation, Monte Carlo Method and Monte Carlo Simulation?

Monte Carlo simulation: Example

Drawing a large number of pseudo-random uniform variables from the interval (0,1], and

assigning values less than 0.50 as heads and greater than or equal to 0.50 as tails, is a *Monte Carlo simulation* of the behavior of repeatedly tossing a coin.